Lecture series on
QCD Exotics in the Heavy Quark Sector
Part II: The single heavy sector

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Outline

Lecture I: **Tools**

→ Lattice QCD

→ Effective field theories (ChPT, HQEFT)

→ Unitarisation

→ Large $N_c$

Lecture II: **The single heavy sector**

→ Goldstone–Boson D-meson scattering

→ The positive parity D-mesons

→ Predictions and tests

Lecture III: **The $\bar{Q}Q$ sector**

→ The XYZ-stories

In this lecture series the **focus is on mesons**
Limiting cases of QCD

\[
\mathcal{L}_{\text{QCD}} = \sum_f \bar{q}_f \left( \gamma_\mu D^\mu - m_f \right) q_f - \frac{1}{4T} \text{Tr} \left( F^{\mu\nu} F_{\mu\nu} \right)
\]

Limit of massless Quarks

\[
\mathcal{L}_{\text{QCD}} = \bar{q}_L \left\{ i\partial + gA^a t^a \right\} q_L + \bar{q}_R \left\{ i\partial + gA^a t^a \right\} q_R + \mathcal{O}\left( m_f / \Lambda_{\text{QCD}} \right)
\]

L and R Quarks decouple + spontaneous symmetry breaking

→ Chiral Perturbation Theory (ChPT)

Limit of infinitely Heavy Quarks

\[
\mathcal{L}_{\text{QCD}} = \bar{q}_f \left\{ iv \cdot \partial + gv \cdot A^a t^a \right\} q_f + \mathcal{O}\left( \Lambda_{\text{QCD}} / m_f \right)
\]

Independent of Heavy Quark Spin and Flavour

→ Heavy Quark Effective Field Theory (HQEFT)

→ Non-Relativistic QCD (NRQCD)
Charmed states

Puzzles:

Why are/is

1. $M(D_{s1}) \& M(D_{s0}^{*})$ so light?

2. $M(D_{s1}) - M(D_{s0}^{*}) \simeq M(D^{*}) - M(D)$?

3. $M(D_{0}^{*}) > M(D_{s0}^{*})$?

$M(D_{1}) \simeq M(D_{s1})$?

Solved by combining unitarized EFTs and Lattice QCD.

Experiment can provide further evidence.
Hadronic Molecules

→ are few-hadron states, bound by the strong force

→ do exist: light nuclei.
  e.g. deuteron as \( pn \) & hypertriton as \( \Lambda d \) bound state

→ are located typically close to relevant continuum threshold;
  e.g., for \( E_B = m_1 + m_2 - M \) and \( \gamma = \sqrt{2\mu E_B} \)

  \[
  \begin{align*}
  E_B^{\text{deuteron}} &= 2.22 \text{ MeV} \quad (\gamma = 45 \text{ MeV}) \\
  E_B^{\text{hypertriton}} &= (0.13 \pm 0.05) \text{ MeV} \quad (\text{to } \Lambda d) \quad (\gamma = 13 \text{ MeV})
  \end{align*}
  \]

→ can be identified in observables (Weinberg compositeness):

\[
\frac{g_{\text{eff}}^2}{4\pi} = \frac{4M^2\gamma}{\mu} (1 - \lambda^2) \rightarrow a = -2 \left( \frac{1 - \lambda^2}{2 - \lambda^2} \right) \frac{1}{\gamma}; \quad r = - \left( \frac{\lambda^2}{1 - \lambda^2} \right) \frac{1}{\gamma}
\]

where \( (1 - \lambda^2) \) = probability to find molecular component in bound state wave function

Are there mesonic molecules?
Size of a Molecule

with \( E = M_{\text{Mol}} \) \& \( E_1 + E_2 = M_1 + M_2 + p^2 / (2\mu) \)
and \( E_B = M_1 + M_2 - M_{\text{Mol}} \).

\[ p \sim \sqrt{2\mu E_B} \]

\[ \rightarrow \text{size of the molecule, } R, \text{ reads } R \sim 1/p \sim 1/\sqrt{2\mu E_B} \]

\[ \text{c.f. H-atom: } E_B = m_e \alpha^2 / 2; \mu = m_e \rightarrow a_0 = 1/(m_e \alpha) \]

On the other hand: confinement radius \( \ll 1 \text{ fm} \)

Molecules extended for \( (\hbar c)^2 / (2\mu) \gtrsim E_B \)

for \( \mu \sim 0.5 \text{ GeV} \) we need \( E_B \sim 40 \text{ MeV} \) or smaller

then external probes couple predominantly via the constituents
Expand in terms of non–interacting quark and meson states

\[ |\Psi\rangle = \left( \frac{\xi |\psi_0\rangle}{\chi(p)} |h_1 h_2\rangle \right), \]

here \( |\psi_0\rangle = \) elementary state and \( |h_1 h_2\rangle = \) two–hadron cont., then \( \xi^2 \) equals probability to find the bare state in the physical state

\[ \rightarrow \xi^2 = 1 - \lambda^2 \text{ is the quantity of interest!} \]

The Schrödinger equation reads

\[ \hat{\mathcal{H}} |\Psi\rangle = E |\Psi\rangle, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}_{hh}^0 \end{pmatrix} \rightarrow \chi(p) = \xi \frac{f(p^2)}{E - p^2 / (2\mu)} \]

introducing the transition form factor \( \langle \psi_0 |\hat{V} |hh\rangle = f(p^2) \),

Note: \( \hat{H}_{hh}^0 \) contains only meson kinetic terms!
Effective Coupling

Therefore

\[ |\Psi\rangle = \xi \left( \frac{|\psi_0\rangle}{f(p^2)/E_B + p^2/(2\mu)} |h_1h_2\rangle \right) , \]

For the normalization of the physical state we get

\[ 1 = \langle \Psi | \Psi \rangle = \xi^2 \left( 1 + \int \frac{f^2(p^2)}{(E_B + p^2/(2\mu))^2} d^3p \right) . \]

This shows that \( \xi^2 = Z \). Using

\[ \int \frac{f^2(p^2) d^3p}{(p^2/(2\mu) + E_B)^2} = \frac{4\pi^2 \mu^2 f(0)^2}{\sqrt{2\mu E_B}} + O \left( \sqrt{E_B \mu} R \right) \]

for \( s \)-waves with \( R \sim 1/\beta = \) range of forces. Using \( 8\pi^2 \mu f(0)^2 = g \)

\[ 1 = \xi^2 \left( 1 + \frac{\mu g/2}{\sqrt{2\mu E_B}} + O \left( \frac{\sqrt{E_B \mu}}{\beta} \right) \right) \]
Thus...

using for residue $g_{\text{eff}}^2/4\pi = Z^2(m_1 + m_2)^2 g$

$$\frac{g_{\text{eff}}^2}{4\pi} = 4(m_1 + m_2)^2(1 - \xi^2) \sqrt{2E_B/\mu} \leq 4(m_1 + m_2)^2 \sqrt{2E_B/\mu}$$

$(1 - \xi^2) = \lambda^2 = \text{molecular component in physical state}$

Note: leading term non–analytic in $E_B$ with clear interpretation

The structure information is hidden in the effective coupling, extracted from experiment, independent of the phenomenology used to introduce the pole(s)

Picture not changed by far away threshold

Equivalent to, e.g.,

V. Baru et al. PLB586 (2004)53

The formalism presented is 'diagnostic' — especially, it does not allow for conclusions on the binding force, it allows one only to study individual states. To go beyond that a dynamical model needs to be employed.

Quantitative interpretation gets lost when states get bound too deeply; we propose to stick to 'the larger the coupling the more molecular the state'. There are striking phenomenological implications from large couplings for they lead to relations between seemingly unrelated reactions rather specific, unusual line shapes.
Formal considerations

Transition from a pos. parity state to a light neg. parity state and

<table>
<thead>
<tr>
<th>transition</th>
<th>a pos. parity $\bar{Q}q$</th>
<th>a neg. parity $\bar{Q}q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>convergent</td>
<td>divergent</td>
</tr>
<tr>
<td>compact state</td>
<td>$N^2$LO</td>
<td>LO</td>
</tr>
<tr>
<td>molecule</td>
<td>LO</td>
<td>NLO</td>
</tr>
</tbody>
</table>

Only those transitions are sensitive to the molecular nature that are dominated by the loops!

M. Cleven et al., PRD87 (2013)074006
Unitraized ChPT


\[
\mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - m_D^2 D D^\dagger
\]

with \( D = (D^0, D^+, D_s^+) \) denoting the \( D \)-mesons, and

\[
\mathcal{D}_\mu = \partial_\mu + \frac{1}{2} \left( u^\dagger \partial_\mu u + u \partial_\mu u^\dagger \right)
\]

where \( u = \exp \left( \frac{\sqrt{2}i\phi}{2F_\pi} \right) \)

The Goldstone boson fields are collected in the matrix

\[
\phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
K^-
\end{pmatrix}
\begin{pmatrix}
\pi^+ \\
-\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^0
\end{pmatrix}
\begin{pmatrix}
K^+ \\
\bar{K}^0 \\
-\frac{2}{\sqrt{6}} \eta
\end{pmatrix}
\]

LO potential parameter free; 1 regulator necessary

At NLO 6 low energy constants enter
Heavy light Systems

→ $\pi/K/\eta - D/D_s$ scattering in ChPT to NLO unitarized

→ controlled quark mass dependence

→ fit LECs to lattice data

$Liu et al. PRD87(2013)014508$

→ $D_s(2317)^*$ emerges as a pole with $M_{D_s(2317)^*} = 2315^{+18}_{-28}$ MeV.
Interpretation

shaded band (dashed line):
full result (best fit)

white band (solid line):
$D_{s0}^*(2317)$ mass fixed to physical value

Liu et al. PRD87(2013)014508

Lattice: Mohler et al., PRL 11(2013)222001

$D_{s0}^*(2317)$: $a = g_{\text{eff}} - g_{\text{eff}} + \mathcal{O}(1/\beta) \simeq \left( \frac{2\lambda^2}{1+\lambda^2} \right) \frac{-1}{\sqrt{2m_K E_B}}$

$a = -(1.05\pm0.36) \text{ fm} \text{ for molecule } (\lambda^2 = 1); \text{ smaller otherwise}$
Isospin breaking (drives decay) via quark masses and charges
The same effective operators lead to

- mass differences, e.g.
  \[ m_{D^+} - m_{D^0} = \Delta m^q + \Delta m^{\text{e.m.}} = ((2.5 \pm 0.2) + (2.3 \pm 0.6)) \text{ MeV} \]
  \[ \pi^0 - \eta \text{ mixing} \quad \rightarrow \quad \text{parameters fixed} \]

- Isospin breaking scattering amplitude
  \[ \text{e.g. } KD \rightarrow \pi^0 D_s \text{ predicted} \]

Specific for molecules!
Measurement of width is decisive, if $D_{s0}$ is molecular or not

Experiment needs very high resolution → PANDA
... and in the $S = 0$ sector

Keeping parameters fixed one gets:

$$E^{\text{free}}_{D\pi}$$
$$E^{\text{free}}_{D\eta}$$
$$E^{\text{free}}_{D_sK}$$
LQCD

![Graph showing energy levels](image)

Poles for

$$m_{\pi} \approx 391 \text{ MeV}: (2264, 0) \text{ MeV} \ [000] \ & \ (2468, 113) \text{ MeV} \ [110]$$

$$m_{\pi} = 139 \text{ MeV}: (2105, 102) \text{ MeV} \ [100] \ & \ (2451, 134) \text{ MeV} \ [110]$$

Questions $c\bar{q}$ nature of lowest lying $0^+ D$ state, $D_0^*(2400)$

SU(3) structure

\[ m(x) = m^{\text{phys}} + x(m - m^{\text{phys}}) \]

\[ m_\phi = 0.49 \text{ GeV}; \quad M_D = 1.95 \text{ GeV} \]

Multiplets: \[ \begin{bmatrix} 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 15 \\ \oplus \end{bmatrix} + \begin{bmatrix} 6 \\ \oplus \end{bmatrix} \]

with \[ \begin{bmatrix} 15 \end{bmatrix} \] repulsive and \[ \begin{bmatrix} 3 \end{bmatrix} \] most attractive

→ 3 poles give observable effect with SU(3)-breaking on

→ At SU(3) symmetric point \( m_\phi \approx 490 \text{ MeV} \): 3 bound and 6 virtual states

→ For \( m_\phi \approx 600 \text{ MeV} \) (SU(3) sym.): even [6]-states get bound

→ Quark Model: \[ \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \end{bmatrix} \] — the [6] is absent
**Observable:** \( B^- \rightarrow D^+ \pi^- \pi^- \)

With the \( \phi D \) amplitude fixed we can calc. production reactions:

Du et al., PRD98(2018)094018

for the \( S' \)-wave (two free para.);

other partial waves from BW-fit

LHCb, PRD94(2016)072001

\[
\langle P_0 \rangle \propto |A_0|^2 + |A_1|^2 + |A_2|^2,
\langle P_2 \rangle \propto \frac{2}{5} |A_1|^2 + \frac{2}{7} |A_2|^2 + \frac{2}{\sqrt{5}} |A_0||A_2| \cos(\delta_2 - \delta_0)
\]

\[
\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}} |A_0||A_1| \cos(\delta_1 - \delta_0)
\]
$D\pi$ S-wave from $B^- \rightarrow D^+ \pi^- \pi^-$

Effect of thresholds enhanced by pole at $\sqrt{s_p} \sim (2451 - i134)$ MeV on nearby unphysical sheet.
Charmed states

Puzzles solved:

1. \( M(D_{s1}) \& M(D_{s0}^*) \) are \( DK \) and \( D^*K \) bound states

2. \( M(D_{s1}) - M(D_{s0}^*) \approx M(D^*) - M(D) \), since spin symmetry gives equal binding

3. Proper mass differences
   \[ M(D_0^*) = 2100 \text{ MeV} \]
   \[ M(D_{s0}^*) = 2317 \text{ MeV} \]
   \[ M(D_1) = 2247 \text{ MeV} \]
   \[ M(D_{s1}) = 2460 \text{ MeV} \]

... further support from experiment eagerly waited for
Roadmap for future studies

fundamental theory and tools

Lattice QCD

QCD

SU(3) structure

EFTs (+unitarization)

theoretical observables

scattering lengths phase shifts

mass distributions, width etc.

spectrum of resonances

spectra of near stable states

more complex observables like Dalitz plots

experimental observables
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