Strong decay of $\Delta(1232)$-isobar

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talk is based on the paper "Δ Isobar Decay in Covariant Quark Model"

by M.A.Ivanov, G.Nurbakova and Z.Tyulemissov

Multiplet of Δ-isobar

$$m_\Delta = 1210 \pm 2 MeV.$$

$$J^P = \frac{3}{2}^+$$

$$\tau_\Delta = (5.63 \pm 0.14) \cdot 10^{-24} s$$

Let $\Delta^{k_1 k_2 k_3}(x)$ be the multiplet of all isospin states of the Δ-isobar. The spinor is

- symmetric with respect to permutation of k_1, k_2, k_3
- satisfied the Rarita-Schwinger equation: $\Delta_\mu \gamma^\mu = 0$

$$\Delta^{111} = \Delta^{++}, \; \Delta^{211} = \frac{1}{\sqrt{3}}\Delta^{+}, \; \Delta^{122} = \frac{1}{\sqrt{3}}\Delta^{0}, \; \Delta^{222} = \Delta^{-}.$$

Table: **Δ DECAY MODES**

	Mode	Fraction(Γ_i/Γ)
Γ_1	$N\pi$	99.4%
Γ_2	$N\gamma$	$0.55 - 0.65\%$
Γ_3	pe^+e^-	$(4.2 \pm 0.7) \cdot 10^{-5}$

Effective Lagrangian

$$\mathcal{L}_\Delta(x) = ig_\Delta \bar{\Delta}_\mu^{k_1k_2k_3}(x)\,(J^\mu)^{k_1k_2k_3}(x) + h.c.$$

where

$$(J^\mu)^{k_1k_2k_3}(x) = \iiint dx_1 dx_2 dx_3\, \delta\left(x - \sum_{i=1}^{3} w_i x_i\right)$$
$$\times \Phi_\Delta\left[\sum_{i<j}(x_i - x_j)^2\right](J^\mu_{3q})^{k_1k_2k_3}(x_1, x_2, x_3),$$
$$(J^\mu_{3q})^{k_1k_2k_3}(x_1, x_2, x_3) = \varepsilon^{a_1a_2a_3}\Gamma_1 q_{a_1}^{k_1}(x_1)\left[q_{a_2}^{k_2}(x_2) C\Gamma_2 q_{a_3}^{k_3}(x_3)\right]$$

where $a_1, a_2, a_3 = 1, 2, 3$ are the color indices; $C = \gamma^0\gamma^2$ is the charge conjugation matrix; $k_1, k_2, k_3 = 1, 2$; and the Lorenz index "μ", corresponding to spin 1,and be either in the matrix Γ_1 or in the matrix Γ_2.

The three-quark current

Let us first consider the index permutation in the diquark

$$\varepsilon^{a_1 a_2 a_3}\Gamma_1 q^{k_1}_{a_1}(x_1)\left[(q^{k_2}_{a_2\alpha_2})(C\Gamma_2)_{\alpha_2\alpha_3} q^{k_3}_{a_3}\right]$$

$$= \varepsilon^{a_1 a_2 a_3}\Gamma_1 q^{k_1}_{a_1}(x_1)\left[(q^{k_3}_{a_2\alpha_2})(C\Gamma_2)_{\alpha_2\alpha_3} q^{k_2}_{a_3\alpha_3}\right]$$

$$= \varepsilon^{a_1 a_2 a_3}\Gamma_1 q^{k_1}_{a_1}(x_1)\left[-q^{k_2}_{a_3\alpha_3}(C\Gamma_2)_{\alpha_2\alpha_3} q^{k_3}_{a_2\alpha_2}\right]$$

$$= \varepsilon^{a_1 a_2 a_3}\Gamma_1 q^{k_1}_{a_1}(x_1)\left[q^{k_2}_{a_2\alpha_3}(C\Gamma_2)^T_{\alpha_3\alpha_2} q^{k_3}_{a_3\alpha_2}\right]$$

$C\Gamma_2 = (C\Gamma_2)^T = -\Gamma_2^T C = -CC\Gamma_2^T C^{-1}$.
We use the following properties:

$$C\Gamma^T C^{-1} = \begin{cases} \Gamma, & \text{for } \Gamma = S, P, A \\ -\Gamma, & \text{for } \Gamma = V, T. \end{cases}$$

where $S \to I$, $P \to \gamma_5$, $V \to \gamma^\mu$, $A \to \gamma^\mu\gamma_5$, $T \to \sigma^{\mu\nu}$.

The Fiertz identity

$$4(\Gamma_1)_{\alpha\alpha_1}(\Gamma_2)_{\beta_2\alpha_3} = \sum_D (\Gamma_1\Gamma^D)_{\alpha\alpha_3}(\Gamma_1\Gamma^D)_{\beta_2\alpha_1},$$

where $\Gamma^D = \{I, \gamma^\mu, \sigma^{\mu\nu}(\mu < \nu), \gamma_5, i\gamma^\mu\gamma_5\}$ is the complete set of basic Dirac matrices. $\Gamma_1 \times \Gamma_2 = I \times \gamma^\mu$, $\Gamma_1 \times \Gamma_2 = \gamma_\nu \times \sigma^{\mu\nu}$

$$\begin{cases} (O_1)_{\alpha\alpha_1}(O_2)_{\alpha_2\alpha_3} \equiv (\tilde{O}_1) \otimes (\tilde{O}_2) \\ (O_1)_{\alpha\alpha_3}(O_2)_{\alpha_2\alpha_1} \equiv (O_1) \otimes (O_2) \end{cases}$$

The three-quark current

Let's use $k_1 \leftrightarrow k_3$, the Fiertz identity and Rarita-Schwinger equation. Finally one obtains

$$\boxed{\Gamma_1 \otimes \Gamma_2 = I \otimes \gamma^\mu - \frac{i}{2}\gamma_\nu \otimes \sigma^{\mu\nu}}$$

The three quark current is rewritten as

$$(J^\mu)^{k_1 k_2 k_3} = \varepsilon^{a_1 a_2 a_3}\left[q_{a_1}^{k_1}\left[q_{a_2}^{k_2} C\gamma^\mu q_{a_3}^{k_3}\right] - \frac{i}{2}\gamma_\nu q_{a_1}^{k_1}\left[q_{a_2}^{k_2} C\sigma^{\mu\nu} q_{a_3}^{k_3}\right]\right]$$

Mass operator

$$S_2(x-y) = i \iint dxdy\, \bar{\Delta}^{++}_{\mu\alpha}(x)\, \Sigma^{\mu\nu}(x-y)\, \Delta^{++}_{\nu\beta}(y),$$

where, without losing generality, we consider the case of the Δ^{++}-isobar with the quantum content $q^{k_1} = q^{k_2} = q^{k_3} = u$

$$\Sigma^{\mu\nu}(x-y) = 36 i g_\Delta^2 \int dxdy \int dx_1 \ldots dx_3 \delta\left(x - \sum_{i=1}^{3} w_i x_i\right)$$

$$\times \Phi_\Delta\left[\sum_{i<j}(x_i - x_j)^2)\right] \int dy_1 \ldots dy_3 \delta\left(y - \sum_{i=1}^{3} w_i y_i\right) \Phi_\Delta\left[\sum_{i<j}(y_i - y_j)^2)\right]$$

$$\times \sum_{m,n=1}^{2} c_{mn} \Gamma_m S(x_1 - y_1) \Gamma_n \mathrm{tr}\left[\tilde{\Gamma}_n S(y_2 - x_2) \tilde{\Gamma}_m S(x_3 - y_3)\right]$$

After using Jacoby coordinates and Fourier transform of the mass operator one obtains

$$\tilde{\Sigma}(p,p') = \delta^{(4)}(p-p')\,\tilde{\Sigma}^{\mu\nu}(p)$$

$$\tilde{\Sigma}^{\mu\nu}(p) = 36 g_\Delta^2 \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} \tilde{\Phi}_\Delta^2\left[-\omega^2\right] \sum_{m,n=1}^{2} c_{mn}\Gamma_m S(k_1+w_1 p)\Gamma_n$$

$$\times \mathrm{tr}\left[\tilde{\Gamma}_n S(k_2-w_2 p)\tilde{\Gamma}_m S(k_2-k_1+w_3 p)\right],$$

where $\omega^2 = 1/2(k_1-k_2)^2 + 1/6(k_1+k_2)^2$.

Compositeness condition

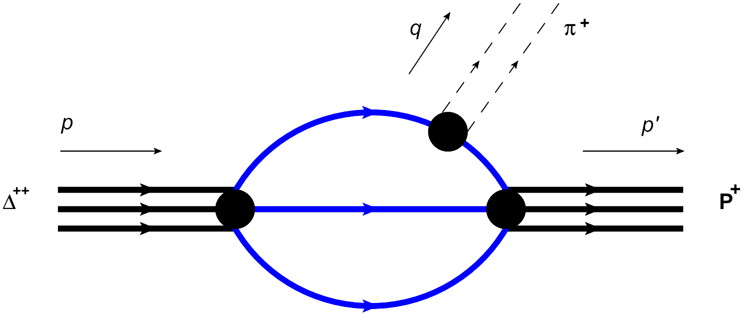
The compositeness condition means that the renormalization constant of the baryon field Z_Δ, which appeared as a result of interaction with its constituents, should be equated to zero, $Z_\Delta = 0$. In the case of the delta isobar, this condition is written as

$$Z_\Delta = 1 - \frac{d}{d\widehat{p}}\Sigma_0(\widehat{p}) = 0,$$

where $\Sigma_0(\widehat{p})$ appears when $\Sigma^{\mu\nu}$ is expanded in the form $g^{\mu\nu}\Sigma_0(\widehat{p})$. For determining strong coupling constant we use identity

$$\frac{d}{dp^\alpha}\Sigma^{\mu\nu} = g^{\mu\nu}\frac{d}{dp^\alpha}\Sigma_0$$

Decay $\Delta^{++} \to p\pi^+$



$$M(\Delta^{++} \to p\pi) = (2\pi)^4 i\,\delta(p - p' - q)\,T(\Delta^{++} \to p\pi),$$

$$T(\Delta^{++} \to p\pi) = G_{\Delta p\pi}\, p'_\mu \bar{u}_p(p', \lambda') u^\mu_\Delta(p, \lambda),$$

Matrix element

- the transition for a spin of 1/2 is

$$\sum_\lambda u(p,\lambda)\bar{u}(p,\lambda) = \hat{p} + m.$$

- the transition for a spin of 3/2 is

$$\sum_\lambda u(p,\lambda)\bar{u}(p,\lambda) = (\hat{p} + m)$$

$$\times \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} + \frac{1}{3}\left(g_{\mu\alpha} - \frac{p_\mu p_\alpha}{m^2}\right)\left(g_{\nu\beta} - \frac{p_\nu p_\beta}{m^2}\right)\gamma^\alpha\gamma^\beta \right]$$

- Let us square the matrix element, sum over polarizations and multiply by 1/4

Finally one can get

$$\frac{1}{4}\sum_{\lambda\lambda'} |M|^2 = \frac{1}{4} G^2_{\Delta p\pi} p^{'}_\mu p^{'}_\nu \sum_{\lambda\lambda'} \left[\bar{u}_p(p',\lambda')u^\mu_\Delta(p,\lambda)\right]\left[\bar{u}^\nu_\Delta(p,\lambda)u_p(p',\lambda')\right]$$

Δ-isobar decay width

$$\Gamma(\Delta^{++} \to p\pi^{+}) = \frac{G^2_{\Delta p\pi}}{24\pi}|\vec{q}|^{3}\left[\left(1+\frac{m_N}{m_\Delta}\right)^2 - \frac{m_\pi^2}{m_\Delta^2}\right]$$

	EXP	OUR	[1]	[2]	[3]	[4]
$G_{\Delta p\pi}$	15.4 ± 2.9	15.2 ± 1.5	17.0	11.14	14.98	14.85

[1] G. V. Efimov, M. A. Ivanov, and V. E. Lyubovitskij, Few Body Syst. 6, 17–43 (1989)

[2] T. Melde, L. Canton, and W. Plessas, Phys. Rev. Lett. 102, 625 (2009)

[3] V. Mader, G. Eichmann, M. Blank, and A. Krassnigg, Phys. Rev. D: Part. Fields 84, 034012 (2011)

[4]T. Sato and T. S. H. Lee, Phys. Rev. C 54, 2600–2684 (1996)

[5]C. Alexandrou, G. Koutsou, J. W. Negele, Y. Proestos, and A. Tsapalis, Phys. Rev. D: Part. Fields 83, 31 (2011)

Strong form factor

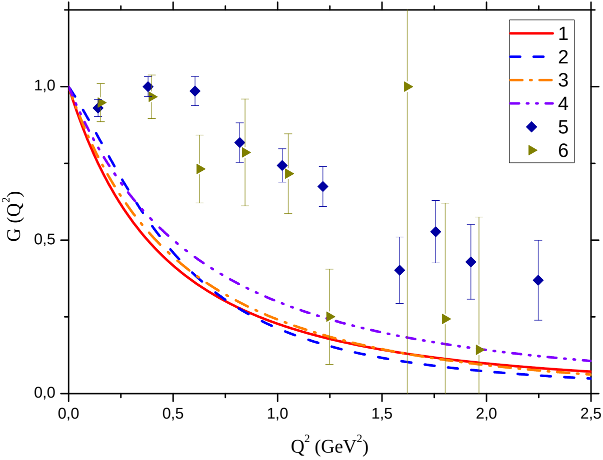


Figure: Strong form factor of the isobar obtained in different theoretical approaches: (1) this study, (2) QCM model [1],(3) RCQM model [2], and (4) [3]; points 5 and 6 are lattice calculations [5] for $m_\pi = 297$ MeV and $m_\pi = 330$ MeV, respectively.

Strong form factor

$$G(Q^2) = \frac{1}{(1 + Q^2/\Lambda_D^2)^2}, \;\; G(q^2) = \frac{1}{(1 + \vec{q}^{\,2}/\lambda_1^2 + \vec{q}^{\,4}/\lambda_2^4)},$$

where $0 \leq Q^2 = -q^2 \leq 2.5 GeV^2$. We use the relation $\vec{q}^{\,2} = q_0^2 + Q^2$. We choose parameter $\Lambda_D = 0.96$ GeV. In [2] parameters $\lambda_1 = 0.594$ GeV, $\lambda_2 = 0.998$ GeV.

Conslusion

- **We construct three-quark current of Δ-isobar based on quantum numbers and the Fierz identity**

- **The matrix element and width of strong decay were calculated.**

- **The dependence of G(Q^2) was constructed in the Euclidean region of the squared transferred pion momentum $Q^2 = -q^2$.**