Lecture series on
QCD Exotics in the Heavy Quark Sector
Part I: Tools

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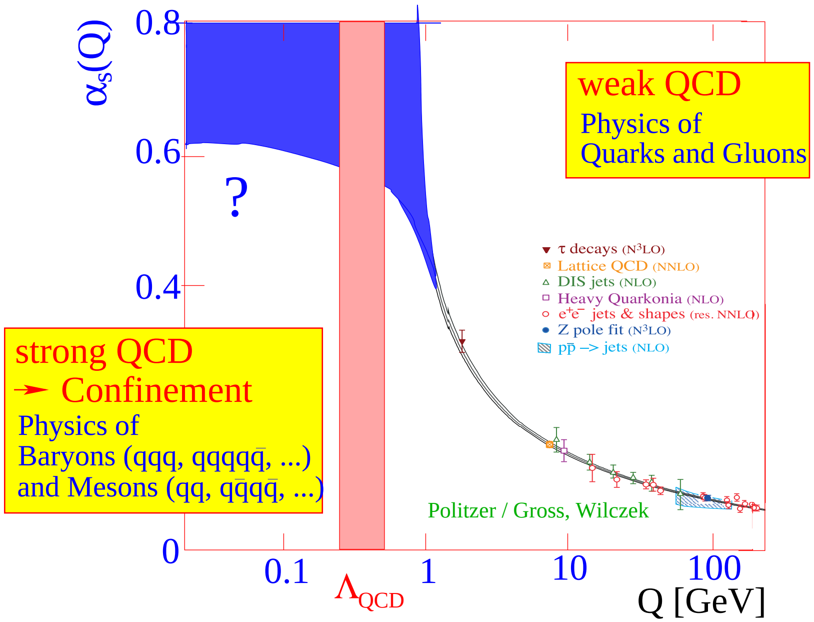
Setting the stage



→ While QCD gets perturbative at large energies, it is non-perturbative at low energies

→ Quarks are confined

→ Spectroscopy is the method of choice to investigate the inner workings of QCD and the formation of matter

→ Although overall providing a good general understanding the quark model has certain short comings

→ The physics of light and heavy quarks is rather different

Outline

Lecture I: Tools

- → Lattice QCD
- → Effective field theories (ChPT, HQEFT)
- → Unitarisation
- → Large N_c

Lecture II: The single heavy sector

- → Goldstone–Boson D-meson scattering
- → The positive parity D-mesons
- → Predictions and tests

Lecture III: The $\bar{Q}Q$ sector

- → The XYZ-stories

In this lecture series the focus is on mesons

Lattice QCD

A ’brute force’ numerical solution of full QCD in the Euclidean

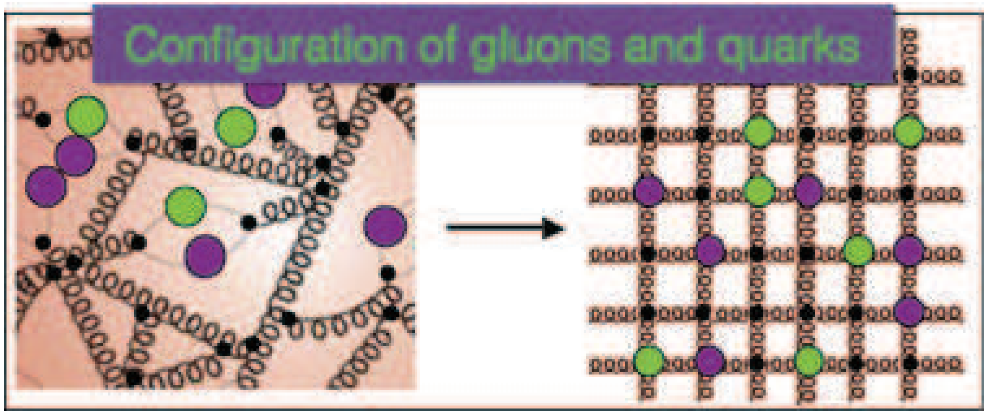


Fig. courtesy of T. Luu

$\rightarrow$ Quarks are located on the sites

$\rightarrow$ Gauge fields on the links

Two point functions

$$\langle \mathcal{O}_j(t')\mathcal{O}_i(t)\rangle = \frac{\int d[U]d[\bar{q}]d[q]\mathcal{O}_j(t')\mathcal{O}_j(t)\, \exp(-S[U,\bar{q},q])}{\int d[U]d[\bar{q}]d[q]\, \exp(-S[U,\bar{q},q])}$$

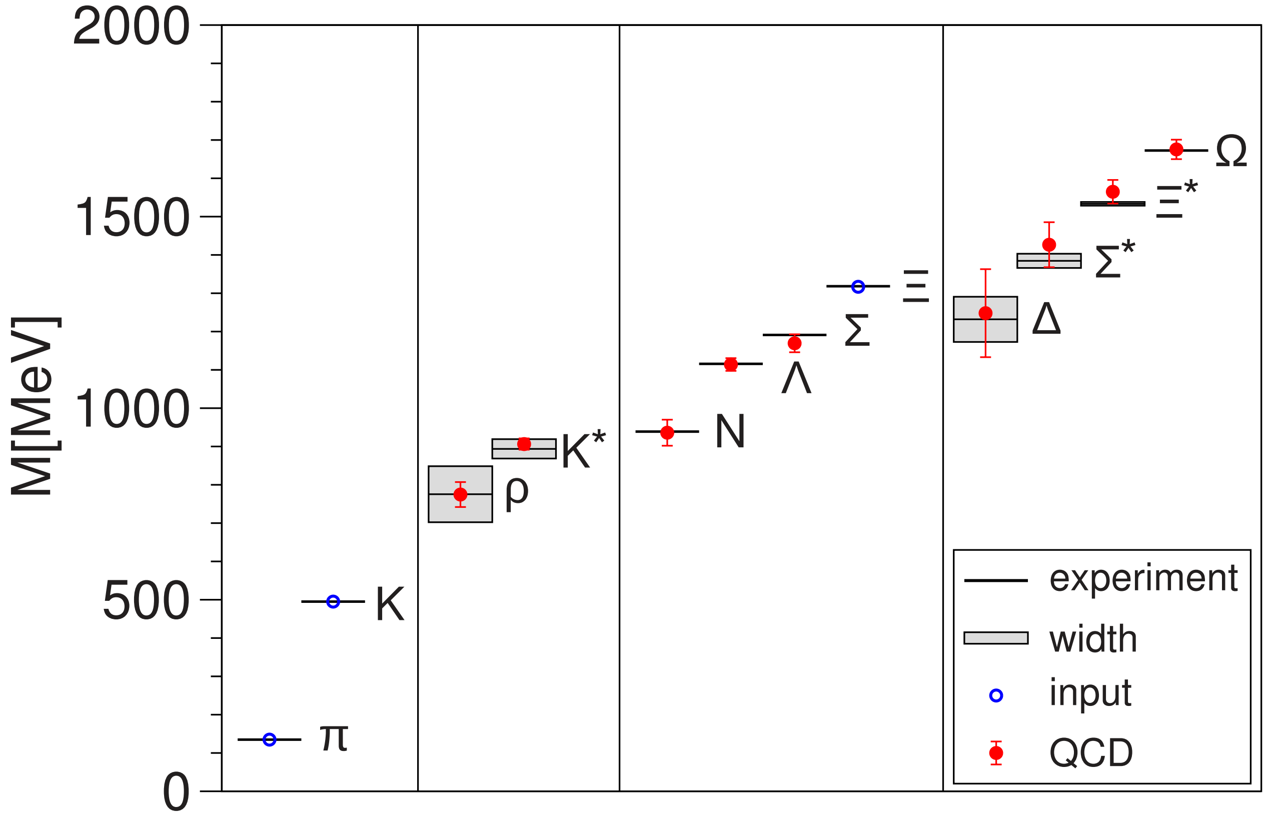
$$= \sum_\alpha C^\alpha_{ji} \exp(-m_\alpha(t'-t))$$

allow one to calculate masses even of excited states

$\rightarrow$ use large enough basis

Lattice QCD: Ground states

Simulations done with finite L, a, m_q (m_π)



Dürr et al., Science 322(2008)1224

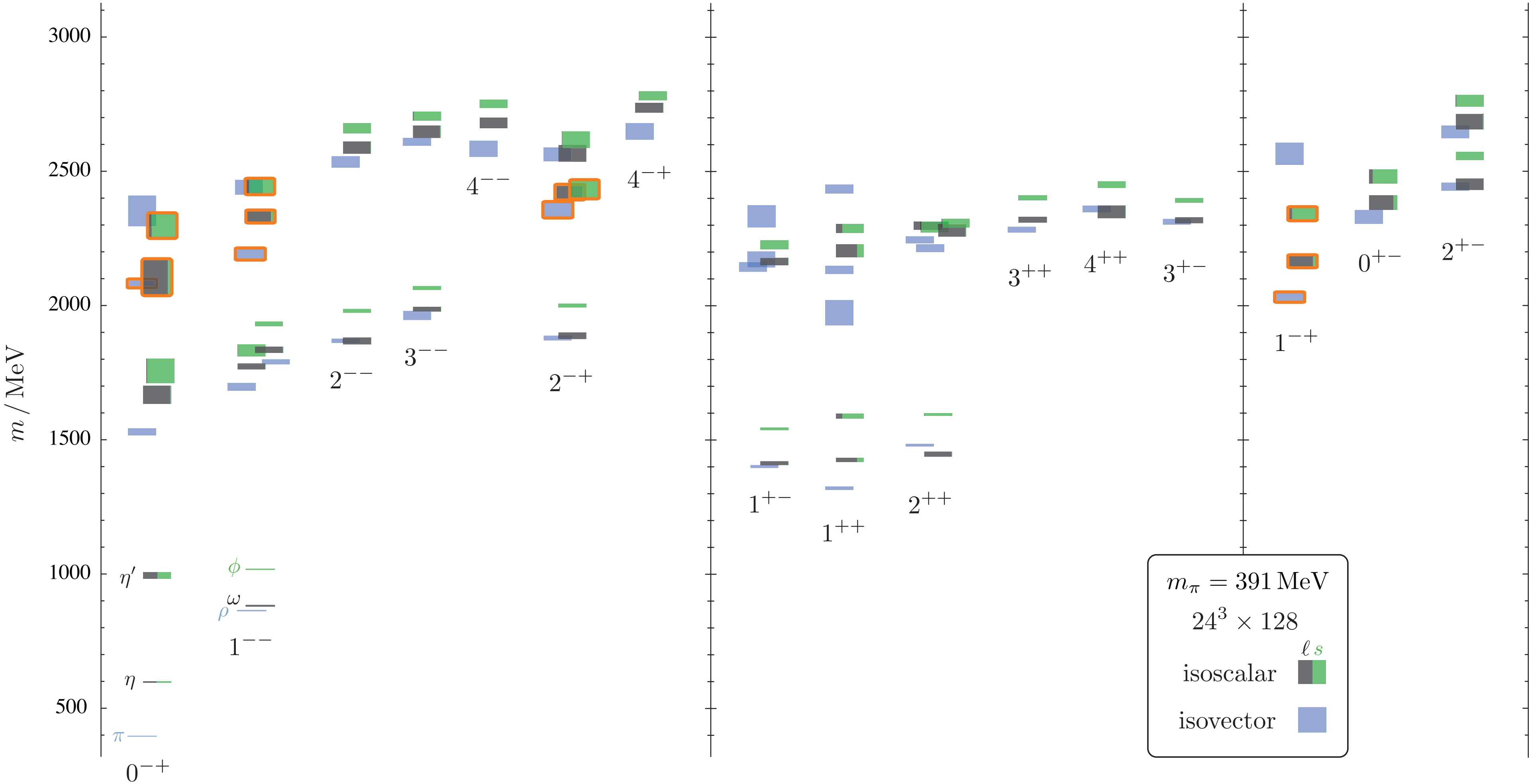
→ light quarks need large volumes $(\lambda_\pi = 1/m_\pi)$

→ computing time (=costs) scale, e.g., as $(L^3T)^{5/4}/(m_\pi^2 a^6)$

→ extrapolations necessary in a, L, (m_π)

Lattice QCD: Excited states

Use of many operators allow extraction of excited states:



Dudek et al., PRD88(2013)094505

At this pion mass / for these operators all states stable

Both quark model states and (crypto)-exotics appear!

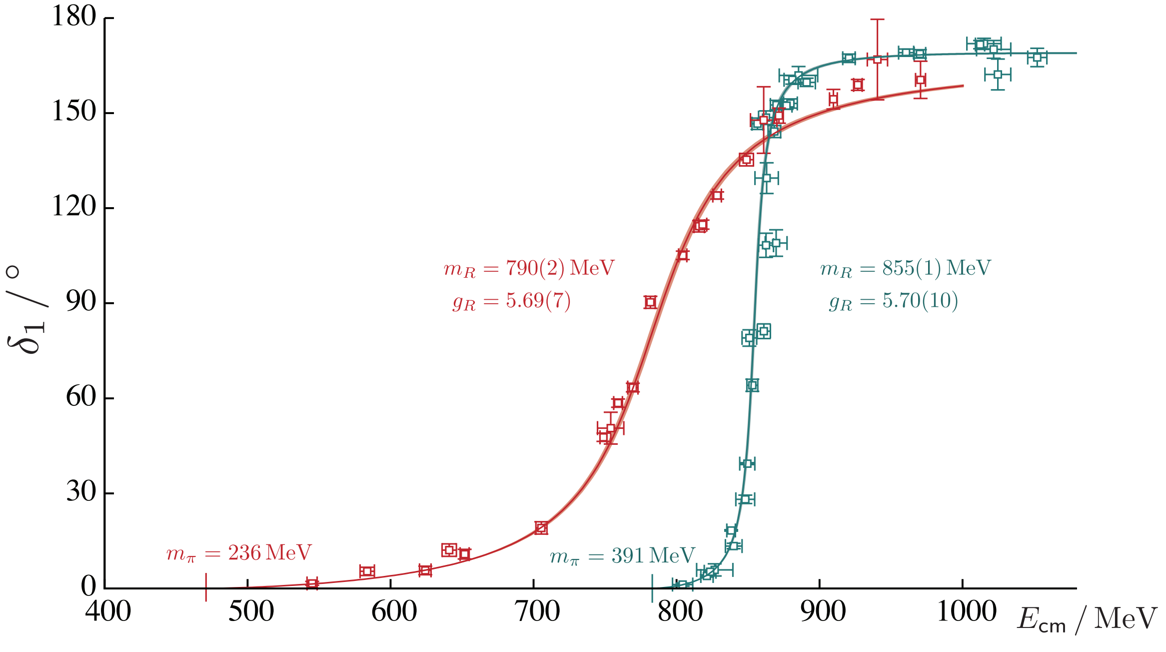
Lattice QCD resonances

Lüscher NPB354(1991)531, Döring et al. EPJA47(2011)139

Volume dependence gives access to resonances via phase shifts

$$p\,\cot(\delta(p)) = \frac{2\pi}{L\sqrt{\pi}} \lim_{\Lambda\to\infty}\left(\sum_{\vec{n}}^{|\vec{n}|<\Lambda} \frac{1}{\vec{n}^{\,2}-\hat{p}^2} - \sqrt{4\pi}\Lambda\right)$$

where $\hat{p} = pL/(2\pi)$ and $\Lambda = q_{\rm max}L/(2\pi)$



Wilson et al., PRD92 (2015) no.9, 094502

$\rho\pi\pi$ coupling g largely m_π independent

C.H. et al., PRL100(2008)152001

Inclusion of inelasticities necessary

Döring et al., EPJA47(2011)139

lowering m_π might become non-trivial

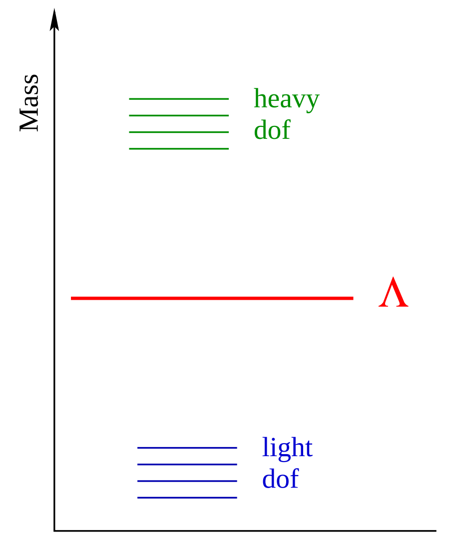
Döring et al., PLB722(2013)185

Simulations technically very demanding

Effective field theories

Weinberg 1979

→ Precondition: separation of scales
low vs. high energy dynamics



- ▷ low-energy dynamics in terms of relevant dof's: $E \sim$ p $\sim Q$
- ▷ high-energy dynamics not resolved → contact interactions

→ Small parameter(s) & power counting

- ▷ Standard QFT: trees + loops → renormalization
- ▷ Expansion in powers of Q over the large scale

$$M = \sum_\nu (Q/\Lambda)^\nu f(Q/\mu, C_i)$$

μ: regularization scale; C_i: low–energy constants
ν bounded from below → controlled expansion

Effect of symmetries I

Let Q be a symmetry charge from Noethers theorem and $U = e^{i\alpha Q}$ and thus

$$U H_0 U^\dagger = H_0 \ .$$

assume two states A and B with

$$U|A\rangle = |B\rangle .$$

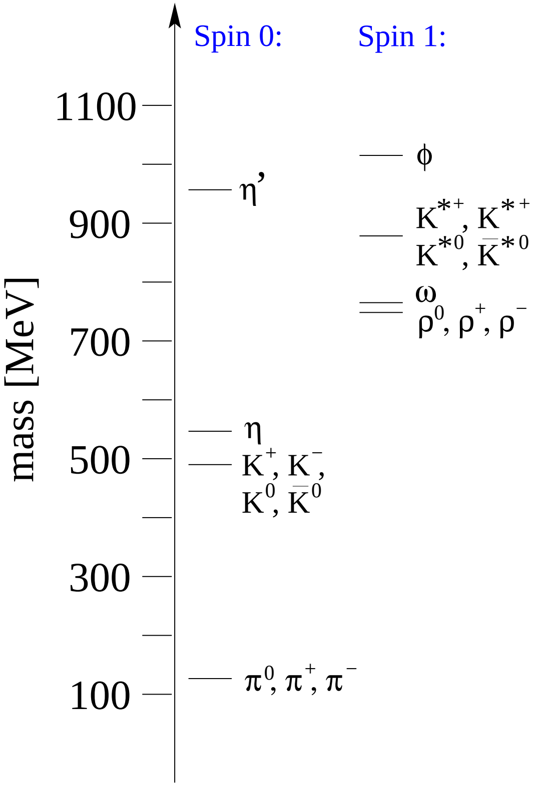
Then one finds

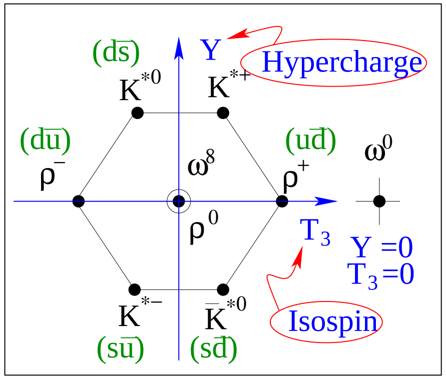
$$E_A = \langle A|H_0|A\rangle = \langle A|U^\dagger U H_0 U^\dagger U|A\rangle = \langle B|H_0|B\rangle = E_B .$$

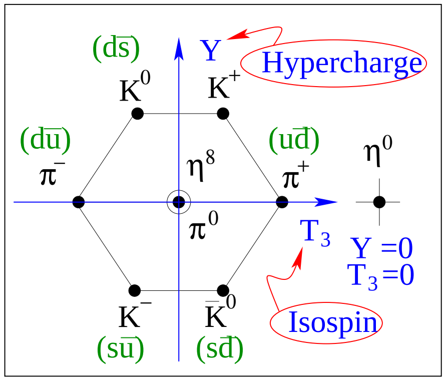
Thus the states that can be reached from A via the symmetry operation are degenerate with A.

This is called an exact symmetry.

$SU(n)_V$







m_{K^+}=494 MeV $\sim u\bar{s}$
m_{K^0}=498 MeV $\sim d\bar{s}$

m_{π^+}=140 MeV $\sim u\bar{d}$
m_{π^0}=135 MeV $\sim u\bar{u} - d\bar{d}$

m_{η}=547 MeV $\sim u\bar{u} + d\bar{d} - 2s\bar{s}$

$m_{\eta'}$=958 MeV $\sim u\bar{u} + d\bar{d} + s\bar{s}$

Note: $Q_u = \frac{2}{3}e\,;\ Q_d = Q_s = -\frac{1}{3}e$

$2(m_{K^+} - m_{K^0})/(m_{K^+} + m_{K^0}) \simeq 2(m_{\pi^+} - m_{\pi^0})/(m_{\pi^+} + m_{\pi^0}) < 3$ %.

Thus: $SU(2)_V$ $(u \leftrightarrow d)$ = Isospin is good symmetry of spectrum.

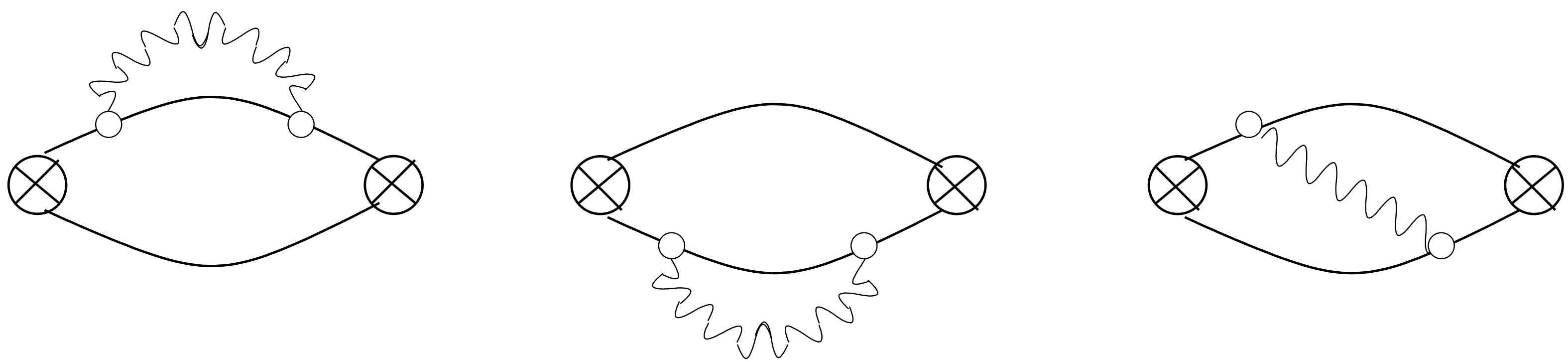
Thus: up and down quark must have very similar properties

What causes Isospin violation?

EM Isospin violation

Natural source of Isospin violation: Different quark charges:

$$Q_u = \frac{2}{3}e \; ; \qquad Q_d = Q_s = -\frac{1}{3}e$$



charged particles heavier than neutrals

→ correct for pions

→ wrong for kaons!

There must be other source of isopion violation

Strong isospin violation

$$\mathcal{L}_{QCD} = \bar{q} i \not{D} q - \bar{q} \mathcal{M} q + \ldots$$

$$\mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} = \frac{m_u + m_d}{2} \hat{1} + (m_u - m_d) \frac{1}{2} \tau_3 \; .$$

$$\mathcal{L}'_{QCD} = \bar{q}' i \not{D} q' - \bar{q}' \mathcal{M} q' + \ldots = \mathcal{L}_{QCD} + \bar{q} \left(\mathcal{M} - \mathcal{M}' \right) q$$

where $q \to q' = U_V q$ and $\mathcal{M}' = U_V \mathcal{M} U_V^\dagger$ with $U_V = \exp(i\theta_V^a \tau^a/2)$;

$\vec{\tau}$ are the Pauli matrices (as used for spin)

$$[\tau^a, \tau^b] = 2i\epsilon^{abc}\tau^c \quad \Longrightarrow \quad \mathcal{M} = \mathcal{M}' \text{ for } m_u = m_d$$

$\rightarrow$ (almost) $SU(2)$ invariant for vanishing (small) $(m_u - m_d)/\Lambda$

Consequences I

→ Isospin violation through $(m_u - m_d)$:

leading strong isospin violation transforms as
third component of an isovector

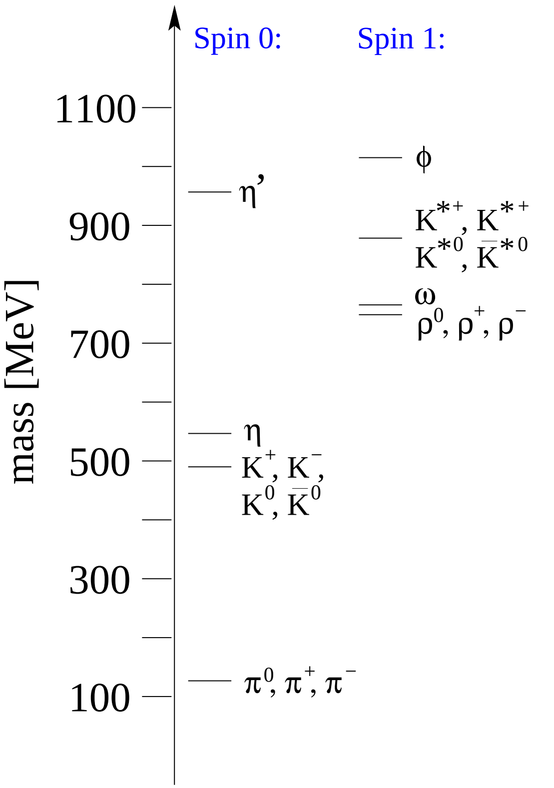
→ Electro–magnetic isospin violation through quark charges;

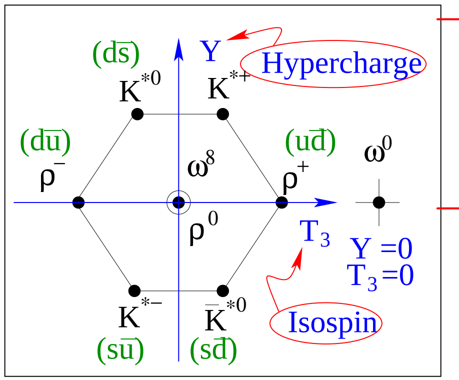
leading net effect transforms as
particular component of a rank 2 tensor

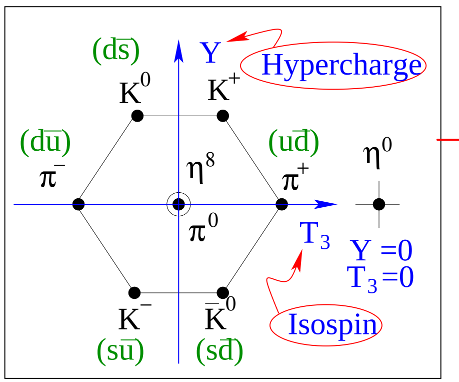
Can be used to disentangle effects

Example

$(m_u - m_d)$ effect transforms as T_3 (to leading order)







→ $m_{\pi^+} - m_{\pi^0} = 4.6$ MeV is electromagn.

$(m_{\pi^+} = m_{\pi^-}$ by CPT)

→ $m_{K^+} - m_{K^0} = -4$ MeV

$\Longrightarrow$ strong and electromagn.

$(m_{K^+} - m_{K^0})^{em} \simeq 1.3$ MeV (Dashen)

$(m_{K^+} - m_{K^0})^{strong} \simeq -5.3$ MeV.

→ $M_p - M_n = -1.3$ MeV

$\Longrightarrow$ strong and electromagn.

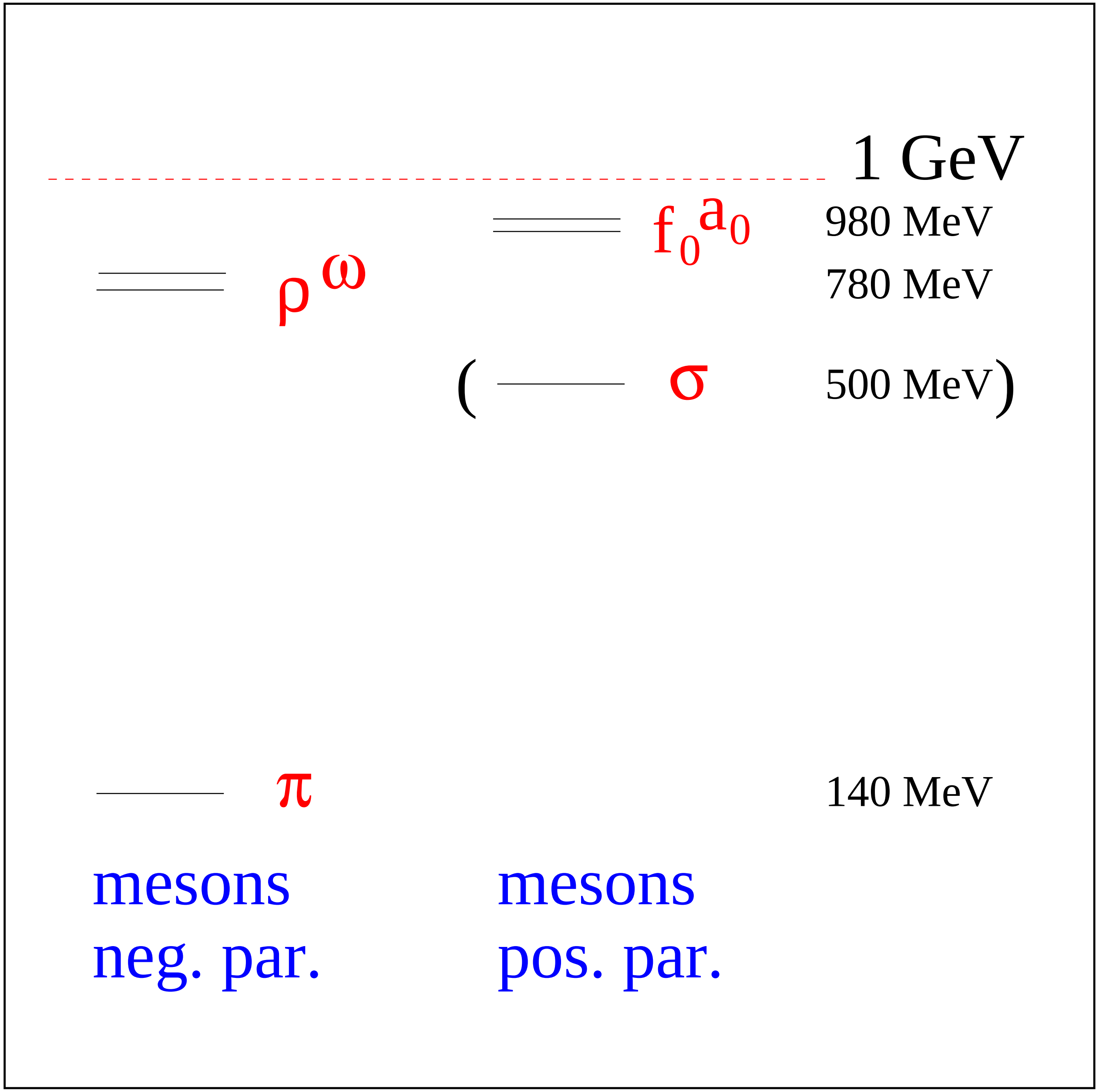
$(M_p - M_n)^{em} \simeq 0.7$ MeV (Cottingham)

$(M_p - M_n)^{strong} = -2$ MeV.

using $K^+ \sim u\bar{s}$, $K^0 \sim d\bar{s}$ and $p \sim uud$, $n \sim ddu$:

implies $m_u < m_d$ and $|\Delta m^{em}| \sim |\Delta m^{strong}|$

Pion properties



Pions very light!

Possible reasons:

→ fine tuning in $q\bar{q}$ interaction

→ (hidden) symmetry!

exploit $m_q = 0$

Symmetries

Let using $q_{R/L} = \frac{1}{2}(1 \pm \gamma_5)q$; then

$$\mathcal{L}_{QCD} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_R \mathcal{M} q_L - \bar{q}_L \mathcal{M} q_R + \dots$$

Thus, if the first n_f elements of $\mathcal{M}$ vanish, we have

$$U(n_f)_L \times U(n_f)_R = SU(n_f)_A \times SU(n_f)_V \times U(1)_V \times U(1)_A$$

where $V = R + L \;\rightarrow\; U_V = \exp(i\Theta_V^a T^a)$ even parity.

$A = R - L \;\rightarrow\; U_A = \exp(i\gamma_5\Theta_A^a T^a)$ odd parity.

$\Longrightarrow$ axial symmetry gives degenerate states of opposite parity but this symmetry is not observed ...

$\rightarrow$ $SU(n_f)_V$ is still conserved for $m_i = m_j \neq 0$ (Isospin).

$\rightarrow$ $U(1)_V$ leads to baryon number conservation.

$\rightarrow$ $U(1)_A$ is broken by quantum anomaly.

Exact vs. Hidden

Nambu; Goldstone

Before we implicitly assumed $U|0\rangle = |0\rangle$.
In field theory picture:

$$|A\rangle = \phi_A|0\rangle \text{ and } |B\rangle = \phi_B|0\rangle \text{ with } U\phi_A U^\dagger = \phi_B \ .$$

Thus we now get

$$\begin{aligned} E_A = \langle A|H_0|A\rangle &= \langle 0|\phi_A^\dagger H_0 \phi_A|0\rangle \\ &= \langle 0|U^\dagger U \phi_A^\dagger U^\dagger U H_0 U^\dagger U \phi_A U^\dagger U|0\rangle \\ &= \langle 0|U^\dagger \phi_B^\dagger H_0 \phi_B U|0\rangle \begin{cases} = E_B & \text{for } U|0\rangle = |0\rangle \\ \neq E_B & \text{for } U|0\rangle \neq |0\rangle . \end{cases} \end{aligned}$$

Then symmetry not reflected in spectrum; it is hidden.

$\rightarrow$ spontaneous symmetry breaking (SSB)

Hidden Symmetries

Let $U|0\rangle^a = \exp\left(i\sum \alpha_k Q_k\right)|0\rangle = |\alpha\rangle$; Then

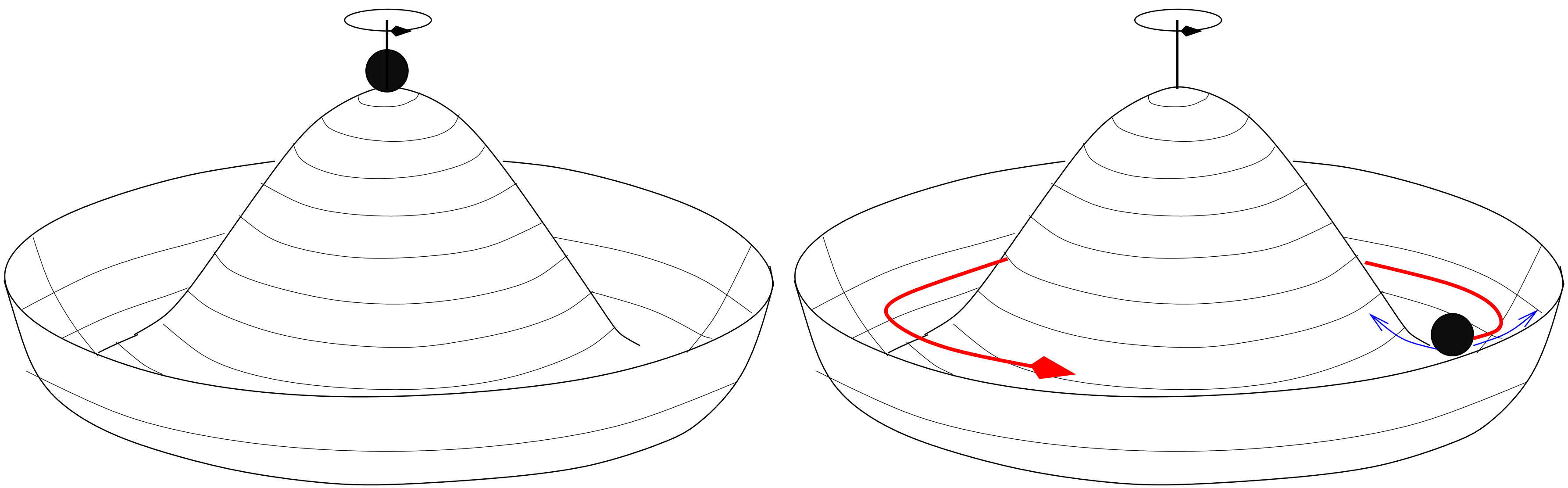
$$E_\alpha = \langle\alpha|H_0|\alpha\rangle = \langle 0|U^\dagger H_0 U|0\rangle = \langle 0|H_0|0\rangle = E_0 \; .$$

Thus, all α states are degenerate with the vacuum

Therefore, for a continuous symmetry

$\rightarrow$ there are massless particles called Goldstone bosons (GB) (Goldstone Theorem) ;

$\rightarrow$ their number agrees to that of broken generators.



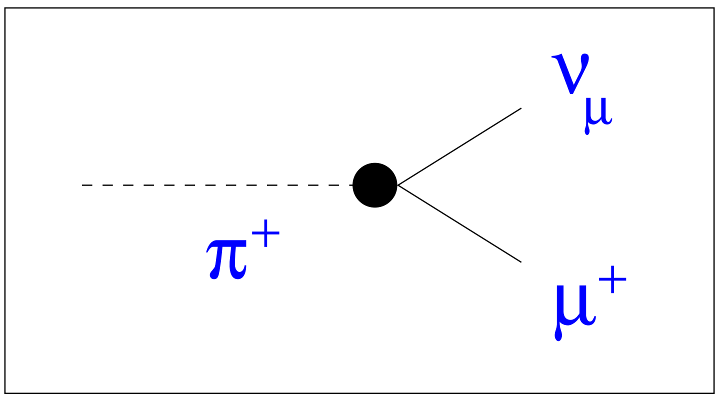
Conjecture: Pions are theses Goldstone-Bosons

Define pion decay constant f_π:

$\langle \pi | Q_A | 0 \rangle \neq 0 \;\rightarrow\;$ Lorentz invariance: $\langle \pi | A^\mu | 0 \rangle =: i f_\pi q^\mu \neq 0$

$f_\pi \neq 0$ is a necessary condition for SSB

Decay constant f_π can be fixed from weak decays:



$$= \frac{1}{\sqrt{2}} G_F \langle 0 | A^-_\alpha | \pi^+ \rangle \bar{\mu}^+ \gamma^\alpha (1 - \gamma_5) \nu_\mu$$

$$\rightarrow \; f_\pi = 92 \text{ MeV}$$

Intimate link between weak matrix elements and strong force!

Goldstone Theorem II

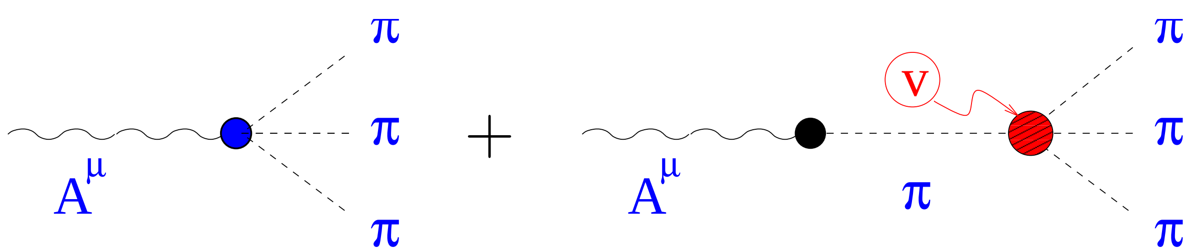
Another statement of the Goldstone theorem:

At vanishing momenta GB do not interact.

Proof: for 4π vertex function V

Currents still conserved: $q_\mu \mathcal{A}^\mu = 0$

$i\mathcal{A}^\mu =$



$$= \quad iR^\mu \quad + \quad i f_\pi q^\mu \left(\frac{i}{q^2}\right) iV$$

$$\Longrightarrow \lim_{q\to 0} q_\mu \mathcal{A}^\mu = -f_\pi \lim_{q\to 0} V = 0$$

Effective Theory for Goldstone Bosons

- collect $\pi^\pm$, π^0, $K^\pm$, K^0, $\bar{K}^0$, η in a common field U:

$$U = \exp\left(\frac{i\phi}{f_\pi}\right) \;,\; \phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

- low energies: expand in powers of momenta = # derivatives:

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

- leading term of the effective Lagrangian is $\mathcal{L}^{(2)}$:

$$\boxed{\mathcal{L}^{(2)} = \frac{f_\pi^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger \rangle}$$

- expand U in powers of ϕ, $\quad U = 1 + i\phi/f_\pi - \phi^2/(2f_\pi^2) + \ldots$

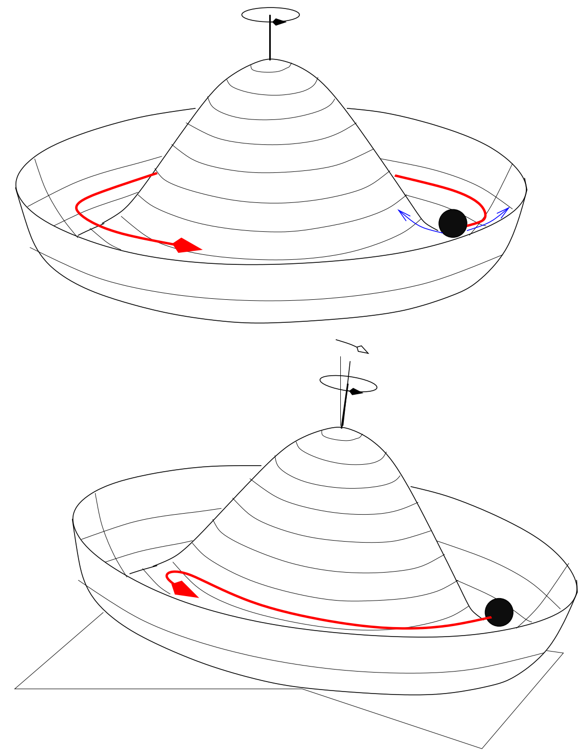
  Normalisation to reproduce canonical kinetic terms

$$\mathcal{L}^{(2)} = \partial_\mu \pi^+ \partial^\mu \pi^- + \partial_\mu K^+ \partial^\mu K^- + \ldots$$

Quark Masses

- in nature, quark masses small, but non-zero
  - ▷ chiral symmetry explicitly broken
  - ▷ if symmetry breaking is weak, perform perturbative expansion in the quark masses

    "Chiral Perturbation Theory"



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}(U, \partial U, \partial^2 U, \ldots, \mathcal{M}) \;, \qquad \mathcal{M} = \text{diag}(m_u, m_d, m_s)$$

- effective Lagrangian (properly generalised) still appropriate too to systematically derive all symmetry relations
- Expansion parameter $m_\pi/\Lambda_\chi \sim p/\Lambda_\chi$ with $\Lambda_\chi \sim 1$ GeV

Selected Results I

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\left[\langle \partial_\mu U \partial^\mu U^\dagger \rangle + 2B\langle \mathcal{M}U^\dagger + \mathcal{M}^\dagger U \rangle\right]$$

- read off mass terms: Gell-Mann–Oakes–Renner relation(s)

$$\begin{aligned}
M^2_{\pi^\pm} &= B(m_u + m_d) + 2Ze^2 f_\pi^2 \\
M^2_{\pi^0} &= B(m_u + m_d) - \mathcal{O}\big((m_u - m_d)^2\big) \\
M^2_{K^\pm} &= B(m_u + m_s) + 2Ze^2 f_\pi^2 \\
M^2_{K^0} &= B(m_d + m_s) \\
M^2_\eta &= \frac{B}{3}(m_u + m_d + 4m_s) + \mathcal{O}\big((m_u - m_d)^2\big)
\end{aligned}$$

- Gell-Mann–Okubo mass formula: $4M_K^2 = 3M_\eta^2 + M_\pi^2$
  $\longrightarrow$ fulfilled in nature at 7% accuracy
- quark mass ratios:

$$\frac{m_u}{m_d} \approx 0.55; \quad \frac{m_s}{m_d} \approx 22 \qquad (1)$$

Selected results II

- isospin decomposition of $\pi\pi$ scattering amplitude:

$$M(\pi^a\pi^b \to \pi^c\pi^d) = \delta_{ab}\delta_{cd}A(s,t,u)+\delta_{ac}\delta_{bd}A(t,u,s)+\delta_{ad}\delta_{bc}A(u,s,t)$$

- calculate $A(s,t,u)$ from $\mathcal{L}^{(2)}$:

$$\boxed{A(s,t,u) = \frac{s-M_\pi^2}{f_\pi^2}}$$

Weinberg, PRL17(1966)1313

$\longrightarrow$ parameter-free prediction
$\longrightarrow$ in accordance with Goldstone theorem:
$\pi\pi$ interaction vanishes at low energies ($s \to 0$, $m_q \to 0$)

$\rightarrow$ Example: $\pi\pi$-isoscalar s–wave scattering length:

$$a_0 = (0.16 + 0.04 + 0.017) \pm 0.009 \text{ at LO, NLO, N}^2\text{LO}$$

Bijnens et al. PLB374(1996)210

Exp.: $a_0 = 0.221 \pm 0.006$ in excellent agreement

Batley et al. [NA48/2 Coll.] EPJC79(2010)635

Heavy Sources: πN scattering

$$\boxed{\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}\left(i\gamma_\mu \mathcal{D}^\mu - m + \frac{g_A}{2}\gamma_\mu\gamma_5 u^\mu\right)\psi} \qquad \psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$\mathcal{D}^\mu = \partial^\mu + \Gamma^\mu \qquad \Gamma^\mu = \frac{1}{2}\left(u^\dagger(\partial^\mu - i\,r^\mu)u + u(\partial^\mu - i\,l^\mu)u^\dagger\right)$$

$$u^\mu = i\left(u^\dagger(\partial^\mu - i\,r^\mu)u - u(\partial^\mu - i\,l^\mu)u^\dagger\right) \qquad u = \sqrt{U}$$

- now, due to spin (Dirac structures), odd powers possible:

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$

new parameters at leading order:

- m: nucleon (baryon) mass in the chiral limit
- g_A: axial vector coupling from neutron beta decay: $g_A = 1.26$

Goldberger–Treiman relation: $\boxed{g_{\pi N} = \frac{g_A m_N}{F_\pi}}$

Unitarization

Unitarity relation: $\text{Im}(t) = \sigma|t|^2$ with $\sigma = \sqrt{1 - 4m_\pi^2/s}$

$\rightarrow$ Perturbative expansion consistent only to given order

$\rightarrow$ s–dependent terms quickly hit unitarity bound

Solution: Unitarization $\rightarrow$ can produce poles

Truong, Dorado, Pelaez, Kaiser, Weise, Oller, Oste, Lutz, Kolomeitsev, Guo, Meißner, C.H., ...

Different methods used (dep. needs to be clarified);

$\rightarrow$ universal picture emerges in many channels!

Example I: $\pi\pi$ scattering from the Inverse Amplitude Method

Idea: write unitarity as

$$\text{Im}(t^{-1}) = -\sigma \qquad \longrightarrow t = \frac{1}{\text{Re}(t^{-1}) - i\sigma}$$

use ChPT to fix $\text{Re}(t^{-1})$ to the required accuracy

Results

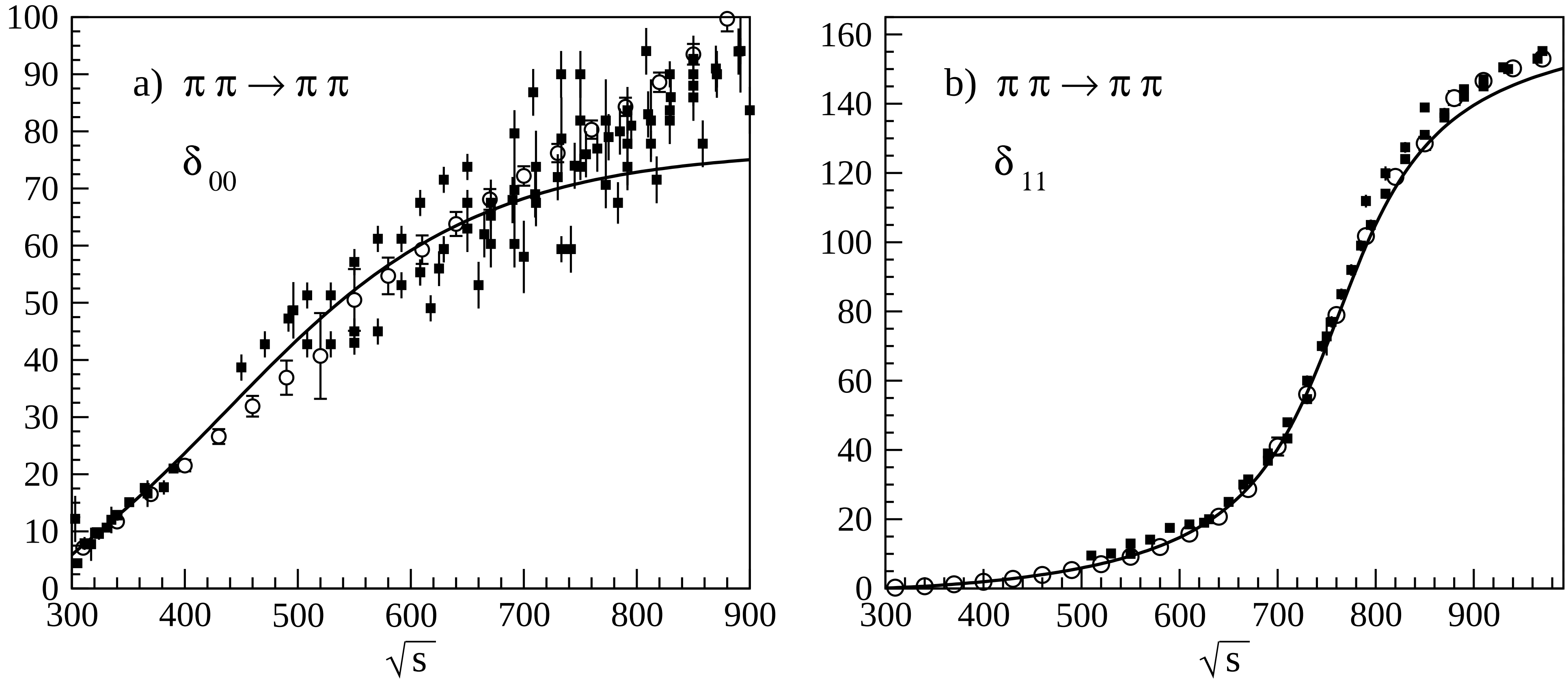
Using ChPT to NLO as input (parameters $\times 10^3$ at $\mu = m_\rho$)

$l_3^r = 0.18 \pm 1.11$; $l_4^r = 6.17 \pm 1.39$ from the literature

Colangelo et al. (2001) & Colangelo et al. (2010)

$l_1^r = -3.7 \pm 0.2$; $l_2^r = 4.3 \pm 0.4$ fit to the data

Nebreda et al. (2011)



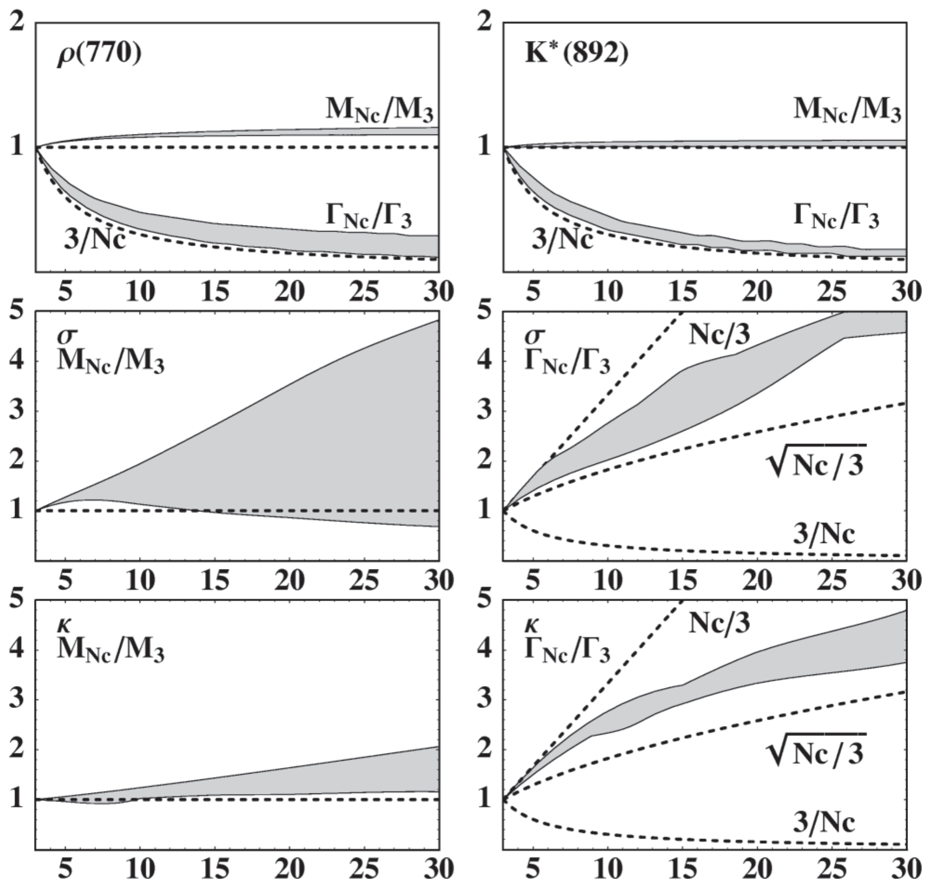
What does this tell about the nature of $f_0(500)$ and $\rho(770)$?

Large N_c

An interesting limit of QCD is $N_c \to \infty$, while $g_s^2 N_c = const.$

$\to$ N_c–scaling of low energy constants known

e.g. Peris & de Rafael. PLB348(1995)539



Pelaez PRL92(2004)102001

$\to$ N_C scaling for states:

Cohen et al. PRD90(2014)036003

- $\bar{q}q$:
  $M \sim \mathcal{O}(1);\ \Gamma \sim \mathcal{O}(1/N_c)$
- $\bar{q}\bar{q}qq$:
  $M \sim \mathcal{O}(1);\ \Gamma \sim \mathcal{O}(1/N_c)$
- gg:
  $M \sim \mathcal{O}(1);\ \Gamma \sim \mathcal{O}(1/N_c^2)$
- $(N_c - 1)\bar{q}q$:
  $M \sim \mathcal{O}(N_c);\ \Gamma \sim \mathcal{O}(1)$

ρ, K^* consistent with $\bar{q}q$; σ, κ do not match to any $\to$ rescattering?

Heavy Quark Symmetry

see, e.g., Neubert Phys. Rep. 245(1994)259

One may derive from the QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = \bar{q}_f \left\{ iv\cdot\partial + gv\cdot A^a t^a \right\} q_f + \mathcal{O}(\Lambda_{\rm QCD}/m_f)$$

At leading order interaction spin and flavor independent!

heavy quark spin and $J_{\rm light}$ of light quarks conserved independently

Terms at $\mathcal{O}(\Lambda_{\rm QCD}/m_f)$ contain

→ kinetic energy of heavy quark

→ term breaking spin symmetry

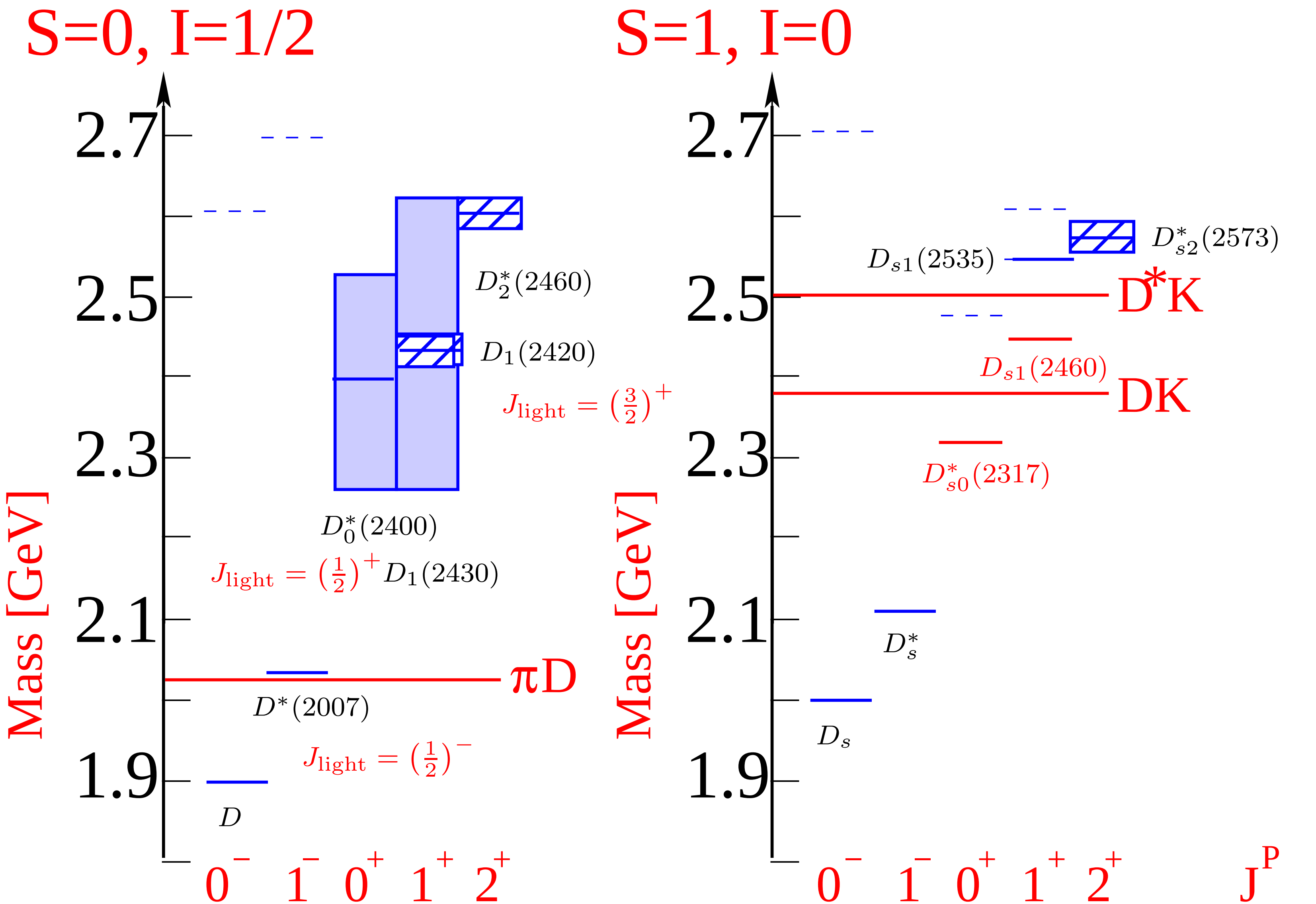
Consequence: mesons form spin multiplets with

$$m_{D^*} - m_D \sim \Lambda_{QCD}\ , \quad m_{B^*}^2 - m_B^2 \simeq m_{D^*}^2 - m_D^2$$

which works nicely - also for excited states

→ Amount of spin symmetry violation important diagnostic tool!

Example: Strange-Charm states



Quark Modell: M. Di Pierro and E. Eichten, PRD **64** (2001) 114004

Note: decay modes of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ either $D_s^{(*)}\pi$ or $D_s^{(*)}\gamma$ $\rightarrow$ narrow

Example: Strange-Charm states

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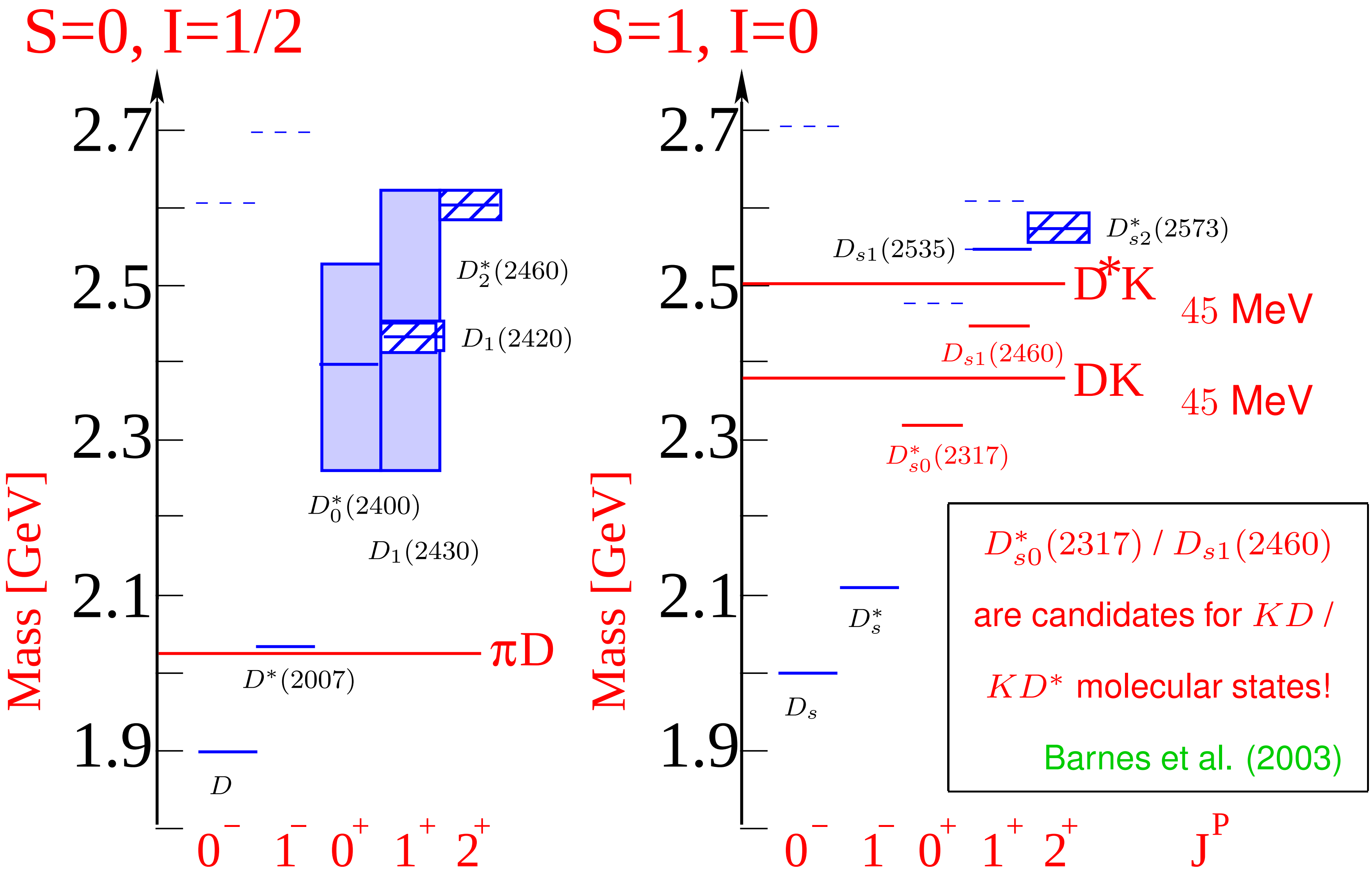

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Note: decay modes of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ either $D_s^{(*)}\pi$ or $D_s^{(*)}\gamma$ $\rightarrow$ narrow

Summary lecture I

There are various options to study QCD at low energies

1. Phenomenological models
   (not part of this talk - will appear later in the series)

2. Lattice QCD

3. Effective Field Theories (ChPT, HQEFT)

Very promising: Combinations of #2 and #3

Next lecture:

→ Discussion of $D_s(2317)$ as hadronic molecule

→ What it takes further support this conjecture