Production of b-hadrons and bottomonia at the LHC.

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https://ssau.ru/science/ni/nip/nil/sltf

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Outline.

1. Introduction
2. Collinear Parton Model (CPM)
3. High Energy Factorization (HEF)
4. Parton Reggeization Approach (PRA)
5. Production of $b\bar{b}-$dijets
6. Production of $B\bar{B}-$pairs
7. Associated production of $\Upsilon(1S)$ and D mesons
8. Conclusions

Introduction: masses and scales

- $m_b(\overline{MS}) = 4.18 \pm 0.03$ GeV
- $m_b(1S) = 4.65 \pm 0.03$ GeV
- $m_b(Potential Model) \sim 4.5 - 5.0$ GeV

$m_b \simeq 4.5$ GeV $\Rightarrow \alpha_S(m_b) = \frac{4\pi}{b_0 \log\left(m_b^2/\Lambda_{QCD}^2\right)} \simeq 0.2,$

$b_0 = 11 - \frac{2}{3}N_F,\ N_F = 4$

Introduction: life times and decay widthes

B mesons

$$m_{B^\pm} = 5279.25 \pm 0.17 \text{ MeV},$$

$$\tau_{B^\pm} = (1.641 \pm 0.008) \times 10^{-12} \text{ sec}, \; b \to c + l^- + \tilde{\nu}_l$$

Upsilon mesons

$$m_{\Upsilon(1S)} = 9460 \pm 0.26 \text{ MeV},$$

$$\Gamma_{tot} = 54.02 \pm 1.25 \text{ keV}, \; \Upsilon(1S) \to ggg \to hadrons,$$

$$\Gamma_{l^+l^-} = 1.340 \pm 0.018 \text{ keV}, \; \Upsilon(1S) \to \gamma^* \to l^+ + l^-$$

$$B_{\tau^+\tau^-} = 0.260, \; B_{\mu^+\mu^-} = 0.248, \; B_{e^+e^-} = 0.238, \; B_{l^+l^-} = \frac{\Gamma_{l^+l^-}}{\Gamma_{tot}}$$

Introduction: life times and decay widthes

$$\tau_{\Upsilon(1S)} = \frac{1}{\Gamma_{tot}} = 1.22 \times 10^{-20} \text{ sec}$$

$$\hbar = c = 1, \qquad 1 \text{ GeV}^{-1} = 6.58 \times 10^{-25} \text{ sec}$$

Introduction: open bottom production

LHC

$\sqrt{S} = 7, \quad 8, \quad 13$ TeV:

B-meson production

$$p + p \to B + X,\ p + p \to B + \bar{B} + X \text{ and } p + p \to B + \bar{B} + jet + X$$

LHCb

Doubly-heavy meson

$$p + p \to B_c + X$$

b-jet production

$$p + p \to jet_b + X, \text{ and } p + p \to jet_b + jet_{\bar{b}} + X$$

Introduction: bottomonium production

$$p + p \to \Upsilon(nS) + X$$

In talk by Anton Karpishkov: "Correlation observables in $\Upsilon(1S)$+D associated production at the LHC within the parton Reggeization approach"

$$p + p \to \Upsilon(1S) + D + X$$

Collinear Parton Model (CPM)

Hard processes are those in which the momentum transfer, μ, is substantial with respect to the QCD scale $\mu >> \Lambda \sim 1$ GeV.

CPM describes special class of hard processes

CPM

1) Proton consists partons (quarks and gluons)
2) Partons are on mass-shell and they have 4-momentum $q_i^\mu = x_i P_i^\mu$, where in c.m.f. one has $P_{1,2} = \frac{\sqrt{S}}{2}(1,0,0,\pm 1)$, $P_i^2 = q_i^2 = 0$, and $\sqrt{S} >> m_p$
3) There is factorization formula, Leading Order (LO) in α_S

$$d\sigma(p+p \to b+X) = \sum_{i,j} \int \int dx_1 dx_2 F_i(x_1,\mu) F_j(x_2,\mu) d\hat{\sigma}(i+j \to b+\bar{b}) + \mathcal{O}\left(\frac{\Lambda}{\mu}\right)^n + \mathcal{O}(\alpha_S)$$

Collinear Parton Model (CPM)

$F_i(x,\mu)$ are Parton Distribution Functions at the factorization scale $\mu_F \sim \mu$ which satisfy DGLAP (Docshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations.

$$\tilde{F}(n,\mu) = \int_0^1 dx x^{n-1} F(x,\mu), \qquad \tilde{F}(n,\mu) \sim \left(\log(\frac{\mu^2}{\Lambda^2})\right)^{f(n)}$$

Approximately, at the law x and $\mu >> \mu_0$

$F_{gluon}(x,\mu) >> F_{sea}(x,\mu) >> F_{valence}(x,\mu)$

At small $\mu_0 \sim \Lambda$, proton consists only valence quarks $p = \{uud\}$, when scale μ grows, $p = \{uud + udu\bar{u} + uudg + uuds\bar{s} + uudc\bar{c} + ...\}$.

In case of b-quark production, $\mu \sim m_{bT} = \sqrt{m_b^2 + p_T^2}$

If $p_T \le m_b$, $i = u, d, s, c$ – scheme with four active flavors
If $p_T >> m_b$, $i = u, d, s, c, b$ – scheme with five active flavors

Collinear Parton Model (CPM)

What do we neglect in CPM?

Transverse momenta of partons, we suggest $|\mathbf{q}_T| \sim \Lambda$

It means we can't describe p_T-spectrum of $b-$quark at law transverse momentum, $0 < p_{bT} \leq \Lambda$.

TMD Parton Model

TMD

Transverse Momentum Dependent factorization, so called Collins-Soper-Sterman (CSS) approach.

$$F(x,\mu) \quad \Rightarrow F(x,\mathbf{q}_T,\mu,\zeta)$$
$$q_i = x_i P_i + q_{iT}, \quad q_{iT} = (0,\mathbf{q}_{iT},0) \quad \Rightarrow q_i^2 = -\mathbf{q}_{iT}^2 \neq 0$$

For off mass-shell partons, QCD amplitudes lost GAUGE INVARIANCE

TMD Parton Model is used for region of small transverse momenta $|\mathbf{q}_{iT}|, p_T << \mu \sim m_b$, and **GI** is restored with error $\sim \mathcal{O}(\frac{p_T}{\mu})$

Collinear Parton Model (CPM)

LO parton processes are

$g + g \to b + \bar{b}$, $q + \bar{q} \to b + \bar{b}$

PHYSICAL REVIEW D VOLUME 18, NUMBER 1 1 JULY 1978

Quantum-chromodynamic estimates for heavy-particle production

John Babcock, Dennis Sivers, and Stephen Wolfram*

High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

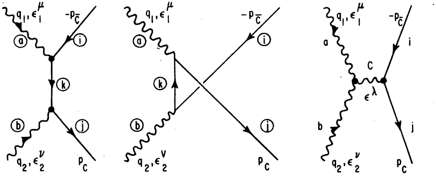
(Received 21 November 1977)

The associated production of hadrons containing heavy quarks is studied in the framework of a model based on quark-gluon color gauge field theory [quantum chromodynamics (QCD)]. We assume that the dominant mechanism for the production of heavy quarks in real and virtual photon beams is $\gamma(Q^2)V \to c\bar{c}$ where V denotes a vector gluon and c an arbitrary heavy quark. For π, p, and $\bar{p}$ beams we consider the mechanisms $VV \to c\bar{c}$ and $q\bar{q} \to c\bar{c}$. The cross sections for the internal subprocesses are calculated at lowest order in the perturbation expansion for QCD. We include a brief discussion of higher-order corrections to our calculation.

Wolfram Mathematica by Stephen Wolfram

With **Mathematica** we can use **FeynCalc**, **FeynArts**, ...

Collinear Parton Model (CPM)



$$M = g^2 \bar{u}(p_c) \left\{ T^a_{ik} T^b_{kj} \frac{\not\epsilon_2[(\not q_1 - \not p_{\bar{c}}) + m]\not\epsilon_1}{(\hat{t} - m^2)} + T^b_{ik} T^a_{kj} \frac{\not\epsilon_1[(\not q_2 - \not p_{\bar{c}}) + m]\not\epsilon_2}{(\hat{u} - m^2)} \right.$$

$$\left. + i f^{abc} C^{\mu\nu\lambda}(-q_1, -q_2, q_1 + q_2) \frac{\epsilon_{1\mu}\epsilon_{2\nu}}{\hat{s}} \gamma_\lambda T^c_{ij} \right\} v(p_{\bar{c}}) ,$$

Collinear Parton Model (CPM)

$$
\begin{aligned}
\left|M(VV \to c\overline{c})(\hat{s},\hat{t},\hat{u})\right|^2 = & \frac{g^4}{(\hat{t}-m^2)^2}\left\langle\tfrac{1}{12}\right\rangle(-2m^4 - 6\hat{t}m^2 - 2\hat{u}m^2 + 2\hat{u}\hat{t}) + \frac{g^4}{(\hat{u}-m^2)^2}\left\langle\tfrac{1}{12}\right\rangle(-2m^4 - 2\hat{t}m^2 - 6\hat{u}m^2 + 2\hat{u}\hat{t}) \\
& + \frac{g^2}{\hat{s}^2}\left\langle\tfrac{3}{16}\right\rangle(-28m^4 + 20\hat{u}m^2 + 20\hat{t}m^2 - 4(\hat{t}+\hat{u})^2 + 4\hat{u}\hat{t}) \\
& + \frac{g^4}{(\hat{t}-m^2)(\hat{u}-m^2)}\left\langle -\tfrac{1}{96}\right\rangle(-8m^4 - 4\hat{t}m^2 - 4\hat{u}m^2) \\
& + \frac{g^4}{(\hat{t}-m^2)\hat{s}}\left\langle +\tfrac{3}{32}\right\rangle(-12m^4 + 4\hat{u}m^2 + 12\hat{t}m^2 - 4\hat{t}^2) \\
& + \frac{g^4}{(\hat{u}-m^2)\hat{s}}\left\langle -\tfrac{3}{32}\right\rangle(+12m^4 - 12\hat{u}m^2 - 4\hat{t}m^2 + 4\hat{u}^2) \qquad (3.3)
\end{aligned}
$$

and

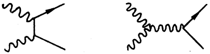
$$
\left|M(q\overline{q} \to c\overline{c})(\hat{s},\hat{t},\hat{u})\right|^2 = \frac{g^4}{\hat{s}^2}\left\langle\tfrac{2}{9}\right\rangle(12m^4 - 8m^2\hat{u} - 8m^2\hat{t} + 2\hat{u}^2 + 2\hat{t}^2), \qquad (3.4)
$$

Collinear Parton Model (CPM)

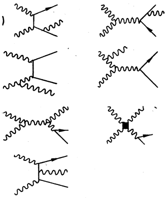
LO+NLO

$\alpha_S = \frac{g^2}{4\pi}, \qquad \sigma^{LO} \sim \alpha_S^2, \qquad \sigma^{NLO} \sim \alpha_S^3$

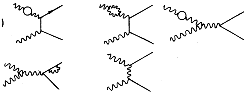
O(g²)



O(g³)



O(g⁴)



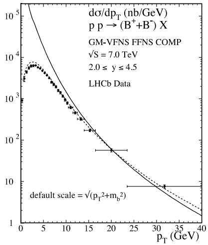
Collinear Parton Model (CPM)

NLO Schemes

Fixed-Flavor-Number-Scheme (FFNS)
Variable-Flavor-Number-Scheme (ZM-VFNS)
General-Mass Variable-Flavor-Number-Scheme (GM-VFNS)

$B-$meson production in NLO CPM

B. A. Kniehl, G. Kramer, I. Schienbein and H. Spiesberger. Eur.Phys.J. C75 (2015) no.3, 140.



Collinear Parton Model (CPM)

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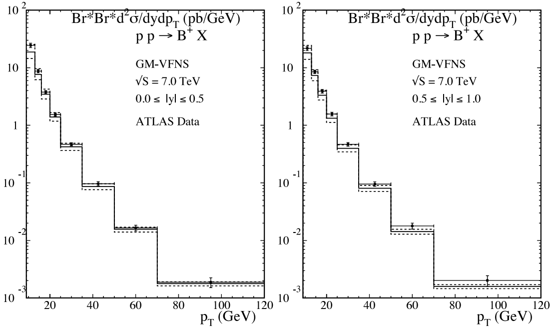
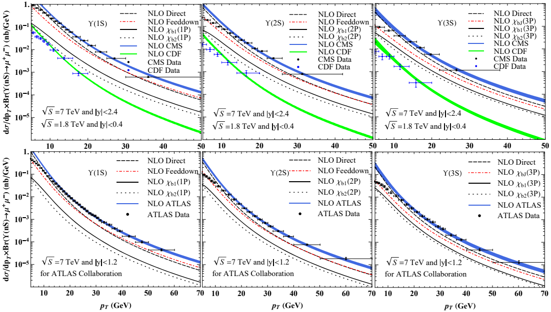


Collinear Parton Model (CPM)

$\Upsilon(nS)$ production in NLO CPM + NRQCD

H. Han, Y.Q. Ma, C. Meng, H.S. Shao, Y.J. Zhang, K.T. Chao. Phys.Rev. D94 (2016) no.1, 014028.

NRQCD - NonRelativistic QCD



Collinear Parton Model (CPM)

Single-scale hard processes are described well in NLO CPM

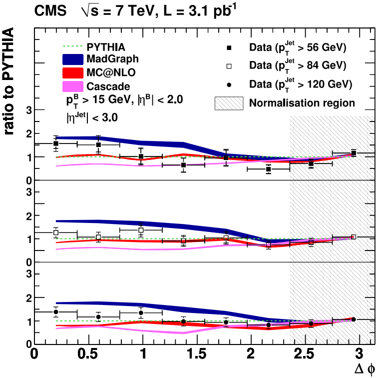
Multi-scale hard processes are under question

Pair-correlations in $b-$hadron production: $b\bar{b}-$dijets, $B\bar{B}$-pairs, $\Upsilon D-$pair production,...
Invariant mass of pairs (M), Azimuthal angle differences ($\triangle\varphi$), Rapidity differences ($\triangle y$), ...

Collinear Parton Model (CPM)

$B\bar{B}$ correlations

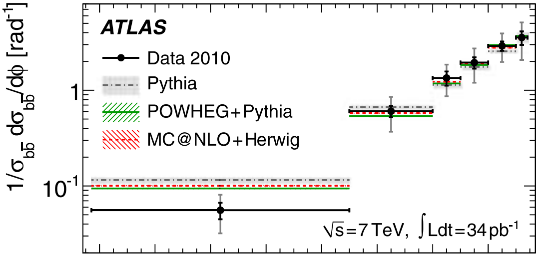
V. Khachatryan et al. [CMS Collaboration], Measurement of $B\bar{B}$ Angular Correlations based on Secondary Vertex Reconstruction at $\sqrt{S} = 7TeV$, JHEP 1103, 136 (2011)



Collinear Parton Model (CPM)

$b\bar{b}$−dijets correlations

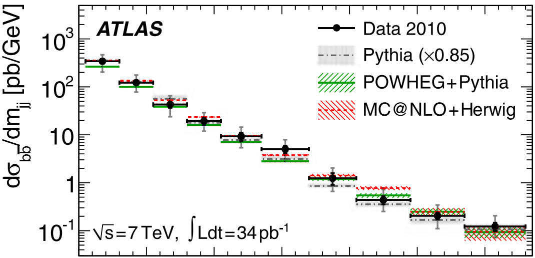
[ATLAS Collaboration] Measurement of the inclusive and dijet cross-sections of b-jets in pp collisions at $\sqrt{S} = 7$ TeV with the ATLAS detector. Eur. Phys. J. C (2011) 71:1846



Collinear Parton Model (CPM)

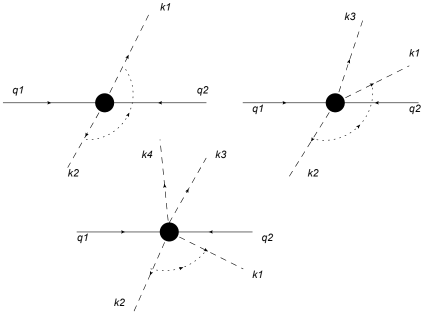
$b\bar{b}$−dijets correlations

[ATLAS Collaboration] Measurement of the inclusive and dijet cross-sections of b-jets in pp collisions at $\sqrt{S} = 7$ TeV with the ATLAS detector. Eur. Phys. J. C (2011) 71:1846



Collinear Parton Model (CPM)

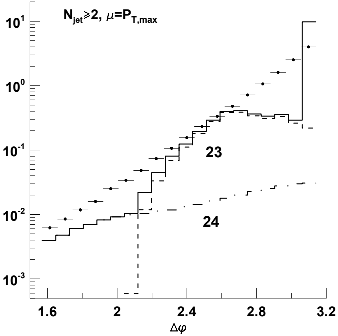
Correlation in azimuthal angle difference is extremely sensitive to high order corrections



Collinear Parton Model (CPM)

2-jets correlation

Azimuthal angle difference normalized spectrum between two most energetic jets, data from CMS Collaboration, $\sqrt{S} = 13$ TeV.



High Energy (k_T-)Factorization

Factorization at High Energy

Hard processes in the Regge kinematics: $\Lambda << \mu << \sqrt{S}$ or $x_i \sim \frac{\mu}{\sqrt{S}} << 1$

At the LHC $\sqrt{S} \sim 10^4$ GeV, $\mu \sim p_{bT} \sim 10^2$ GeV, for many processes $x_i < 10^{-2}$

First works:

- V. S. Fadin and L. N. Lipatov, High-Energy Production of Gluons in a QuasimultiRegge Kinematics, JETP Lett. **49** (1989) 352.
- S. Catani, M. Ciafaloni, F. Hautmann, High-energy factorization and small x heavy flavor production, Nucl. Phys. B366 (1991) 135188.
- J. C. Collins, R. K. Ellis, Heavy quark production in very high-energy hadron collisions, Nucl. Phys. B360 (1991) 330.
- E. M. Levin, M. G. Ryskin, Yu. M. Shabelski, A. G. Shuvaev, Heavy quark production in semihard nucleon interactions, Sov. J. Nucl. Phys. 53 (1991) 657.

High Energy (k_T-)Factorization

k_T-factorization formula

$$d\sigma^{\rm KT} = \sum_{i,j} \int\limits_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_i(x_1, t_1, \mu^2) \int\limits_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_j(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{ij}^{\rm KT}$$

Off mass-shell parton cross section

$$d\hat{\sigma}^{\rm KT} = d\hat{\sigma}^{\rm KT}\left(\mathbf{q}_{T1}, \mathbf{q}_{T2}\right)$$

Unintegrated PDFs

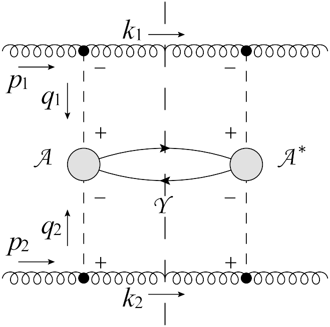
$$\Phi(x, t, \mu^2), \qquad t = \mathbf{q}_T^2 = -q_T^2$$

Parton Reggeization Approach (PRA)

Factorization formula + unPDFs + off mass-shell matrix elements

- M. A. Nefedov, V. A. Saleev and A. V. Shipilova, Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach, Phys. Rev. D **87** (2013) no.9, 094030.
- M. Nefedov and V. Saleev, Diphoton production at the Tevatron and the LHC in the NLO approximation of the parton Reggeization approach, Phys. Rev. D **92** (2015) no.9, 094033.
- A. V. Karpishkov, M. A. Nefedov and V. A. Saleev, $B\bar{B}$ angular correlations at the LHC in parton Reggeization approach merged with higher-order matrix elements, Phys. Rev. D **96** (2017) no.9, 096019.
- M. Nefedov and V. Saleev, On the one-loop calculations with Reggeized quarks, Mod. Phys. Lett. A **32** (2017) no.40, 1750207.
- M. Nefedov. Hard processes in the Parton Reggeization Approach. PhD Thesis, Samara-Dubna, 2016.

LO factorization formula in the PRA



$n^+ = (1, 0, 0, -1)$, $n^- = (1, 0, 0, +1)$, $q^\pm = (qn^\pm)$

Auxiliary hard CPM subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_\mathcal{A}) + g(k_2),$$

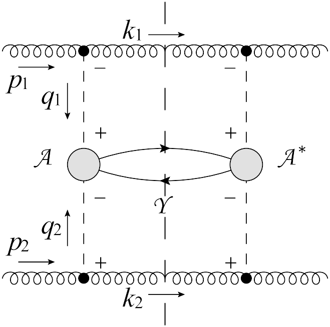
where $p_1^2 = 0$, $p_1^- = 0$, $p_2^2 = 0$, $p_2^+ = 0$.
Kinematic variables ($0 < z_{1,2} < 1$):

$$z_1 = \frac{p_1^+ - k_1^+}{p_1^+}, \quad z_2 = \frac{p_2^- - k_2^-}{p_2^-},$$

Two limits where $\overline{|\mathcal{M}|^2}$ factorizes:

1. **Collinear limit:** $\mathbf{q}_{T1,2}^2 \ll \mu^2$, $z_{1,2}$ – arbitrary,
2. **Multi-Regge limit:** $z_{1,2} \ll 1$, $\mathbf{q}_{T1,2}^2$ – arbitrary.

LO factorization formula in the PRA



Auxiliary hard CPM subprocess:

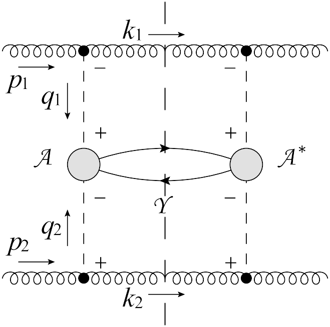
$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2),$$

1 **Collinear limit:** $\mathbf{q}^2_{T1,2} \ll \mu^2$, $z_{1,2}$ – arbitrary:

$$\overline{|\mathcal{M}|^2}_{\rm CL} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{gg}(z_1) P_{gg}(z_2) \frac{\overline{|\mathcal{A}_{CPM}|^2}}{z_1 z_2},$$

where $\overline{|\mathcal{A}_{CPM}|^2}$ – amplitude $g + g \to \mathcal{Y}$ with **on-shell** initial-state partons, $P_{gg}(z)$ – DGLAP $g \to g$ splitting function.

LO factorization formula in the PRA



Auxiliary hard CPM subprocess:

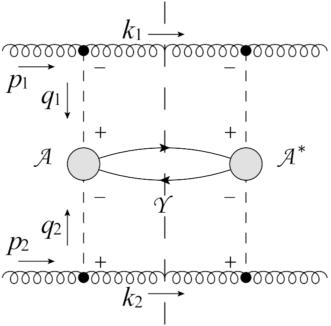
$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2),$$

2 **Multi-Regge limit:** $z_{1,2} \ll 1$ ($\Leftrightarrow \Delta y_{1,2} \gg 1$), $\mathbf{q}_{T1,2}^2$ – arbitrary:

$$\overline{|\mathcal{M}|^2}_{\mathrm{MRK}} \simeq \frac{4g_s^4}{\mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2} \tilde{P}_{gg}(z_1) \tilde{P}_{gg}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $\tilde{P}_{gg}(z) = 2C_A/z$ and $\overline{|\mathcal{A}_{PRA}|^2}$ is the **gauge-invariant** amplitude $R_+(q_1) + R_-(q_2) \to \mathcal{Y}$ with **Reggeized** (**off-shell**) partons in the initial state.

LO factorization formula in the PRA



Auxiliary hard CPM subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_\mathcal{A}) + g(k_2),$$

Modified MRK approximation: $\underline{z_{1,2}}$ and $\underline{\mathbf{q}^2_{T1,2}}$ – arbitrary:

$$\overline{|\mathcal{M}|^2}_{\text{mMRK}} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{gg}(z_1) P_{gg}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $q^2_{1,2} = \mathbf{q}^2_{T1,2}/(1 - z_{1,2})$, has correct **collinear** and **Multi-Regge** limits!

Factorization formula in the PRA

Substituting the $\overline{|\mathcal{M}|^2}_{\text{mMRK}}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \tilde{\Phi}_g(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \tilde{\Phi}_g(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\text{PRA}},$$

where $x_1 = q_1^+/P_1+$, $x_2 = q_2^-/P_2^-$, $\tilde{\Phi}(x, t, \mu^2)$ – "tree-level" **unintegrated PDFs**, the partonic cross-section in PRA is:

$$d\hat{\sigma}_{\text{PRA}} = \frac{\overline{|\mathcal{A}_{PRA}|^2}}{2Sx_1x_2} \cdot (2\pi)^4 \delta\left(\frac{1}{2}\left(q_1^+ n_- + q_2^- n_+\right) + q_{T1} + q_{T2} - P_{\mathcal{A}}\right) d\Phi_{\mathcal{A}}.$$

Note the usual **flux-factor** Sx_1x_2 for **off-shell** initial-state partons.

LO unintegrated PDF

The "tree-level" unPDF:

$$\tilde{\Phi}_g(x,t,\mu^2) = \frac{1}{t}\frac{\alpha_s}{2\pi}\int\limits_x^1 dz\ P_{gg}(z)\cdot\frac{x}{z}f_g\left(\frac{x}{z},\mu^2\right).$$

contains collinear divergence at $t \to 0$ and IR divergence at $z \to 1$.

In the "dressed" unPDF collinear divergence is regulated by Sudakov formfactor $T(t,\mu^2)$:

$$\Phi_i(x,t,\mu^2) \quad = \quad \frac{T_i(t,\mu^2,x)}{t}\times\frac{\alpha_s(t)}{2\pi}\int\limits_x^1 dz\ \theta_z^{\text{cut}} P_{ij}(z)\frac{x}{z}f_j\left(\frac{x}{z},t\right)$$

where: $\theta_z^{\text{cut}} = \theta\left((1-\Delta_{KMR}(t,\mu^2)) - z\right)$, and the Kimber-Martin-Ryskin(KMR) **cut condition** [KMR, 2001]:

$$\Delta_{KMR}(t,\mu^2) = \frac{\sqrt{t}}{\sqrt{\mu^2}+\sqrt{t}},$$

follows from the **rapidity ordering** between the last emission and the hard subprocess.

Factorization formula in the PRA

LO unintegrated PDF in the PRA

$$\Phi_i(x,t,\mu^2) = \frac{T_i(t,\mu^2,x)}{t} \times \frac{\alpha_s(t)}{2\pi} \int\limits_x^1 dz\; \theta_z^{\rm cut} P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z},t\right)$$

$$= \boxed{\frac{\partial}{\partial t}\left[T_i(t,\mu^2,x)\cdot x f_i(x,t)\right]} \leftarrow \text{derivative form of unPDF}$$

⇒ **LO normalization condition:**

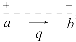
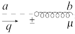
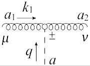
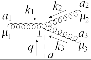
$$\boxed{\int_0^{\mu^2} dt\; \Phi_i(x,t,\mu^2) = x f_i(x,\mu^2)} \leftarrow \textbf{Holds exactly!}$$

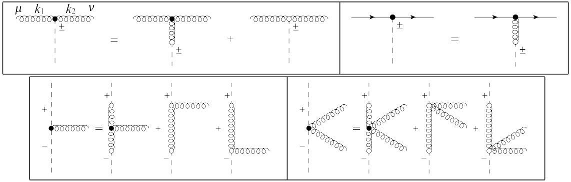
Because $T(0,\mu^2,x)=0$ and $T(\mu^2,\mu^2,x)=1$.

Gauge-invariant off-shell amplitudes

$\overline{|\mathcal{A}_{\rm PRA}|^2}$ is obtained from Lipatov's **gauge-invariant effective theory for MRK processes in QCD** [Lipatov 1995; Lipatov, Vyazovsky, 2001].

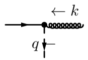
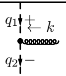
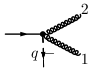
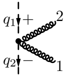
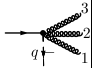
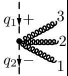
Some Feynman rules for **Reggeized gluons:**

 $= \frac{-i\delta_{ab}}{2q^2}$	 $= (-iq^2) n^{\mp}_{\mu} \delta_{ab}$
	$g_s f_{aa_1a_2} \left(n^{\mp}_{\mu} n^{\mp}_{\nu}\right) \frac{q^2}{k_1^{\mp}}$
	$i g_s^2 \left(n^{\mp}_{\mu_1} n^{\mp}_{\mu_2} n^{\mp}_{\mu_3}\right) \frac{q^2}{k_3^{\mp}} \left[\frac{f_{aba_1} f_{ba_2a_3}}{k_1^{\mp}} + \frac{f_{aba_2} f_{ba_1a_3}}{k_2^{\mp}}\right]$



Gauge-invariant off-shell amplitudes

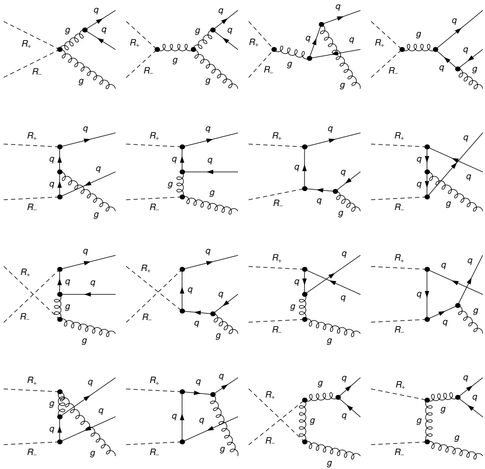
Some Feynman rules for **Reggeized quarks**:

	$ig_sT^a\left(\gamma_\mu+\hat{q}\frac{n^+_\mu}{k^+}\right)$		$ig_sT^a\left(\gamma_\mu+\hat{q}_2\frac{n^+_\mu}{k^+}+\hat{q}_1\frac{n^-_\mu}{k^-}\right)$
	$ig_s^2(n^+_{\mu_1}n^+_{\mu_2})\hat{q}\left[\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+}+\frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right]$		$ig_s^2\left[\hat{q}_2(n^+_{\mu_1}n^+_{\mu_2})\left(\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+}+\frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right)-\hat{q}_1(n^-_{\mu_1}n^-_{\mu_2})\left(\frac{T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-}+\frac{T^{a_1}T^{a_2}}{k_2^-(k_1+k_2)^-}\right)\right]$
	$ig_s^3\hat{q}(n^+_{\mu_1}n^+_{\mu_2}n^+_{\mu_3})\left[\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+}+(1\leftrightarrow2\leftrightarrow3)\right]$		
	$ig_s^3\left[\hat{q}_2(n^+_{\mu_1}n^+_{\mu_2}n^+_{\mu_3})\left(\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+}+(1\leftrightarrow2\leftrightarrow3)\right)+\hat{q}_1(n^-_{\mu_1}n^-_{\mu_2}n^-_{\mu_3})\left(\frac{T^{a_3}T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-(k_1+k_2+k_3)^-}+(1\leftrightarrow2\leftrightarrow3)\right)\right]$		

Implementation in FeynArts.

Model-file "ReggeQCD"

The Feynman rules of Lipatov's EFT, up to the processes with **4** final particles are implemented in **model-file ReggeQCD for the package FeynArts.**



Production of $b\bar{b}$–dijets

V. Saleev and A. Shipilova, Inclusive b-jet and $b\bar{b}$-dijet production at the LHC via Reggeized gluons, Phys. Rev. D 86 (2012) 034032.

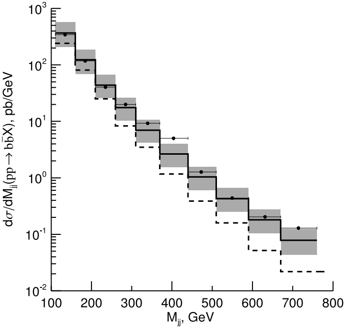
$$R + R \to b + \bar{b}$$

ATLAS Collaboration, G. Aadet al., Eur. Phys. J. C 71, 1846 (2011).

Cone condition for b-jets

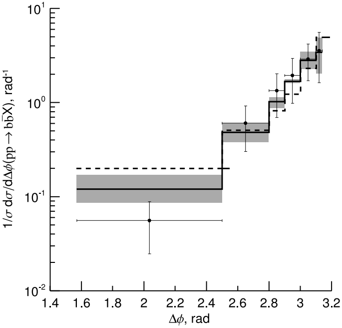
$$R_{b\bar{b}} = \sqrt{(y_b - y_{\bar{b}})^2 + (\phi_b - \phi_{\bar{b}})^2} < R = 0.4$$

Production of $b\bar{b}$−dijets



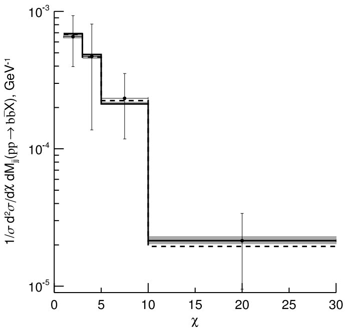
The $b\bar{b}$-dijet cross-section as a function of dijet invariant mass M_{jj} for b-jets with $p_T > 40$ GeV, $|y| < 2.1$. The data are from ATLAS Collaboration, the solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

Production of $b\bar{b}$–dijets



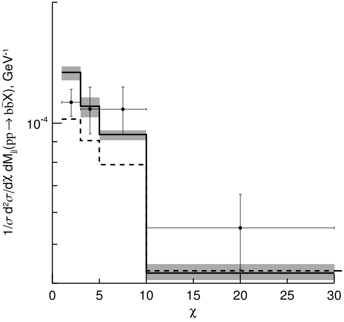
The $b\bar{b}$-dijet cross-section as a function of the azimuthal angle difference between the two jets for b-jets with $p_T > 40$ GeV, $|y| < 2.1$ and a dijet invariant mass of $M_{jj} < 110$ GeV. The data are from ATLAS Collaboration , the solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

Production of $b\bar{b}$–dijets



The $b\bar{b}$-dijet cross-section as a function of $\chi = \exp|y_b - y_{\bar{b}}|$ for b-jets with $p_T > 40$ GeV, $|y| < 2.1$ and $|y_{boost}| = \frac{1}{2}|y_1 + y_2| < 1.1$, for dijet invariant mass range $110 < M_{jj} < 370$ GeV. The data are from ATLAS Collaboration, the solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

Production of $b\bar{b}$–dijets



The $b\bar{b}$-dijet cross-section as a function of $\chi = \exp|y_b - y_{\bar{b}}|$ for b-jets with $p_T > 40$ GeV, $|y| < 2.1$ and $|y_{boost}| = \frac{1}{2}|y_1 + y_2| < 1.1$, for dijet invariant mass range $370 < M_{jj} < 850$ GeV. The data are from ATLAS Collaboration , the solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF. The shaded bands indicate the theoretical uncertainties in the case of KMR unintegrated PDF.

Production of $B\bar{B}-$pairs

Results are published in

- A. V. Karpishkov, M. A. Nefedov, V. A. Saleev and A. V. Shipilova, B-meson production in the Parton Reggeization Approach at Tevatron and the LHC," Int. J. Mod. Phys. A **30** (2015) no.04n05, 1550023
- A. V. Karpishkov, M. A. Nefedov and V. A. Saleev, $B\bar{B}$ angular correlations at the LHC in parton Reggeization approach merged with higher-order matrix elements, Phys. Rev. D **96** (2017) no.9, 096019

$$R + R \to i(\to B) + j(\to \bar{B})$$

Fragmentation model

$$\frac{d\sigma(p+p\to B+X)}{dp_{BT}dy} = \sum_i \int_0^1 \frac{dz}{z} D_{i\to B}(z,\mu^2) \frac{d\sigma(p+p\to i(p_i)+X)}{dp_{iT}dy_i},$$

where $D_{i\to B}(z,\mu^2)$ is the fragmentation function for producing the B-meson from the parton i, created at the hard scale μ, the fragmentation parameter z is defined through the relation $p_i = p_B/z$, with p_B and p_i to be B-meson and i-parton four-momenta, correspondingly, and their rapidities $y_B = y_i$.

Production of $B\bar{B}-$pairs

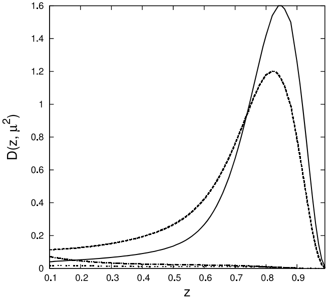
$D_{i\to B}(z,\mu^2)$, $\mu_0 = m_b$

Nonperturbative input $D_{i\to B}(z,\mu_0^2) = az^b(1-z)^c$ is evaluated in all orders of perturbative series, resums large logarithms $\alpha_S \log(\mu^2/m_b^2)$ through the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations.

$D_{i\to B}(z,\mu^2)$, $i = b, c, s, d, u, g$

B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, *Phys. Rev. D* **77**, 014011 (2008).
$D_{i\to B}(z,\mu_0^2)$ were extracted from the experimental data for the reaction $e^+e^- \to B + X$ provided by the ALEPH and OPAL Collaborations at the CERN LEP1 collider.

Production of $B\bar{B}-$pairs



The fragmentation function $D(z, \mu^2)$ of b-quarks and gluons into B mesons from Ref. [KKSS] at the $\mu^2 = 100$ GeV^2 (solid curve for b-quark, pair-dotted for gluon) and $\mu^2 = 1000$ GeV^2 (dashed line for b-quark, dash-dotted for gluon)

Production of $B\bar{B}$−pairs

V. Khachatryan *et al.* [CMS Collaboration], Measurement of $B\bar{B}$ Angular Correlations based on Secondary Vertex Reconstruction at $\sqrt{s} = 7$ TeV, JHEP **1103**, 136 (2011)

$$\frac{d\sigma}{d\triangle\phi}, \qquad \frac{d\sigma}{d\triangle R}, \qquad R = \sqrt{(\phi_B - \phi_{\bar{B}})^2 + (y_B - y_{\bar{B}})^2}$$

In this experiment, the events with at least one jet having $p_T^{\rm jet} > p_{TL}^{\rm min}$ has been recorded in pp-collisions at the $\sqrt{S} = 7$ TeV, and the semileptonic decays of B-hadrons where reconstructed in this events, through the decay vertices, displaced w. r. t. the primary pp-collision vertex. The B-hadron is required to have $p_{TB} > p_{TB}^{\rm min} = 15$ GeV, while three data-samples are presented in the Ref. [CMS2011] for three values of $p_{TL}^{\rm min} = 56$, 84 and 120 GeV. The rapidities of B-hadrons are constrained to be $|y_B| < y_B^{\rm max} = 2$, while the leading jet is searched in somewhat wider domain $|y_{\rm jet}| < y_{\rm jet}^{\rm max} = 3$.

Production of $B\bar{B}-$pairs

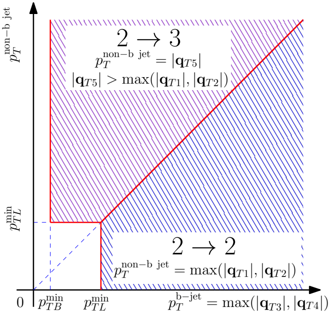
$RR \to b\bar{b}$ and $RR \to b\bar{b}g$ contributions

The LO ($O(\alpha_s^2)$) subprocess, which we will take into account is:

$$R_+(q_1) + R_-(q_2) \to b(q_3)(\to B(p_{TB})) + \bar{b}(q_4)(\to \bar{B}(p_{T\bar{B}}))$$

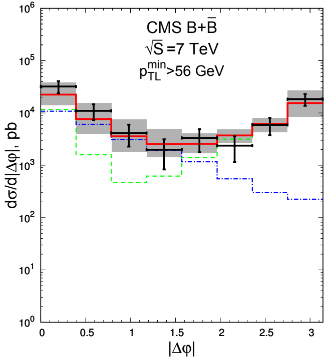
The NLO ($O(\alpha_s^3)$) subprocess is

$$R_+(q_1) + R_-(q_2) \to b(q_3)(\to B(p_{TB})) + \bar{b}(q_4)(\to \bar{B}(p_{T\bar{B}})) + g(q_5)$$

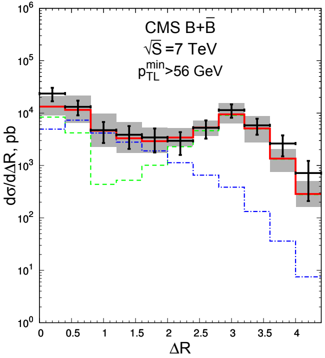


The kinematic cuts for $2 \to 2$ and $2 \to 3$ contributions in the space of transverse momentum of the leading b-jet in the event ($p_T^{\text{b-jet}}$) vs. transverse momentum of the leading light-quark/gluon jet in the same event ($p_T^{\text{non-b jet}}$)

Production of $B\bar{B}-$pairs

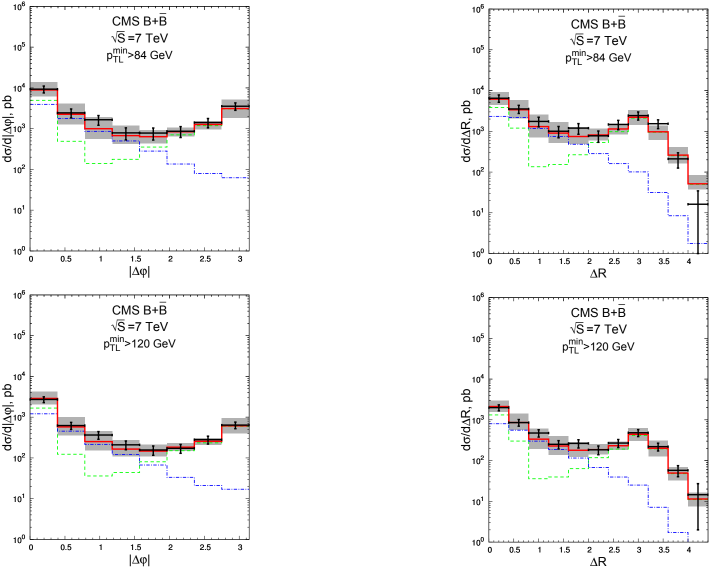


Comparison of the predictions for $\Delta\phi$-spectra of $B\bar{B}$-pairs with the CMS data. Dashed line – contribution of the LO subprocess, dash-dotted line – contribution of the NLO subprocess, solid line – sum of LO and NLO contributions.



Comparison of the predictions for ΔR-spectra of $B\bar{B}$-pairs with the CMS data. Dashed line – contribution of the LO subprocess, dash-dotted line – contribution of the NLO subprocess, solid line – sum of LO and NLO contributions.

Production of $B\bar{B}$−pairs



Production of $B\bar{B}-$pairs

In the present part of my talk, the example of $B\bar{B}$-azimuthal decorrelations is used to show, how the contributions of $2 \to 2$ and $2 \to 3$ processes in PRA can be consistently taken together to describe multiscale correlational observables in a presence of experimental constraints on additional QCD radiation.

Υ production in the Nonrelativistic QCD.

Estimate of the heavy quark velocity:

$$\frac{m_Q v^2}{2} \sim \frac{\alpha_s(1/r)}{r}$$

Mean radius and velocity are related: $r \sim \frac{1}{m_Q v} \Rightarrow$

$$v \sim \alpha_s(m_Q v)$$

Then for $m_Q = 1.5$ GeV, $v \simeq 0.3$, $m_Q = 4.7$ GeV, $v \simeq 0.1$.

NRQCD Lagrangian:[G. T. Bodwin, E. Braaten, G. P. Lepage, Phys. Rev. **D51** (1995) 1125]

$$L_{NRQCD} = L_{Heavy} + L_{Light} + \delta L$$

$$L_{Heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m_Q} \right) \psi + \chi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m_Q} \right) \chi$$

$$L_{Light} = -\frac{1}{2} Tr \left[F_{\mu\nu} F^{\mu\nu} \right] + \sum_f \bar{q}_f \; i\hat{D} \; q_f$$

Where $\hat{D} = \gamma^\mu D_\mu$, $D_\mu = \partial_\mu + i g_s A_\mu$

Nonrelativistic QCD. Velocity scaling.

L_{Heavy} is $O(M^4 v^5)$, δL contains higher order corrections in v:

$$\delta L = \frac{c_1}{8m_Q^3}\left[\psi^\dagger\,(\mathbf{D})^2\,\psi - \chi^\dagger\,(\mathbf{D})^2\,\chi\right] + ...$$

Velocity scaling rules:

Operator	Scaling
$\psi,\ \chi$	$(m_Q v)^{3/2}$
$\mathbf{D}$	$m_Q v$
$D_t, g_s A^0$	$m_Q v^2$
$g_s\mathbf{A},\ g_s\mathbf{E}/m_Q$	$m_Q v^3$
$g_s\mathbf{B}/m_Q$	$m_Q v^4$

$\langle H|\int d^3\mathbf{x}\psi^\dagger(x)\psi(x)|H\rangle \sim 1,\ V \sim r^3 \sim \frac{1}{(m_Q v)^3} \ \Rightarrow\ \psi^\dagger\psi \sim (m_Q v)^3$

Factorization of production amplitude:

$$\mathcal{A}\left[g+g\rightarrow\mathcal{H}+X\right]=\sum_{n}\mathcal{A}\left[g+g\rightarrow n\right]\langle n|\mathcal{H}+X\rangle$$

Where: $n=Q\bar{Q}\left[{}^{2S+1}L_J^{(1,8)}\right],\ Q\bar{Q}g,\ \ldots$

Inclusive production rate:

$$|\mathcal{A}\left[g+g\rightarrow\mathcal{H}+X\right]|^2=\sum_{n}|\mathcal{A}\left[g+g\rightarrow n\right]|^2\times$$
$$\times\langle 0|\mathcal{O}^{\mathcal{H}}\left[n\right]|0\rangle$$

NRQCD factorization.

Finally:

$$|\mathcal{A}\left[gg \to \mathcal{H}(P) + X\right]|^2 =$$

$$= \sum_n \frac{\langle \mathcal{O}^{\mathcal{H}}\left[n\right]\rangle}{N_{col}N_{pol}} \left| C_{ij}^{(1,8)} \Pi\left[n\right] \mathcal{A}_{ij}\left[gg \to Q(P/2+q) + \bar{Q}(P/2-q)\right] \right|^2_{q=0}$$

$Q\bar{Q}$-Fock states (LO in v^2):

$$n = {}^3S_1^{(1,8)},\ {}^1S_0^{(1,8)},\ {}^3P_J^{(1,8)}\ ,$$

$N_{pol} = 2J_{\mathcal{H}} + 1$, $N_{col} = 2N_c$ for $^{(1)}$ and $N_c^2 - 1$ for $^{(8)}$.

NRQCD factorization.

Color projectors:

$$C_{ij}^{(1)} = \frac{\delta_{ij}}{\sqrt{N_c}},\ C_{ij}^{(8)} = \sqrt{2}T_{ij}^{a}$$

Spin-orbital projectors:

$$\Pi_0 = (8m_Q^3)^{-1/2}\left(\hat{P}/2 - \hat{q} - m\right)\gamma_5\left(\hat{P}/2 + \hat{q} + m\right)$$

$$\Pi_1^{\alpha} = (8m_Q^3)^{-1/2}\left(\hat{P}/2 - \hat{q} - m\right)\gamma_{\alpha}\left(\hat{P}/2 + \hat{q} + m\right)$$

$$\Pi\left[{}^1S_0\right] = \Pi_0,\ \Pi\left[{}^3S_1\right] = \varepsilon_{\alpha}(P)\Pi_1^{\alpha}$$

$$\Pi\left[{}^1P_1\right] = \varepsilon^{\beta}(P)\frac{\partial}{\partial q^{\beta}}\Pi_0,\ \Pi\left[{}^3P_J\right] = \varepsilon_{\alpha\beta}^{(J)}(P)\frac{\partial}{\partial q_{\beta}}\Pi_1^{\alpha}$$

Long-distance matrix elements (LDMEs, NMEs, ...).

Color-singlet NMEs:

$$\left\langle \mathcal{O}^{\mathcal{H}_J}\left[{}^3S_1^{(1)}\right]\right\rangle = 2N_c(2J+1)\frac{1}{4\pi}|R(0)|^2,$$
$$\left\langle \mathcal{O}^{\mathcal{H}_J}\left[{}^3P_J^{(1)}\right]\right\rangle = 2N_c(2J+1)\frac{3}{4\pi}|R'(0)|^2.$$

Radial wavefunction $R(0)$ or it's derivative in the origin $R'(0)$ is known from the potential models [E. J. Eichten and C. Quigg, Phys. Rev. D 52 (1995) 1726].

Multiplicative relations, proven in LO in v^2:

$$\begin{aligned} \langle \mathcal{O}^{\mathcal{H}}[{}^3P_J^{(1,8)}]\rangle &= (2J+1)\langle \mathcal{O}^{\mathcal{H}}[{}^3P_0^{(1,8)}]\rangle, \\ \langle \mathcal{O}^{\mathcal{H}_J}[{}^3S_1^{(8)}]\rangle &= (2J+1)\langle \mathcal{O}^{\mathcal{H}}[{}^3S_1^{(8)}]\rangle, \end{aligned}$$

Color-octet NMEs may be obtained using nonperturbative techniques or by a fit. So, the main task is to calculate the hard scattering matrix element:

$$\mathcal{A}\left[pp \to Q\bar{Q} + X\right].$$

State of the art.

- **NLO, fixed order in α_s** calculations of charmonium and bottomonium production in the Collinear Parton Model(CPM) are avaliable [M. Butenschoen, B. A. Kniehl, Phys. Rev. D **84** (2011) 051501; Y. -Q. Ma, K. Wang, K. -T. Chao, Phys. Rev. Lett. **106** (2011) 042002]. But they are applicable only in the region of high $p_T > M$, because of appearance of large logs $\alpha_s^m \log^{2m-1}\left(M^2/p_T^2\right)$.
- **Resummation procedures**[P. Sun, C.-P. Yuan, F. Yuan, arXiv:hep-ph/1210.3432] usually works in the region $\Lambda_{QCD} \ll p_T \ll M$, and requires **matching** with high p_T region. So, the approach which describes low and high p_T regions on a same grounds is needed.
- **Non-complete** $NNLO^{\star}$ calculations in **Color-singlet model** [P. Artoisenet , J. Campbell , J.P. Lansberg , F. Maltoni , F. Tramontano, Phys. Rev. Lett. **101** (2008) 152001] show, that the NNLO corrections in CPM can be large. So, the role of Color-octet production mechanism is disputable.
- **Color evaporation model** is an alternative to NRQCD with smaller number of free parameters. It assumes, that any $Q\bar{Q}$ pair with invariant mass near the open-flavour meson production treshold, evolves into the heavy quarkonium $\mathcal{H}$ with some process and energy-independent probability $F_{\mathcal{H}}$. The model provides rather poor but stable quality description of the experimental data.

Υ production in PRA

$2 \to 1$ and $2 \to 2$ processes.

Expressions for $2 \to 1$ and $2 \to 2$ subprocesses are derived in [B. A. Kniehl, V. A. Saleev, D. V. Vasin, Phys. Rev. D**73** (2006) 074022; Phys. Rev. D**74** (2006) 014024]

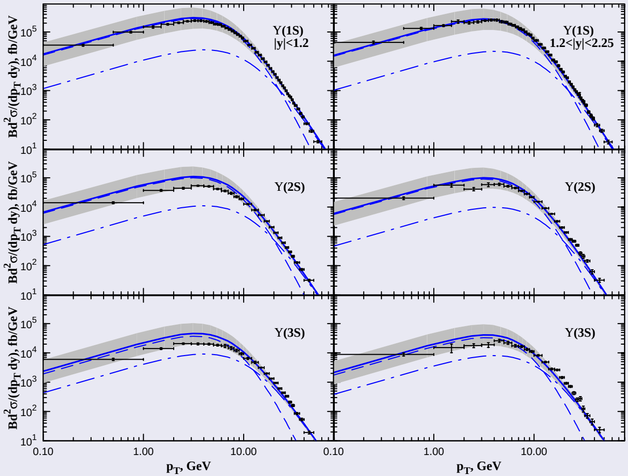
Bottomonium production at the LHC and Tevatron.

For the details see [M. A. Nefedov, V. A. Saleev, A. V. Shipilova, Phys. Rev. D**88** (2013) 014003]

NME	Fit LO PRA
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^3S_1^{(1)}\right]\right\rangle \times$ GeV^{-3}	9.28
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	2.31 ± 0.25
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^1S_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0 ± 0.05
$\left\langle \mathcal{O}^{\Upsilon(1S)}\left[{}^3P_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-5}	0.0 ± 0.38
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^3S_1^{(1)}\right]\right\rangle \times$ GeV^{-3}	4.62
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	1.51 ± 0.17
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^1S_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0 ± 0.01
$\left\langle \mathcal{O}^{\Upsilon(2S)}\left[{}^3P_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-5}	0.0 ± 0.03
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(1)}\right]\right\rangle \times$ GeV^{-3}	3.54
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	1.24 ± 0.13
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^1S_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0 ± 0.01
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3P_0^{(8)}\right]\right\rangle \times 10^2$ GeV^{-5}	0.0 ± 0.02
$\left\langle \mathcal{O}^{\chi(1P)}\left[{}^3P_0^{(1)}\right]\right\rangle \times$ GeV^{-5}	2.03
$\left\langle \mathcal{O}^{\chi(1P)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0
$\left\langle \mathcal{O}^{\chi(2P)}\left[{}^3P_0^{(1)}\right]\right\rangle \times$ GeV^{-5}	2.36
$\left\langle \mathcal{O}^{\chi(2P)}\left[{}^3S_1^{(8)}\right]\right\rangle \times 10^2$ GeV^{-3}	0.0

Inclusive $\Upsilon(nS)$ production at the LHC (ATLAS).

$\sqrt{S} = 7$ TeV.



Production of $\Upsilon(1S)D-$pairs

Talk by Anton Karpishkov

Associated production of prompt $\Upsilon(1S)$ and $D^{+,0}$ mesons has been proposed as a golden channel for the search of Double Parton Scattering, because Single Parton Scattering contribution to the cross-section is believed to be negligible on a basis of leading order calculations in the Collinear Parton Model. We study this process in the leading order approximation of the Parton Reggeization Approach. Hadronization of $b\bar{b}$-pair into bottomonium states is described within framework of the NRQCD-factorization approach while production of D mesons is described in the fragmentation model with scale-dependent fragmentation functions. We have found good agreement with LHCb data for various normalized differential distributions, except for the case of spectra on azimuthal angle differences at the small $\Delta\varphi$ values. Crucially, the total cross-section in our Single Parton Scattering model accounts for more than one half of observed cross-section, thus dramatically shrinking the room for Double Parton Scattering mechanism.