# First Order Phase Transition From Hyper Nuclear Matter to Deconfined Quark Matter

Mahboubeh Shahrbaf  Motlagh

**Collaborators: David Blaschke, Ana Gabriela Grunfeld , Hamid Reza Moshfegh**

Helmholtz International Summer School(HISS 2019)
«Quantum Field Theory at the Limits: From Strong Fields to Heavy Quarks»
*25 Jul. 2019-JINR-Dubna*

Uniwersytet Wrocławski

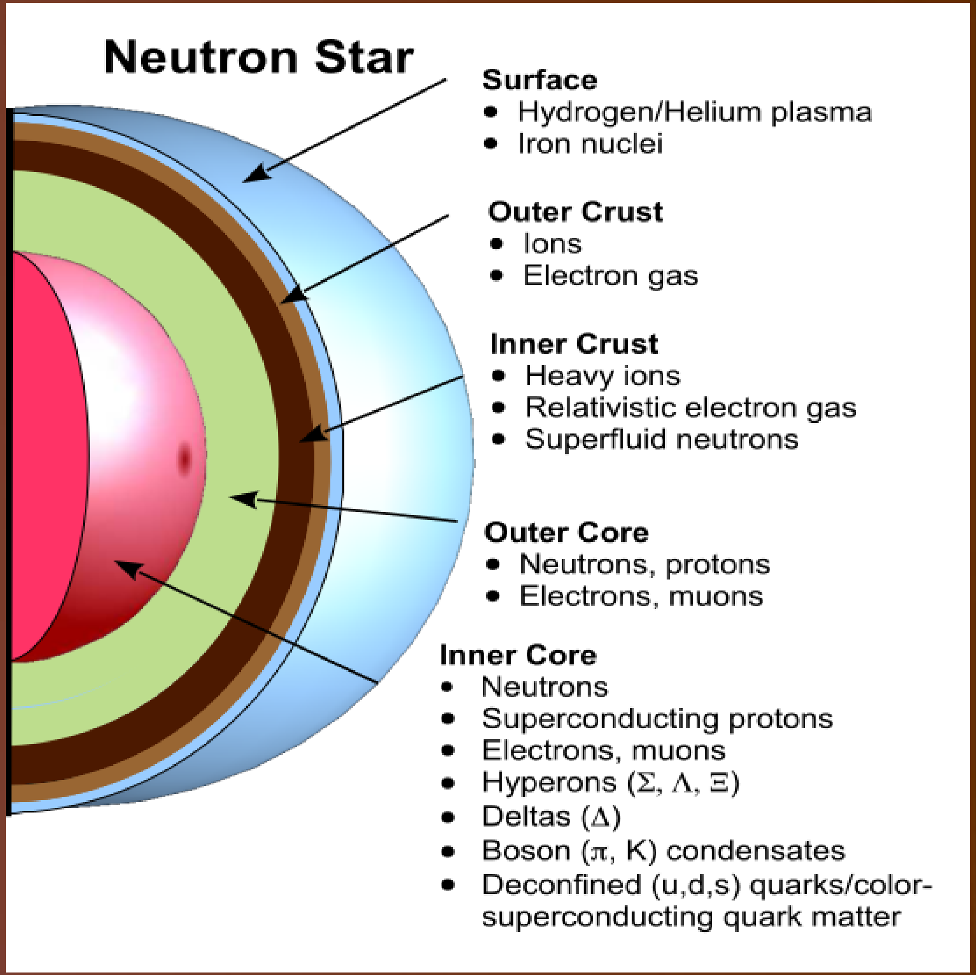
**Nuclear Matter**

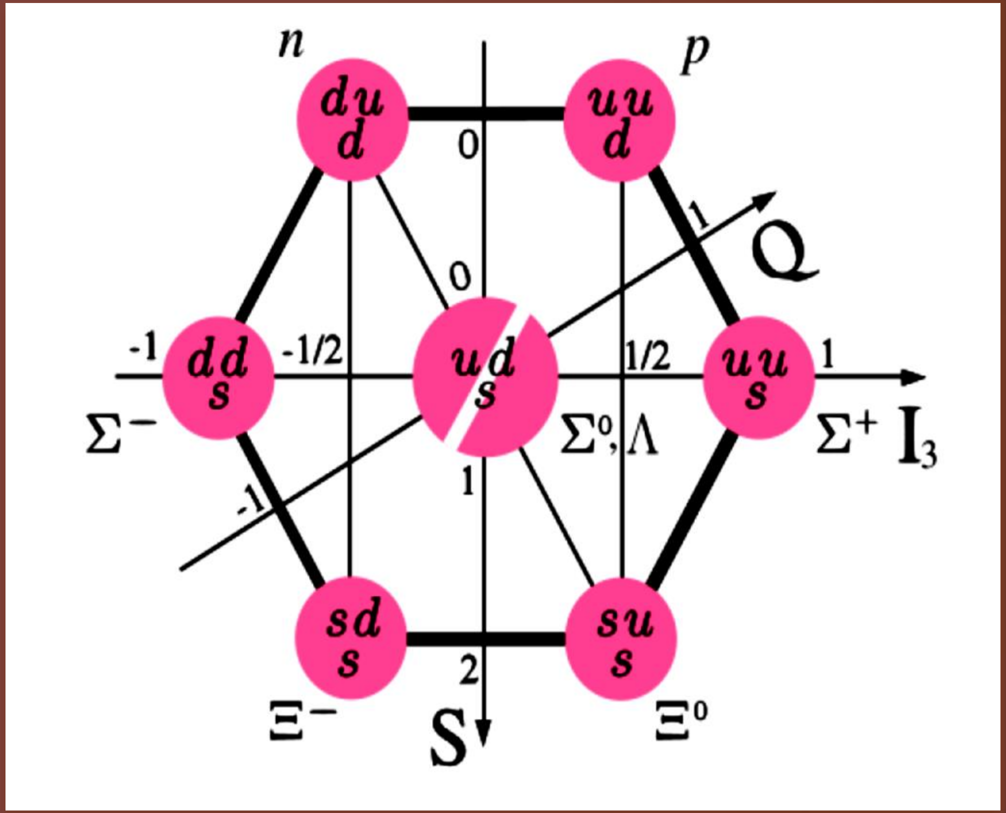
The number of nucleons is supposed to be infinite

Coulomb interaction is disregarded because of the strong interaction between nucleons

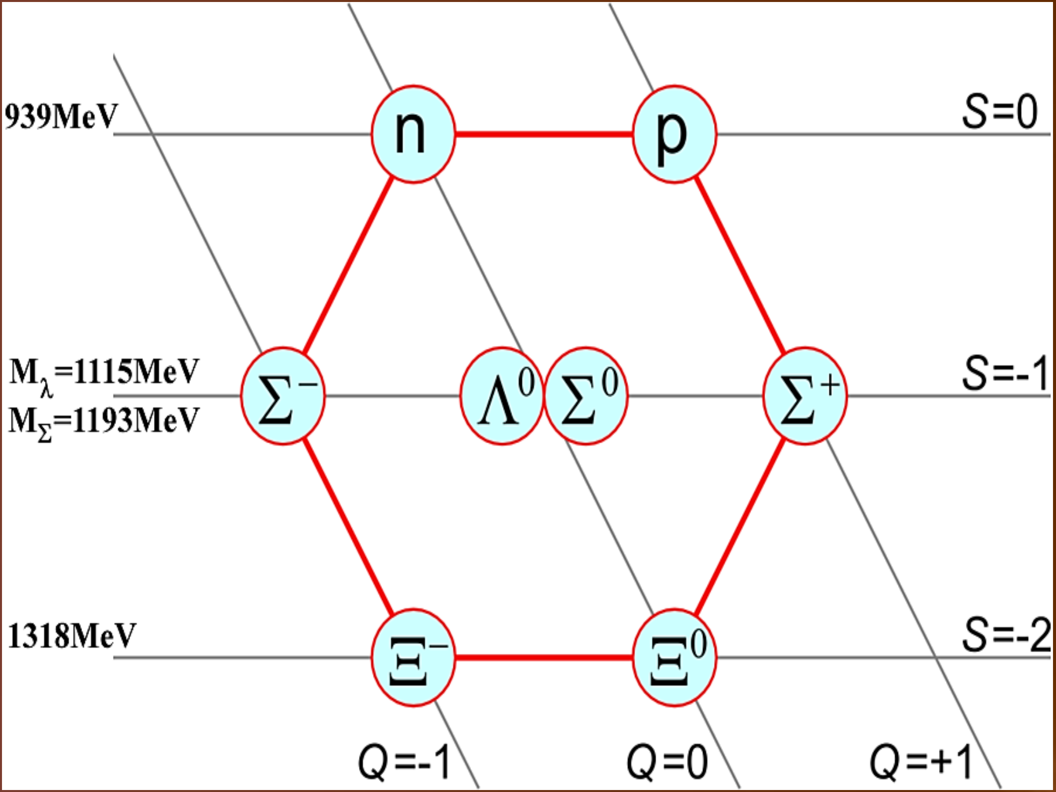
The density of nuclear matter is supposed to be finite

$$\rho = \lim_{N,V\to\infty} \frac{N}{V}$$

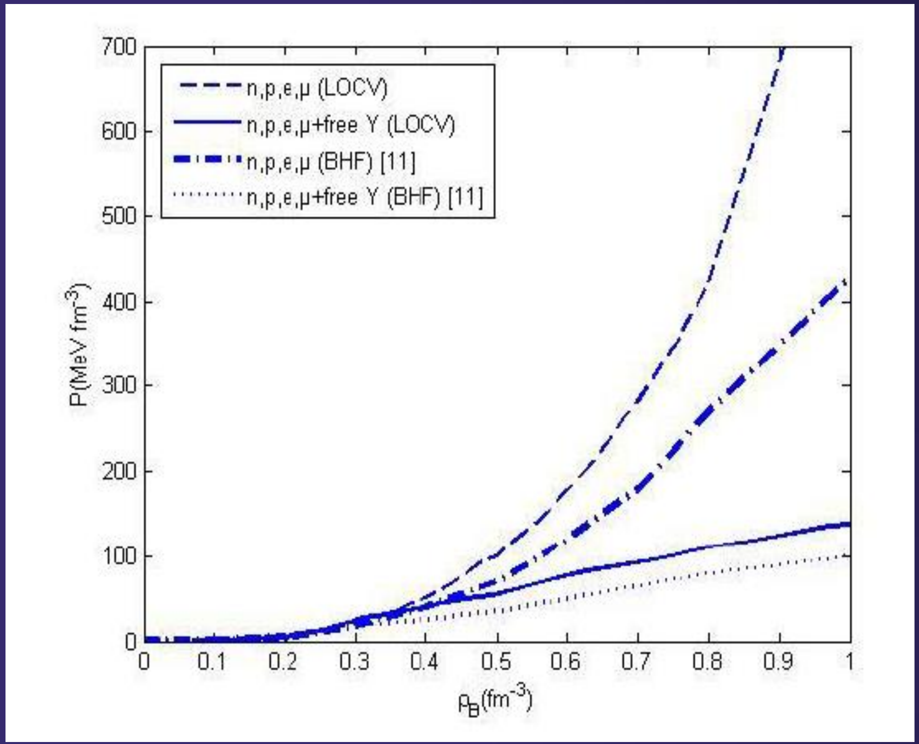


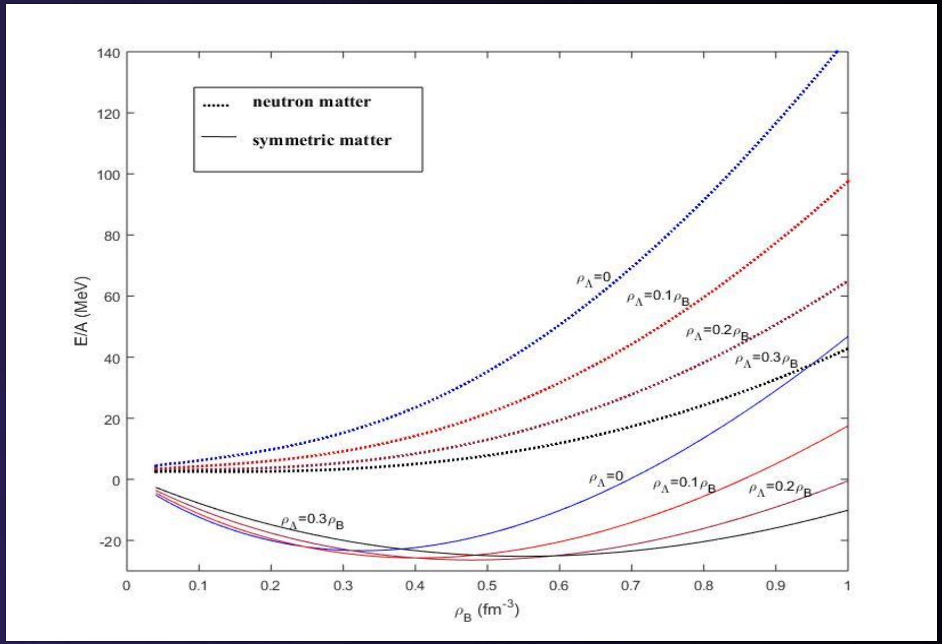


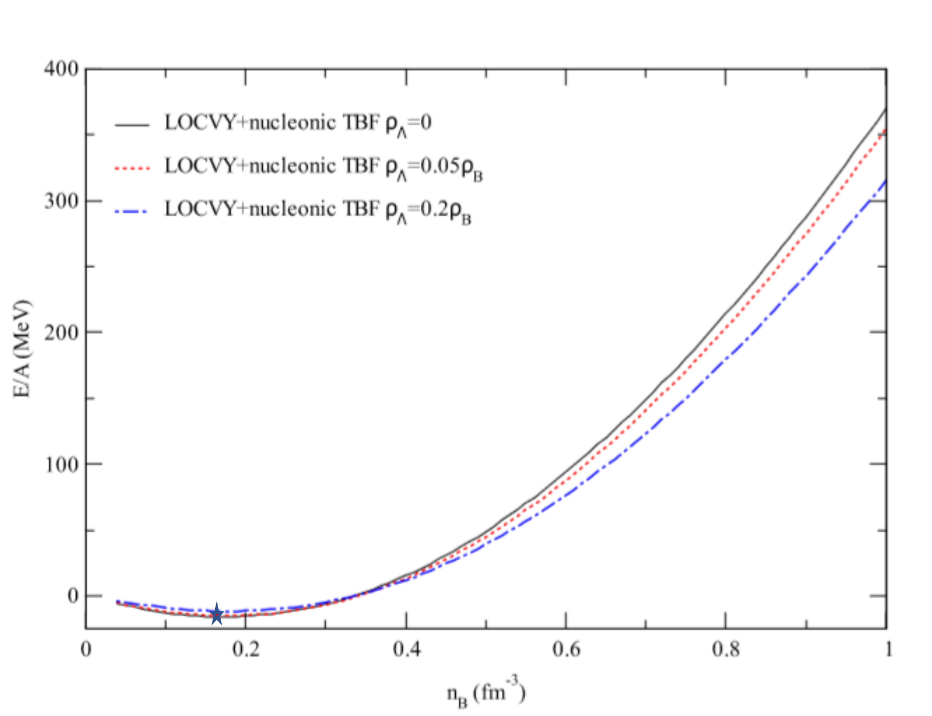
n
p
du d
uu d
0
0
1
Q
-1
dd s
-1/2
ud s
1/2
uu s
1
Σ⁻
Σ⁰, Λ
Σ⁺
I3
-1
1
sd s
2
su s
Ξ⁻
S
Ξ⁰

939MeV
n
p
S=0
Mλ=1115MeV
MΣ=1193MeV
Σ⁻
Λ⁰
Σ⁰
Σ⁺
S=-1
1318MeV
Ξ⁻
Ξ⁰
S=-2
Q=-1
Q=0
Q=+1

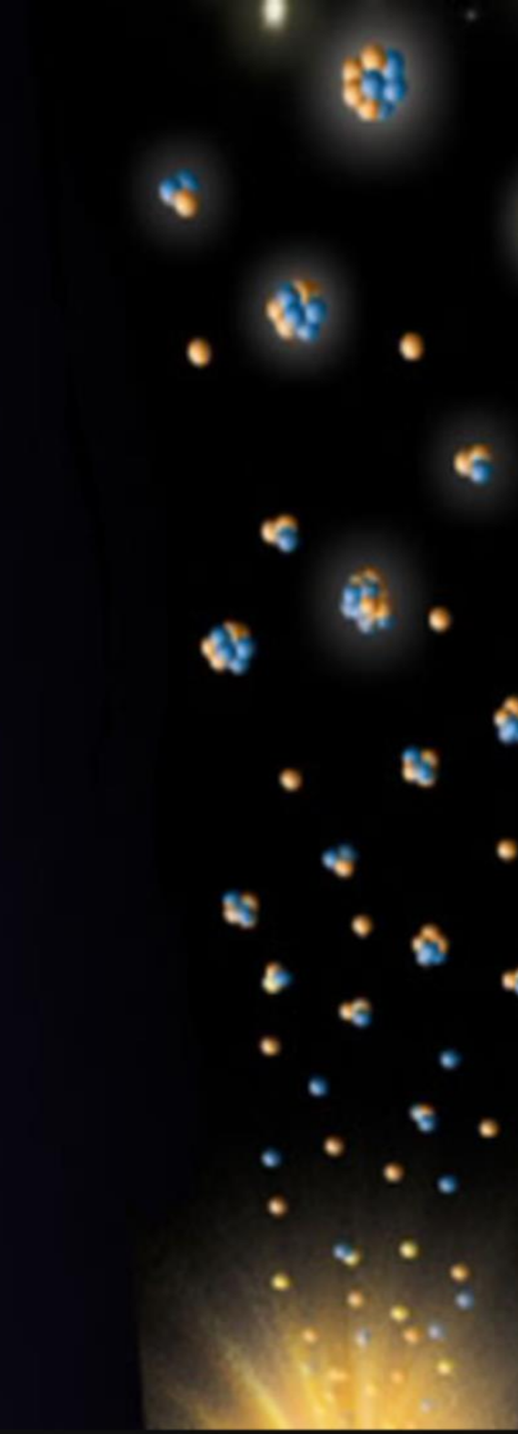


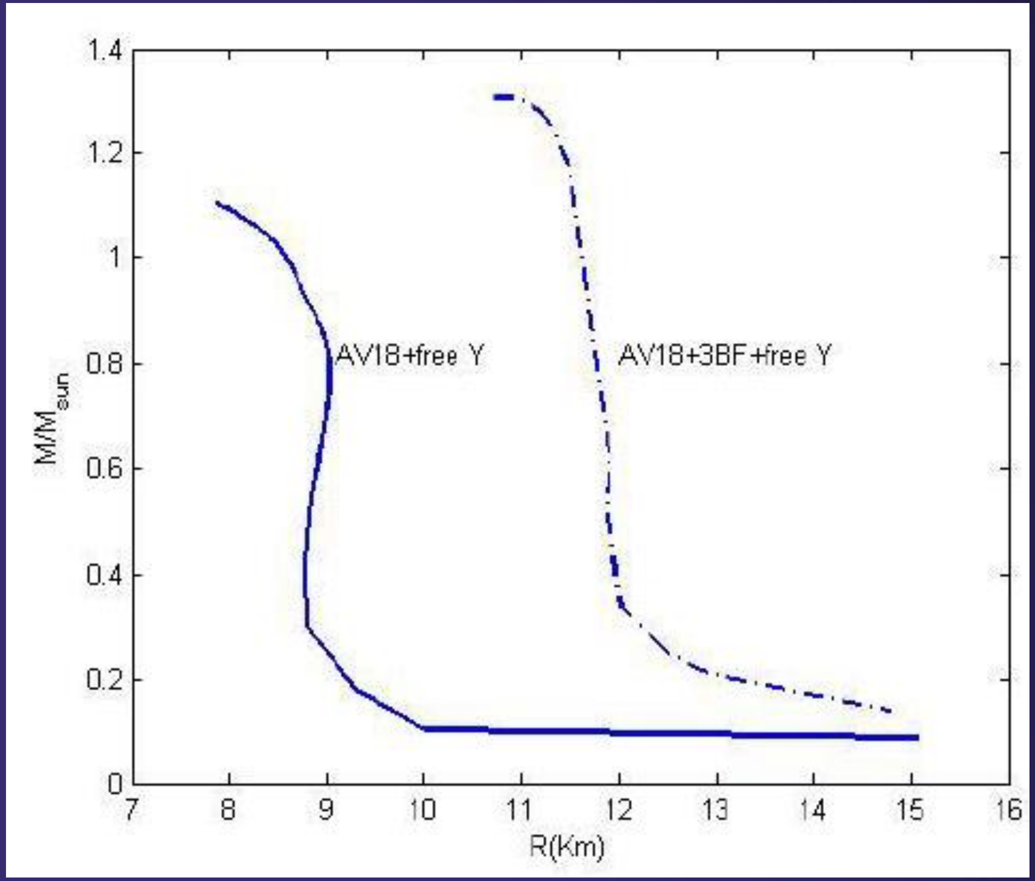




M. Baldo, G. F. Burgio, and H. J. Schulze, Physical Review C 61, 055801 (2000)

| $\rho_0(fm^{-3})$ | 0.1748 |
|---|---|
| $E_0/_A(MeV)$ | $-15.58$ |
| $E_{sym}(MeV)$ | 39.9 |
| $K_0$ | 295.77 |






For PSRJ0740+6620

$Mmax = 2.17^{+0.11}_{-0.10} M\odot$

for the binary neutron star merger GW170817

R($1.6M\odot$) > 10.7 km

&

R($1.4M\odot$) < 13.6 km

# Hyperon Puzzle

# Phase Transition From Hyper Nuclear Matter to Deconfined Quark Matter as a Solution to the Hyperon Puzzle

**Initiation of a new collaboration that joins different domains of state-of-the-art expertise**

LOCV
For hadronic phase

nl-NJL
For quark phase

# LOCV method
## Lowest Order Constrained Variational method

$$Hamiltonian\ of\ nuclear\ matter: H = \sum_i \frac{{P_i}^2}{2m_i} + \sum_{i\neq j} V(ij)$$

Trial wave function : $\boldsymbol{\Psi(1 \dots A) = F(1 \dots A)\Phi(1 \dots A)}$

$$\boldsymbol{E = \langle H \rangle = \frac{1}{N}\frac{\langle \Psi|H|\Psi\rangle}{\langle \Psi|\Psi\rangle} = E_1 + E_{MB} \cong E_1 + E_2}$$

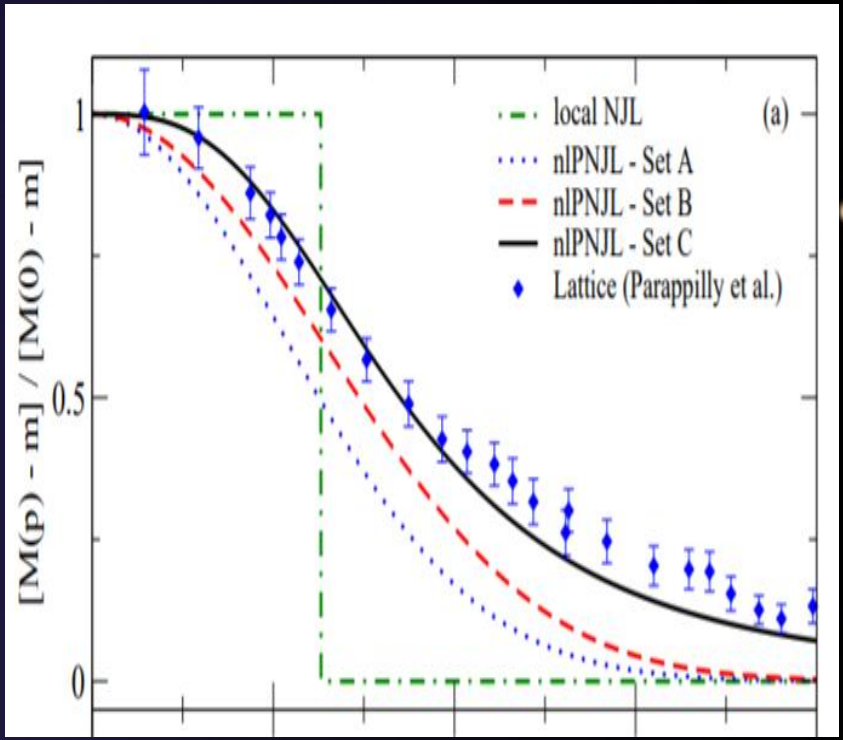
**Characteristics**

- A pure variational method in configuration space
- Generalized to finite temperature
- Calculation of correlation functions
- Using both central and tensor correlation functions
- Energy per baryon and correlation functions are state-dependent
- Using normalization condition as the only constraint

# nl-NJL model
## Nonlocal Nambu–Jona-Lasinio model

**Characteristics**

- nonlocal covariant extension of the NJL model
- quark fields interact via nonlocal (momentum dependent) vertices
- Nonlocal interactions regularize the model in such a way there is not need to introduce sharp cutoffs




G.A. Contrera, A. G. Grunfeld and D. Blaschke, EPJ A 52 (2016)

nl-NJL model

- Constant coefficients (model A)
- density-dependent coefficients (model B)

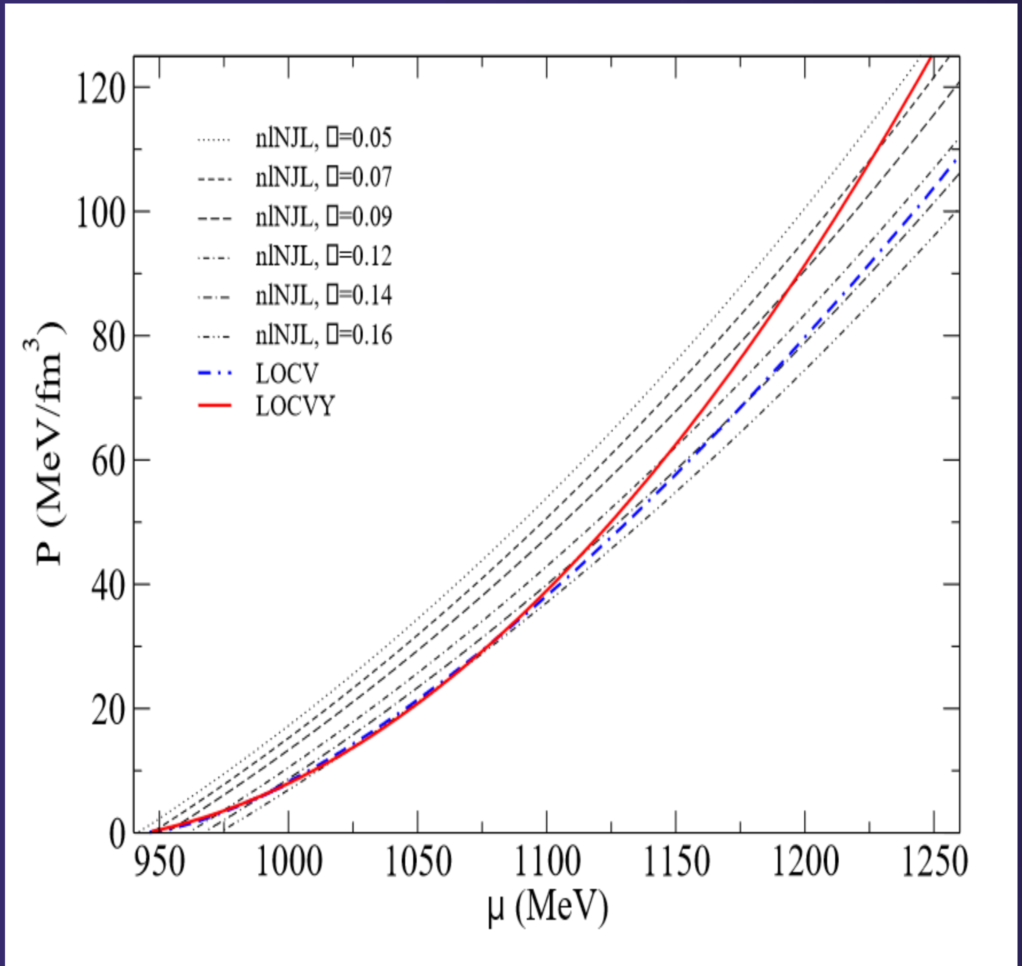
# First Order Phase Transition (PT) by a Maxwell construction

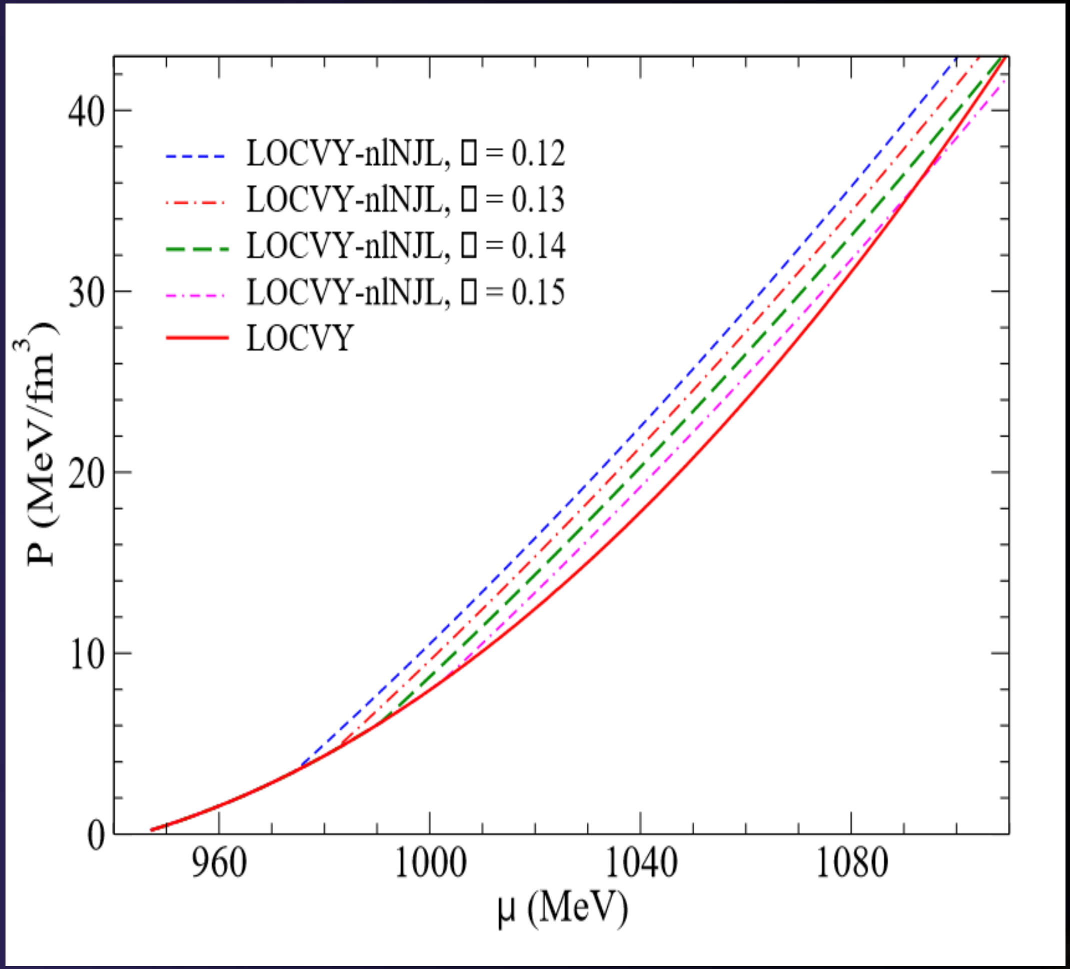
$$\mu H = \mu Q = \mu_c$$

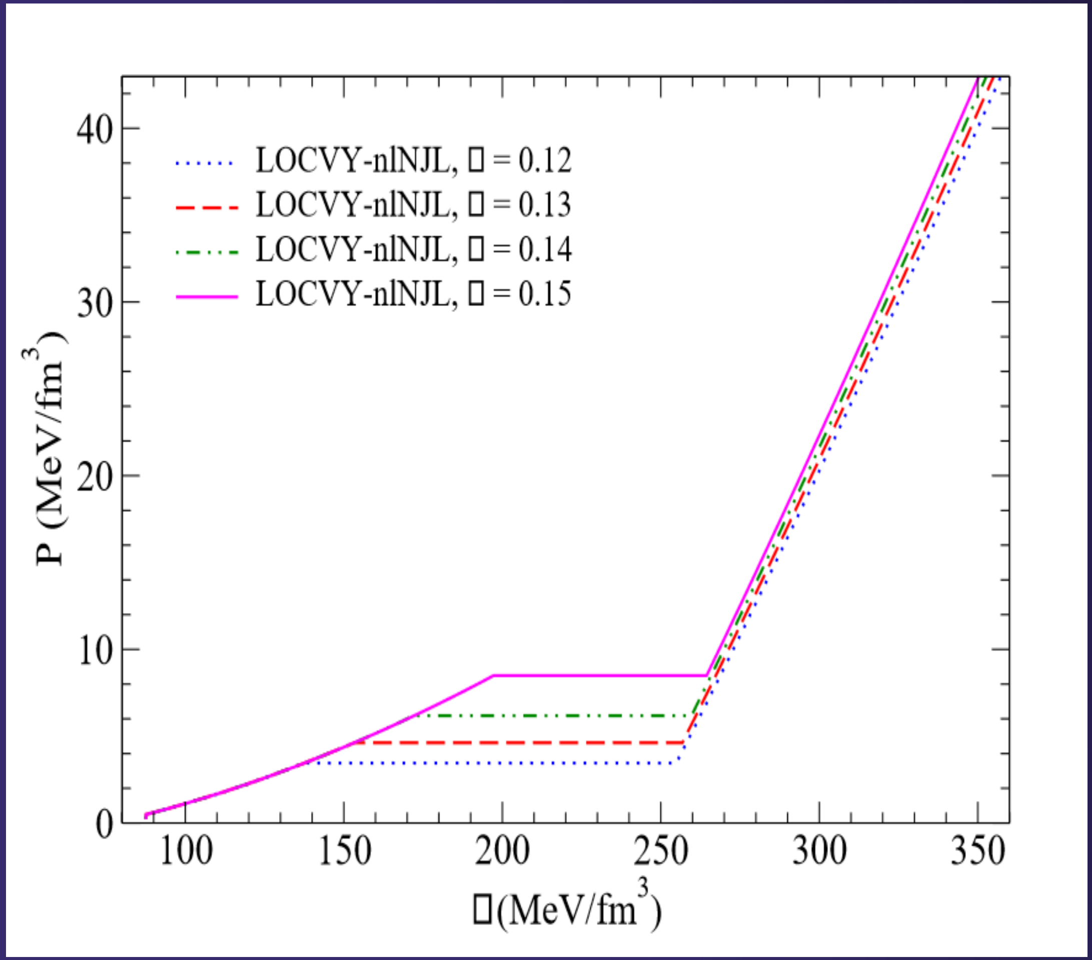
$$TH = TQ = T_c$$

$$P_H(\mu B,\mu e) = P_H(\mu B,\mu e) = p_c$$

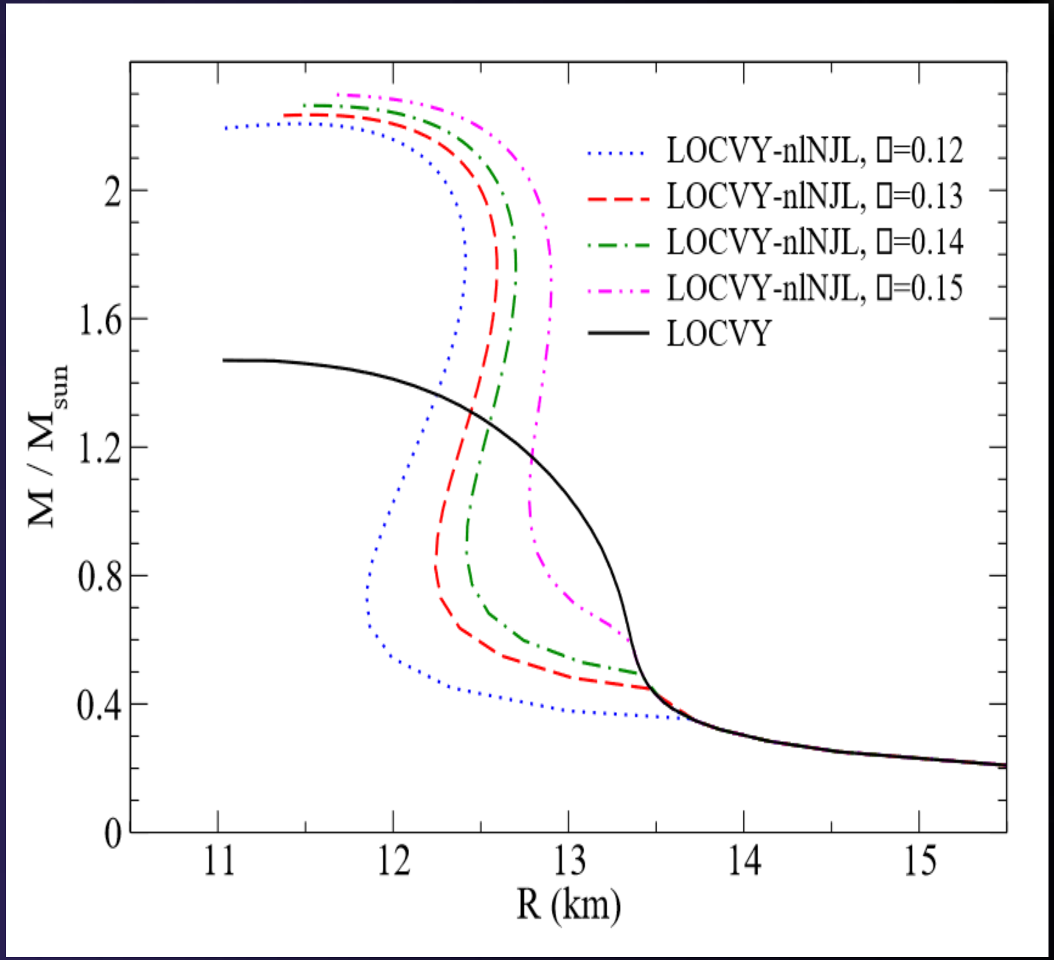
# Model A


nlNJL, □=0.05
nlNJL, □=0.07
nlNJL, □=0.09
nlNJL, □=0.12
nlNJL, □=0.14
nlNJL, □=0.16
LOCV
LOCVY
P (MeV/fm$^3$)
μ (MeV)
0
20
40
60
80
100
120
950
1000
1050
1100
1150
1200
1250


LOCVY-nlNJL, □ = 0.12
LOCVY-nlNJL, □ = 0.13
LOCVY-nlNJL, □ = 0.14
LOCVY-nlNJL, □ = 0.15
LOCVY
P (MeV/fm$^3$)
μ (MeV)
0
10
20
30
40
960
1000
1040
1080

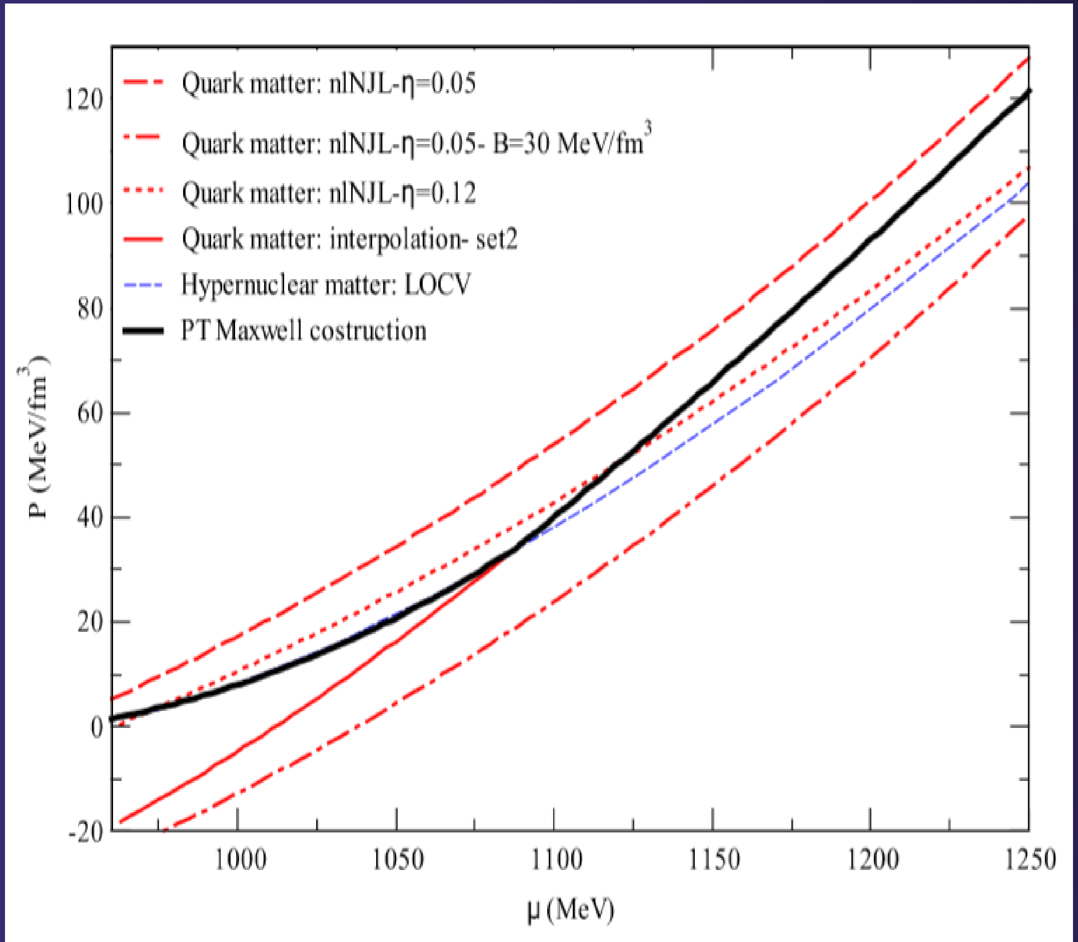


P (MeV/fm^3)
0
10
20
30
40
100
150
200
250
300
350
(MeV/fm^3)
LOCVY-nlNJL, = 0.12
LOCVY-nlNJL, = 0.13
LOCVY-nlNJL, = 0.14
LOCVY-nlNJL, = 0.15

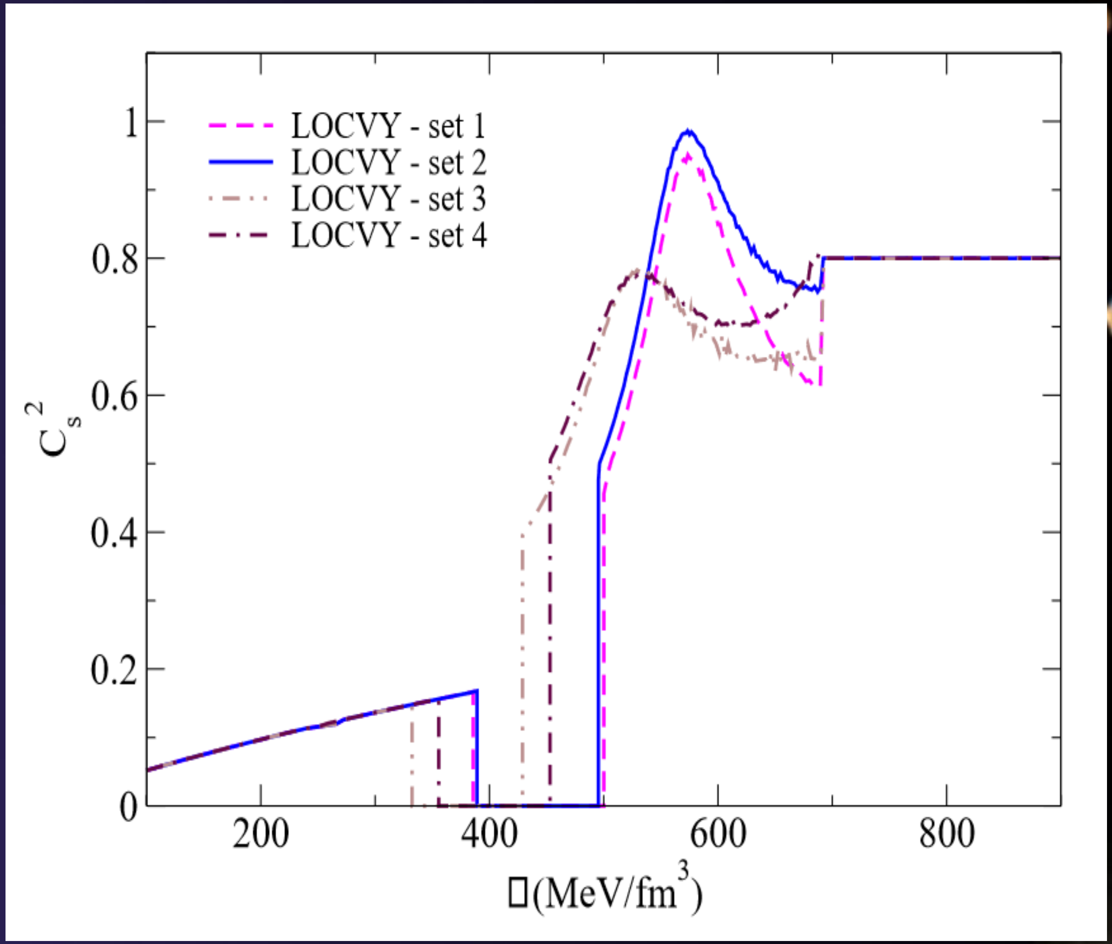



M / M_sun
0
0.4
0.8
1.2
1.6
2
11
12
13
14
15
R (km)
LOCVY-nlNJL, =0.12
LOCVY-nlNJL, =0.13
LOCVY-nlNJL, =0.14
LOCVY-nlNJL, =0.15
LOCVY

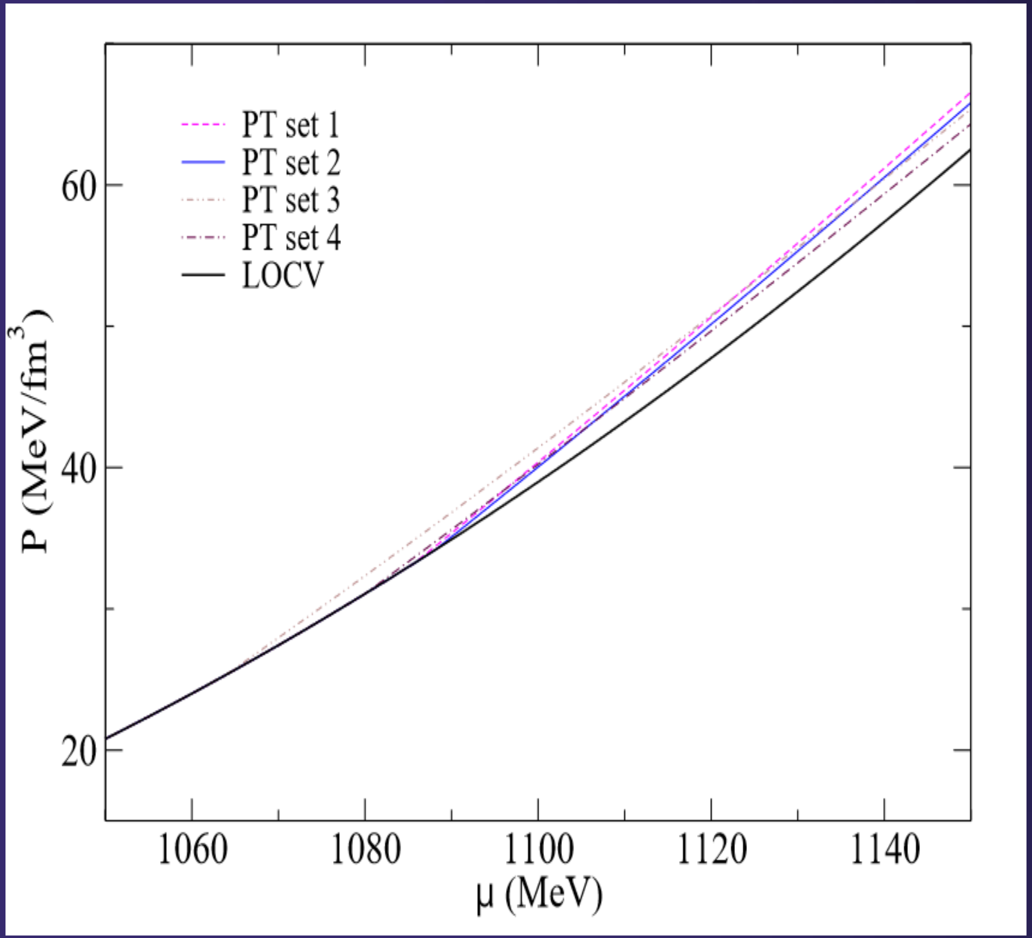
# Model B



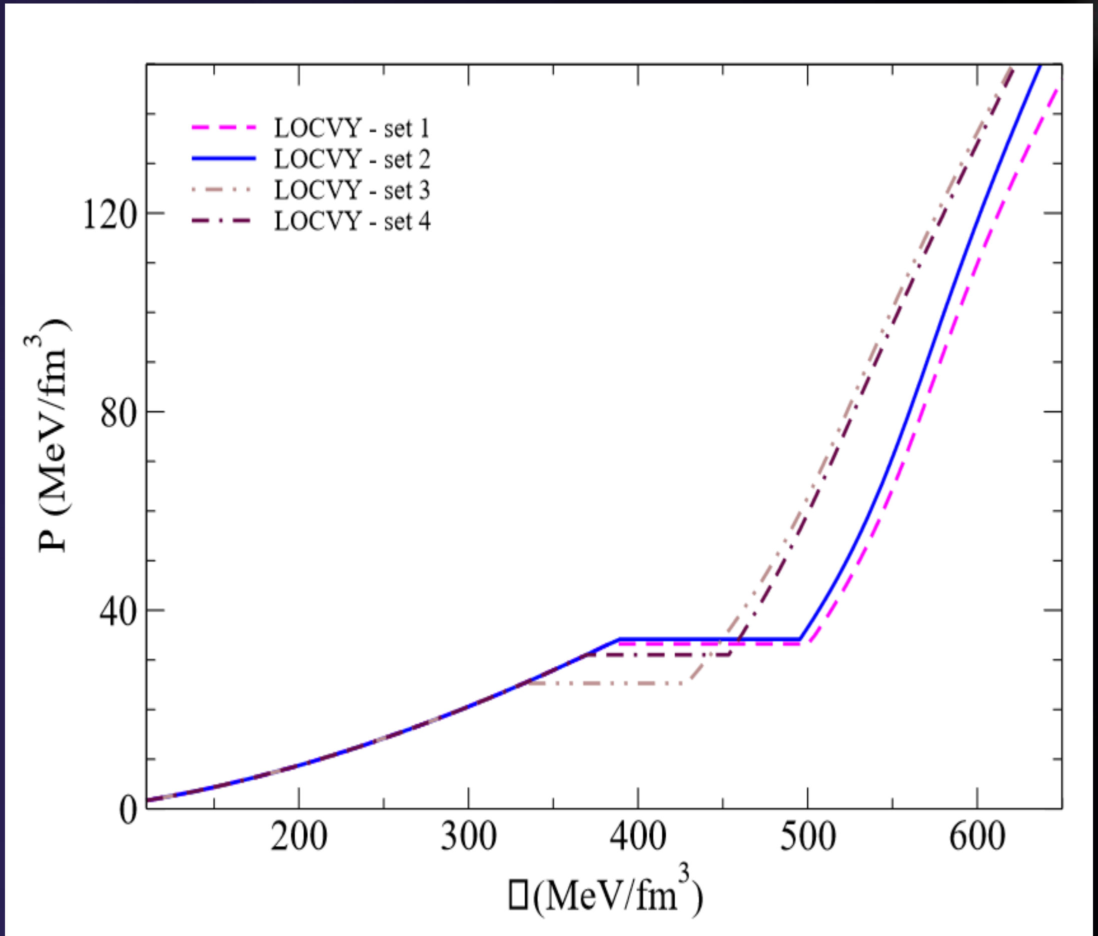
Quark matter: nlNJL-η=0.05
Quark matter: nlNJL-η=0.05- B=30 MeV/fm3
Quark matter: nlNJL-η=0.12
Quark matter: interpolation- set2
Hypernuclear matter: LOCV
PT Maxwell costruction
P (MeV/fm3)
μ (MeV)
120
100
80
60
40
20
0
-20
1000
1050
1100
1150
1200
1250

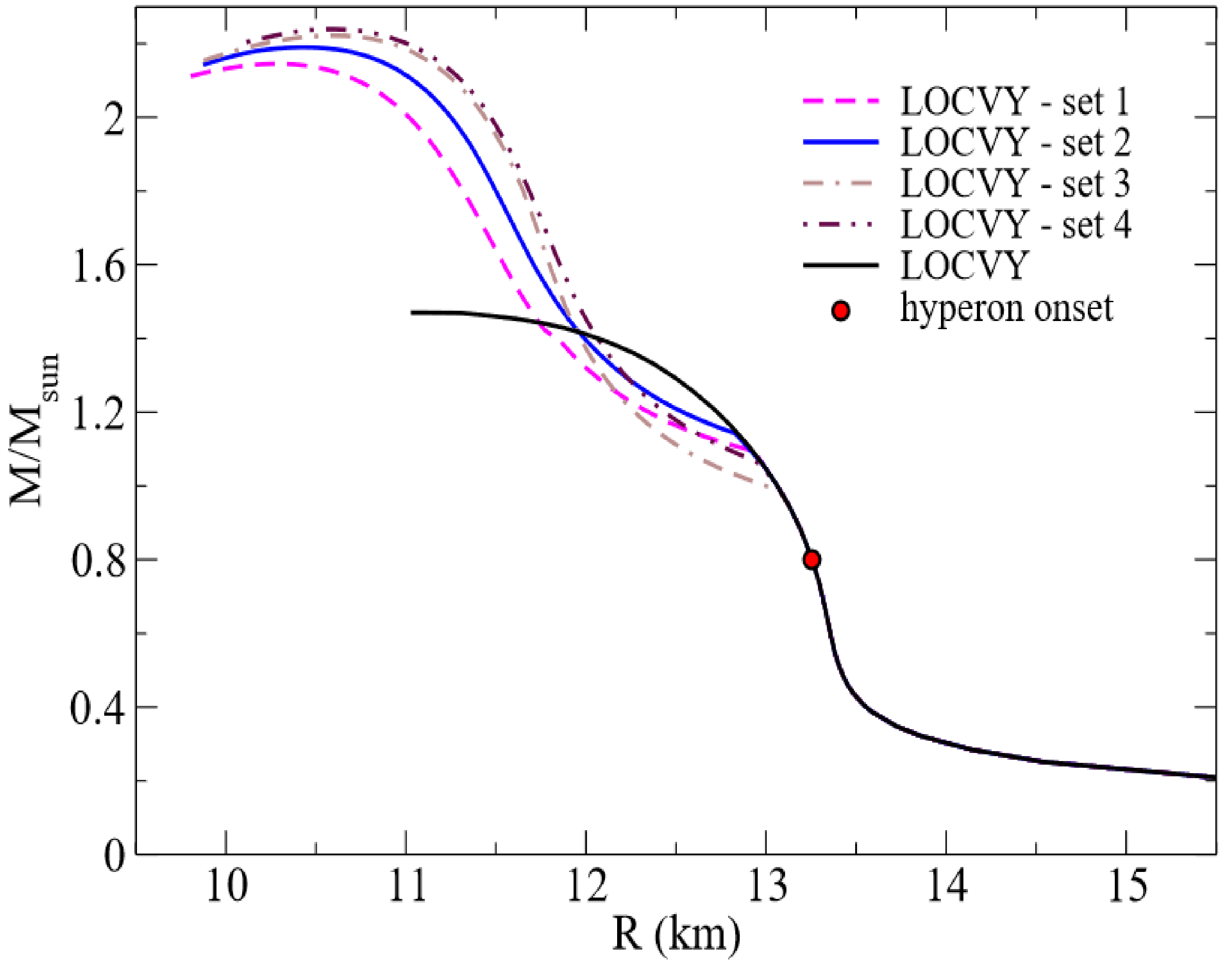
LOCVY - set 1
LOCVY - set 2
LOCVY - set 3
LOCVY - set 4
Cs2
□(MeV/fm3)
1
0.8
0.6
0.4
0.2
0
200
400
600
800

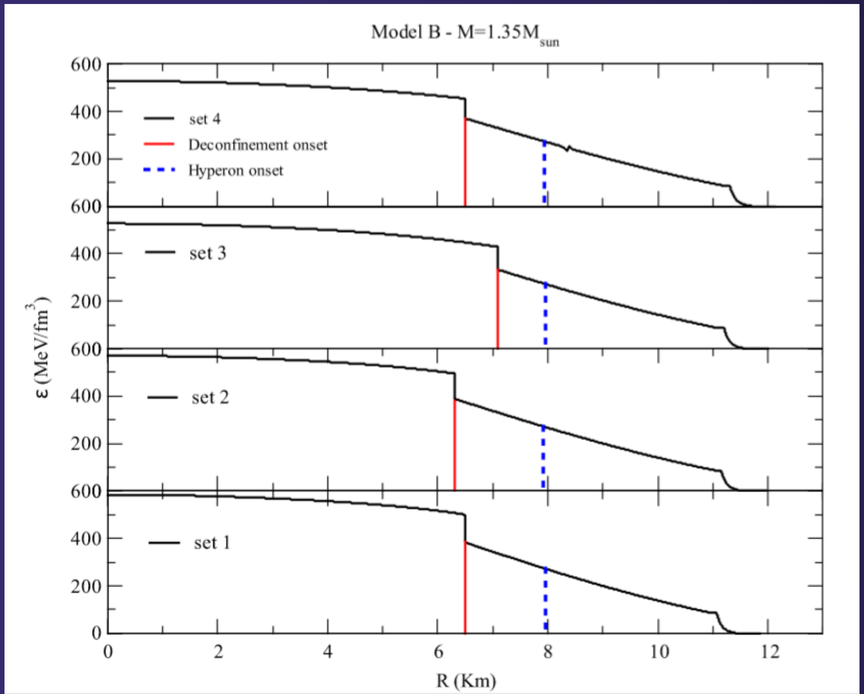


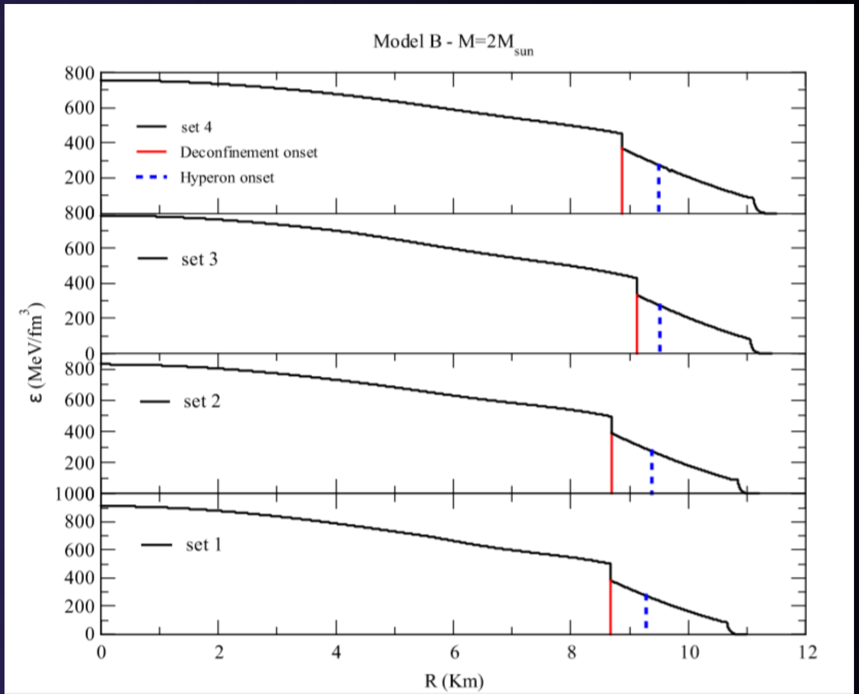
PT set 1
PT set 2
PT set 3
PT set 4
LOCV
P (MeV/fm$^3$)
μ (MeV)
20
40
60
1060
1080
1100
1120
1140

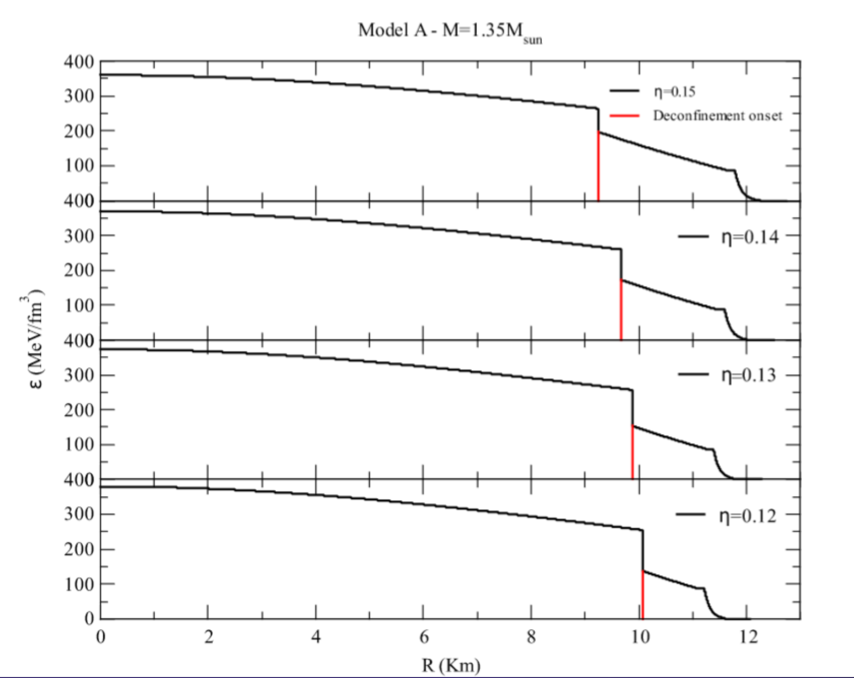



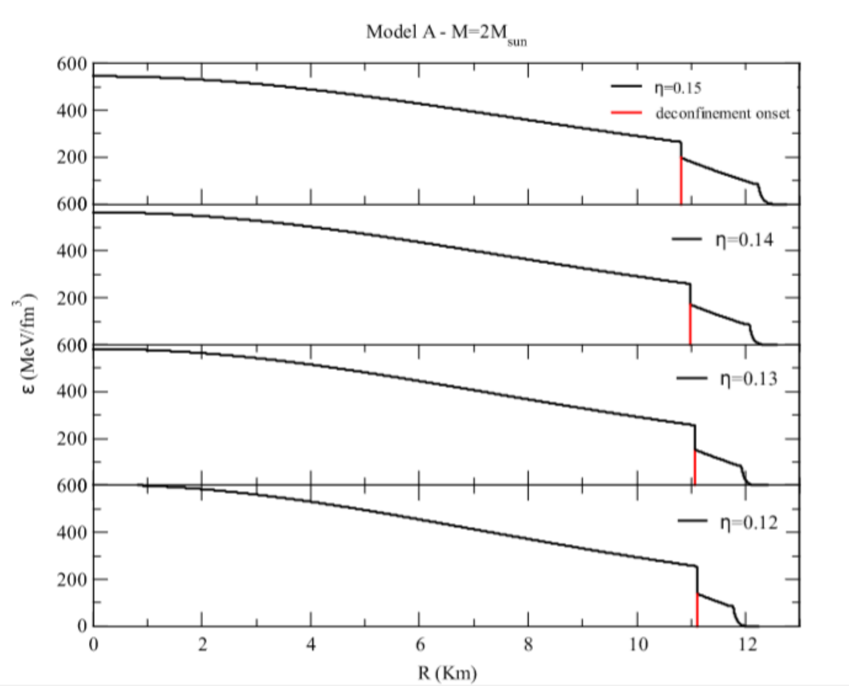
LOCVY - set 1
LOCVY - set 2
LOCVY - set 3
LOCVY - set 4
P (MeV/fm$^3$)
(MeV/fm$^3$)
0
40
80
120
200
300
400
500
600

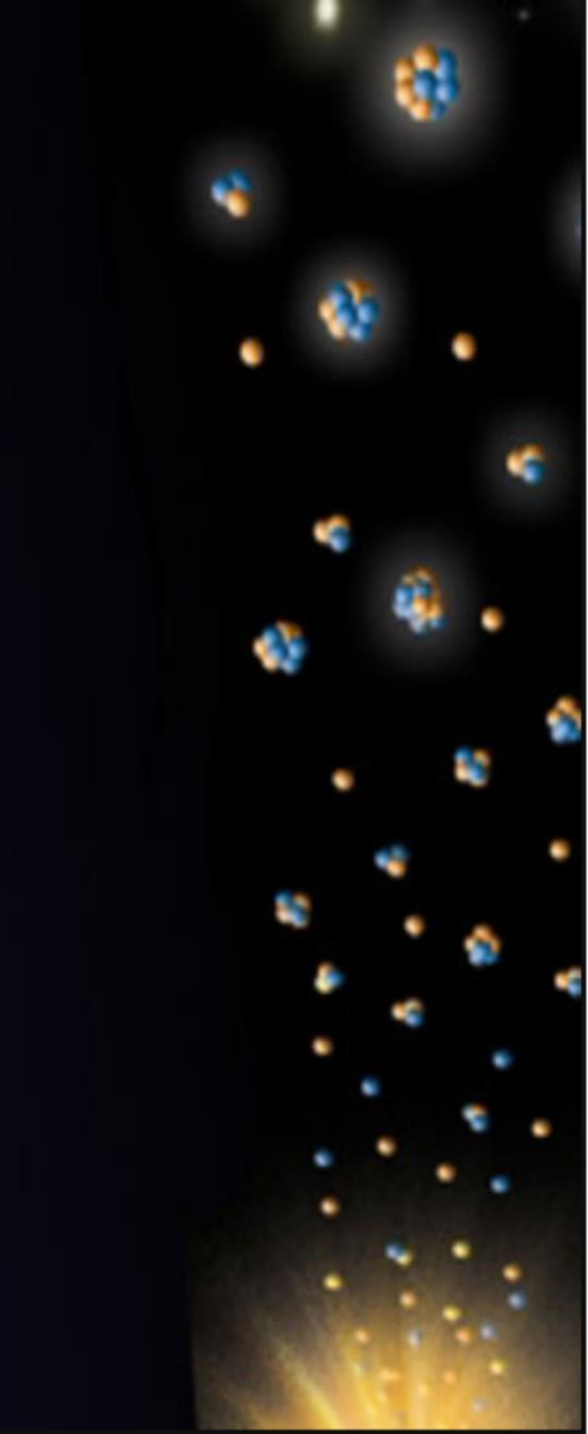

LOCVY - set 1
LOCVY - set 2
LOCVY - set 3
LOCVY - set 4
LOCVY
hyperon onset
M/M_sun
R (km)
0
0.4
0.8
1.2
1.6
2
10
11
12
13
14
15


Model B - M=1.35M$_{sun}$
set 4
Deconfinement onset
Hyperon onset
set 3
set 2
set 1
ε (MeV/fm$^3$)
R (Km)
0 2 4 6 8 10 12
0 200 400 600

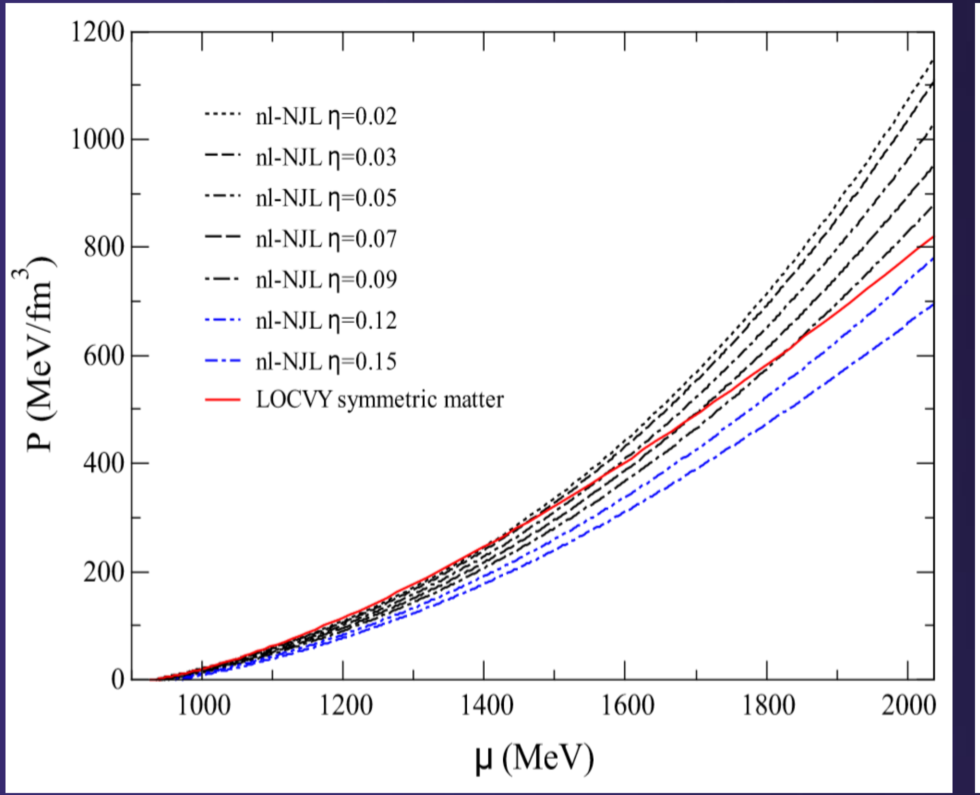

Model B - M=2M$_{sun}$
set 4
Deconfinement onset
Hyperon onset
set 3
set 2
set 1
ε (MeV/fm$^3$)
R (Km)
0 2 4 6 8 10 12
0 200 400 600 800 1000

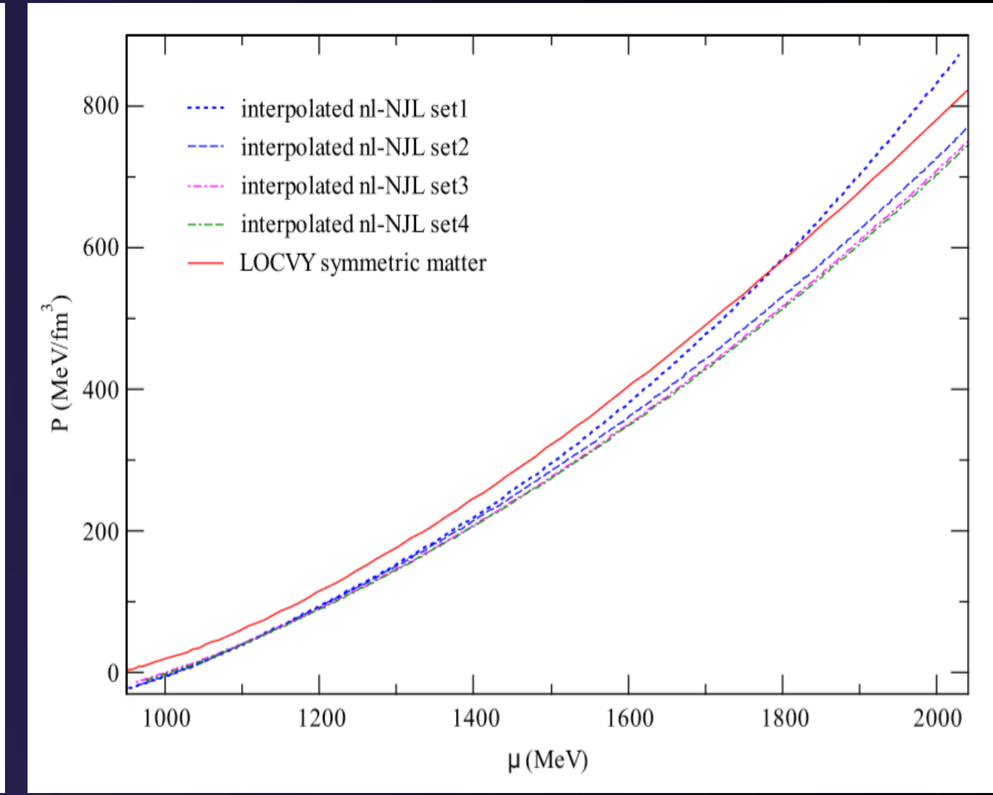

Model A - M=1.35M$_{sun}$
η=0.15
Deconfinement onset
η=0.14
η=0.13
η=0.12
ε (MeV/fm$^3$)
R (Km)
0 2 4 6 8 10 12
0 100 200 300 400


Model A - M=2M$_{sun}$
η=0.15
deconfinement onset
η=0.14
η=0.13
η=0.12
ε (MeV/fm$^3$)
R (Km)
0 2 4 6 8 10 12
0 200 400 600



# Symmetric Matter




**Main results:**

1. Model A : PT in Symmetric matter for η<0.09 while for this cases there is no PT in CS matter
2. Model B : PT in both CS matter and symmetric matter for set 1
3. We have a large difference in critical density for the onset of deconfinement in CS matter and symmetric matter. Onset density for CS matter lies at n=0.38 $fm^{-3}$ while for symmetric matter it is at n=0.95 $fm^{-3}$.


Thank you

# LOCV Method:Lowest Order Constrained Variational Method

$$f(ij) = \sum_{\alpha.p=1}^{3} f_{\alpha}^{p}(ij) O_{\alpha}^{p}(ij)$$

$$\alpha = \{J, L, S, T, T_z\}$$

$p$=1 $for$ $s$=0

$s$=1 with $L$=$J$

$p$=2,3 $for$ $s$=1 with $J$=$L\pm1$

$$O_{\alpha}^{p}(ij) = 1,\ \frac{1}{6}(S_{12} + 4P_t),\ \frac{1}{6}(2P_t - S_{12})$$

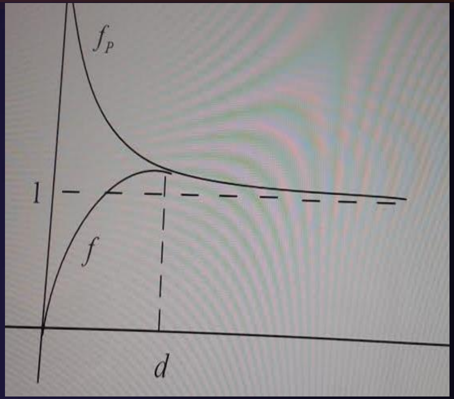
$$S_{12} = 3(\sigma_1.\hat{r})(\sigma_2.\hat{r}) - \sigma_1.\sigma_2$$

$$\left|ij\right\rangle = \left|k_1, {}^1\!/_2, m_{\sigma_1}, \tfrac{1}{2}, m_{\tau_1}, k_2, {}^1\!/_2, m_{\sigma_2}, \tfrac{1}{2}, m_{\tau_2}\right\rangle$$

**The only constraint in LOCV method is renormalization condition of wave functions**

$$\langle \Psi | \Psi \rangle = 1 - \sum_{ij} \langle ij | F_p^2 - F^2 | ij - ji \rangle \quad : \quad \chi = \frac{1}{N} \sum_{ij} \langle ij | F_p^2 - F^2 | ij - ji \rangle = 0$$

$$F_P = \begin{cases} \left(1 - \frac{9}{2}\left(\frac{J_l(K_f r)}{K_f r}\right)^2\right)^{-\frac{1}{2}} & T_z = \pm 1 \\ 1 & T_z = 0 \end{cases}$$




$$E_2 = \int dr \left[G\left(f'^2(r)\right) + S(f(r)) - \lambda(f(r))\right] = \int dr\, L(f'(r), f(r)), \delta E_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial f} - \frac{\partial}{\partial r}\frac{\partial \mathcal{L}}{\partial f'} = 0$$

H. Moshfegh and M. Modarres, Journal of Physics G: Nuclear and Particle Physics **24**, 821 (1998)