

# Correlation observables in $\Upsilon + D$ associated production at the LHC within the parton Reggeization approach

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## Motivation: SPS vs DPS

- A study of correlation observables (such as  $\Delta\varphi$  and  $\Delta y$ ) is a sensitive tool for probing so-called Double Parton Scattering (DPS) mechanism. Another way to test DPS is investigating at high energies the total cross-section of the process, for which contribution of Single Parton Scattering (SPS) is as small as possible, due to suppression by high power of  $\alpha_s$ .
- Recently in [Aaij R. et al. [LHCb Collaboration], JHEP 07 (2016) 052] different spectra of associated  $\Upsilon(1S) + D$  production at  $\sqrt{S} = 7$  and 8 TeV have been presented. Theoretical predictions for that process within SPS [A. V. Berezhnoy, A. K. Likhoded, Int. J. Mod. Phys. A 30 (2015) 1550125.] had given a small value of total cross-section while the experimental result had been at least 10 times more. So that had been interpreted as a clear signal of DPS mechanism.
- But due to low hard scales the SPS cross-section could receive unexpectedly large higher-order QCD corrections. Moreover, at low- $x$  plenty of phase space is available for emission of additional relatively hard partons, what can mimic the behaviour of DPS, thus weakening the case for DPS-dominance in this process.

# Sketch of the Parton Reggeization Approach (PRA)

## Model process and Sudakov's decomposition

We can derive the factorization formula in PRA, considering the following auxilliary hard subprocess:

$$g(p_1) + g(p_2) \rightarrow g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2), \quad (1)$$

where  $p_1^2 = p_2^2 = k_1^2 = k_2^2 = 0$ ,  $M_{\mathcal{A}}^2 = P_{\mathcal{A}}^2$ .

We use the Sudakov (light-cone) components of any four-momentum  $k$ :

$$k^\mu = \frac{1}{2} \left( k^+ n_-^\mu + k^- n_+^\mu \right) + k_T^\mu,$$

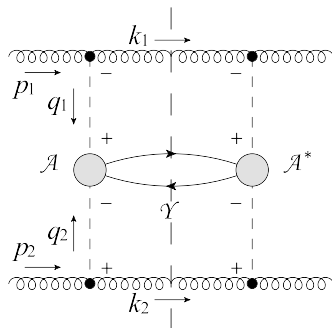
where  $n_\pm^\mu = (n^\pm)^\mu = (1, 0, 0, \mp 1)^\mu$ ,  $n_\pm^2 = 0$ ,  $n_+ n_- = 2$ ,

$k^\pm = k_\pm = (n_\pm k) = k^0 \pm k^3$ ,  $n_\pm k_T = 0$ , so that  $p_1^- = p_2^+ = 0$  and

$s = (p_1 + p_2)^2 = p_1^+ p_2^- > 0$ . Then the dot-product of two four-vectors  $k$  and  $q$  in this notation is equal to:

$$(kq) = \frac{1}{2} (k^+ q_- + k^- q_+) - \mathbf{k}_T \mathbf{q}_T.$$

## Multi-Regge Kinematics (MRK)



The limit of **Multi-Regge Kinematics** (MRK) for the subprocess (1) is defined as:

$$\Delta y_1 = y(k_1) - y(P_A) \gg 1, \quad \Delta y_2 = y(P_A) - y(k_2) \gg 1, \quad (2)$$

$$\mathbf{k}_{T1}^2 \sim \mathbf{k}_{T2}^2 \sim M_{TA}^2 \sim \mu^2 \ll s, \quad (3)$$

where rapidity for the four-momentum  $k$  is equal to  $y(k) = \log(k^+/k^-)/2$ .

MRK limit of QCD amplitudes can be obtained using **Lipatov's EFT for MRK processes in QCD** [L. N. Lipatov, Nucl. Phys. B **452**, 369 (1995)].

## The field content of the effective theory.

Light-cone derivatives:

$$x^\pm = n^\pm x = x^0 \pm x^3, \quad \partial_\pm = 2 \frac{\partial}{\partial x^\mp}$$

Lagrangian of the effective theory  $L = L_{kin} + \sum_y (L_{QCD} + L_{ind})$ ,  $v_\mu = v_\mu^a t^a$ ,

$[t^a, t^b] = f^{abc} t^c$ . The rapidity space is sliced into the subintervals, corresponding to the groups of final-state particles, close in rapidity. Each subinterval in rapidity ( $1 \ll \eta \ll Y$ ) has its own set of QCD fields:

$$L_{QCD} = -\frac{1}{2} \text{tr} [G_{\mu\nu}^2], \quad G_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + g [v_\mu, v_\nu].$$

Different rapidity intervals communicate via the gauge-invariant fields of Reggeized gluons ( $A_\pm = A_\pm^a t^a$ ) with the kinetic term:

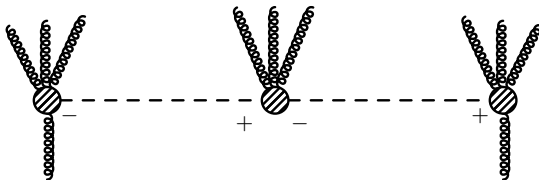
$$L_{kin} = -\partial_\mu A_+^a \partial^\mu A_-^a,$$

and the kinematical constraint:

$$\partial_- A_+ = \partial_+ A_- = 0 \Rightarrow$$

$$A_+ \text{ has } k_- = 0 \text{ and } A_- \text{ has } k_+ = 0.$$

## The effective action for high energy processes in QCD.



Particles and Reggeons interact via *induced interactions*:

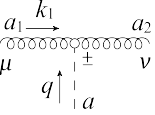
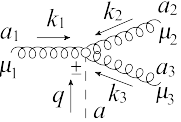
$$L_{ind} = - \operatorname{tr} \left\{ \frac{1}{g} \partial_+ \left[ P \exp \left( -\frac{g}{2} \int_{-\infty}^{x^-} dx'^- v_+(x') \right) \right] \cdot \partial_\sigma \partial^\sigma A_-(x) + \right. \\ \left. + \frac{1}{g} \partial_- \left[ P \exp \left( -\frac{g}{2} \int_{-\infty}^{x^+} dx'^+ v_-(x') \right) \right] \cdot \partial_\sigma \partial^\sigma A_+(x) \right\}$$

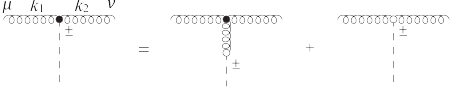

Wilson lines lead to the infinite chain of the induced vertices:

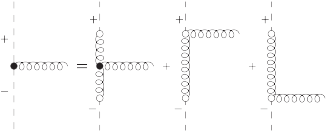
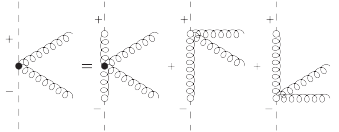
$$L_{ind} = \operatorname{tr} \left\{ \left[ v_+ - g v_+ \partial_+^{-1} v_+ + g^2 v_+ \partial_+^{-1} v_+ \partial_+^{-1} v_+ - \dots \right] \partial_\sigma \partial^\sigma A_- + \right. \\ \left. + \left[ v_- - g v_- \partial_-^{-1} v_- + g^2 v_- \partial_-^{-1} v_- \partial_-^{-1} v_- - \dots \right] \partial_\sigma \partial^\sigma A_+ \right\}$$



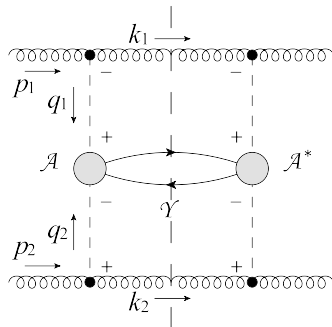
## Feynman rules

$\frac{+}{a} \xrightarrow[q]{-} \frac{-}{b} = \frac{-i\delta_{ab}}{2q^2}$	$\frac{a}{\xrightarrow[q]{-}} \xrightarrow[\pm]{\text{gluon}} \frac{b}{\mu} = (-iq^2)n_{\mu}^{\mp}\delta_{ab}$
	$g_s f_{aa_1 a_2} \left( n_{\mu}^{\mp} n_{\nu}^{\mp} \right) \frac{q^2}{k_1^{\mp}}$
	$ig_s^2 \left( n_{\mu_1}^{\mp} n_{\mu_2}^{\mp} n_{\mu_3}^{\mp} \right) \frac{q^2}{k_3^{\mp}} \left[ \frac{f_{aba_1} f_{ba_2 a_3}}{k_1^{\mp}} + \frac{f_{aba_2} f_{ba_1 a_3}}{k_2^{\mp}} \right]$

	
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## Factorization formula for the PRA



In the MRK-limit we obtained the following formula in  $k_T$ -factorized form [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys. Rev. D **96** (2017) 096019]:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_g(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_g(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\text{PRA}},$$

where the partonic cross-section in PRA is given by:

$$d\hat{\sigma}_{\text{PRA}} = \frac{|\mathcal{A}_{\text{PRA}}|^2}{2Sx_1x_2} \cdot (2\pi)^4 \delta \left( \frac{1}{2} \left( q_1^+ n_- + q_2^- n_+ \right) + q_{T1} + q_{T2} - P_A \right) d\Phi_{\mathcal{A}}.$$

## Nonintegrated Parton Distribution Functions (nPDFs)

The tree-level “unintegrated PDFs” (unPDFs) are:

$$\tilde{\Phi}_g(x, t, \mu^2) = \frac{1}{t} \frac{\alpha_s}{2\pi} \int_x^1 dz P_{gg}(z) \frac{x}{z} f_g\left(\frac{x}{z}, \mu^2\right),$$

which have the collinear divergence at  $t_{1,2} \rightarrow 0$  and infrared (IR) divergence at  $z_{1,2} \rightarrow 1$ . It regularizes at  $z_{1,2} < 1 - \Delta_{KMR}(t_{1,2}, \mu^2)$ , where

$\Delta_{KMR}(t, \mu^2) = \sqrt{t}/(\sqrt{\mu^2} + \sqrt{t})$ , and  $\mu^2 \sim M_{TA}^2$ .

The collinear singularity is regularized by the Sudakov formfactor:

$$T_i(t, \mu^2) = \exp \left[ - \int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} \sum_{j=q, \bar{q}, g} \int_0^1 dz z \cdot P_{ji}(z) \theta(1 - \Delta_{KMR}(t', \mu^2) - z) \right].$$

The final form of our unPDF is:

$$\Phi_i(x, t, \mu^2) = T_i(t, \mu^2) \frac{\alpha_s(t)}{2\pi} \sum_{j=q, \bar{q}, g} \int_x^1 dz P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z}, \mu^2\right) \theta(1 - \Delta_{KMR}(t, \mu^2) - z).$$

The KMR unPDF satisfies the following normalization condition:

$$\int_0^{\mu^2} dt \Phi_i(x, t, \mu^2) = x f_i(x, \mu^2).$$

## Ingredients of the parton Reggeization approach

- Factorization formula in the Regge limit of QCD:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2 \mathbf{q}_{T1}}{\pi} \Phi_g(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2 \mathbf{q}_{T2}}{\pi} \Phi_g(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\text{PRA}}.$$

- Unintegrated parton distribution functions in KMR model:

$$\Phi_i(x, t, \mu^2) = T_i(t, \mu^2) \frac{\alpha_s(t)}{2\pi} \sum_{j=q, \bar{q}, g} \int_x^1 dz P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z}, \mu^2\right) \theta(1 - \Delta_{\text{KMR}}(t, \mu^2) - z).$$

- Partonic amplitudes with initial-state reggeized quarks and gluons in the Lipatov's EFT:

$$L = L_{kin} + \sum_y (L_{QCD} + L_{ind}).$$

# Associated production of $\Upsilon(1S)$ and $D^{0/+}$ mesons at the LHCb

## Basics of NRQCD factorization

The NRQCD framework [G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev. D* **51**, 1125 (1995)] describes heavy quarkonia in terms of Fock state decompositions. In case of orthoquarkonium state the wave function can be written as power series expansion in the velocity parameter  $v \sim 1/\ln M_Q$ :

$$|\mathcal{H}\rangle = \mathcal{O}(v^0)|Q\bar{Q}[{}^3S_1^{(1)}]\rangle + \mathcal{O}(v)|Q\bar{Q}[{}^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)|Q\bar{Q}[{}^1S_0^{(8)}]g\rangle \quad (4)$$

$$+\mathcal{O}(v^2)|Q\bar{Q}[{}^3S_1^{(1,8)}]gg\rangle + \dots \quad (5)$$

In the NRQCD effects of short and long distances are separated, and then the cross-section of heavy-quarkonium production via a partonic subprocess  $a + b \rightarrow \mathcal{H} + X$  can be presented in a factorized form:

$$d\hat{\sigma}(a + b \rightarrow \mathcal{H} + X) = \sum_n d\hat{\sigma}(a + b \rightarrow Q\bar{Q}[n] + X) \times \langle \mathcal{O}^{\mathcal{H}}[n] \rangle, \quad (6)$$

where  $n$  denotes the set of quantum numbers of the  $Q\bar{Q}$  pair, and its nonperturbative transitions into  $\mathcal{H}$  is described by the NMEs  $\langle \mathcal{O}^{\mathcal{H}}[n] \rangle$ .

In the general case, the partonic cross-section of quarkonium production from the  $Q\bar{Q}$  Fock state  $n = {}^{2S+1}L_J^{(1,8)}$  has the form:

$$d\hat{\sigma}(a + b \rightarrow Q\bar{Q}[{}^{2S+1}L_J^{(1,8)}] \rightarrow \mathcal{H}) = d\hat{\sigma}(a + b \rightarrow Q\bar{Q}[{}^{2S+1}L_J^{(1,8)}]) \times \frac{\langle \mathcal{O}^{\mathcal{H}}[{}^{2S+1}L_J^{(1,8)}] \rangle}{N_{col}N_{pol}},$$

where  $N_{col} = 2N_c$  for color-singlet state,  $N_{col} = N_c^2 - 1$  for color-octet state, and  $N_{pol} = 2J + 1$ .

## Amplitude of specified state

The definition of partonic cross-section of  $Q\bar{Q}[{}^{2S+1}L_J^{(1,8)}]$  production is following:

$$d\hat{\sigma}(a + b \rightarrow Q\bar{Q}[{}^{2S+1}L_J^{(1,8)}]) = \frac{1}{2x_1x_2S} \overline{|\mathcal{A}(a + b \rightarrow Q\bar{Q}[{}^{2S+1}L_J^{(1,8)}])|^2} d\Phi. \quad (7)$$

The projectors on spin-zero and spin-one states:

$$\Pi_0 = \frac{1}{8m^3} \left( \frac{\hat{p}}{2} - \hat{q} - m \right) \gamma^5 \left( \frac{\hat{p}}{2} + \hat{q} + m \right), \Pi_1^\alpha = \frac{1}{8m^3} \left( \frac{\hat{p}}{2} - \hat{q} - m \right) \gamma^\alpha \left( \frac{\hat{p}}{2} + \hat{q} + m \right).$$

The projectors on color-singlet and color-octet states:

$$C_1 = \frac{\delta_{ij}}{\sqrt{N_c}}, \quad C_8 = \sqrt{2}T_{ij}^a,$$

respectively, where  $T^a$  with  $a = 1, \dots, N_c^2 - 1$  are the generators of the color gauge group  $SU(N_c)$ .

For example, the amplitude of  $Q\bar{Q}$  production in state  ${}^3S_1^{(1,8)}$  is:

$$\mathcal{A}(a + b \rightarrow Q\bar{Q}[{}^3S_1^{(1,8)}]) = \text{Tr}[C_{1,8}\Pi_1^\alpha \times \mathcal{A}(a + b \rightarrow Q\bar{Q})\varepsilon_\alpha(p)]_{q=0},$$

where  $\varepsilon_\alpha(p)$  is the polarization 4-vector of a spin-one particle with momentum  $p^\mu$  and mass  $M = p^2$ . And the polarization sum then is:

$$\sum_{J_z} \varepsilon_\alpha(p)\varepsilon_{\alpha'}^*(p) = \mathcal{P}_{\alpha\alpha'}(p) = -g_{\alpha\alpha'} + \frac{p_\alpha p'_{\alpha'}}{M^2}.$$

## Fragmentation approach

In the fragmentation approach [B. Mele, P. Nason, 1991], the cross-section of the inclusive production of  $D$ -meson is related with the partonic cross-section as follows:

$$\frac{d\sigma}{dp_T dy} \left( p + p \rightarrow D^{0/+}(p) + X \right) = \sum_a \int_0^1 \frac{dz}{z} D^{0/+}(z, \mu^2) \frac{d\hat{\sigma}}{dp_T dy} (p + p \rightarrow a(p/z) + X)$$

where  $D^{0/+}(z, \mu^2)$ -fragmentation function for the meson  $D^{0/+}$ . In our calculations we use the LO set of FFs BKK05 by [B. A. Kniehl, G. Kramer *et. al.*] fitted on the  $e^+e^-$  annihilation data.

In case of direct production of  $\Upsilon(1S)$  we take into account the following partonic subprocesses:

$$\mathcal{R}(q_1) + \mathcal{R}(q_2) \rightarrow \Upsilon(1S)(p_1) + g(k) [\rightarrow D(p_2)], \quad (8)$$

$$\mathcal{R}(q_1) + \mathcal{R}(q_2) \rightarrow \Upsilon(1S)(p_1) + c(k) [\rightarrow D(p_2)] + \bar{c}(p_3), \quad (9)$$

where  $q_1^2 = -\mathbf{q}_{T1}^2 = -t_1$ ,  $q_2^2 = -\mathbf{q}_{T2}^2 = -t_2$ .

But we add also contribution of feed-down decays into ground state of bottomonium:

$$\mathcal{R}(q_1) + \mathcal{R}(q_2) \rightarrow \Upsilon(3S, 2S)(p_1) + g(k) [\rightarrow D(p_2)], \quad (10)$$

$$\mathcal{R}(q_1) + \mathcal{R}(q_2) \rightarrow \chi_{bJ}(2P, 1P)(p_1) + g(k) [\rightarrow D(p_2)]. \quad (11)$$



# Leading PRA subprocesses in $\Upsilon(1S) + D$ production

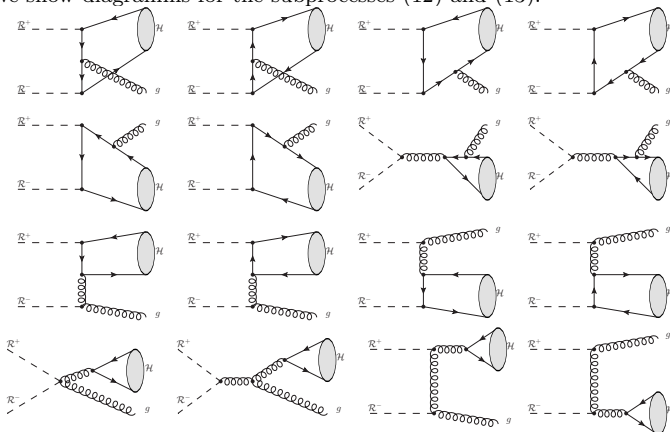
At the quark level we consider following PRA subprocesses:

$$\mathcal{R} + \mathcal{R} \rightarrow b\bar{b}[{}^3S_1^{(1,8)}] + g [\rightarrow D], \quad (12)$$

$$\mathcal{R} + \mathcal{R} \rightarrow b\bar{b}[{}^3P_{0,1,2}^{(1)}] + g [\rightarrow D], \quad (13)$$

$$\mathcal{R} + \mathcal{R} \rightarrow b\bar{b}[{}^3S_1^{(1,8)}] + c[\rightarrow D] + \bar{c}. \quad (14)$$

Here we show diagrams for the subprocesses (12) and (13):



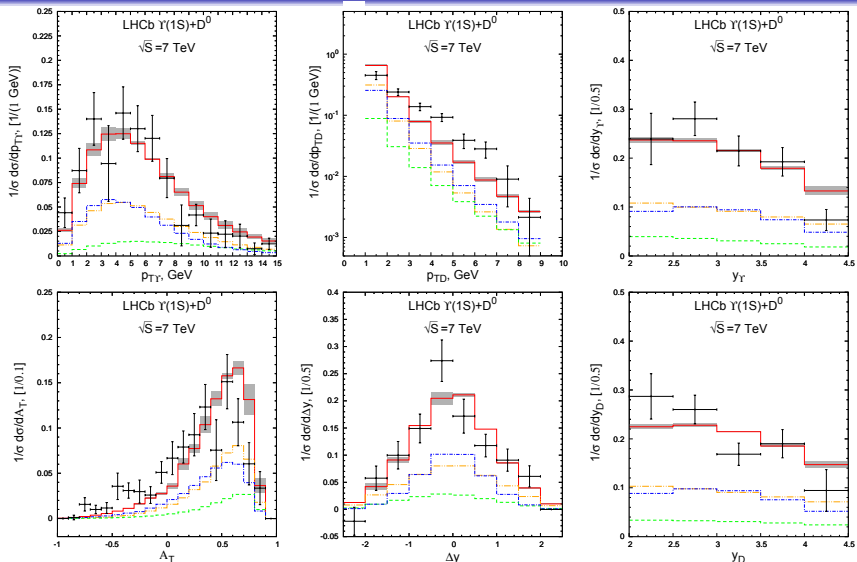
LHCb data of  $\Upsilon(1S) + D^0$  production,  $2.0 < y_{\Upsilon(D)} < 4.5$ ,  $\sqrt{S} = 7$  TeV

Figure 1 :  $p_{T\Upsilon}$ ,  $p_{TD}$ ,  $y_{\Upsilon}$ ,  $A_T$ ,  $\Delta y$  and  $y_D$  spectra of  $\Upsilon(1S)D^0$  pair. LHCb data are from [LHCb Collab. R. Aaij *et al.*, JHEP **1607**, 052 (2016)].

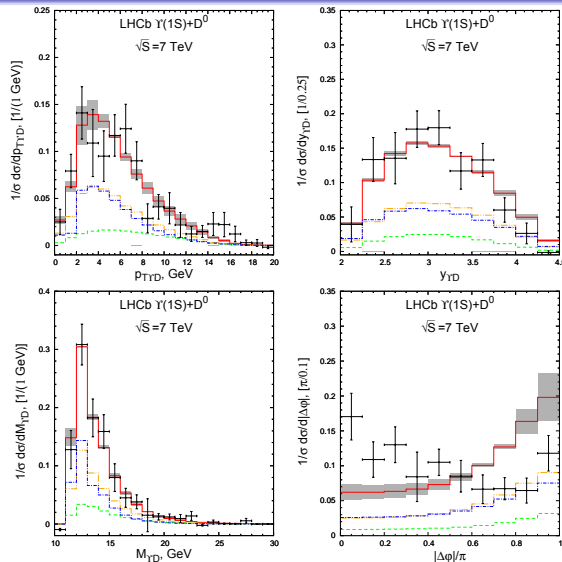
LHCb data of  $\Upsilon(1S) + D^0$  production,  $2.0 < y_{\Upsilon(D)} < 4.5$ ,  $\sqrt{S} = 7$  TeV

Figure 2 :  $p_{T(D)}$ ,  $y_{D}$ ,  $M_{YD}$  and  $\Delta\phi$  spectra of  $\Upsilon(1S)D^0$  pair. LHCb data are from [LHCb Collab. R. Aaij *et al.*, JHEP **1607**, 052 (2016)].

The total cross sections of  $\Upsilon(1S)D^{0/+}$  production at the LHCb

Kinematical region:

$$2.0 < y_{\Upsilon} < 4.5, 0 < p_{T\Upsilon} < 15 \text{ GeV}/c, 2.0 < y_D < 4.5, 1 < p_{TD} < 20 \text{ GeV}/c$$

$\sqrt{S} = 7 \text{ TeV}$	$B_{\mu^+\mu^-} \times \sigma^{\Upsilon(1S)D^0}, \text{ pb}$	$B_{\mu^+\mu^-} \times \sigma^{\Upsilon(1S)D^+}, \text{ pb}$
Direct contributions:	51	20
$R + R \rightarrow \Upsilon[{}^3S_1^{(1)}] + g$	37	15
$R + R \rightarrow \Upsilon[{}^3S_1^{(8)}] + g$	14	5
Sum of feed-down contributions	40	16
Total cross section, LO PRA	$91^{+48}_{-41}$	$36^{+19}_{-16}$
$R + R \rightarrow \Upsilon[{}^3S_1^{(1)}] + c + \bar{c}$	14	5
$R + R \rightarrow \Upsilon[{}^3S_1^{(8)}] + c + \bar{c}$	6	3
Total cross section, experiment	$155 \pm 28$	$82 \pm 24$
$\sqrt{S} = 8 \text{ TeV}$	$B_{\mu^+\mu^-} \times \sigma^{\Upsilon(1S)D^0}, \text{ pb}$	$B_{\mu^+\mu^-} \times \sigma^{\Upsilon(1S)D^+}, \text{ pb}$
Direct contributions:	61	24
$R + R \rightarrow \Upsilon[{}^3S_1^{(1)}] + g$	44	18
$R + R \rightarrow \Upsilon[{}^3S_1^{(8)}] + g$	17	6
Sum of feed-down contributions	49	19
Total cross section, LO PRA	$108^{+56}_{-48}$	$42^{+22}_{-19}$
$R + R \rightarrow \Upsilon[{}^3S_1^{(1)}] + c + \bar{c}$	16	6
$R + R \rightarrow \Upsilon[{}^3S_1^{(8)}] + c + \bar{c}$	8	3
Total cross section, experiment	$250 \pm 39$	$80 \pm 21$

## Conclusions

- Using **ReggeQCD** model-file for **FeynArts** we obtained matrix elements of  $\mathcal{R}\mathcal{R} \rightarrow \Upsilon[{}^3S_1^{(1,8)}] + g$ ,  $\mathcal{R}\mathcal{R} \rightarrow \Upsilon[{}^3P_{0,1,2}^{(1)}] + g$  and calculated different spectra of  $\Upsilon(1S) + D^{0/+}$  production.
- We have found a good agreement with LHCb data for various normalized differential distributions, except for the case of spectra on azimuthal angle differences at the small  $\Delta\varphi$  values.
- In the mentioned above kinematical region the total cross-section in our Single Parton Scattering model accounts for more than one half of observed cross-section, thus dramatically shrinking the room for Double Parton Scattering mechanism.

For more details see our recent paper [Karpishkov A. V., Nefedov M. A., Saleev V. A., *Evidence in favor of Single Parton Scattering mechanism in  $\Upsilon$  and  $D$  associated production at the LHC*, Phys. Rev. D **99**, 096021 (2019)].

Thank you for your attention!