Nonleptonic decays of doubly charmed baryons

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Lecture is based on the series of papers performed in collaboration with T. Gutsche, J.G. Körner, V.E. Lyubovitskij and Z. Tyulemissov

Light baryons in SU(3): $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8'} \oplus \mathbf{1}$

- **SU(3) (u,d,s): M. Gell-Mann, G.Zweig, 1964**
- **Octet 8: ground state with $J^P = \frac{1}{2}^+$**

$$B^i_j = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}$$

- **Quark content (notation from Fayyazuddin & Riazuddin)**

$p \to \frac{1}{\sqrt{2}}[u,d]u$ $\quad$ $n \to \frac{1}{\sqrt{2}}[u,d]d$

$\Sigma^0 \to \frac{1}{2}([d,s]u + [u,s]d)$ $\quad$ $\Sigma^+ \to \frac{1}{\sqrt{2}}[u,s]u$ $\quad$ $\Sigma^- \to \frac{1}{\sqrt{2}}[d,s]d$

$\Lambda^0 \to \frac{1}{\sqrt{12}}(2[u,d]s - [d,s]u - [s,u]d)$ $\quad$ $\Xi^- \to \frac{1}{\sqrt{2}}[d,s]s$ $\quad$ $\Xi^0 \to \frac{1}{\sqrt{2}}[s,u]s$

Relativistic *qqq*–currents in SU(3)

- Relativistic *qqq*–state is defined by

$$|qqq> = |color, flavor, spin, space>$$

- More precisely

$$B_j^k \to R_j^{k;\,j_1,j_2,j_3}\; q_{j_1}^{a_1} q_{j_2}^{a_2} q_{j_3}^{a_3} \varepsilon_{a_1,a_2,a_3}$$

  Here $j = (\alpha, m)$; a_i, α_i, m_i are the color, spinor and flavor indices.

- The *qqq*–currents are symmetric under permutation of all quarks in the case of exact $SU_F(3)$-symmetry.
- There exist two independent currents for a baryon octet with quantum numbers $J^P = \frac{1}{2}^+$.
- The original way to construct the relativistic symmetric three-quark currents was suggested in the paper by

  G.V. Efimov, M.A. Ivanov, V.E. Lyubovitskij, Z. Phys. C47, 583-594 (1990)

Basis of Dirac matrices

Basis of sixteen 4×4 matrices:

Scalar S	I	**1**
Vector V	γ^μ	**4**
Tensor T	$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ $(\mu < \nu)$	**6**
Pseudoscalar P	$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$	**1**
Axial A	$i\gamma^\mu\gamma_5$	**4**
		16 total

Normalization and Completeness

- **Normalization:**

$$\mathrm{tr}(\Gamma^C\Gamma^D) = 4\delta_{CD}, \qquad (C, D = S, V, T, P, A),$$

$$\mathrm{tr}(I_4 I_4) = 4 \qquad \mathrm{tr}(\gamma^\mu\gamma^\nu) = 4\,g^{\mu\nu}$$

$$\mathrm{tr}(\sigma^{\mu\nu}\sigma^{\alpha\beta}) = 4\,(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha})$$

$$\mathrm{tr}(\gamma_5\gamma_5) = 4 \qquad \mathrm{tr}(i\gamma_5\gamma^\mu \cdot i\gamma_5\gamma^\nu) = 4\,g^{\mu\nu}$$

- **Any product of Dirac matrices can be decomposed into 16 independent matrices:**

$$\Gamma = \sum_{D=S,V,T,P,A} C^D\Gamma^D, \qquad 4\,C^D = \mathrm{tr}(\Gamma\Gamma^D).$$

Normalization and Completeness

- **Completeness:**

$$\sum_{D=S,V,T,P,A} \Gamma^D_{\alpha_1\alpha_2}\Gamma^D_{\alpha_3\alpha_4} = 4\,\delta_{\alpha_1\alpha_4}\delta_{\alpha_3\alpha_2}$$

- **Fierz transformation:**

$$4\,\Gamma^{(1)}_{\alpha_1\alpha_2}\Gamma^{(2)}_{\alpha_3\alpha_4} = 4\,\delta_{\beta_2\alpha_2}\delta_{\beta_4\alpha_4}\,\Gamma^{(1)}_{\alpha_1\beta_2}\Gamma^{(2)}_{\alpha_3\beta_4} = \sum_D \Gamma^D_{\beta_2\alpha_4}\Gamma^D_{\beta_4\alpha_2}\,\Gamma^{(1)}_{\alpha_1\beta_2}\Gamma^{(2)}_{\alpha_3\beta_4}$$

$$= \sum_D (\Gamma^{(1)}\Gamma^D)_{\alpha_1\alpha_4}(\Gamma^{(2)}\Gamma^D)_{\alpha_3\alpha_2}.$$

Fierz transformation: applications

Short notation:

$$\Gamma^{(1)}_{\alpha_1\alpha_2}\Gamma^{(2)}_{\alpha_3\alpha_4} = \widetilde{\Gamma^{(1)}} \otimes \widetilde{\Gamma^{(2)}}, \quad \text{and} \quad \Gamma^{(1)}_{\alpha_1\alpha_4}\Gamma^{(2)}_{\alpha_3\alpha_2} = \Gamma^{(1)} \otimes \Gamma^{(2)}$$

Some examples:

$$
\begin{aligned}
4\,\widetilde{\gamma^\mu} \otimes \widetilde{\gamma_\mu} &= +4\,I \otimes I - 2\,\gamma^\mu \otimes \gamma_\mu - 2\,\gamma^\mu\gamma_5 \otimes \gamma_\mu\gamma_5 - 4\,\gamma_5 \otimes \gamma_5 \\
4\,\widetilde{\gamma^\mu\gamma_5} \otimes \widetilde{\gamma_\mu\gamma_5} &= -4\,I \otimes I - 2\,\gamma^\mu \otimes \gamma_\mu - 2\gamma^\mu\gamma_5 \otimes \gamma_\mu\gamma_5 + 4\,\gamma_5 \otimes \gamma_5 \\
4\,\widetilde{\gamma^\mu} \otimes \widetilde{\gamma_\mu\gamma_5} &= -4\,I \otimes \gamma_5 + 4\,\gamma_5 \otimes I - 2\gamma^\mu \otimes \gamma_\mu\gamma_5 - 2\,\gamma^\mu\gamma_5 \otimes \gamma_\mu \\
4\,\widetilde{\gamma^\mu\gamma_5} \otimes \widetilde{\gamma_\mu} &= +4\,I \otimes \gamma_5 - 4\,\gamma_5 \otimes I - 2\gamma^\mu \otimes \gamma_\mu\gamma_5 - 2\,\gamma^\mu\gamma_5 \otimes \gamma_\mu
\end{aligned}
$$

$$4\,\widetilde{I} \otimes \widetilde{\gamma_5} = +I \otimes \gamma_5 + \gamma_5 \otimes I - \gamma^\mu \otimes \gamma_\mu\gamma_5 + \gamma^\mu\gamma_5 \otimes \gamma_\mu + \tfrac{1}{2}\,\sigma^{\mu\nu}\gamma_5 \otimes \sigma_{\mu\nu}$$

$$4\,\widetilde{\gamma_5} \otimes \widetilde{I} = +I \otimes \gamma_5 + \gamma_5 \otimes I + \gamma^\mu \otimes \gamma_\mu\gamma_5 - \gamma^\mu\gamma_5 \otimes \gamma_\mu + \tfrac{1}{2}\,\sigma^{\mu\nu}\gamma_5 \otimes \sigma_{\mu\nu}$$

$$4\,\widetilde{\sigma_{\mu\nu}\gamma_5} \otimes \widetilde{\sigma^{\mu\nu}} = +12\,I \otimes \gamma_5 + 12\,\gamma_5 \otimes I - 2\,\sigma^{\mu\nu}\gamma_5 \otimes \sigma_{\mu\nu}$$

Fierz transformation: applications

- **Weak matrices with Left/Right chirality:**

$$O^{\mu}_{L/R} = \gamma^{\mu}(I \mp \gamma_5)$$

- **Fierz transformation:**

$$\widetilde{O_{L/R}} \otimes \widetilde{O_{L/R}} = -\,O_{L/R} \otimes O_{L/R}, \qquad \widetilde{O_{L/R}} \otimes \widetilde{O_{R/L}} = 2\,(I \pm \gamma_5) \otimes (I \mp \gamma_5)$$

- **It significantly simplifies the calculation:**

$$\begin{aligned} \mathrm{tr}(\Gamma_1 \mathrm{O}_{\mathrm{L/R}} \Gamma_2 \mathrm{O}_{\mathrm{L/R}}) &= -\,\mathrm{tr}(\Gamma_1 \mathrm{O}_{\mathrm{L/R}}) \cdot \mathrm{tr}(\Gamma_2 \mathrm{O}_{\mathrm{L/R}}) \\ \left(\Gamma_1 O_R \Gamma_2 O_L \Gamma_3\right)^{\alpha\beta} &= 2\left(\Gamma_1 (I - \gamma_5) \Gamma_3\right)^{\alpha\beta} \cdot \mathrm{tr}\left[\Gamma_2 (\mathrm{I} + \gamma_5)\right] \end{aligned}$$

$SU(n)$-matrices

- For $SU(n)$ there are $m = n^2 - 1$ generators $t^a = \frac{1}{2}\lambda^a$:

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c, \qquad \mathrm{tr}(\lambda^a\lambda^b) = 2\delta^{ab}, \quad (a, b = 1, \ldots, m)$$

- For completeness one has to add the unit matrix $\lambda^0 = \sqrt{\frac{2}{n}}\, I_n$.
- Any $n \times n$ matrix can be decomposed into basis matrices:

$$M = \sum_{a=0}^{m} M^a \lambda^a, \qquad M^a = \frac{1}{2}\mathrm{tr}(\lambda^a M).$$

- Completeness:

$$\sum_{a=0}^{m} \lambda^a_{m_1 m_2}\lambda^a_{m_3 m_4} = 2\,\delta_{m_1 m_4}\delta_{m_3 m_2}$$

$SU(n)$-matrices

- Some corollaries from completeness:

$$\sum_{a=1}^{m} \lambda^a_{m_1 m_2} \lambda^a_{m_3 m_4} = 2\,\delta_{m_1 m_4}\delta_{m_3 m_2} - \frac{2}{n}\delta_{m_1 m_2}\delta_{m_3 m_4}$$

$$\sum_{a=1}^{m} \lambda^a_{m_1 m_2} \lambda^a_{m_3 m_4} = \frac{2(n^2-1)}{n^2}\,\delta_{m_1 m_4}\delta_{m_3 m_2} - \frac{1}{n}\sum_{a=1}^{m} \lambda^a_{m_1 m_4} \lambda^a_{m_3 m_2}$$

- Applications:

$$\sum_{a=1}^{m} \mathrm{tr}\,(\lambda^a M_1 \lambda^a M_2) = -\frac{2}{n}\mathrm{tr}\,(M_1 M_2) + 2\,\mathrm{tr}\,(M_1)\,\mathrm{tr}\,(M_2)$$

$$\sum_{a=1}^{m} \mathrm{tr}\,(\lambda^a M_1)\,\mathrm{tr}\,(\lambda^a M_2) = 2\,\mathrm{tr}\,(M_1 M_2) - \frac{2}{n}\mathrm{tr}\,(M_1)\,\mathrm{tr}\,(M_2)$$

- The final expression may be written as

$$J^{km} = \varepsilon^{km_2 m_1} \delta^{mm_3} \Gamma_1 \, q^{a_1}_{m_1} \, (q^{a_2}_{m_2} C \Gamma_2 q^{a_3}_{m_3}) \varepsilon_{a_1 a_2 a_3}$$

where $\Gamma_1 \otimes \Gamma_2 = \gamma_\mu \gamma_5 \otimes \gamma^\mu$ or $\sigma_{\mu\nu}\gamma_5 \otimes \sigma^{\mu\nu}$

- The matrix $C = \gamma^0 \gamma^2$ is the usual charge conjugation matrix:

$$C^T = -C, \quad C^{-1} = C, \quad C^\dagger = C$$

$$C \Gamma^T C^{-1} = \begin{cases} +\Gamma & S, P, A \\ -\Gamma & V, T \end{cases}$$

- One can check that the diquark with identical flavors exists for $\Gamma_2 = \gamma^\mu$ and $\sigma^{\mu\nu}$ only:

$$(u^{a_2} C \Gamma_2 u^{a_3}) \varepsilon_{a_1 a_2 a_3} = -(u^{a_3} \, (C \Gamma_2)^T \, u^{a_2}) \varepsilon_{a_1 a_2 a_3}$$

$$= \quad -(u^{a_3} C \underbrace{C^{-1} \Gamma_2^T C^T}_{+\Gamma_2} u^{a_2}) \varepsilon_{a_1 a_2 a_3} = +(u^{a_2} C \Gamma_2 u^{a_3}) \varepsilon_{a_1 a_2 a_3}$$

Three-quark currents for baryons with $J^P = \frac{1}{2}^+$

- **Isotopic components for vector currents**

  (tensor Levi-Civita $\varepsilon_{a_1 a_2 a_3}$ is factorized out)

$$p \to \gamma_\mu \gamma_5 d_{a_1} (u_{a_2} C \gamma^\mu u_{a_3}) \qquad n \to \gamma_\mu \gamma_5 u_{a_1} (d_{a_2} C \gamma^\mu d_{a_3})$$

$$\Sigma^+ \to \gamma_\mu \gamma_5 s_{a_1} (u_{a_2} C \gamma^\mu u_{a_3}) \qquad \Sigma^- \to \gamma_\mu \gamma_5 s_{a_1} (d_{a_2} C \gamma^\mu d_{a_3})$$

$$\Sigma^0 \to \sqrt{2} \gamma_\mu \gamma_5 s_{a_1} (u_{a_2} C \gamma^\mu d_{a_3})$$

$$\Lambda^0 \to \sqrt{\tfrac{2}{3}} \{ \gamma_\mu \gamma_5 u_{a_1} (d_{a_2} C \gamma^\mu s_{a_3}) - \gamma_\mu \gamma_5 d_{a_1} (u_{a_2} C \gamma^\mu s_{a_3}) \}$$

$$\Xi^- \to \gamma_\mu \gamma_5 d_{a_1} (s_{a_2} C \gamma^\mu s_{a_3}) \qquad \Xi^0 \to \gamma_\mu \gamma_5 u_{a_1} (s_{a_2} C \gamma^\mu s_{a_3})$$

- **The obtained expressions are coincided with those from**

  B.L. Ioffe, Z. Phys. C18, 67 (1983); L.J. Reinders, H. Rubinstein, S. Yazaki, Phys.Rep., 127 (1985)

Charmed baryons

- **SU(4) (u,d,s,c): B.J. Bjorken, S.L. Glashow (1964)**
- **GIM mechanism: S.L. Glashow, J. Iliopoulos, L. Maiani (1970)**
  - **Problem: flavor changing neutral current at tree level**

$$Q_L^u = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}$$

$$\begin{aligned} \bar{Q}_L^u \gamma^\mu \frac{\tau^3}{2} Q_L^u &= \frac{1}{2}\Big(\bar{u}_L\gamma^\mu u_L - \cos^2\theta_C \bar{d}_L\gamma^\mu d_L - \sin^2\theta_C \bar{s}_L\gamma^\mu s_L \\ &\quad - \sin\theta_C\cos\theta_C[\bar{d}_L\gamma^\mu s_L + \bar{s}_L\gamma^\mu d_L]\Big) \end{aligned}$$

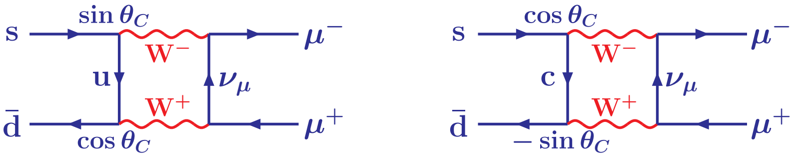
  - **Solution: extra duplet with charm quark**

$$Q_L^c = \begin{pmatrix} c \\ -d\sin\theta_C + s\cos\theta_C \end{pmatrix}$$

$$\begin{aligned} \bar{Q}_L^c \gamma^\mu \frac{\tau^3}{2} Q_L^c &= \frac{1}{2}\Big(\bar{c}_L\gamma^\mu c_L - \sin^2\theta_C \bar{d}_L\gamma^\mu d_L - \cos^2\theta_C \bar{s}_L\gamma^\mu s_L \\ &\quad + \sin\theta_C\cos\theta_C[\bar{d}_L\gamma^\mu s_L + \bar{s}_L\gamma^\mu d_L]\Big) \end{aligned}$$

$$\bar{Q}_L^u \gamma^\mu \frac{\tau^3}{2} Q_L^u + Q_L^c \gamma^\mu \frac{\tau^3}{2} Q_L^c = \frac{1}{2}\left(\bar{u}_L\gamma^\mu u_L + \bar{c}_L\gamma^\mu c_L - \bar{d}_L\gamma^\mu d_L - \bar{s}_L\gamma^\mu s_L\right)$$

Weak decay $K_L^0 \to \mu^+\mu^-$ goes via sum of one-loop diagrams with u-quark and c-quark



$$M(K \to \mu^+\mu^-) \propto \frac{g_2^4 \sin\theta_c \cos\theta_C}{M_W^2} \frac{m_c^2 - m_u^2}{M_W^2}$$

Experiment $\longrightarrow m_c \approx 1.5$ GeV.

Charmed baryons

Charmed baryons in one gluon exchange model:

A. De Rújula, H. Georgi, S.L. Glashow, 1975.

- Aim: hadron spectroscopy in the gauge model of weak, electromagnetic, and strong interactions.
- The model involves four types of fractionally charged quarks, each in three colors, coupling to massless gauge gluons.
- The asymptotic freedom of the model was used to justify that the short-range quark-quark interaction may be taken to be Coulomb-like for the calculation of hadron masses.
- Many successful quark-model mass relations for the low-lying hadrons have been rederived.
- The masses of charmed mesons and baryons have been predicted.

Charmed baryons in SU(4): $\mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} = \mathbf{20}_S \oplus \mathbf{20}_M \oplus \mathbf{20}'_M \oplus \mathbf{4}_A$

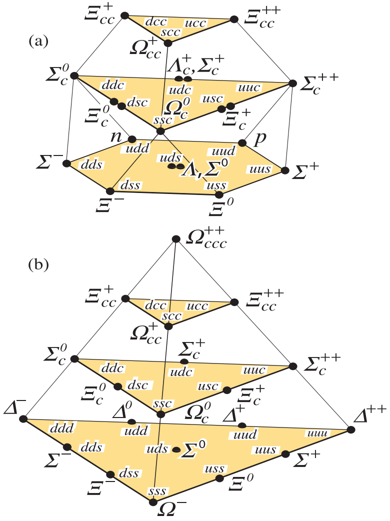


Figure 15.4: SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

Charmed baryons

- There are now more precise results on the decays of single-charmed baryons

$$\Lambda_c^+ \to p\phi, \ \ \Lambda\pi^+, \ \ \Sigma^+\pi^0; \qquad \Xi_c^+ \to p\bar{K}^{*\,0}(892)$$

- Three weakly decaying baryons with C=2 expected:

$$\begin{array}{lllll} \Xi_{cc}^{++} & = & ccu \ \ \text{and} & \Xi_{cc}^{+} = ccd & \text{isospin doublet} \\ \Omega_{cc}^{+} & = & ccs & & \text{isospin singlet} \end{array}$$

- In 2005 the SELEX Coll. reported on the observation of double-charmed baryon Ξ_{cc}^+ with a mass of 3518 ± 3 MeV.

- However, other Collaborations (*BABAR*, Belle, LHCb) found no evidence for the Ξ_{cc}^+ nor the Ξ_{cc}^{++} states in the conjectured mass region of ~ 3500 MeV.

Charmed baryons

- Recently the LHCb Collaboration discovered the double charm state Ξ_{cc}^{++} in the invariant mass spectrum of the final state particles $(\Lambda_c^+ K^- \pi^+ \pi^+)$.

- The extracted mass was given as $M_{\Xi_{cc}^{++}} = (3621.40 \pm 0.78)$ MeV

  i.e. ~ 100 MeV heavier than the mass of the original SELEX double charm baryon but in agreement with the value predicted by one gluon exchange model of de Rujula, Georgi and Glashow.

- Measurement of the lifetime: $\tau_{\Xi_{cc}^{++}} = 0.256 \pm 0.027$ ps

- Observation of the decay: $\Xi_{cc}^{++} \to \Xi_c^+ + \pi^+$

Double-charmed baryons

Tremendous theoretical activities in describing doubly heavy baryon:

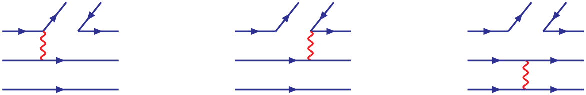
- Likhoded, Kiselev et al. (nonrelativistic potential model, nonrelativistic QCD SR)
- Faustov, Galkin et al. (relativistic quark model)
- Dhir, Sharma at al. (effective quark mass scheme)
- Chang, Li et al. (non-relativistic harmonic oscillator model)
- Karliner, Rosner et al. (masses in naive quark model)
- Hernández, Nieves et al. (nonrelativistic quark model)
- Aliev, Azizi et al. (QCD SR)
- Ivanov, Körner, Lyubovitskij et al. (quark confinement model)
- ...

Nonleptonic two-body weak decays of baryons

- **Ground states of baryons with $J^P = \frac{1}{2}^+$ can decay only weakly.**
- **Two-body decays of baryons have five different quark topologies:**

Ia Ib

Tree diagrams



IIa IIb III

W-exchange diagrams

Effective low-energy theory for weak decays–preliminaries

G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996)

- Main idea is to derive an effective low-energy theory describing the weak interactions of quarks.
- The formal framework is using the operator product expansion (OPE).
- An example: the tree-level W-exchange amplitude for $c \to s u \bar{d}$:

$$A = -\frac{g_2^2}{8} V_{cs}^* V_{ud} \, (\bar{s} O^\mu c) \left[\frac{-g_{\mu\nu}}{M_W^2 - k^2} \right] (\bar{u} O^\nu d)$$

$$= -\frac{g_2^2}{8\, M_W^2} V_{cs}^* V_{ud} \, (\bar{s} O^\mu c)\, (\bar{u} O_\mu d) + \mathcal{O}\left(\frac{k^2}{M_W^2} \right)$$

Since the momentum transfer $|k| << M_W$ the terms of order $\mathcal{O}(k^2/M_W^2)$ may be neglected.

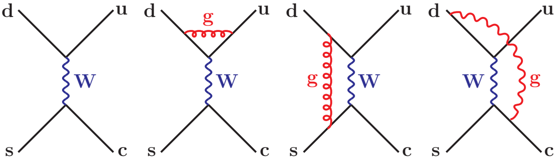
- The leading term can be obtained from an effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{\text{tree}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \, (\bar{s}_a O^\mu c_a)(\bar{u}_b O_\mu d_b), \qquad \frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8\, M_W^2}$$

One-loop QCD corrections

Tree and one-loop diagrams in full and effective theory:

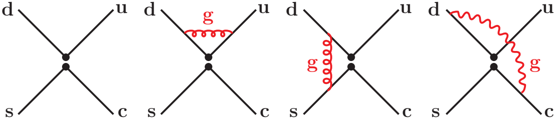
Current-current diagrams



tree

OCD one-loop

Effective Theory



One-loop QCD corrections

Including QCD corrections, the effective Hamiltonian is generalized to

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[C_1(\mu) Q_1 + C_2(\mu) Q_2 \right],$$

$$Q_1 \equiv (\bar{s}_a O^\mu c_b)(\bar{u}_b O_\mu d_a), \qquad Q_2 \equiv (\bar{s}_a O^\mu c_a)(\bar{u}_b O_\mu d_b)$$

- Perturbation theory: calculation of Wilson coefficients $C(\mu_W)$ at $\mu \approx M_W)$ to some order in α_S.

$$C_1 = -3 \frac{\alpha_S}{4\pi} \ln \frac{M_W^2}{\mu^2}, \qquad C_2 = 1 + \frac{\alpha_S}{4\pi} \ln \frac{M_W^2}{\mu^2}$$

- RG-improved Perturbation theory: evaluation of the Wilson coefficients from μ_W down to the low-energy scale μ.
- Nonperturbative regime: calculation of hadronic matrix elements $< Q(\mu) >$ at the low-energy scale μ by using nonperturbative methods.

Covariant Constituent Quark Model

- The CCQM is based on a phenomenological, nonlocal relativistic Lagrangian describing the coupling of a hadron to its constituents:

$$\mathcal{L}_{\rm int} = g_H \cdot H(x) \cdot J_H(x)$$

- Quark currents

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1)\, \Gamma_M \, q^a_{f_2}(x_2) \qquad \text{Meson}$$

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3 \, F_B(x; x_1, x_2, x_3) \qquad \text{Baryon}$$

$$\times \, \Gamma_1 \, q^{a_1}_{f_1}(x_1) \left[\varepsilon^{a_1 a_2 a_3} q^{T\, a_2}_{f_2}(x_2) C \, \Gamma_2 \, q^{a_3}_{f_3}(x_3) \right]$$

$$J_T(x) = \int dx_1 \ldots \int dx_4 \, F_T(x; x_1, \ldots, x_4) \qquad \text{Tetraquark}$$

$$\times \left[\varepsilon^{a_1 a_2 c} q^{T\, a_1}_{f_1}(x_1) \, C\Gamma_1 \, q^{a_2}_{f_2}(x_2) \right] \cdot \left[\varepsilon^{a_3 a_4 c} \bar{q}^{T\, a_3}_{f_3}(x_3) \, \Gamma_2 C \, \bar{q}^{a_4}_{f_4}(x_4) \right]$$

The vertex functions and quark propagators

- Translational invariance for the vertex function

$$F_H(x+a, x_1+a, x_2+a) = F_H(x, x_1, x_2)\,, \qquad \forall a\,.$$

- Our choice:

$$F_B(x, x_1, \ldots, x_n) \;=\; \delta^{(4)}\Big(x - \sum_{i=1}^{n} w_i x_i\Big)\;\Phi_H\Big(\sum_{i<j}(x_i - x_j)^2\Big)$$

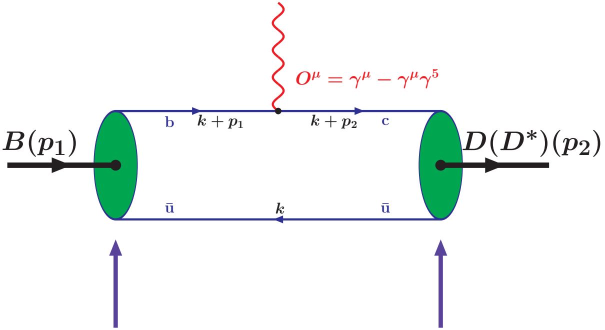
where $w_i = m_i / \sum_i m_i$.

- The quark propagators

$$S_q(x_1 - x_2) = \int \frac{d^4k}{(2\pi)^4 i}\,\frac{e^{-ik(x_1-x_2)}}{m_q - \not{k}}$$

- Infrared confinement (cutting an analog of the proper time on the upper limit)

Heavy quark limit in $B - D$ transition



$$\phi_B\left(-(k + w_u\, p_1)^2\right) \quad \phi_{D(D^*)}\left(-(k + w'_u\, p_2)^2\right)$$

$$w_u = \frac{m_u}{m_u + m_b} \qquad w'_u = \frac{m_u}{m_u + m_c}$$

Heavy quark limit: $m_H = m_Q + E, \quad m_Q \to \infty; \quad \Lambda_B = \Lambda_D = \Lambda_{D^*}.$

Isgur-Wise function

$$\frac{1}{m_i - \not{k} - \not{p}_i} \to -\frac{1+\not{v}_i}{2} \cdot \frac{1}{kv_i + E}, \qquad v_i = \frac{p_i}{m_i}$$

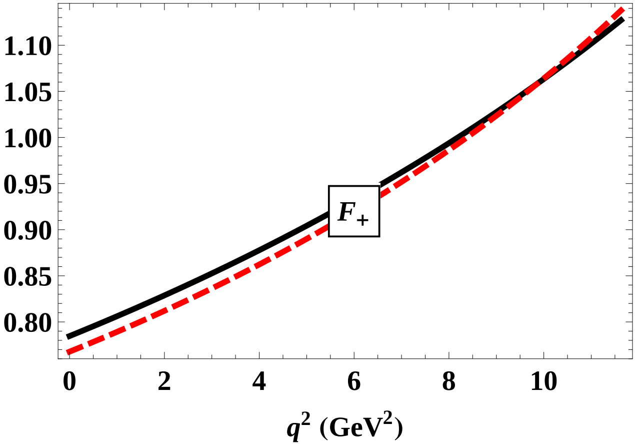
$$\begin{aligned} M^{\mu}_{BD}(p_1, p_2) &= f_+(q^2)(p_1 + p_2)^{\mu} + f_-(q^2)(p_1 - p_2)^{\mu}\,, \\ f_{\pm} &= \pm\frac{m_1 \pm m_2}{2\sqrt{m_1 m_2}} \cdot \xi(w)\,, \qquad w = v_1 \cdot v_2. \end{aligned}$$

the Isgur-Wise function is equal to

$$\xi(w) = \frac{J_3(E,w)}{J_3(E,1)}, \qquad J_3(E,w) = \int\limits_0^1 \frac{d\tau}{W} \int\limits_0^{\infty} du\, \widetilde{\Phi}^2(z) \frac{m_u + \sqrt{u/W}}{m_u^2 + z}$$

where $W = 1 + 2\tau(1-\tau)(w-1)$, $z = u - 2E\sqrt{u/W}$.

Isgur-Wise function



Some applications

- Semileptonic, nonleptonic and rare $B\,(B_s)$–decays
- Exclusive semileptonic $D\,(D_s)$–decays
- Semileptonic decay $\Lambda_b \to \Lambda_c + \tau^- + \bar{\nu_\tau}$
- Heavy-to-light semileptonic decays of Λ_b and Λ_c
- Rare decays $\Lambda_b \to \Lambda + \ell^+\ell^-$
- Polarization effects in the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-) + J/\psi(\to \ell^+\ell^-)$
- Strong and radiative decays of the tetraquark state $X(3872)$. Four-quark structure of $Z_c(3900)$, $Z_b(10610)$, $Z_b'(10650)$ exotic states
- Analyzing new physics in the decays $\bar{B}^0 \to D^{(*)}\tau^-\bar{\nu}_\tau$ (lecture by Chien-Thang Tran)
- Study of B_c decays (lecture by Aidos Issadykov)

Some double charmed baryon decays

We will consider the decays that belong to the same topology class:

$$\Xi_{cc}^{++} \to \Xi_c^+ (\Xi_c'^+) + \pi^+(\rho^+) \qquad \text{T-Ia and W-IIb}$$

$$\Omega_{cc}^{+} \to \Xi_c^+ (\Xi_c'^+) + \bar{K}^0(K^{*\,0}) \qquad \text{T-Ib and W-IIb}$$

Quantum numbers and interpolating currents:

Baryon	J^P	Interpolating current	Mass (MeV)
Ξ_{cc}^{++}	$\frac{1}{2}^+$	$\varepsilon_{abc}\,\gamma^\mu\gamma_5\,u^a(c^b C\gamma_\mu c^c)$	3620.6
Ω_{cc}^{+}	$\frac{1}{2}^+$	$\varepsilon_{abc}\,\gamma^\mu\gamma_5\,s^a(c^b C\gamma_\mu c^c)$	3710.0
$\Xi_c'^{+}$	$\frac{1}{2}^+$	$\varepsilon_{abc}\,\gamma^\mu\gamma_5\,c^a(u^b C\gamma_\mu s^c)$	2577.4
Ξ_c^{+}	$\frac{1}{2}^+$	$\varepsilon_{abc}\,c^a(u^b C\gamma_5 s^c)$	2467.9

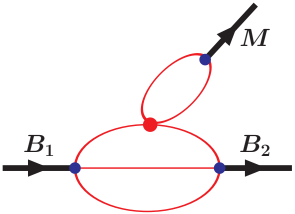
Körner-Pati-Woo (KPW) theorem

J.G. Körner, Nucl. Phys. B25, 282 (1971); J.C. Pati and C.H. Woo, Phys. Rev. D3, 2920 (1971)

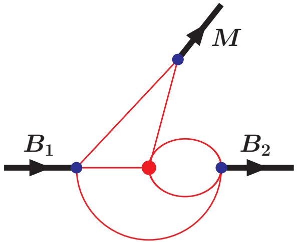
The W–exchange contributions to the above decays fall into two classes:

- The decays with a $\Xi_c^{\prime +}$-baryon containing a symmetric $\{us\}$ diquark described by the interpolating current $\varepsilon_{abc}\,(u^b C\gamma_\mu s^c)$.
- The W–exchange contribution is strongly suppressed due to the KPW theorem which states that the contraction of the flavor antisymmetric current-current operator with a flavor symmetric final state configuration is zero in the $SU(3)$ limit.
- The decays with a Ξ_c^+-baryon containing a antisymmetric $[us]$ diquark described by the interpolating current $\varepsilon_{abc}\,(u^b C\gamma_5 s^c)$.
- In this case the W–exchange contribution is not a priori suppressed.

Matrix elements



tree diagrams Ia, Ib



W-exchange diagram IIb

$$< B_2\, M | \mathcal{H}_{\rm eff} | B_1 >= \frac{G_F}{\sqrt{2}} V_{cs} V^{\dagger}_{ud}\, \bar{u}(p_2) \Big(12\, C_T\, M_T + 12\, (C_1 - C_2)\, M_W \Big) u(p_1).$$

$$C_T = \begin{cases} C_T = +(C_2 + \xi C_1) & \text{charged meson} \\ C_T = -(C_1 + \xi C_2) & \text{neutral meson} \end{cases}$$

The factor of $\xi = 1/N_c$ is set to zero in the numerical calculations.

Tree-diagram contribution: factorization

The contribution from the tree diagram factorizes into two pieces:

$$M_T = M_T^{(1)} \cdot M_T^{(2)}$$

$$M_T^{(1)} = N_c\, g_M \int \frac{d^4k}{(2\pi)^4 i} \widetilde{\Phi}_M(-k^2)\, \mathrm{tr}\left[O_L S_d(k - w_d q)\Gamma_M S_{s(u)}(k + w_{s(u)} q)\right]$$

$$\begin{aligned} M_T^{(2)} &= g_{B_1} g_{B_2} \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} \widetilde{\Phi}_{B_1}\Big(-\vec{\Omega}_1^{\,2}\Big)\widetilde{\Phi}_{B_2}\Big(-\vec{\Omega}_2^{\,2}\Big) \\ &\times \Gamma_1 S_c(k_2)\gamma^\mu S_c(k_1 - p_1) O_R S_{u(s)}(k_1 - p_2)\widetilde{\Gamma}_2 S_{s(u)}(k_1 - k_2)\gamma_\mu \gamma_5 \end{aligned}$$

The $M_T^{(1)}$ is related to the leptonic decay constants:

$$M_T^{(1)} = \begin{cases} -f_P \cdot q & \text{pseudoscalar meson} \\ +f_V m_V \cdot \epsilon_V & \text{vector meson} \end{cases}$$

W-exchange diagram contribution: no factorization

$$
\begin{aligned}
M_W &= g_{B_1} g_{B_2} g_M \int \frac{d^4 k_1}{(2\pi)^4 i} \int \frac{d^4 k_2}{(2\pi)^4 i} \int \frac{d^4 k_3}{(2\pi)^4 i} \widetilde{\Phi}_{B_1}\Big(-\vec{\Omega}_1^{\,2}\Big) \widetilde{\Phi}_{B_2}\Big(-\vec{\Omega}_2^{\,2}\Big) \widetilde{\Phi}_M(-P^2) \\
&\times 2\Gamma_1 S_c(k_1)\gamma^\mu S_c(k_2)(1-\gamma_5) S_d(k_2-k_1+p_2)\Gamma_M S_{s(u)}(k_2-k_1+p_1)\gamma_\mu\gamma_5 \\
&\times \mathrm{tr}\Big[S_{u(s)}(k_3)\widetilde{\Gamma}_2 S_{s(u)}(k_3-k_1+p_2)(1+\gamma_5)\Big]
\end{aligned}
$$

Here $\Gamma_1 \otimes \widetilde{\Gamma}_2 = I \otimes \gamma_5$ for $B_2 = \Xi_c^+$ and $-\gamma_\nu\gamma_5 \otimes \gamma^\nu$ for $B_2 = \Xi_c'^{\,+}$.

To verify the KPW theorem in the case of $B_2 = \Xi_c'^{\,+}$ we use the identity

$$
\mathrm{tr}\Big[S_u(k_3)\gamma_\nu S_s(k_3-k_1+p_2)\Big] = -\,\mathrm{tr}\Big[S_s(-k_3+k_1-p_2)\gamma_\nu S_u(-k_3)\Big]
$$

Then by shifting $k_3 \to -k_3 + k_1 - p_2$ one gets the same expression with opposite sign and $u \leftrightarrow s$ interchange. Thus, if $m_u = m_s$ then $M_W \equiv 0$.

It directly confirms the KPW–theorem.

Evaluation of the diagrams

- Use the Schwinger representation of the propagator:

$$\frac{m + \not{k}}{m^2 - k^2} = (m + \not{k}) \int\limits_0^\infty d\alpha \, \exp[-\alpha(m^2 - k^2)]$$

- Choose a simple Gaissian form for the vertex function

$$\Phi(-K^2) = \exp\left(K^2/\Lambda^2\right)$$

where the parameter Λ characterizes the hadron size.

- We imply that the loop integration k proceed over Euclidean space:

$$k^0 \to e^{i\frac{\pi}{2}} k_4 = ik_4 \,, \qquad k^2 = (k^0)^2 - \vec{k}^2 \to -k_E^2 \leq 0 \,.$$

- We also put all external momenta p to Euclidean space:

$$p^0 \to e^{i\frac{\pi}{2}} p_4 = ip_4 \,, \qquad p^2 = (p^0)^2 - \vec{p}^2 \to -p_E^2 \leq 0$$

so that the quadratic momentum form in the exponent becomes negative-definite and the loop integrals are absolutely convergent.

Evaluation of the diagrams

- Convert the loop momemta in the numerator into derivatives over external momenta:

$$k_i^\mu e^{2kr} = \frac{1}{2}\frac{\partial}{\partial r_{i\,\mu}} e^{2kr},$$

- Move the derivatives outside of the loop integrals.
- Calculate the scalar loop integral:

$$\prod_{i=1}^{n} \int \frac{d^4 k_i}{i\pi^2} e^{kAk+2kr} = \prod_{i=1}^{n} \int \frac{d^4 k_i^E}{\pi^2} e^{-k_E A k_E - 2k_E r_E} = \frac{1}{|A|^2} e^{-rA^{-1}r}$$

where a symmetric $n \times n$ real matrix A is positive-definite.

- Use the identity

$$P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) e^{-rA^{-1}r} = e^{-rA^{-1}r} P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1}r\right)$$

to move the exponent to the left.

Evaluation of the diagrams

- Employ the commutator

$$[\frac{\partial}{\partial r_{i\,\mu}}, r_{j\,\nu}] = \delta_{ij}\, g_{\mu\nu}$$

to make differentiation in

$$P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1} r\right)$$

for any polynomial P. The necessary commutations of the differential operators are done by a FORM program.

- One obtains

$$\Pi = \int\limits_0^\infty d^n\alpha\, F(\alpha_1, \ldots, \alpha_n)\,,$$

where F stands for the whole structure of a given diagram.

The set of Schwinger parameters α_i can be turned into a simplex by introducing an additional t–integration via the identity

$$1 = \int_0^\infty dt\, \delta(t - \sum_{i=1}^{n} \alpha_i)$$

leading to

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n\alpha\, \delta\left(1 - \sum_{i=1}^{n} \alpha_i\right) F(t\alpha_1, \ldots, t\alpha_n)\,.$$

Infrared confinement

- Cut off the upper integration at $1/\lambda^2$

$$\Pi^c = \int\limits_0^{1/\lambda^2} dt t^{n-1} \int\limits_0^1 d^n\alpha\, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big)\, F(t\alpha_1, \ldots, t\alpha_n)$$

- The infrared cut-off has removed all possible thresholds in the quark loop diagram.

- We take the cut-off parameter λ to be the same in all physical processes.

T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij,
Phys. Rev. D81, 034010 (2010)

- An example of a scalar one–loop two–point function:

$$\Pi_2(p^2) = \int \frac{d^4 k_E}{\pi^2} \frac{e^{-s\, k_E^2}}{[m^2 + (k_E + \frac{1}{2}p_E)^2][m^2 + (k_E - \frac{1}{2}p_E)^2]}$$

where the numerator factor $e^{-s\, k_E^2}$ comes from the product of nonlocal vertex form factors of Gaussian form. $k_E,\ p_E$ are Euclidean momenta ($p_E^2 = -\, p^2$).

- Doing the loop integration one obtains

$$\Pi_2(p^2) = \int\limits_0^\infty dt \frac{t}{(s+t)^2} \int\limits_0^1 d\alpha\ \exp\left\{ -\, t\, [m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2 \right\}$$

A branch point at $p^2 = 4m^2$.

Infrared confinement

- By introducing a cut-off in the t–integration one obtains

$$\Pi_2^c(p^2) = \int\limits_0^{1/\lambda^2} dt \frac{t}{(s+t)^2} \int\limits_0^1 d\alpha \, \exp\left\{ -t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2 \right\}$$

where the one–loop two–point function $\Pi_2^c(p^2)$ no longer has a branch point at $p^2 = 4m^2$.

- The confinement scenario also allows to include all possible both two-quark and multi-quark resonance states in our calculations.

Invariant and helicity amplitudes

The transition amplitudes in terms of invariant amplitudes:

$$< B_2\, P|\mathcal{H}_{\rm eff}|B_1 > \;=\; \frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud}\, \bar{u}(p_2)\, (A + \gamma_5\, B)\, u(p_1)$$

$$< B_2\, V|\mathcal{H}_{\rm eff}|B_1 > \;=\; \frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud}$$

$$\times \quad \bar{u}(p_2)\, \epsilon^*_{V\delta} \left(\gamma^\delta V_\gamma + p_1^\delta V_p + \gamma_5 \gamma^\delta V_{5\gamma} + \gamma_5 p_1^\delta\, V_{5p}\right) u(p_1)$$

The invariant amplitudes in terms of helicity amplitudes:

$$H^V_{\frac{1}{2}\,t} = \sqrt{Q_+}\, A \qquad H^A_{\frac{1}{2}\,t} = \sqrt{Q_-}\, B$$

$$H^V_{\frac{1}{2}\,0} = +\sqrt{Q_-/q^2}\left(m_+\, V_\gamma + \tfrac{1}{2} Q_+\, V_p\right) \qquad H^V_{\frac{1}{2}\,1} = -\sqrt{2Q_-}\, V_\gamma$$

$$H^A_{\frac{1}{2}\,0} = +\sqrt{Q_+/q^2}\left(m_-\, V_{5\gamma} + \tfrac{1}{2} Q_-\, V_{5p}\right) \qquad H^A_{\frac{1}{2}\,1} = -\sqrt{2Q_+}\, V_{5\gamma}$$

Here $m_\pm = m_1 \pm m_2$, $Q_\pm = m_\pm^2 - q^2$ **and** $|\mathbf{p_2}| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/(2m_1)$.

The parity relations: $H^V_{-\lambda_2,-\lambda_M} = +H^V_{\lambda_2,\lambda_M}\,,\; H^A_{-\lambda_2,-\lambda_M} = -H^A_{\lambda_2,\lambda_M}$

The two-body decay widths read

$$\Gamma\Big(B_1 \to B_2 + P(V)\Big) = \frac{G_F^2}{32\pi}\,|V_{cs}^* V_{ud}|^2\,\frac{|\mathbf{p}_2|}{m_1^2}\,\mathcal{H}_{P(V)}$$

$$\mathcal{H}_P = \left|H_{\frac{1}{2}\,t}\right|^2 + \left|H_{-\frac{1}{2}\,t}\right|^2,$$

$$\mathcal{H}_V = \left|H_{\frac{1}{2}\,0}\right|^2 + \left|H_{-\frac{1}{2}\,0}\right|^2 + \left|H_{\frac{1}{2}\,1}\right|^2 + \left|H_{-\frac{1}{2}\,-1}\right|^2,$$

where $H = H^V - H^A$.

$\Omega_{cc}^{+} \to \Xi_c'^{+} + \bar{K}^0(\bar{K}^{*0})$

Helicity	Tree diagram	W diagram	total
$H^V_{\frac{1}{2}\,t}$	0.20	−0.01	0.19
$H^A_{\frac{1}{2}\,t}$	0.25	−0.01	0.24
$\Gamma(\Omega_{cc}^{+} \to \Xi_c'^{+} + \bar{K}^0) = 0.15 \cdot 10^{-13}$ GeV			
$H^V_{\frac{1}{2}\,0}$	−0.25	0.04×10^{-1}	−0.25
$H^A_{\frac{1}{2}\,0}$	−0.50	0.01	−0.49
$H^V_{\frac{1}{2}\,1}$	0.27	−0.01	0.26
$H^A_{\frac{1}{2}\,1}$	0.56	0.04×10^{-2}	0.56
$\Gamma(\Omega_{cc}^{+} \to \Xi_c'^{+} + \bar{K}^{*0}) = 0.74 \cdot 10^{-13}$ GeV			

$\Omega_{cc}^{+} \to \Xi_{c}^{+} + \bar{K}^{0}(\bar{K}^{*\,0})$

Helicity	Tree diagram	W diagram	total
$H^{V}_{\frac{1}{2}\,t}$	-0.35	1.06	0.71
$H^{A}_{\frac{1}{2}\,t}$	-0.10	0.31	0.21
$\Gamma(\Omega_{cc}^{+} \to \Xi_{c}^{+} + \bar{K}^{0}) = 0.95 \cdot 10^{-13}$ GeV			
$H^{V}_{\frac{1}{2}\,0}$	0.50	-0.69	-0.19
$H^{A}_{\frac{1}{2}\,0}$	0.18	-0.45	-0.27
$H^{V}_{\frac{1}{2}\,1}$	-0.11	-0.24	-0.35
$H^{A}_{\frac{1}{2}\,1}$	-0.18	0.66	0.48
$\Gamma(\Omega_{cc}^{+} \to \Xi_{c}^{+} + \bar{K}^{*\,0}) = 0.62 \cdot 10^{-13}$ GeV			

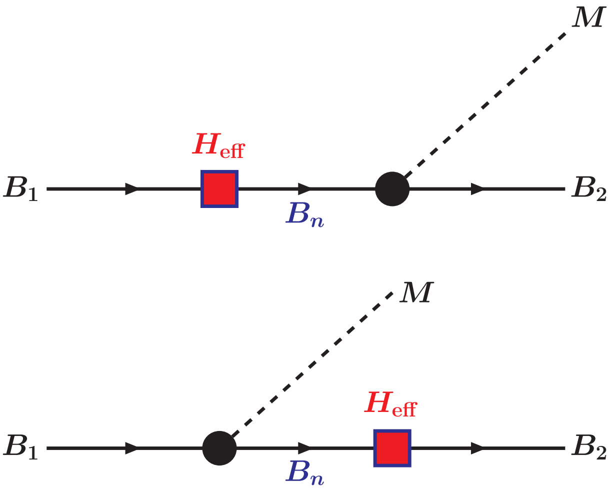
$\Xi_{cc}^{++} \to \Xi_c'^{+} + \pi^{+}(\rho^{+})$

Helicity	Tree diagram	W diagram	total
$H^V_{\frac{1}{2}\,t}$	-0.38	-0.01	-0.39
$H^A_{\frac{1}{2}\,t}$	-0.55	-0.02	-0.57
$\Gamma(\Xi_{cc}^{++} \to \Xi_c'^{+} + \pi^{+}) = 0.82 \cdot 10^{-13}$ GeV			
$H^V_{\frac{1}{2}\,0}$	0.60	0.04×10^{-1}	0.61
$H^A_{\frac{1}{2}\,0}$	1.20	0.01	1.21
$H^V_{\frac{1}{2}\,1}$	-0.49	-0.01	-0.50
$H^A_{\frac{1}{2}\,1}$	-1.27	0.01×10^{-1}	-1.27
$\Gamma(\Xi_{cc}^{++} \to \Xi_c'^{+} + \rho^{+}) = 4.27 \cdot 10^{-13}$ GeV			

$\Xi_{cc}^{++} \to \Xi_c^{+} + \pi^{+}(\rho^{+})$

Helicity	Tree diagram	W diagram	total
$H^V_{\frac{1}{2}\,t}$	-0.70	0.99	0.29
$H^A_{\frac{1}{2}\,t}$	-0.21	0.30	0.09
$\Gamma(\Xi_{cc}^{++} \to \Xi_c^{+} + \pi^{+}) = 0.18 \cdot 10^{-13}$ GeV			
$H^V_{\frac{1}{2}\,0}$	1.17	-0.70	0.47
$H^A_{\frac{1}{2}\,0}$	0.45	-0.44	0.003
$H^V_{\frac{1}{2}\,1}$	-0.20	-0.23	-0.43
$H^A_{\frac{1}{2}\,1}$	-0.41	0.62	0.21
$\Gamma(\Xi_{cc}^{++} \to \Xi_c^{+} + \rho^{+}) = 0.63 \cdot 10^{-13}$ GeV			

Pole Model



Comparison with other approaches. Abbr.: M=NRQM, T=HQET

Mode	Width (in 10^{-13} GeV)					
	our	Dhir	Jiang	Wang	Yu	Likhoded
$\Omega_{cc}^{+} \to \Xi_c^{\prime +} + \bar{K}^0$	0.15	0.31 (M) 0.59 (T)				
$\Omega_{cc}^{+} \to \Xi_c^{+} + \bar{K}^0$	0.95	0.68 (M) 1.08 (T)				
$\Omega_{cc}^{+} \to \Xi_c^{\prime +} + \bar{K}^{*0}$	0.74		$2.64^{+2.72}_{-1.79}$			
$\Omega_{cc}^{+} \to \Xi_c^{+} + \bar{K}^{*0}$	0.62		$1.38^{+1.49}_{-0.95}$			
$\Xi_{cc}^{++} \to \Xi_c^{\prime +} + \pi^+$	0.82	1.40 (M) 1.93 (T)		1.10		
$\Xi_{cc}^{++} \to \Xi_c^{+} + \pi^+$	0.18	1.71 (M) 2.39 (T)		1.57	1.58	2.25
$\Xi_{cc}^{++} \to \Xi_c^{\prime +} + \rho^+$	4.27		$4.25^{+0.32}_{-0.19}$	4.12	3.82	
$\Xi_{cc}^{++} \to \Xi_c^{+} + \rho^+$	0.63		$4.11^{+1.37}_{-0.86}$	3.03	2.76	6.70

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Estimating uncertainties in the decay widths

- The only free parameter in our approach is the size parameter Λ_{cc}.
- We have chosen $\Lambda_{cc} = \Lambda_c = 0.8675$ GeV.
- To estimate the uncertaintity caused by the choice of the size parameter we allow the size parameter to vary from 0.6 to 1.135 GeV.
- We evaluate the mean $\bar{\Gamma} = \sum \Gamma_i / N$ and the mean square deviation $\sigma^2 = \sum (\Gamma_i - \bar{\Gamma})^2 / N$.
- The rate errors amount to $6 - 15\%$.

Mode	Width (in 10^{-13} GeV)
$\Omega_{cc}^{+} \to \Xi_c'^{+} + \bar{K}^0$	0.14 ± 0.01
$\Omega_{cc}^{+} \to \Xi_c'^{+} + \bar{K}^{*\,0}$	0.72 ± 0.06
$\Omega_{cc}^{+} \to \Xi_c^{+} + \bar{K}^0$	0.87 ± 0.13
$\Omega_{cc}^{+} \to \Xi_c^{+} + \bar{K}^{*\,0}$	0.58 ± 0.07
$\Xi_{cc}^{++} \to \Xi_c'^{+} + \pi^{+}$	0.77 ± 0.05
$\Xi_{cc}^{++} \to \Xi_c'^{+} + \rho^{+}$	4.08 ± 0.29
$\Xi_{cc}^{++} \to \Xi_c^{+} + \pi^{+}$	0.16 ± 0.02
$\Xi_{cc}^{++} \to \Xi_c^{+} + \rho^{+}$	0.59 ± 0.04

Summary and outlook

- We have made an ab initio three-loop quark model calculation of the W-exchange contribution to the nonleptonic two-body decays of the doubly charmed baryons Ξ_{cc}^{++} and Ω_{cc}^{+}.
- The W-exchange contributions appear in addition to the factorizable graph contributions and are not suppressed in general.
- We made use of the covariant confined quark model to calculate the factorizable graph as well as the W-exchange contribution.
- We have calculated helicity amplitudes and quantitavely compare the factorizable graph and W-exchange contributions. Finally, we compared the calculated decay widths with those from other theoretical approaches when they are available.

Summary and outlook

- We now have the tools at hand to calculate all Cabibbo favored and Cabibbo suppressed nonleptonic two-body decays of the double charm ground state baryons Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+}. These would also include the $1/2^+ \to 3/2^+ + P(V)$ nonleptonic decays not treated in this paper.

- Of particular interest are the modes

$$
\begin{aligned}
\Xi_{cc}^{+} &\to \Sigma^{(*)+} + D^{(*)0} \\
\Xi_{cc}^{+} &\to \Xi^{(*)0} + D_s^{(*)+} \\
\Omega_{cc}^{+} &\to \Xi^{(*)0} + D^{(*)+}
\end{aligned}
$$

  They proceed only due to a single W-exchange contribution.

- Three modes involving the final state $3/2^+$ baryons (Σ^{*+} and Ξ^{*0}) are forbidden due to the KPW theorem. It would be interesting to check on this prediction of the quark model.