

Standard Model

Effective Field Theory

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Introduction

The Standard Model does not describe *all* physics up to *infinitely high* energies (or down to *infinitely small* distances).

At least, quantum gravity becomes important at the Planck scale.

SM does not explain dark matter and baryon asymmetry of the Universe.

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At least, quantum gravity becomes important at the Planck scale.

SM does not explain dark matter and baryon asymmetry of the Universe.

What can appear at some very short distance scale?

- ▶ Supersymmetry
- ▶ Compositeness
- ▶ Extra dimensions
- ▶ Strings / supersyrings / branes
- ▶ Something we cannot imagine at the moment

Introduction

We can construct scenarios for new physics searches based on some known variants of the next theory (more fundamental than SM).

I. e. we can investigate some finite number of directions of departure from SM, but an infinite number of directions which we cannot now imagine remain unexplored — this is a set of measure 0.

What we need is a systematic model-independent approach for searching of some *absolutely unknown* new physics at small distances. It produces new local interactions of the SM fields.

SMEFT

$$L = L_{\text{SM}}^{(4)} + \frac{1}{\Lambda} \sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} Q_i^{(6)} + \dots$$

SM works well at the energy scale being investigated now
 \Rightarrow the new physics scale $\Lambda \gg$ the SM energy scale.

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A theory at scales $\geq \Lambda$

- ▶ $SU(3) \times SU(2) \times U(1)$ is a subgroup of its gauge group
- ▶ Contains all SM degrees of freedom (either as elementary or as composite particles)
- ▶ Reduces to SM at scales $Q \ll \Lambda$ (unless couplings of some fields to SM ones becomes very small)

Standard Model

Matter fields

	$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	$q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$
Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

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Indices:

- ▶ Weak isospin I : fundamental i, j ; adjoint I, J
- ▶ Color: fundamental α, β ; adjoint A, B
- ▶ Generation: not written

$$D_\mu = \partial_\mu + ig t^A G_\mu^A + ig_2 t^I W_\mu^I + ig_1 Y B_\mu$$

$$\varphi_i^+ = \varphi^{i*} \quad \tilde{\varphi}^i = \varepsilon^{ij} \varphi_j^+ \quad \varphi^+ \overleftarrow{D}_\mu \varphi = (D_\mu \varphi)^+ \varphi$$

$$\varphi^+ \overleftrightarrow{D}_\mu \varphi = \varphi^+ (D_\mu - \overleftarrow{D}_\mu) \varphi \quad \varphi^+ \overleftrightarrow{D}_\mu^I \varphi = \varphi^+ (t^I D_\mu - \overleftarrow{D}_\mu t^I) \varphi$$

Standard Model

$$\begin{aligned} L_{\text{SM}}^{(4)} = & -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} \\ & + (D_\mu\varphi)^\dagger (D^\mu\varphi) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2 \\ & + i(\bar{l}\not{D}l + \bar{e}\not{D}e + \bar{u}\not{D}u + \bar{d}\not{D}d) \\ & - (\bar{l}y_e e\varphi + \bar{q}y_u u\tilde{\varphi} + \bar{q}y_d d\varphi + \text{h.c.}) \end{aligned}$$

All but 1 operators have dimension 4.

Standard Model

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All but 1 operators have dimension 4.

We can add ν_R and y_ν — neutrino (dirac) masses and mixings.

Standard Model

Full derivatives of gauge-invariant operators are irrelevant.
Gauge-invariant operators = full derivatives of
gauge-variant ones

$$\tilde{G}_{\mu\nu}^A G^{A\mu\nu} = 4\varepsilon^{\mu\nu\rho\sigma} (G_\nu^A \partial_\rho G_\sigma^A - \frac{1}{3} g f^{ABC} G_\nu^A G_\rho^B G_\sigma^C)$$

and $\tilde{W}_{\mu\nu}^I W^{I\mu\nu}$ don't affect Feynman rules and EOMs, but
lead to topological nonperturbative effects.

Dimension 5

Only 1 operator (Weinberg)

$$Q_{\nu\nu}^{(5)} = (\tilde{\varphi}^+ l)^T C (\tilde{\varphi}^+ l) = (\varepsilon_{ij} \varphi^i l^j)^T C (\varepsilon_{mn} \varphi^m l^n)$$

T — transposition of the Dirac bispinor, C — the charge conjugation matrix, 2 generation indices. After spontaneous symmetry breaking gives newtrino masses and mixings.

Dimension 6

B. Grzadkowski, M. Iskrzyński, M. Misiak and J. Rosiek (2010): Warsaw basis (M. Iskrzyński M.Sc. thesis) > 900 citations

Matter fields, $X_{\mu\nu} \in \{G_{\mu\nu}, W_{\mu\nu}, B_{\mu\nu}\}$, D_μ

EOM: 0 contribution to S -matrix elements (eliminated by field redefinitions). EOMs at the leading order in $1/\Lambda$

$$D^2\varphi^i = m^2\varphi^i - \lambda(\varphi^+\varphi)\varphi^i - \bar{e}y_e^+l^i + \varepsilon^{ij}\bar{q}_j y_u u - \bar{d}y_d^+q^i$$

$$D^\nu G_{\nu\mu}^A = g(\bar{q}\gamma_\mu t^A q + \bar{u}\gamma_\mu t^A u + \bar{d}\gamma_\mu t^A d)$$

$$D^\nu W_{\nu\mu}^I = g_2(\varphi^+ i\overleftrightarrow{D}_\mu^I \varphi + \bar{l}\gamma_\mu t^I l + \bar{q}\gamma_\mu t^I q)$$

$$D^\nu B_{\nu\mu} = g_1(Y_\varphi\varphi^+ i\overleftrightarrow{D}_\mu \varphi + \sum Y_\psi \bar{\psi}\gamma_\mu \psi)$$

Bosonic

- ▶ $SU(2)$: even number of φ
- ▶ Lorentz: even number of D

$$\begin{array}{ccccc} X^3 & & & & \\ X^2\varphi^2 & & X^2D^2 & & \\ X\varphi^4 & & XD^4 & & X\varphi^2D^2 \\ \varphi^6 & & \varphi^4D^2 & & \varphi^2D^4 \end{array}$$

With 1 \tilde{X} — CP odd

$X\varphi^4$ — no scalar operators

XD^4 — always $[D_\mu, D_\nu] \rightarrow X_{\mu\nu}$

$\varphi^2 D^4$

- ▶ all D 's act on one φ
- ▶ $\varepsilon_{\mu\nu\rho\sigma}$ leads to $[D_\mu, D_\nu]$
- ▶ The ordering of D_μ is irrelevant $\Rightarrow D^2\varphi$ — EOM \rightarrow lower classes

$\varphi^2 D^4$

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- ▶ The ordering of D_μ is irrelevant $\Rightarrow D^2\varphi$ — EOM \rightarrow lower classes

$X\varphi^2 D^2$

- ▶ X or \tilde{X} , no ε
- ▶ each D acts on its own $\varphi \rightarrow$ by parts
- ▶ both D 's act on a single field: $[D_\mu, D_\nu]$
- ▶ one D acts on X and the other one on φ : EOM for X

$X^2 D^2$

- ▶ both D 's act on a single X
- ▶ X or \tilde{X} , no ε
- ▶ both D 's are contracted with a single X : $[D_\mu, D_\nu]$
- ▶ one D is contracted with one X , the other D — with the other X : reorder D 's, EOM $D_\mu X^{\mu\nu}$
- ▶ the D 's are contracted with each other: $D^2 X$ EOM

X^3

- ▶ X, Y, Z ; 1 may be dual; $X^\lambda{}_\mu Y^\mu{}_\nu Z^\nu{}_\lambda$
- ▶ $g^{\alpha\beta} X_{\alpha\mu} X_{\beta\nu} Z^{\mu\nu} = 0$, $g^{\alpha\beta} X_{\alpha\mu} \tilde{X}_{\beta\nu} = \frac{1}{4} g_{\mu\nu} X_{\alpha\beta} \tilde{X}^{\alpha\beta}$
- ▶ 3 different tensors — ε^{ABC} or ε^{IJK}

$$Q_G = f^{ABC} G^{A\lambda}{}_\mu G^{B\mu}{}_\nu G^{C\nu}{}_\lambda \quad Q_{\tilde{G}} = f^{ABC} \tilde{G}^{A\lambda}{}_\mu G^{B\mu}{}_\nu G^{C\nu}{}_\lambda$$
$$Q_W = f^{IJK} W^{I\lambda}{}_\mu W^{J\mu}{}_\nu W^{K\nu}{}_\lambda \quad Q_{\tilde{W}} = f^{IJK} \tilde{W}^{I\lambda}{}_\mu W^{J\mu}{}_\nu W^{K\nu}{}_\lambda$$

$$X^2 \varphi^2$$

Singlet or triplet: $\varphi^+ \varphi$ or $\varphi^+ t^I \varphi$. $\varphi^+ t^I \tilde{\varphi}$ has a non-zero Y .

$$Q_{\varphi G} = \varphi^+ \varphi G_{\mu\nu}^A G^{A\mu\nu}$$

$$Q_{\varphi \tilde{G}} = \varphi^+ \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$$

$$Q_{\varphi W} = \varphi^+ \varphi W_{\mu\nu}^I W^{I\mu\nu}$$

$$Q_{\varphi \tilde{W}} = \varphi^+ \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$$

$$Q_{\varphi B} = \varphi^+ \varphi B_{\mu\nu} B^{\mu\nu}$$

$$Q_{\varphi \tilde{B}} = \varphi^+ \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$$

$$Q_{\varphi WB} = \varphi^+ t^I \varphi W_{\mu\nu}^I B^{\mu\nu}$$

$$Q_{\varphi \tilde{W}B} = \varphi^+ t^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$$

φ^6

- ▶ $Y = 0 \Rightarrow (\varphi^+)^3 \varphi^3$. Signlets and triplets.
- ▶ 0 triplets
- ▶ 2 triplets

$$\begin{array}{c}
 \begin{array}{c} \rightarrow \bullet \rightarrow \\ \leftarrow \bullet \leftarrow \end{array} \text{---} \text{wavy line} \text{---} \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \\
 = T_F \left[\begin{array}{c} \square \downarrow \\ \square \uparrow \end{array} - \frac{1}{N} \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \right] \\
 t^I{}^i{}_j t^I{}^k{}_l = \frac{1}{2} [\delta_l^i \delta_j^k - \frac{1}{2} \delta_j^i \delta_l^k] \\
 (\varphi^+ t^I \varphi)^2 = \frac{1}{4} (\varphi^+ \varphi)^2
 \end{array}$$

- ▶ 3 triplets

$$\varepsilon^{IJK} (\varphi^+ t^I \varphi) (\varphi^+ t^J \varphi) (\varphi^+ t^K \varphi) = 0$$

$$Q_\varphi = (\varphi^+ \varphi)^3$$

$\varphi^4 D^2$

- ▶ $Y = 0 \Rightarrow (\varphi^+)^2 \varphi^2$; 2 D 's are contracted
- ▶ both D 's act on a single $\varphi \Rightarrow$ EOM
- ▶ both D 's act on both φ 's or both φ^+ 's \Rightarrow by parts
- ▶ one D acts on φ , and the other one on φ^+ : 2 isospin structures
 - ▶ (isospin Fierz)

$$\begin{aligned} & (\varphi^+ t^I \varphi) [(D_\mu \varphi)^+ t^I (D^\mu \varphi)] \\ &= \frac{1}{2} (\varphi^+ D_\mu \varphi)^* (\varphi^+ D^\mu \varphi) - \frac{1}{4} (\varphi^+ \varphi) [(D_\mu \varphi)^+ (D^\mu \varphi)] \end{aligned}$$

- ▶ EOM

$$\partial^2 (\varphi^+ \varphi) = 2 (D_\mu \varphi)^+ (D^\mu \varphi) + \varphi^+ D^2 \varphi + (D^2 \varphi)^+ \varphi$$

$$Q_{\varphi\Box} = (\varphi^+ \varphi) \partial^2 (\varphi^+ \varphi) \quad Q_{\varphi D} = (\varphi^+ D_\mu \varphi) (\varphi^+ D^\mu \varphi)$$

2-fermion

Left $\psi \in \{l, e^c, q, u^c, d^c\}$

	2	3
$\bar{\psi}_1 \gamma_\mu \psi_2$	φD	$XD, \varphi^2 D, D^3$
$\psi_1^T C \psi_2$	φ^2, D^2	$\varphi^3, \varphi D^2$
$\psi_1^T C \sigma_{\mu\nu} \psi_2$	X, D^2	$X\varphi, \varphi D^2$

2-fermion

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$\psi_1^T C \sigma_{\mu\nu} \psi_2$	X, D^2	$X\varphi, \varphi D^2$

Dimension 5

- ▶ $\psi_1^T C y \psi_2$: $Y \neq 0$
- ▶ $\bar{\psi}_1 \gamma_\mu \psi_2$: $Y \neq \pm \frac{1}{2}$

$\psi^2 X, \psi^2 D^2, \psi^2 \varphi D$ — empty

$\psi^2 \varphi^2$: $Y_{\varphi^2} = \pm 1$, only l^2 , a single isospin structure \Rightarrow
Weinberg operator

Dimension 6

- ▶ Scalar and tensor currents $\bar{\psi}_1\psi_2$, $\bar{\psi}_1\sigma_{\mu\nu}\psi_2$: the number of φ is odd \Rightarrow the current $I = \frac{1}{2}$
- ▶ $\bar{\psi}_1\gamma_\mu\psi_2$: the number of φ is even \Rightarrow the current $I = 0, 1$. Both left or both right \Rightarrow no currents with C .

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$\psi^2 D^3$

- ▶ $\bar{\psi}_1\gamma_\mu\psi_2$
- ▶ D acting on $\bar{\psi} \Rightarrow$ by parts
- ▶ $[D, D] \rightarrow X$
- ▶ $\bar{\psi}D^2\not{D}\psi \Rightarrow$ EOM

$\psi^2 \varphi D^2$

- ▶ $\bar{\psi}\psi$ or $\bar{\psi}\sigma_{\mu\nu}\psi$
- ▶ D acts on $\bar{\psi} \Rightarrow$ by parts
- ▶ $\bar{\psi}\sigma^{\mu\nu}\psi D_\mu D_\nu\varphi, \varphi\bar{\psi}\sigma^{\mu\nu}D_\mu D_\nu\psi \Rightarrow \psi^2 X\varphi$
- ▶ $\bar{\psi}\psi D^2\varphi \Rightarrow$ EOM
- ▶ $\varphi\bar{\psi}D^2\psi: \not{D}^2 = D^2 - \frac{i}{2}\sigma^{\mu\nu}[D_\mu, D_\nu]$

$$(D_\mu\varphi)\bar{\psi}\sigma^{\mu\nu}D_\nu\psi = \frac{i}{2}(D_\mu\varphi)\bar{\psi}(\gamma^\mu\not{D} - \not{D}\gamma^\mu)\psi$$

$$(D_\mu\varphi)\bar{\psi}D^\mu\psi = \frac{1}{2}(D_\mu\varphi)\bar{\psi}(\gamma^\mu\not{D} + \not{D}\gamma^\mu)\psi$$

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$$(D_\mu\varphi)\bar{\psi}\gamma^\mu\not{D}\psi \quad \text{EOM}$$

$$(D_\mu\varphi)\bar{\psi}\not{D}\gamma^\mu\psi \rightarrow -(D_\mu\varphi)\bar{\psi}\overleftarrow{\not{D}}\gamma^\mu\psi - (D_\nu D_\mu\varphi)\bar{\psi}\gamma^\nu\gamma^\mu\psi$$

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$$(D_\mu D_\nu\varphi)\bar{\psi}\gamma^\mu\gamma^\nu\psi = (D^2\varphi)\bar{\psi}\psi - \frac{i}{2}([D_\mu, D_\nu]\varphi)\bar{\psi}\sigma^{\mu\nu}\psi$$

$\psi^2 X D$

- ▶ X may be dual, no $\varepsilon_{\mu\nu\rho\sigma}$
- ▶ $\bar{\psi}\gamma^\mu\psi \Rightarrow D$ contracted with X
- ▶ D acts on $X \Rightarrow$ EOM
- ▶ D acts on $\bar{\psi} \Rightarrow$ by parts

$$\begin{aligned} X^{\mu\nu} \bar{\psi} \gamma_\mu D_\nu \psi &= \frac{1}{2} X^{\mu\nu} \bar{\psi} (\gamma_\mu \gamma_\nu \not{D} + \gamma_\mu \not{D} \gamma_\nu) \psi \\ &= \frac{1}{2} X^{\mu\nu} \bar{\psi} (\gamma_\mu \gamma_\nu \not{D} - \not{D} \gamma_\mu \gamma_\nu) \psi + X^{\mu\nu} \bar{\psi} \gamma_\nu D_\mu \psi \end{aligned}$$

the last term is our LHS with $-$

$$\begin{aligned} &= \frac{1}{4} X^{\mu\nu} \bar{\psi} (\gamma_\mu \gamma_\nu \not{D} - \not{D} \gamma_\mu \gamma_\nu) \psi \\ &\rightarrow \frac{1}{4} X^{\mu\nu} \bar{\psi} (\gamma_\mu \gamma_\nu \not{D} + \overset{\leftarrow}{\not{D}} \gamma_\mu \gamma_\nu) \psi + \frac{1}{4} (D^\rho X^{\mu\nu}) \bar{\psi} \gamma_\rho \gamma_\mu \gamma_\nu D^\rho \psi \end{aligned}$$

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$$\begin{aligned} &= \frac{1}{4} X^{\mu\nu} \bar{\psi} (\gamma_\mu \gamma_\nu \not{D} - \not{D} \gamma_\mu \gamma_\nu) \psi \\ &\rightarrow \frac{1}{4} X^{\mu\nu} \bar{\psi} (\gamma_\mu \gamma_\nu \not{D} + \overset{\leftarrow}{\not{D}} \gamma_\mu \gamma_\nu) \psi + \frac{1}{4} (D^\rho X^{\mu\nu}) \bar{\psi} \gamma_\rho \gamma_\mu \gamma_\nu D^\rho \psi \\ \gamma_\rho \gamma_\mu \gamma_\nu &= g_{\rho\mu} \gamma_\nu - g_{\rho\nu} \gamma_\mu + g_{\mu\nu} \gamma_\rho - i \varepsilon_{\rho\mu\nu\sigma} \gamma^\sigma \gamma_5 \\ (D^\rho X^{\mu\nu}) \bar{\psi} \gamma_\rho \gamma_\mu \gamma_\nu D^\rho \psi &= 2(D_\nu X^{\mu\nu}) \bar{\psi} \gamma_\mu \psi - 2i(D_\nu \tilde{X}^{\mu\nu}) \bar{\psi} \gamma_\mu \gamma_5 \psi \end{aligned}$$

$$\psi^2 \varphi^3$$

- ▶ $\bar{\psi}_1 \psi_2$ with $I = \frac{1}{2}$ — the same as in Yukawa terms in $L_{\text{SM}}^{(4)}$. The number of φ and $\tilde{\varphi}$ is given by Y .
- ▶ there are 2 ways to combine 3 isospins $\frac{1}{2}$ to $\frac{1}{2}$. One of them is 0: $\varphi^+ \tilde{\varphi} = \varepsilon^{ij} \varphi_i^+ \varphi_j^+ = 0$, $\varepsilon_{ij} \varphi^i \varphi^j = 0$.

Yukawa terms in $L_{\text{SM}}^{(4)}$ times $\varphi^+ \varphi$.

$$Q_{e\varphi} = (\varphi^+ \varphi)(\bar{l}_e \varphi)$$

$$Q_{u\varphi} = (\varphi^+ \varphi)(\bar{q}_u \tilde{\varphi})$$

$$Q_{d\varphi} = (\varphi^+ \varphi)(\bar{q}_d \varphi)$$

$\psi^2 X \varphi$

- ▶ $\bar{\psi}_1 \sigma^{\mu\nu} \psi_2$ with $I = \frac{1}{2}$
- ▶ $Y = 0 \Rightarrow$ fermion–higgs combinations like in $L_{\text{SM}}^{(4)}$
- ▶ $W_{\mu\nu}^I, G_{\mu\nu}^A$ contracted with t^I, t^A — unique way
- ▶ Dual B, W, G — nothing new: $\varepsilon_{\mu\nu\rho\sigma} \sigma^{\rho\sigma} = \gamma_5 \sigma_{\mu\nu}$, γ_5 absorbed into ψ

$$Q_{eW} = (\bar{l} \sigma^{\mu\nu} e) t^I \varphi W_{\mu\nu}^I$$

$$Q_{eB} = (\bar{l} \sigma^{\mu\nu} e) \varphi B_{\mu\nu}$$

$$Q_{uG} = (\bar{q} \sigma^{\mu\nu} t^A u) \tilde{\varphi} G_{\mu\nu}^A$$

$$Q_{uW} = (\bar{q} \sigma^{\mu\nu} u) t^I \tilde{\varphi} W_{\mu\nu}^I$$

$$Q_{uB} = (\bar{q} \sigma^{\mu\nu} u) \tilde{\varphi} B_{\mu\nu}$$

$$Q_{dG} = (\bar{q} \sigma^{\mu\nu} t^A d) \varphi G_{\mu\nu}^A$$

$$Q_{dW} = (\bar{q} \sigma^{\mu\nu} d) t^I \varphi W_{\mu\nu}^I$$

$$Q_{dB} = (\bar{q} \sigma^{\mu\nu} d) \varphi B_{\mu\nu}$$

$\psi^2 \varphi^2 D$

- ▶ $\bar{\psi}_1 \gamma^\mu \psi_2$; D acts on $\psi \Rightarrow$ EOM
- ▶ higgs part: $I = 0, 1$; color-singlet \Rightarrow fermion part too
- ▶ $Y = 0 \Rightarrow$ the number of φ, φ^+
- ▶ $\varphi^+(D_\mu + \overleftarrow{D}_\mu)\varphi, \varphi^+(t^I D_\mu + \overleftarrow{D}_\mu t^I)\varphi$: by parts, EOM.
 $\overleftrightarrow{D}_\mu, \overleftrightarrow{D}_\mu^I$ remain
- ▶ $\tilde{\varphi}^+(D_\mu + \overleftarrow{D}_\mu)\varphi = 0, \tilde{\varphi}^+ \overleftrightarrow{D}_\mu \varphi = 2\tilde{\varphi}^+ D_\mu \varphi$, the Hermitian conjugate operator is independent

$$Q_{\varphi l}^{(1)} = (\varphi^+ i \overleftrightarrow{D}_\mu \varphi) (\bar{l} \gamma^\mu l) \quad Q_{\varphi l}^{(3)} = (\varphi^+ i \overleftrightarrow{D}_\mu^I \varphi) (\bar{l} \gamma^\mu t^I l)$$

$$Q_{\varphi e} = (\varphi^+ i \overleftrightarrow{D}_\mu \varphi) (\bar{e} \gamma^\mu e)$$

$$Q_{\varphi q}^{(1)} = (\varphi^+ i \overleftrightarrow{D}_\mu \varphi) (\bar{q} \gamma^\mu q) \quad Q_{\varphi q}^{(3)} = (\varphi^+ i \overleftrightarrow{D}_\mu^I \varphi) (\bar{q} \gamma^\mu t^I q)$$

$$Q_{\varphi u} = (\varphi^+ i \overleftrightarrow{D}_\mu \varphi) (\bar{u} \gamma^\mu u) \quad Q_{\varphi d} = (\varphi^+ i \overleftrightarrow{D}_\mu \varphi) (\bar{d} \gamma^\mu d)$$

$$Q_{\varphi ud} = (\tilde{\varphi}^+ i \overleftrightarrow{D}_\mu \varphi) (\bar{u} \gamma^\mu d)$$

4-fermion

Left $\psi \in \{l, e^c, q, u^c, d^c\}$. Scalar products of the currents $\bar{\psi}_1 \gamma_\mu \psi_2$, $\psi_1^T C \psi_2$, $\psi_1^T C \sigma_{\mu\nu} \psi_2$ never contain $\psi^3 \bar{\psi}$ or $\psi \bar{\psi}^3$.

4-fermion

Left $\psi \in \{l, e^c, q, u^c, d^c\}$. Scalar products of the currents $\bar{\psi}_1 \gamma_\mu \psi_2$, $\psi_1^T C \psi_2$, $\psi_1^T C \sigma_{\mu\nu} \psi_2$ never contain $\psi^3 \bar{\psi}$ or $\psi \bar{\psi}^3$.

Search for all $Y = 0$ combinations (computer code)

- ▶ reduce to products of $Y = 0$ currents
- ▶

$$\begin{array}{llll} \bar{l} e^c d^c q & qu^c qd^c & le^c qu^c & (B \text{ conserving}) \\ qqql & d^c u^c u^c e^c & qq\bar{u}^c \bar{e}^c & ql\bar{u}^c \bar{d}^c \quad (B \text{ violating}) \end{array}$$

and their Hermitian conjugates

Products of $Y = 0$ currents

- Fierz

$$(\bar{L}_1 \gamma_\mu L_2)(\bar{L}_3 \gamma_\mu L_4) = (\bar{L}_1 \gamma_\mu L_4)(\bar{L}_3 \gamma_\mu L_2)$$

$\Rightarrow Y = 0$ currents

- to usual isospin singlets: $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$, $(\bar{L}L)(\bar{R}R)$
- isospin: singlets, triplets; color: singlets, octets

The diagram shows a four-fermion interaction on the left, where two fermions (top) and two antifermions (bottom) meet at a central vertex connected by a wavy line. This is equated to a Fierz transformation T_F of two terms in square brackets. The first term consists of two fermion lines forming a loop with a downward arrow, and two antifermion lines forming a loop with an upward arrow. The second term is a single four-fermion contact term with a coefficient of $-\frac{1}{N}$.

$$t^I_j t^{Ik}_l = \frac{1}{2} [\delta_l^i \delta_j^k - \frac{1}{2} \delta_j^i \delta_l^k]$$

$$t^{A\alpha}_\beta t^{A\gamma}_\delta = \frac{1}{2} [\delta_\delta^\alpha \delta_\beta^\gamma - \frac{1}{3} \delta_\beta^\alpha \delta_\delta^\gamma]$$

$(\bar{L}L)(\bar{L}L)$

$$(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^{\mu t^I} l_s) = \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s)$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^\mu t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^\mu q_s) \\&+ \frac{1}{2}(\bar{q}_{pi\alpha} \gamma_\mu q_q^{i\beta})(\bar{q}_{rj\beta} \gamma^\mu q_s^{j\alpha})\end{aligned}$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^\mu t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^\mu q_s) \\&+ \frac{1}{2}(\bar{q}_{pi\alpha} \gamma_\mu q_s^{j\alpha})(\bar{q}_{rj\beta} \gamma^\mu q_q^{i\beta})\end{aligned}$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^\mu t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^\mu q_s) \\&+ (\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^\mu t^I q_q) + \frac{1}{4}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q)\end{aligned}$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^{\mu t^I} l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^{\mu t^I} l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^{\mu t^I} l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^{\mu t^A} q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^{\mu t^A} q_s) \\&+ (\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^{\mu t^I} q_q) + \frac{1}{4}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^{\mu t^I} q_q) \\(\bar{q}_p \gamma_\mu t^I t^A q_q)(\bar{q}_r \gamma^{\mu t^I t^A} q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu t^I q_q)(\bar{q}_r \gamma^{\mu t^I} q_s) \\&+ \frac{1}{2}(\bar{q}_{pi\alpha} \gamma_\mu t^{Ii} q_q^{j\beta})(\bar{q}_{rk\beta} \gamma^{\mu t^{Ik}} q_s^{l\alpha})\end{aligned}$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^\mu t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^\mu q_s) \\&+ (\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^\mu t^I q_q) + \frac{1}{4}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q) \\(\bar{q}_p \gamma_\mu t^I t^A q_q)(\bar{q}_r \gamma^\mu t^I t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu t^I q_q)(\bar{q}_r \gamma^\mu t^I q_s) \\&- \frac{1}{8}(\bar{q}_{pi\alpha} \gamma_\mu q_q^{i\beta})(\bar{q}_{rj\beta} \gamma^\mu q_s^{j\alpha}) + \frac{1}{4}(\bar{q}_{pi\alpha} \gamma_\mu q_q^{j\beta})(\bar{q}_{rj\beta} \gamma^\mu q_s^{i\alpha})\end{aligned}$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^\mu t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^\mu q_s) \\&+ (\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^\mu t^I q_q) + \frac{1}{4}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q) \\(\bar{q}_p \gamma_\mu t^I t^A q_q)(\bar{q}_r \gamma^\mu t^I t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu t^I q_q)(\bar{q}_r \gamma^\mu t^I q_s) \\&- \frac{1}{8}(\bar{q}_{pi\alpha} \gamma_\mu q_s^{j\alpha})(\bar{q}_{rj\beta} \gamma^\mu q_q^{i\beta}) + \frac{1}{4}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q)\end{aligned}$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^{\mu t^I} l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^{\mu t^A} q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^\mu q_s) \\&+ (\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^{\mu t^I} q_q) + \frac{1}{4}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q) \\(\bar{q}_p \gamma_\mu t^I t^A q_q)(\bar{q}_r \gamma^{\mu t^I t^A} q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu t^I q_q)(\bar{q}_r \gamma^{\mu t^I} q_s) \\&- \frac{1}{4}(\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^{\mu t^I} q_q) + \frac{3}{16}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q)\end{aligned}$$

$(\bar{L}L)(\bar{L}L)$

$$\begin{aligned}(\bar{l}_p \gamma_\mu t^I l_q)(\bar{l}_r \gamma^\mu t^I l_s) &= \frac{1}{2}(\bar{l}_p \gamma_\mu l_s)(\bar{l}_r \gamma^\mu l_q) - \frac{1}{4}(\bar{l}_p \gamma_\mu l_q)(\bar{l}_r \gamma^\mu l_s) \\(\bar{q}_p \gamma_\mu t^A q_q)(\bar{q}_r \gamma^\mu t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu q_q)(\bar{q}_r \gamma^\mu q_s) \\&+ (\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^\mu t^I q_q) + \frac{1}{4}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q) \\(\bar{q}_p \gamma_\mu t^I t^A q_q)(\bar{q}_r \gamma^\mu t^I t^A q_s) &= -\frac{1}{6}(\bar{q}_p \gamma_\mu t^I q_q)(\bar{q}_r \gamma^\mu t^I q_s) \\&- \frac{1}{4}(\bar{q}_p \gamma_\mu t^I q_s)(\bar{q}_r \gamma^\mu t^I q_q) + \frac{3}{16}(\bar{q}_p \gamma_\mu q_s)(\bar{q}_r \gamma^\mu q_q)\end{aligned}$$

$$Q_u = (\bar{l} \gamma_\mu l)(\bar{l} \gamma^\mu l)$$

$$Q_{qq}^{(1)} = (\bar{q} \gamma_\mu q)(\bar{q} \gamma^\mu q)$$

$$Q_{lq}^{(1)} = (\bar{l} \gamma_\mu l)(\bar{q} \gamma^\mu q)$$

$$Q_{qq}^{(3)} = (\bar{q} \gamma_\mu t^I q)(\bar{q} \gamma^\mu t^I q)$$

$$Q_{lq}^{(3)} = (\bar{l} \gamma_\mu t^I l)(\bar{q} \gamma^\mu t^I q)$$

$$(\overline{RR})(\overline{RR})$$

$$\begin{aligned}(\overline{u}_p \gamma_\mu t^A u_q)(\overline{u}_r \gamma_\mu t^A u_s) &= \frac{1}{2}(\overline{u}_p \gamma_\mu u_s)(\overline{u}_r \gamma_\mu u_q) - \frac{1}{6}(\overline{u}_p \gamma_\mu u_q)(\overline{u}_r \gamma_\mu u_s) \\(\overline{d}_p \gamma_\mu t^A d_q)(\overline{d}_r \gamma_\mu t^A d_s) &= \frac{1}{2}(\overline{d}_p \gamma_\mu d_s)(\overline{d}_r \gamma_\mu d_q) - \frac{1}{6}(\overline{d}_p \gamma_\mu d_q)(\overline{d}_r \gamma_\mu d_s)\end{aligned}$$

$$Q_{ee} = (\overline{e} \gamma_\mu e)(\overline{e} \gamma^\mu e)$$

$$Q_{uu} = (\overline{u} \gamma_\mu u)(\overline{u} \gamma^\mu u)$$

$$Q_{eu} = (\overline{e} \gamma_\mu e)(\overline{u} \gamma^\mu u)$$

$$Q_{ud}^{(1)} = (\overline{u} \gamma_\mu u)(\overline{d} \gamma^\mu d)$$

$$Q_{dd} = (\overline{d} \gamma_\mu d)(\overline{d} \gamma^\mu d)$$

$$Q_{ed} = (\overline{e} \gamma_\mu e)(\overline{d} \gamma^\mu d)$$

$$Q_{ud}^{(8)} = (\overline{u} \gamma_\mu t^A u)(\overline{d} \gamma^\mu t^A d)$$

$(\bar{L}L)(\bar{R}R)$

$$Q_{le} = (\bar{l}\gamma_{\mu}l)(\bar{e}\gamma^{\mu}e)$$

$$Q_{lu} = (\bar{l}\gamma_{\mu}l)(\bar{u}\gamma^{\mu}u)$$

$$Q_{qe} = (\bar{q}\gamma_{\mu}q)(\bar{e}\gamma^{\mu}e)$$

$$Q_{qu}^{(1)} = (\bar{q}\gamma_{\mu}q)(\bar{u}\gamma^{\mu}u)$$

$$Q_{qd}^{(1)} = (\bar{q}\gamma_{\mu}q)(\bar{d}\gamma^{\mu}d)$$

$$Q_{ld} = (\bar{l}\gamma_{\mu}l)(\bar{d}\gamma^{\mu}d)$$

$$Q_{qu}^{(8)} = (\bar{q}\gamma_{\mu}t^A q)(\bar{u}\gamma^{\mu}t^A u)$$

$$Q_{qd}^{(8)} = (\bar{q}\gamma_{\mu}t^A q)(\bar{d}\gamma^{\mu}t^A d)$$

Exceptional $\bar{L}^2 L^2$

- ▶ 1 scalar $(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$
- ▶ $SU(2)$: 2 singlets, 2 doublets \Rightarrow 1 structure
- ▶ $SU(3)$: $\psi^\alpha \bar{\psi}_\beta$ (B preserving) or $\psi^\alpha \psi^\beta \psi^\gamma$ (B violating)
 \Rightarrow 1 structure

Exceptional L^4

- ▶ 2 scalars 1×1 , $\sigma_{\mu\nu} \times \sigma^{\mu\nu}$

$$\begin{aligned} & (\psi_1^T C \sigma_{\mu\nu} \psi_2) (\psi_3^T C \sigma^{\mu\nu} \psi_4) = \\ & 4(\psi_1^T C \psi_2) (\psi_3^T C \psi_4) + 8(\psi_1^T C \psi_4) (\psi_3^T C \psi_2) \end{aligned}$$

(choose the rhs everywhere except $le^c qu^c$ where we want color-singlet currents). $qu^c qd^c$, $qqql$, $d^c u^c u^c e^c$: 2 pairings — just permutations of generation indices.

- ▶ Isospin and color: 2 structures for $qu^c qd^c$, $qqql$; 1 for $d^c u^c u^c e^c$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$

$$Q_{ledq} = (\bar{l}e)(\bar{d}q)$$

$$Q_{quqd}^{(1)} = \varepsilon^{ij}(\bar{q}_i u)(\bar{q}_j d)$$

$$Q_{quqd}^{(8)} = \varepsilon^{ij}(\bar{q}_i t^A u)(\bar{q}_j t^A d)$$

$$Q_{lequ}^{(1)} = \varepsilon^{ij}(\bar{l}_i e)(\bar{q}_j u)$$

$$Q_{lequ}^{(3)} = \varepsilon^{ij}(\bar{l}_i \sigma_{\mu\nu} e)(\bar{q}_j \sigma^{\mu\nu} u)$$

B violating

$$Q_{duq} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ij} [(d^\alpha)^T C u^\beta] [(q^{i\gamma})^T C l^j]$$

$$Q_{qqu} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ij} [(q^{i\alpha})^T C q^{j\beta}] [(u^\gamma)^T C e]$$

$$Q_{qqq}^{(1)} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}\varepsilon_{kl} [(q^{i\alpha})^T C q^{j\beta}] [(q^{k\gamma})^T C l^l]$$

$$Q_{qqq}^{(3)} = \varepsilon_{\alpha\beta\gamma} t_{ij}^I t_{kl}^I [(q^{i\alpha})^T C q^{j\beta}] [(q^{k\gamma})^T C l^l]$$

$$Q_{duu} = \varepsilon_{\alpha\beta\gamma} [(d^\alpha)^T C u^\beta] [(u^\gamma)^T C e]$$

Summary

Not counting generations and Hermitian conjugated operators

$\#\psi$	
0	15
2	19
4	25
	59

Summary

Not counting generations and Hermitian conjugated operators

$\#\psi$	
0	15
2	19
4	25 + 5
	64

Higgs sector

$$L_H = (D_\mu \varphi)^\dagger (D^\mu \varphi) + m^2 (\varphi^\dagger \varphi) - \frac{\lambda}{2} (\varphi^\dagger \varphi)^2 + C_\varphi (\varphi^\dagger \varphi)^3 \\ + C_{\varphi \square} (\varphi^\dagger \varphi) \partial^2 (\varphi^\dagger \varphi) + C_{\varphi D} (\varphi^\dagger D_\mu \varphi)^\dagger (\varphi^\dagger D^\mu \varphi)$$

$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\Phi^0) \end{pmatrix}$$

Higgs sector

$$L_H = (D_\mu \varphi)^\dagger (D^\mu \varphi) + m^2 (\varphi^\dagger \varphi) - \frac{\lambda}{2} (\varphi^\dagger \varphi)^2 + C_\varphi (\varphi^\dagger \varphi)^3 \\ + C_{\varphi\Box} (\varphi^\dagger \varphi) \partial^2 (\varphi^\dagger \varphi) + C_{\varphi D} (\varphi^\dagger D_\mu \varphi)^* (\varphi^\dagger D^\mu \varphi)$$

$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\Phi^0) \end{pmatrix}$$

$$v = m \sqrt{\frac{2}{\lambda}} \left[1 + \frac{3}{2} \frac{m^2}{\lambda^2} c_\varphi \right]$$

Higgs sector

$$L_H = (D_\mu \varphi)^\dagger (D^\mu \varphi) + m^2 (\varphi^\dagger \varphi) - \frac{\lambda}{2} (\varphi^\dagger \varphi)^2 + C_\varphi (\varphi^\dagger \varphi)^3 \\ + C_{\varphi\Box} (\varphi^\dagger \varphi) \partial^2 (\varphi^\dagger \varphi) + C_{\varphi D} (\varphi^\dagger D_\mu \varphi)^\dagger (\varphi^\dagger D^\mu \varphi)$$

$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\Phi^0) \end{pmatrix}$$

$$v = m \sqrt{\frac{2}{\lambda}} \left[1 + \frac{3m^2}{2\lambda^2} C_\varphi \right]$$

$$L_H^{(2)} = \frac{1}{2} \left[1 + 2v^2 \left(\frac{1}{4} C_{\varphi D} - C_{\varphi\Box} \right) \right] (\partial_\mu H) (\partial^\mu H)$$

$$+ \left(\frac{m^2}{2} - \frac{3}{4} \lambda v^2 + \frac{15}{8} C_\varphi v^4 \right) H^2$$

$$+ \frac{1}{2} \left(1 + \frac{1}{2} C_{\varphi D} v^2 \right) (\partial_\mu \Phi^0) (\partial^\mu \Phi^0) + (\partial_\mu \Phi^-) (\partial^\mu \Phi^+)$$

Higgs sector

$$h = Z_h H \quad G^0 = Z_{G^0} \Phi^0 \quad G^\pm = \Phi^\pm$$

$$Z_h = 1 + \left(\frac{1}{4} C_{\varphi D} - C_{\varphi \square} \right) v^2$$

$$Z_{G^0} = 1 + \frac{1}{4} C_{\varphi D} v^2 \Phi^0$$

Higgs sector

$$h = Z_h H \quad G^0 = Z_{G^0} \Phi^0 \quad G^\pm = \Phi^\pm$$

$$Z_h = 1 + \left(\frac{1}{4} C_{\varphi D} - C_{\varphi \square} \right) v^2$$

$$Z_{G^0} = 1 + \frac{1}{4} C_{\varphi D} v^2 \Phi^0$$

$$L_h^{(2)} = \frac{1}{2} (\partial_\mu h) (\partial^\mu h) - \frac{M_h^2}{2} h^2$$

$$M_h^2 = 2m^2 \left[1 - \frac{m^2}{\lambda^2} (3C_\varphi + \lambda(C_{\varphi D} - 4C_{\varphi \square})) \right]$$

Gauge sector: QCD

$$L_{\text{QCD}} = -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} + C_{\varphi G}(\varphi^\dagger \varphi)G_{\mu\nu}^A G^{A\mu\nu}$$

Gauge sector: QCD

$$L_{\text{QCD}} = -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} + C_{\varphi G}(\varphi^\dagger \varphi)G_{\mu\nu}^A G^{A\mu\nu}$$

$$\bar{G}_\mu^A = Z_G G_\mu^A \quad \bar{g} = Z_G^{-1} g$$

$$Z_G = 1 - C_{\varphi G} v^2$$

$$L_{\text{QCD}}^{(2)} = -\frac{1}{4}\bar{G}_{\mu\nu}^A \bar{G}^{A\mu\nu}$$

Gauge sector: electroweak

$$\begin{aligned} L_{\text{EW}} = & -\frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) \\ & + C_{\varphi W}(\varphi^\dagger\varphi)W_{\mu\nu}^I W^{I\mu\nu} + C_{\varphi B}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} \\ & + C_{\varphi WB}(\varphi^\dagger t^I\varphi)W_{\mu\nu}^I B^{\mu\nu} + C_{\varphi D}(\varphi^\dagger D_\mu\varphi)^\dagger(\varphi^\dagger D^\mu\varphi) \end{aligned}$$

Gauge sector: electroweak

$$\begin{aligned}L_{\text{EW}} = & -\frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) \\ & + C_{\varphi W}(\varphi^\dagger\varphi)W_{\mu\nu}^I W^{I\mu\nu} + C_{\varphi B}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} \\ & + C_{\varphi WB}(\varphi^\dagger t^I\varphi)W_{\mu\nu}^I B^{\mu\nu} + C_{\varphi D}(\varphi^\dagger D_\mu\varphi)^\dagger(\varphi^\dagger D^\mu\varphi)\end{aligned}$$

$$\bar{W}_\mu^I = Z_W W_\mu^I \quad \bar{g}_2 = Z_W^{-1} g_2$$

$$\bar{B}_\mu = Z_B B_\mu \quad \bar{g}_1 = Z_B^{-1} g_1$$

$$Z_W = 1 - C_{\varphi W} v^2 \quad Z_B = 1 - C_{\varphi B} v^2$$

Gauge sector: electroweak

$$\begin{aligned}L_{\text{EW}} = & -\frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) \\ & + C_{\varphi W}(\varphi^\dagger\varphi)W_{\mu\nu}^I W^{I\mu\nu} + C_{\varphi B}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} \\ & + C_{\varphi WB}(\varphi^\dagger t^I\varphi)W_{\mu\nu}^I B^{\mu\nu} + C_{\varphi D}(\varphi^\dagger D_\mu\varphi)^\dagger(\varphi^\dagger D^\mu\varphi)\end{aligned}$$

$$\bar{W}_\mu^I = Z_W W_\mu^I \quad \bar{g}_2 = Z_W^{-1} g_2$$

$$\bar{B}_\mu = Z_B B_\mu \quad \bar{g}_1 = Z_B^{-1} g_1$$

$$Z_W = 1 - C_{\varphi W} v^2 \quad Z_B = 1 - C_{\varphi B} v^2$$

$$\begin{aligned}L_{\text{EW}}^{(2)} = & -\frac{1}{4} \sum_{I=1,2} \bar{W}_{\mu\nu}^I \bar{W}^{I\mu\nu} - \frac{1}{4} (\bar{W}_{\mu\nu}^3 \bar{B}_{\mu\nu}) \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} \bar{W}^{3\mu\nu} \\ \bar{B}^{\mu\nu} \end{pmatrix} \\ & + \frac{\bar{g}_2^2 v^2}{8} \sum_{I=1,2} \bar{W}_\mu^I \bar{W}^{I\mu} + \frac{v^2}{8} Z_{G^0}^2 (\bar{W}_\mu^3 \bar{B}_\mu) \begin{pmatrix} \bar{g}_2^2 & -\bar{g}_1 \bar{g}_2 \\ -\bar{g}_1 \bar{g}_2 & \bar{g}_1^2 \end{pmatrix} \begin{pmatrix} \bar{W}^{3\mu} \\ \bar{B}^\mu \end{pmatrix}\end{aligned}$$

$$\epsilon = 2C_{\varphi WB} v^2$$

Gauge sector: electroweak

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (\bar{W}_{\mu}^1 \mp i\bar{W}_{\mu}^2) \quad M_W^2 = \frac{1}{2} \bar{g}_2 v$$

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$$\begin{pmatrix} \bar{W}_{\mu}^3 \\ \bar{B}_{\mu} \end{pmatrix} = X \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & -\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 1 \end{pmatrix} \begin{pmatrix} \cos \bar{\theta} & \sin \bar{\theta} \\ -\sin \bar{\theta} & \cos \bar{\theta} \end{pmatrix}$$

$$\tan \bar{\theta} = \frac{\bar{g}_1}{\bar{g}_2} + \frac{\epsilon}{2} \left(1 - \frac{\bar{g}_1^2}{\bar{g}_2^2} \right)$$

$$M_Z = \frac{1}{2} \sqrt{\bar{g}_1^2 + \bar{g}_2^2} v \left(1 + \frac{\bar{g}_1 \bar{g}_2}{\bar{g}_1^2 + \bar{g}_2^2} \epsilon \right) Z_{G^0}$$

$$M_A = 0$$

Gauge–glodstone mixing

$$\begin{aligned} & (D_\mu\varphi)^+(D^\mu\varphi) + C_{\varphi D}(\varphi^+D_\mu\varphi)(\varphi^+D^\mu\varphi) \Rightarrow \\ & -i\frac{\bar{g}_2v}{2\sqrt{2}}\bar{W}_\mu^1\partial^\mu(\Phi^+ - \Phi^-) + \frac{\bar{g}_2v}{2\sqrt{2}}\bar{W}_\mu^2\partial^\mu(\Phi^+ + \Phi^-) \\ & -\frac{\bar{g}_2v}{2}Z_{G^0}^2\bar{W}_\mu^3\partial^\mu\Phi^0 + \frac{\bar{g}_1v}{2}Z_{G^0}^2\bar{B}_\mu\partial^\mu\Phi^0 \end{aligned}$$

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Fermion sector

$$\begin{aligned} L_f = & i(\bar{l}' \not{D} l' + \bar{e}' \not{D} e' + \bar{q}' \not{D} q' + \bar{u}' \not{D} u' + \bar{d}' \not{D} d') \\ & - (\bar{l}' y_e e' \varphi + \bar{q}' y_u u' \tilde{\varphi} + \bar{q}' y_d d' \varphi + \text{h.c.}) \\ & + (C_{\nu\nu} (\tilde{\varphi}^+ l')^T C (\tilde{\varphi}^+ l') + \text{h.c.}) \\ & + (\varphi^+ \varphi) (\bar{l}' C_{e\varphi} e' \varphi + \bar{q}' C_{u\varphi} u' \tilde{\varphi} + \bar{q}' C_{d\varphi} d' \varphi + \text{h.c.}) \end{aligned}$$

Fermion sector

$$\begin{aligned}L_f &= i(\bar{l}' \not{D} l' + \bar{e}' \not{D} e' + \bar{q}' \not{D} q' + \bar{u}' \not{D} u' + \bar{d}' \not{D} d') \\&\quad - (\bar{l}' y_e e' \varphi + \bar{q}' y_u u' \tilde{\varphi} + \bar{q}' y_d d' \varphi + \text{h.c.}) \\&\quad + (C_{\nu\nu}(\tilde{\varphi}^+ l')^T C(\tilde{\varphi}^+ l') + \text{h.c.}) \\&\quad + (\varphi^+ \varphi)(\bar{l}' C_{e\varphi} e' \varphi + \bar{q}' C_{u\varphi} u' \tilde{\varphi} + \bar{q}' C_{d\varphi} d' \varphi + \text{h.c.}) \\L_m &= -\frac{1}{2} \nu_L^T C M'_\nu - \bar{e}'_L M'_e e'_R - \bar{u}'_L M'_u u'_R - \bar{d}'_L M'_d d'_R + \text{h.c.} \\M'_\nu &= -C_{\nu\nu} v^2 & M'_e &= \frac{v}{\sqrt{2}} \left(y_e - C_{e\varphi} \frac{v^2}{2} \right) \\M'_u &= \frac{v}{\sqrt{2}} \left(y_u - C_{u\varphi} \frac{v^2}{2} \right) & M'_d &= \frac{v}{\sqrt{2}} \left(y_d - C_{d\varphi} \frac{v^2}{2} \right)\end{aligned}$$

Fermion sector

$$\psi'_X = U_{\psi_X} \psi_X \quad \psi = \nu, e, u, d \quad X = L, R$$

$$U_{\nu_L}^T M'_\nu U_{\nu_L} = M_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$$

$$U_{e_L}^+ M'_e U_{e_R} = M_e = \text{diag}(m_e, m_\mu, m_\tau)$$

$$U_{u_L}^+ M'_u U_{u_R} = M_u = \text{diag}(m_u, m_c, m_t)$$

$$U_{d_L}^+ M'_d U_{d_R} = M_d = \text{diag}(m_d, m_s, m_b)$$