Lecture series on
QCD Exotics in the Heavy Quark Sector
Part II: The single heavy sector

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Outline

Lecture I: Tools

→ Lattice QCD

→ Effective field theories (ChPT, HQEFT)

→ Unitarisation

→ Large N_c

Lecture II: The single heavy sector

→ Goldstone–Boson D-meson scattering

→ The positive parity D-mesons

→ Predictions and tests

Lecture III: The $\bar{Q}Q$ sector

→ The XYZ-stories

In this lecture series the focus is on mesons

Limiting cases of QCD

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{q}_f \left(\gamma_\mu D^\mu - m_f\right) q_f - \frac{1}{4T}\text{Tr}\left(F^{\mu\nu} F_{\mu\nu}\right)$$

Limit of massless Quarks

Weinberg/ Gasser, Leutwyler

$$\mathcal{L}_{\rm QCD} = \bar{q}_L \left\{ i\partial\!\!\!/ + g A\!\!\!/^a t^a \right\} q_L + \bar{q}_R \left\{ i\partial\!\!\!/ + g A\!\!\!/^a t^a \right\} q_R + \mathcal{O}(m_f/\Lambda_{\rm QCD})$$

L and R Quarks decouple + spontaneous symmetry breaking

$\rightarrow$ Chiral Perturbation Theory (ChPT)

Limit of infinitely Heavy Quarks

Isgur, Wise, Manohar, Caswell/Lepage

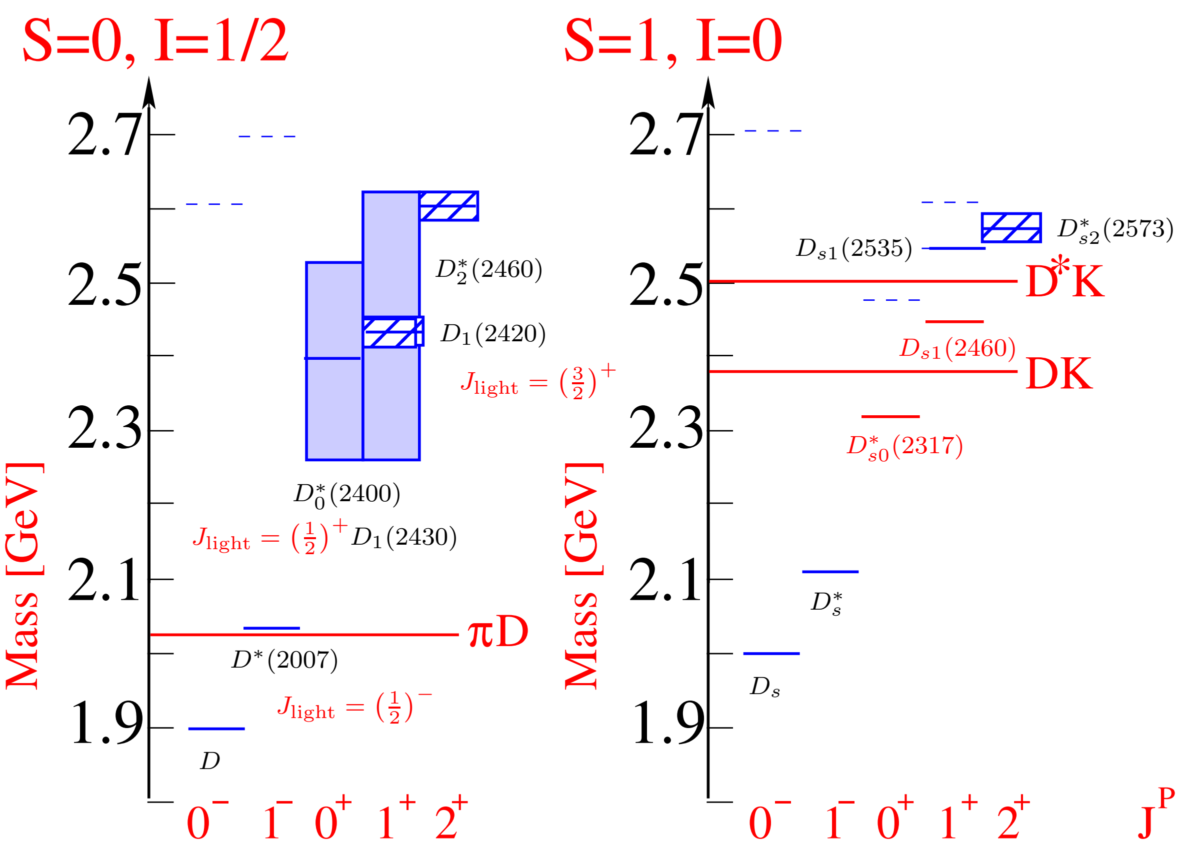
$$\mathcal{L}_{\rm QCD} = \bar{q}_f \left\{ i v \cdot \partial + g v \cdot A^a t^a \right\} q_f + \mathcal{O}(\Lambda_{\rm QCD}/m_f)$$

Independent of Heavy Quark Spin and Flavour

$\rightarrow$ Heavy Quark Effective Field Theory (HQEFT)

$\rightarrow$ Non-Relativistic QCD (NRQCD)

Charmed states



QM: Di Pierro and Eichten, PRD64(2001)114004

Puzzles:

Why are/is

1. $M(D_{s1})\&M(D^*_{s0})$ so light?
2. $M(D_{s1})-M(D^*_{s0}) \simeq M(D^*)-M(D)$?
3. $M(D^*_0) > M(D^*_{s0})$? $M(D_1) \simeq M(D_{s1})$?

Solved by combining unitarized EFTs and Lattice QCD.

Experiment can provide further evidence.

Hadronic Molecules

→ are few-hadron states, bound by the strong force

→ do exist: light nuclei.
e.g. deuteron as pn & hypertriton as Λd bound state

→ are located typically close to relevant continuum threshold;
e.g., for $E_B = m_1 + m_2 - M$ and $\gamma = \sqrt{2\mu E_B}$

- ▷ $E_B^{\rm deuteron} = 2.22$ MeV ($\gamma = 45$ MeV)
- ▷ $E_B^{\rm hypertriton} = (0.13 \pm 0.05)$ MeV (to Λd) ($\gamma = 13$ MeV)

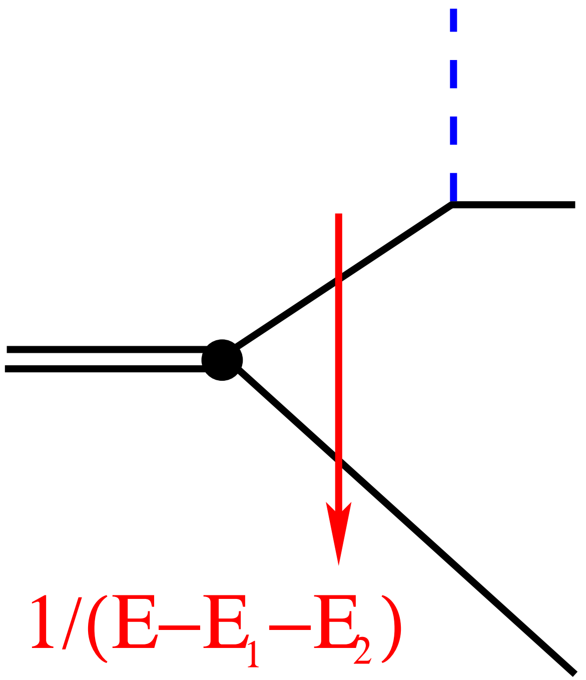
→ can be identified in observables (Weinberg compositeness):

$$\frac{g_{\rm eff}^2}{4\pi} = \frac{4M^2\gamma}{\mu}(1-\lambda^2) \;\rightarrow\; a = -2\left(\frac{1-\lambda^2}{2-\lambda^2}\right)\frac{1}{\gamma}\,; \quad r = -\left(\frac{\lambda^2}{1-\lambda^2}\right)\frac{1}{\gamma}$$

where $(1-\lambda^2)$=probability to find molecular component in bound state wave function

Are there mesonic molecules?

Size of a Molecule



with $E\!=\!M_{\rm Mol.}$ & $E_1\!+\!E_2\!=\!M_1\!+\!M_2\!+\!p^2/(2\mu)$

and $E_B = M_1 + M_2 - M_{\rm Mol.}$

$\longrightarrow p \sim \sqrt{2\mu E_B}$

$\longrightarrow$ size of the molecule, R, reads

$$R \sim 1/p \sim 1/\sqrt{2\mu E_B}$$

c.f. H-atom: $E_B = m_e\alpha^2/2; \mu = m_e \to a_0 = 1/(m_e\alpha)$

On the other hand: confinement radius $\ll$ 1 fm

Molecules extended for $(\hbar c)^2/(2\mu) \gtrsim E_B$

for $\mu \sim$ 0.5 GeV we need $E_B \sim 40$ MeV or smaller

then external probes couple predominantly via the constituents

Non–rel. QM

S. Weinberg PR 130(1963)776, V. Baru et al. PLB586 (2004)53

Expand in terms of non–interacting quark and meson states

$$|\Psi\rangle = \begin{pmatrix} \xi|\psi_0\rangle \\ \chi(\mathbf{p})|h_1h_2\rangle \end{pmatrix},$$

here $|\psi_0\rangle$ = elementary state and $|h_1h_2\rangle$ = two–hadron cont., then ξ^2 equals probability to find the bare state in the physical state

$\rightarrow \ \xi^2 = 1-\lambda^2$ is the quantity of interest!

The Schrödinger equation reads

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle, \ \ \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}^0_{hh} \end{pmatrix} \longrightarrow \ \chi(p) = \xi \, \frac{f(p^2)}{E-p^2/(2\mu)}$$

introducing the transition form factor $\langle\psi_0|\hat{V}|hh\rangle = f(p^2)$,

Note: $\hat{H}^0_{hh}$ contains only meson kinetic terms!

Effective Coupling

Therefore

$$|\Psi\rangle = \xi \begin{pmatrix} |\psi_0\rangle \\ -\frac{f(p^2)}{E_B+p^2/(2\mu)}|h_1 h_2\rangle \end{pmatrix} ,$$

For the normalization of the physical state we get

$$1 = \langle\Psi|\Psi\rangle = \xi^2 \left(1 + \int \frac{f^2(p^2)}{(E_B + p^2/(2\mu))^2} d^3p\right) \ .$$

This shows that $\xi^2 = Z$. Using

$$\int \frac{f^2(p^2) d^3p}{(p^2/(2\mu) + E_B)^2} = \frac{4\pi^2\mu^2 f(0)^2}{\sqrt{2\mu E_B}} + \mathcal{O}\left(\sqrt{E_B\mu}R\right)$$

for s–waves with $R \sim 1/\beta$= range of forces. Using $8\pi^2\mu f(0)^2 = g$

$$1 = \xi^2 \left(1 + \frac{\mu g/2}{\sqrt{2\mu E_B}} + \mathcal{O}\left(\frac{\sqrt{E_B\mu}}{\beta}\right)\right)$$

Thus...

using for residue $g_{\rm eff}^2/4\pi = Z2(m_1+m_2)^2 g$

$$\frac{g_{\rm eff}^2}{4\pi} = 4(m_1+m_2)^2(1-\xi^2)\sqrt{2E_B/\mu} \le 4(m_1+m_2)^2\sqrt{2E_B/\mu}$$

$(1-\xi^2)$ = λ^2 = molecular component in physical state

Note: leading term non–analytic in E_B with clear interpretation

The structure information is hidden in the effective coupling, extracted from experiment, independent of the phenomenology used to introduce the pole(s)

Picture not changed by far away threshold V. Baru et al. PLB586 (2004)53

Equivalent to, e.g., D. Morgan NPA543 (1992)632, N. Törnqvist PRD51 (1995)5312

Disclaimers

→ The formalism presented is 'diagnostic' — especially,

- ▷ it does not allow for conclusions on the binding force
- ▷ it allows one only to study individual states.

To go beyond that a dynamical model needs to be employed.

→ Quantitative interpretation gets lost when states get bound too deeply; we propose to stick to

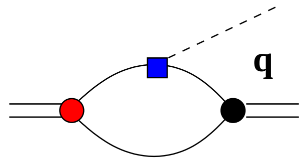
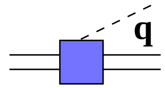
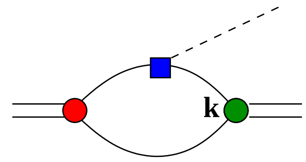
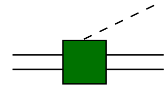
'the larger the coupling the more molecular the state'

There are striking phenomenological implications from large couplings for they lead to

- ▷ relations between seemingly unrelated reactions
- ▷ rather specific, unusual line shapes.

Formal considerations

Transition from a pos. parity state to a light neg. parity state and

transition	a pos. parity $\bar{Q}q$		a neg. parity $\bar{Q}q$	
	 convergent		 divergent	
compact state	N^2LO	LO	N^2LO	LO
molecule	LO	NLO	LO	LO

Only those transitions are sensitive to the molecular nature that are dominated by the loops!

M. Cleven et al., PRD87 (2013)074006

Unitraized ChPT

Kolomeitsev/ Lutz PLBB582(2004)39; Guo et al. PLB641(2006)278
Gamermann et al. PRD76(2007)074016; Guo et al. PLB666(2008)251

$$\mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - m_D^2 D D^\dagger$$

with $D = (D^0, D^+, D_s^+)$ denoting the D–mesons, and

$$\mathcal{D}_\mu = \partial_\mu + \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right) \text{ where } u = \exp\left(\frac{\sqrt{2} i \phi}{2 F_\pi}\right)$$

The Goldstone boson fields are collected in the matrix

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} .$$

LO potential parameter free; 1 regulator necessary

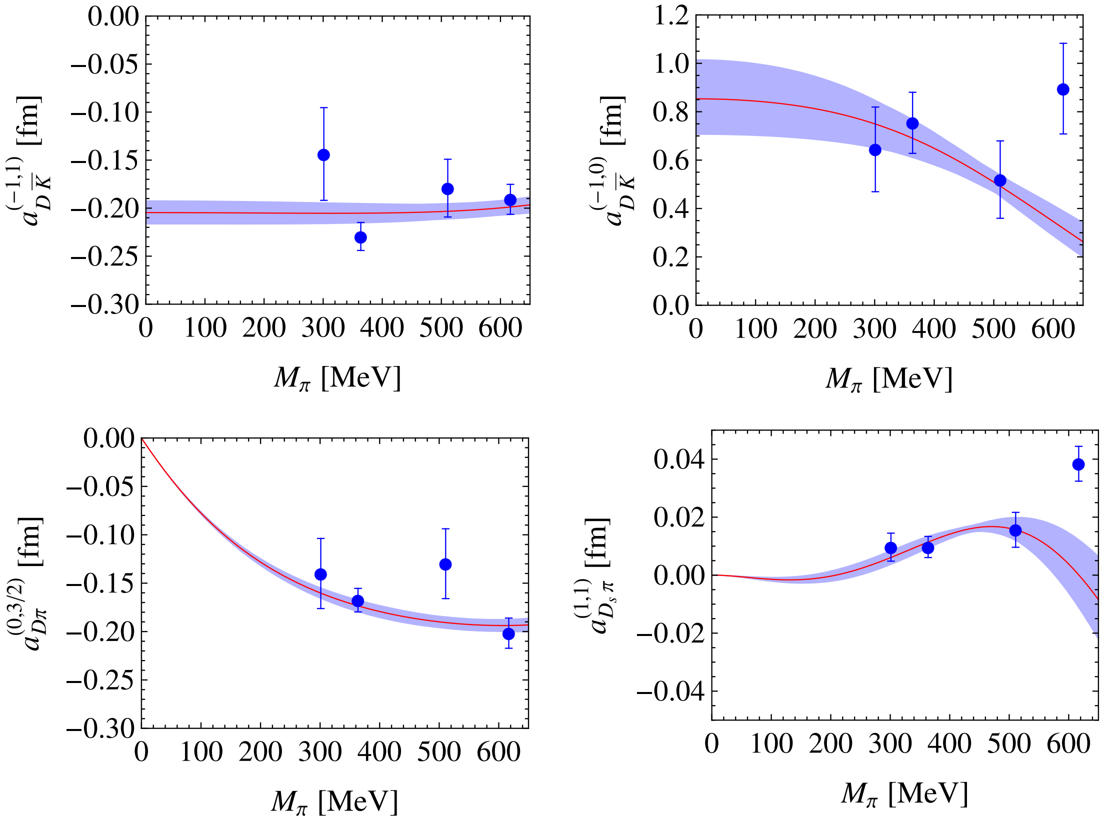
At NLO 6 low energy constants enter

Heavy light Systems

→ $\pi/K/\eta$–D/D_s scattering in ChPT to NLO unitarized

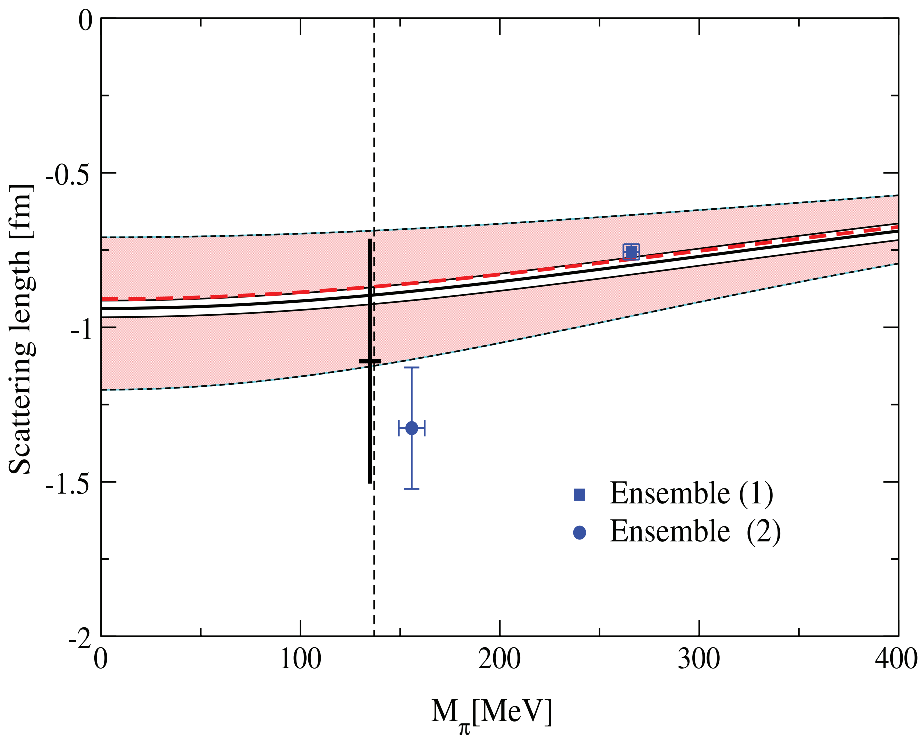
→ controlled quark mass dependence

→ fit LECs to lattice data   Liu et al. PRD87(2013)014508



→ $D^*_{s0}(2317)$ emerges as a pole with $M_{D^*_{s0}} = 2315^{+18}_{-28}$ MeV.

Interpretation

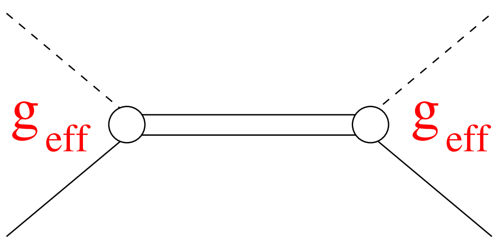


shaded band (dashed line):
full result (best fit)

white band (solid line):
$D^*_{s0}(2317)$ mass fixed to physical value

Liu et al. PRD87(2013)014508

Lattice: Mohler et al., PRL 11(2013)222001

$D^*_{s0}(2317)$: $a =$ $g_{\rm eff}$  $g_{\rm eff}$ $+\mathcal{O}(1/\beta) \simeq \left(\frac{2\lambda^2}{1+\lambda^2}\right)\frac{-1}{\sqrt{2m_K E_B}}$

$a = -(1.05\pm 0.36)$ fm for molecule $(\lambda^2 = 1)$; smaller otherwise

Exp. Test: Hadronic decays

Faessler et al. PRD76(2007)014005; Lutz, Soyeur NPA813(2008)14; Guo et al., PLB666 (2008)251

Isospin breaking (drives decay) via quark masses and charges

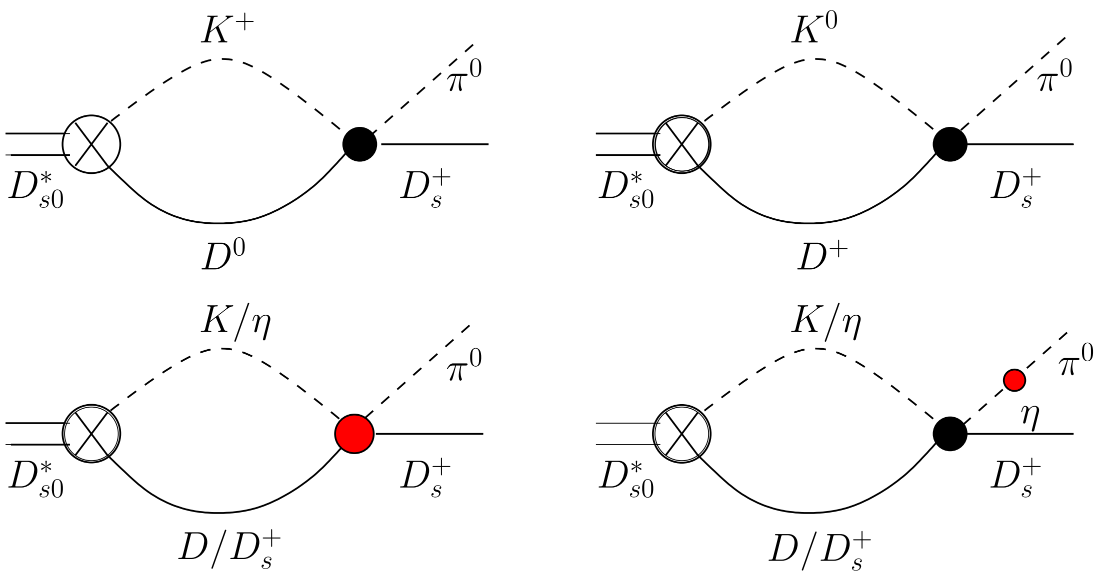
The same effective operators lead to

→ mass differences, e.g.

- ▷ $m_{D^+} - m_{D^0} = \Delta m^{\rm q} + \Delta m^{\rm e.m.} = ((2.5 \pm 0.2) + (2.3 \pm 0.6))$ MeV
- ▷ $\pi^0 - \eta$ mixing ⟶ parameters fixed

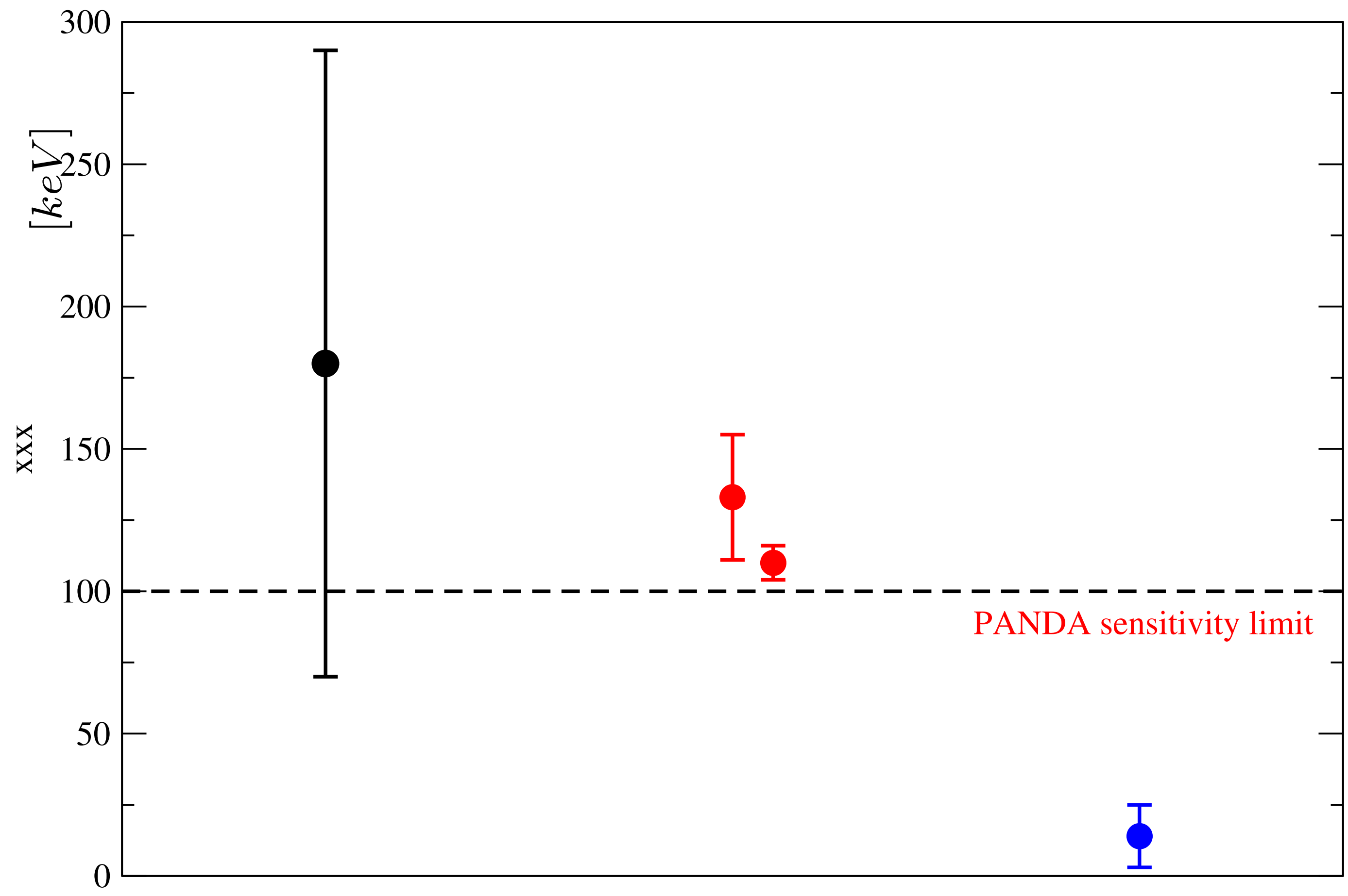
→ Isospin breaking scattering amplitude

- ▷ e.g. $KD \to \pi^0 D_s$ predicted



Specific for molecules!

Hadronic width



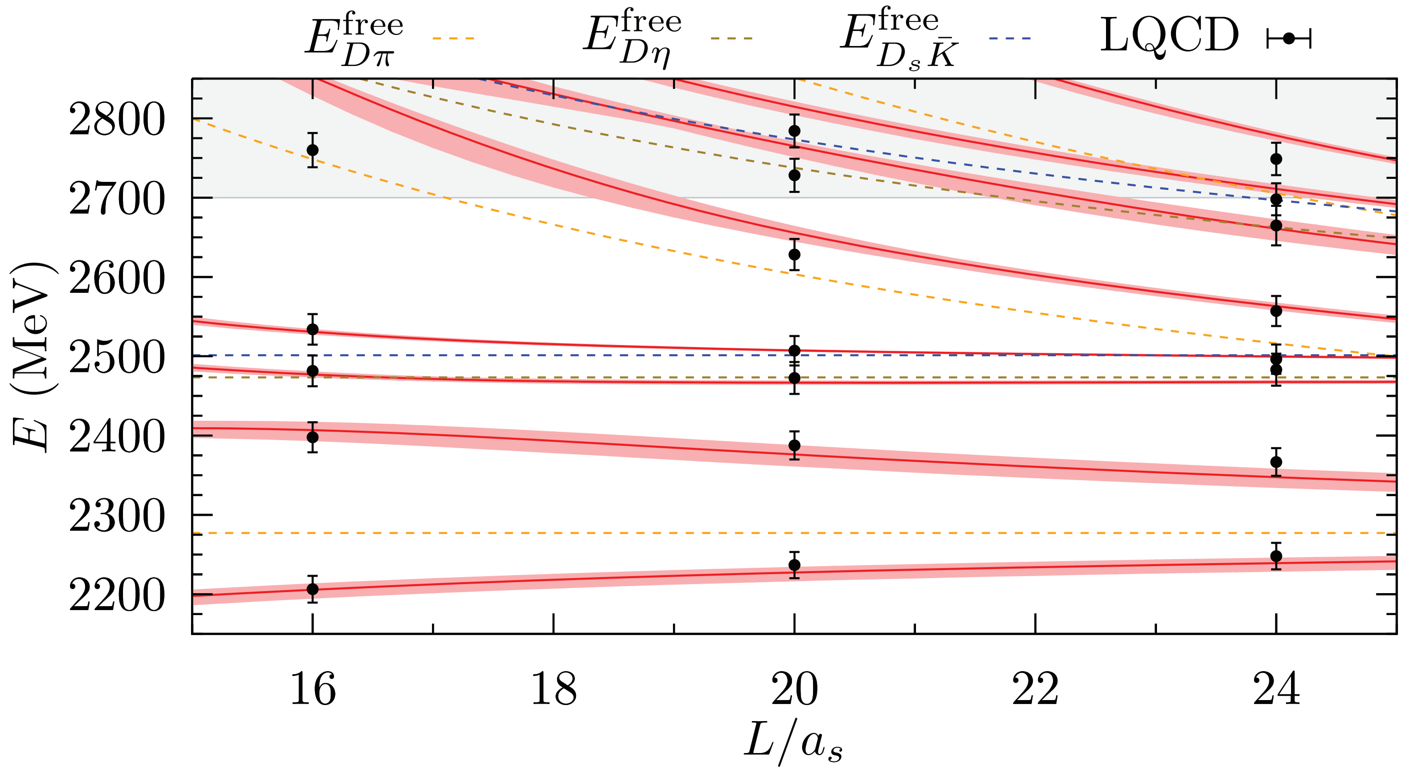
F.K. Guo et al., PLB666(2008)251; L. Liu et al. PRD87(2013)014508;
X.Y. Guo et al., PRD98(2018)014510 and, e.g., P. Colangelo and F. De Fazio, PLB570(2003)180

Measurement of width is decisive, if D^*_{s0} is molecular or not

Experiment needs very high resolution $\to$ PANDA

... and in the S = 0 sector

Keeping parameters fixed one gets:



Albaladejo et al., PLB767(2017)465; Lattice: Moir et al. [Had.Spec.Coll.] JHEP1610(2016)011

Poles for

→ $m_\pi \simeq 391$ MeV: $(2264, \quad 0)$ MeV [000] & $(2468, 113)$ MeV [110]

→ $m_\pi = 139$ MeV: $(2105, 102)$ MeV [100] & $(2451, 134)$ MeV [110]

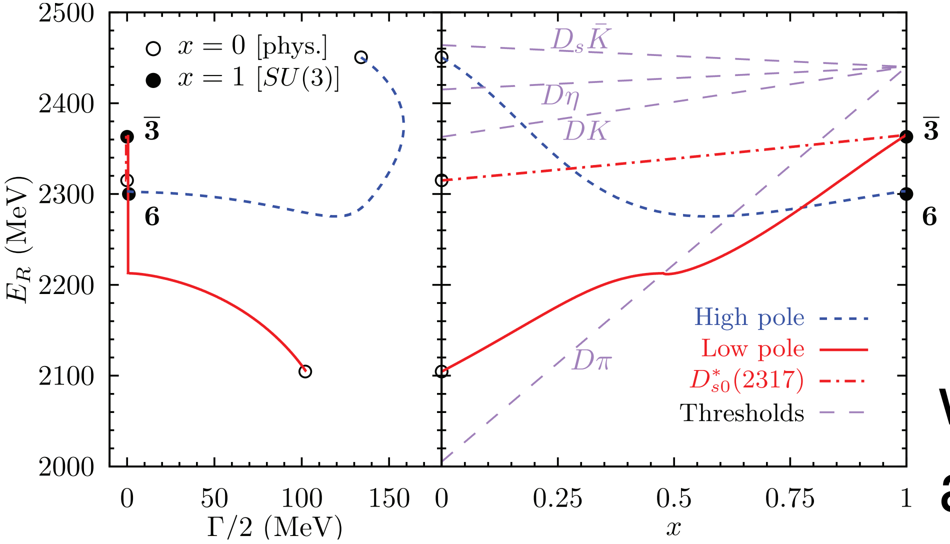
Questions $c\bar{q}$ nature of lowest lying 0^+ D state, $D_0^*(2400)$

SU(3) structure

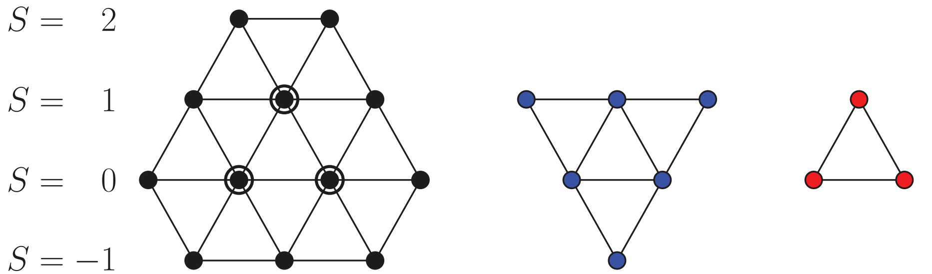
Albaladejo et al., PLB767(2017)465

$$m(x) = m^{\rm phy} + x(m - m^{\rm phy})$$

$m_\phi = 0.49$ GeV; $M_D = 1.95$ GeV



Multiplets: $[\bar{3}] \otimes [8] = [\overline{15}] \oplus [6] \oplus [\bar{3}]$



with $[\overline{15}]$ repulsive
and $[\bar{3}]$ most attractive

→ 3 poles give observable effect with SU(3)-breaking on

→ At $SU(3)$ symmetric point $m_\phi \simeq 490$ MeV: 3 bound and 6 virtual states

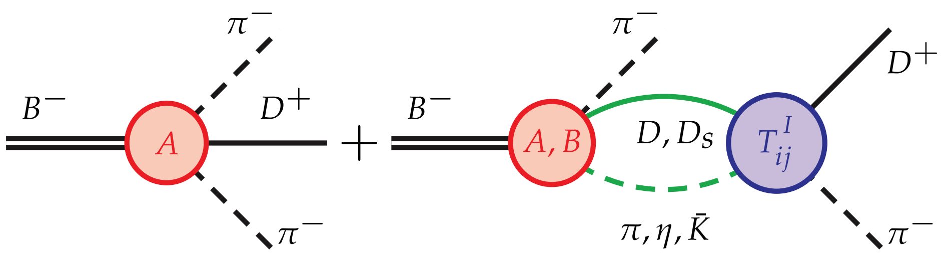
→ For $m_\phi \simeq 600$ MeV (SU(3) sym.): even [6]-states get bound

→ Quark Model: $[\bar{3}] \otimes [1] = [\bar{3}]$ — the $[6]$ is absent

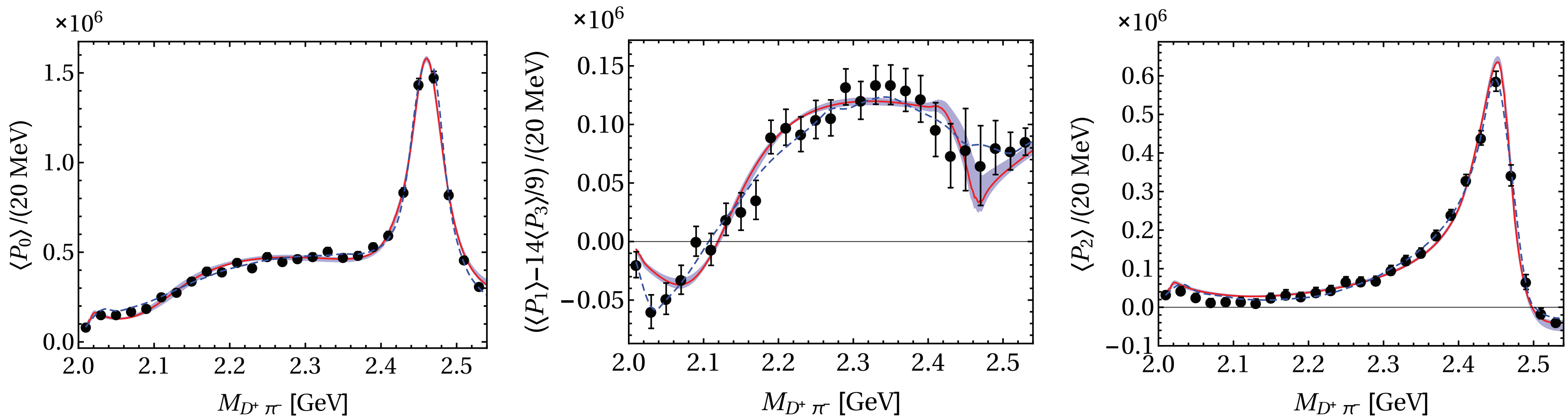
Observable: $B^- \to D^+\pi^-\pi^-$

With the ϕD amplitude fixed we can calc. production reactions:

Du et al., PRD98(2018)094018



for the S-wave (two free para.);
other partial waves from BW-fit

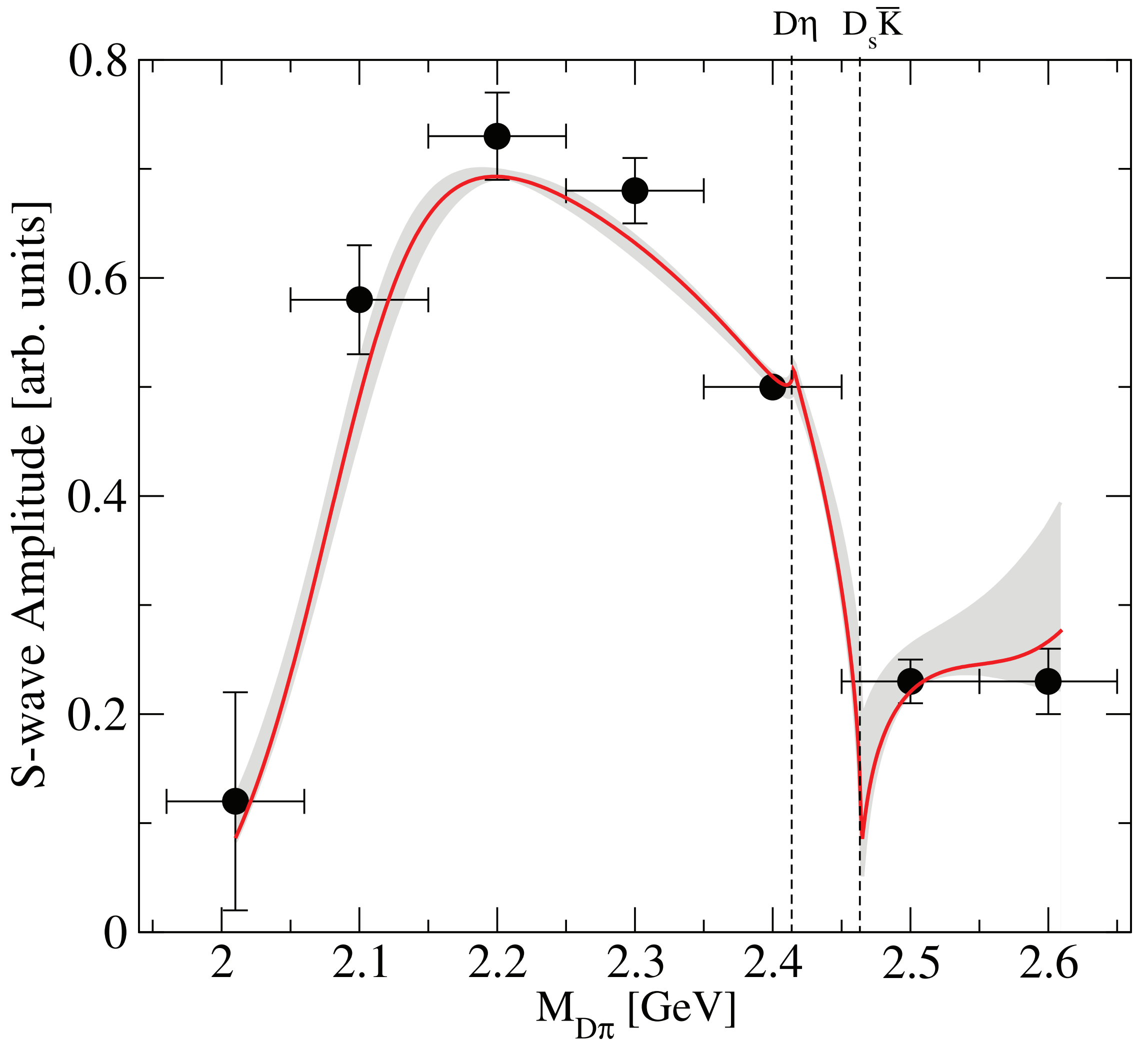


LHCb, PRD94(2016)072001

$$\langle P_0\rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2\,,\ \langle P_2\rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2|\cos(\delta_2 - \delta_0)$$

$$\langle P_{13}\rangle \equiv \langle P_1\rangle - \frac{14}{9}\langle P_3\rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1|\cos(\delta_1 - \delta_0)$$

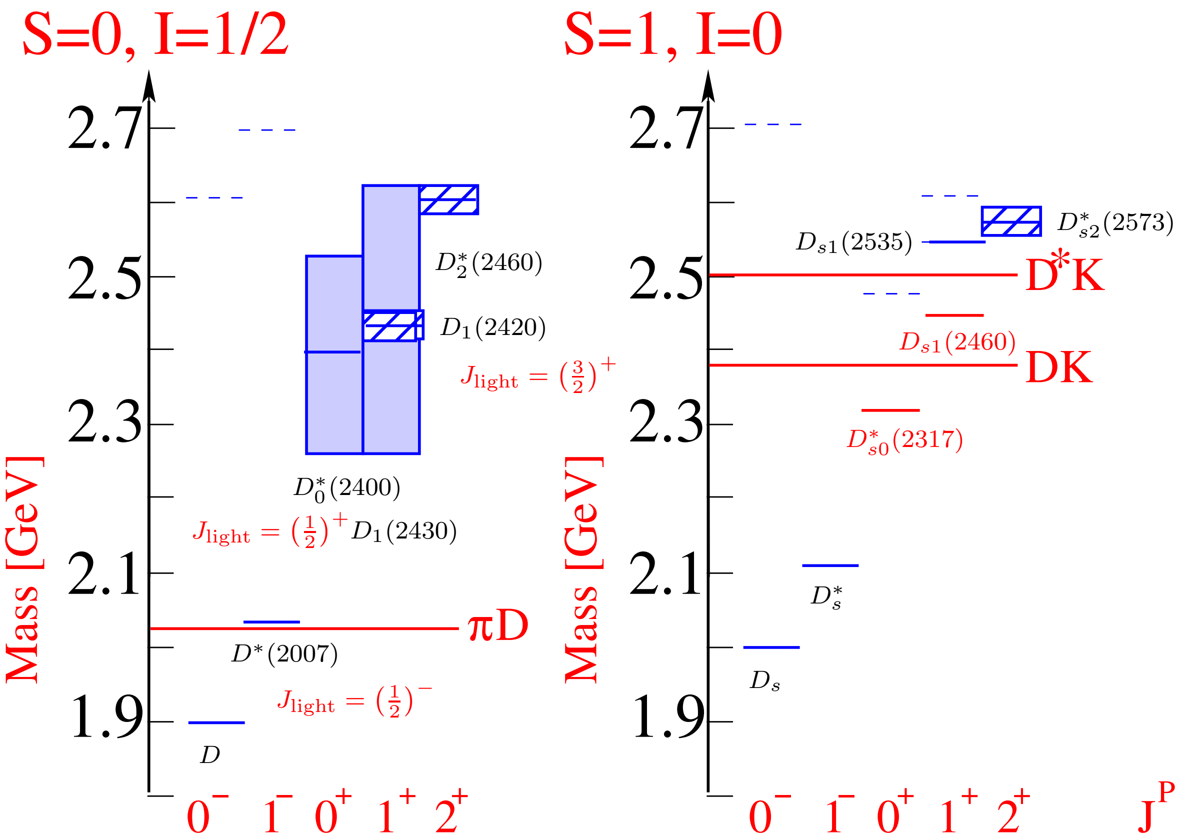
$D\pi$ **S-wave from** $B^- \to D^+\pi^-\pi^-$



Effect of thresholds enhanced by pole at $\sqrt{s_p} \sim (2451 - i134)$ MeV

on nearby unphysical sheet

Charmed states



QM: Di Pierro and Eichten, PRD64(2001)114004

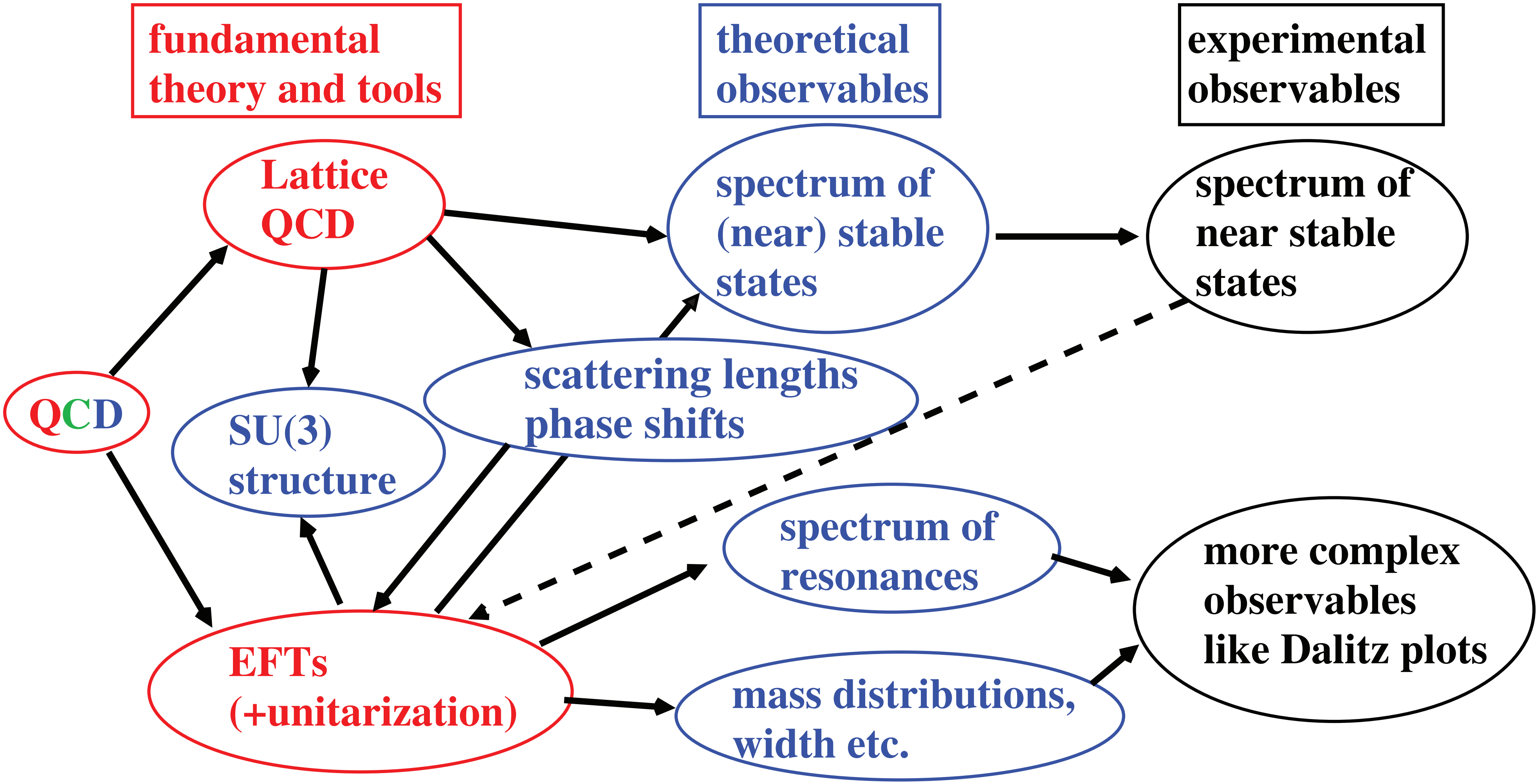
... further support from experiment eagerly waited for

Puzzles solved:

1. $M(D_{s1})\&M(D_{s0}^*)$ are DK and D^*K bound states
2. $M(D_{s1})-M(D_{s0}^*)$ $\simeq M(D^*)-M(D)$, since spin symmetry gives equal binding
3. Proper mass differences
   $M(D_0^*) = 2100$ MeV
   $M(D_{s0}^*) = 2317$ MeV

   $M(D_1) = 2247$ MeV
   $M(D_{s1}) = 2460$ MeV

Roadmap for future studies



Roadmap for future studies