Study of B_c decays

Aidos Issadykov & Mikhail A. Ivanov

in collaboration with:

S. Dubnicka, A.Z. Dubnickova, A. Liptaj

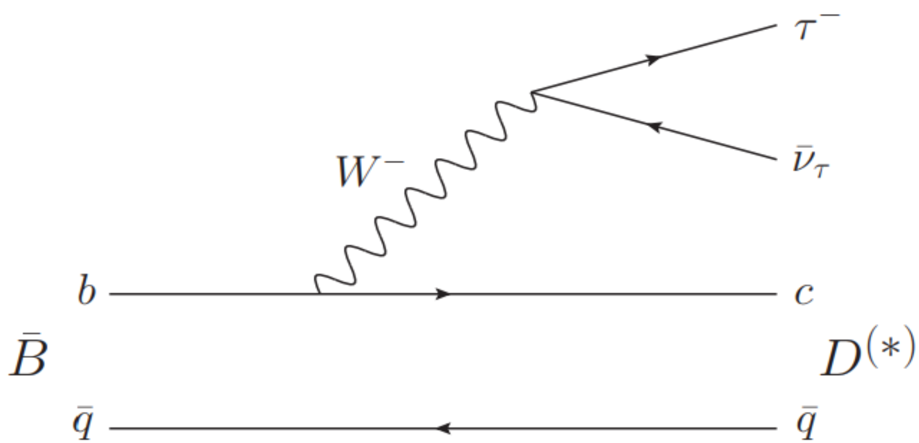
JOINT INSTITUTE FOR NUCLEAR RESEARCH


NOBEL

Outline

- Introduction
- Semileptonic decays
- Rare decays
- Conclusion

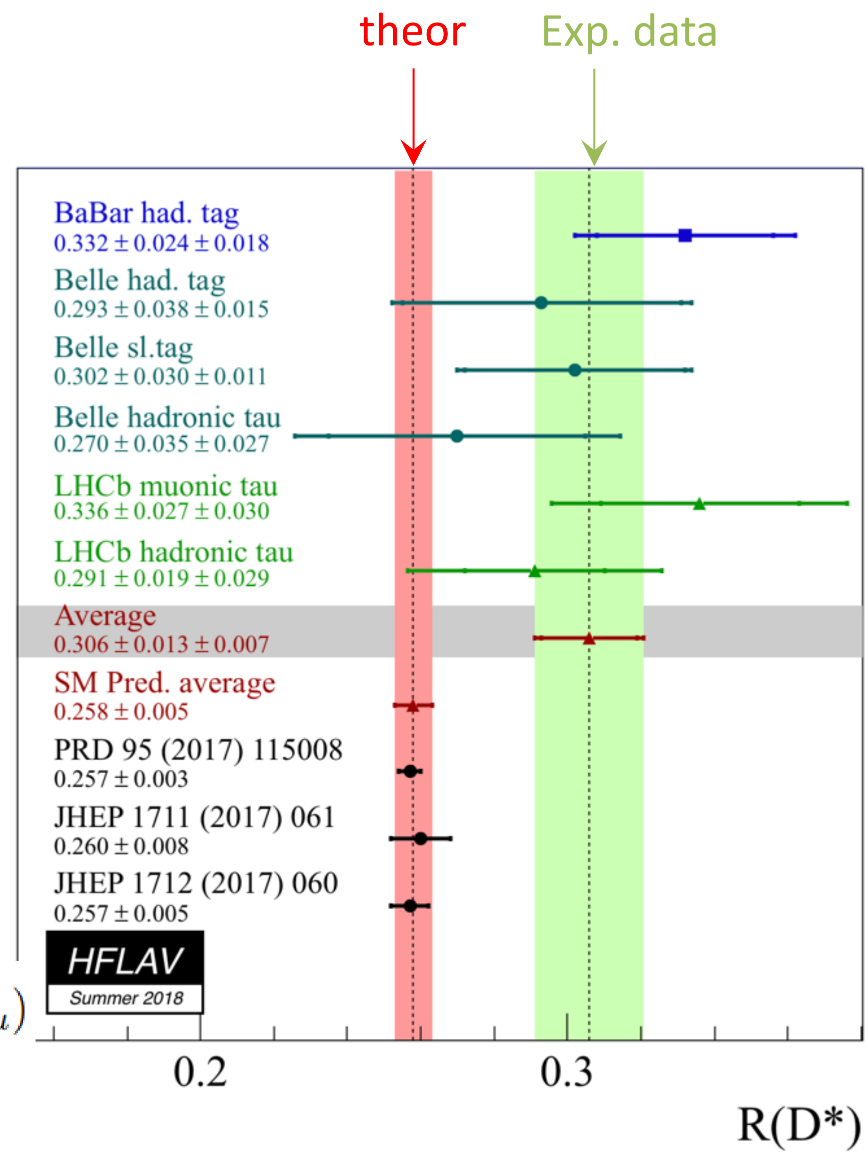
Semileptonic decays of B meson



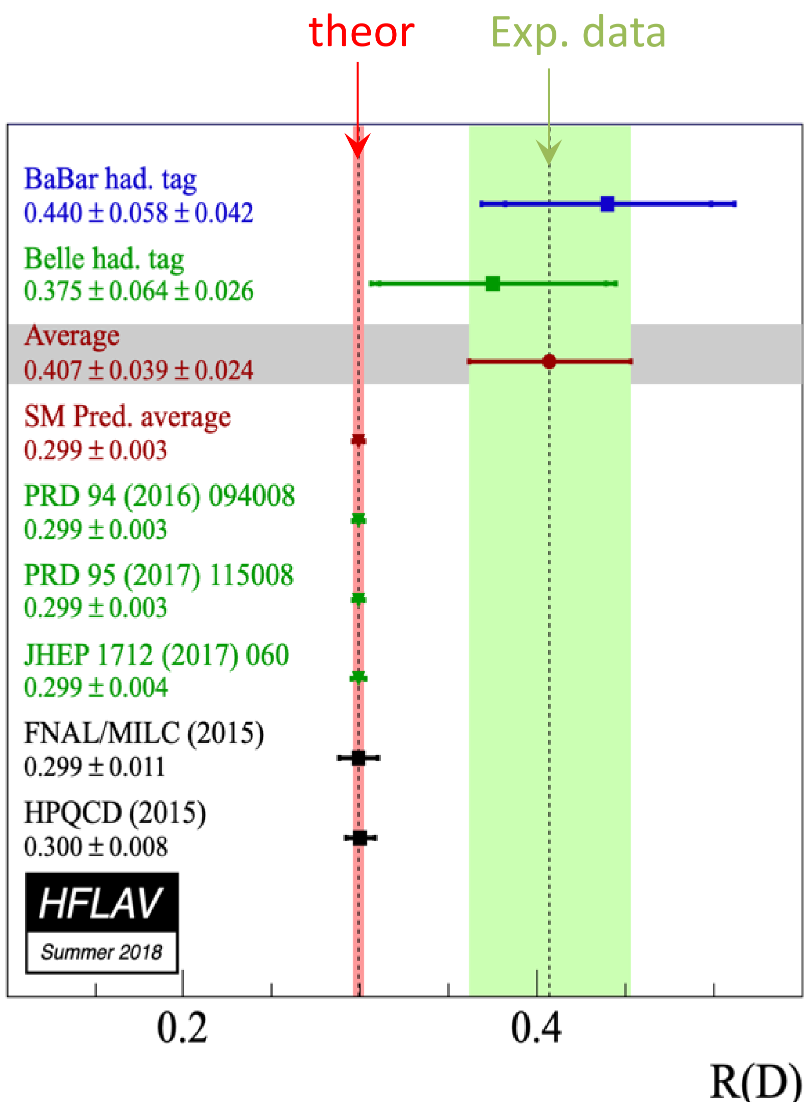
Tree-level decays b → cνℓ:

- abundant
- very well known in the SM
- violation of the lepton universality in tau sector

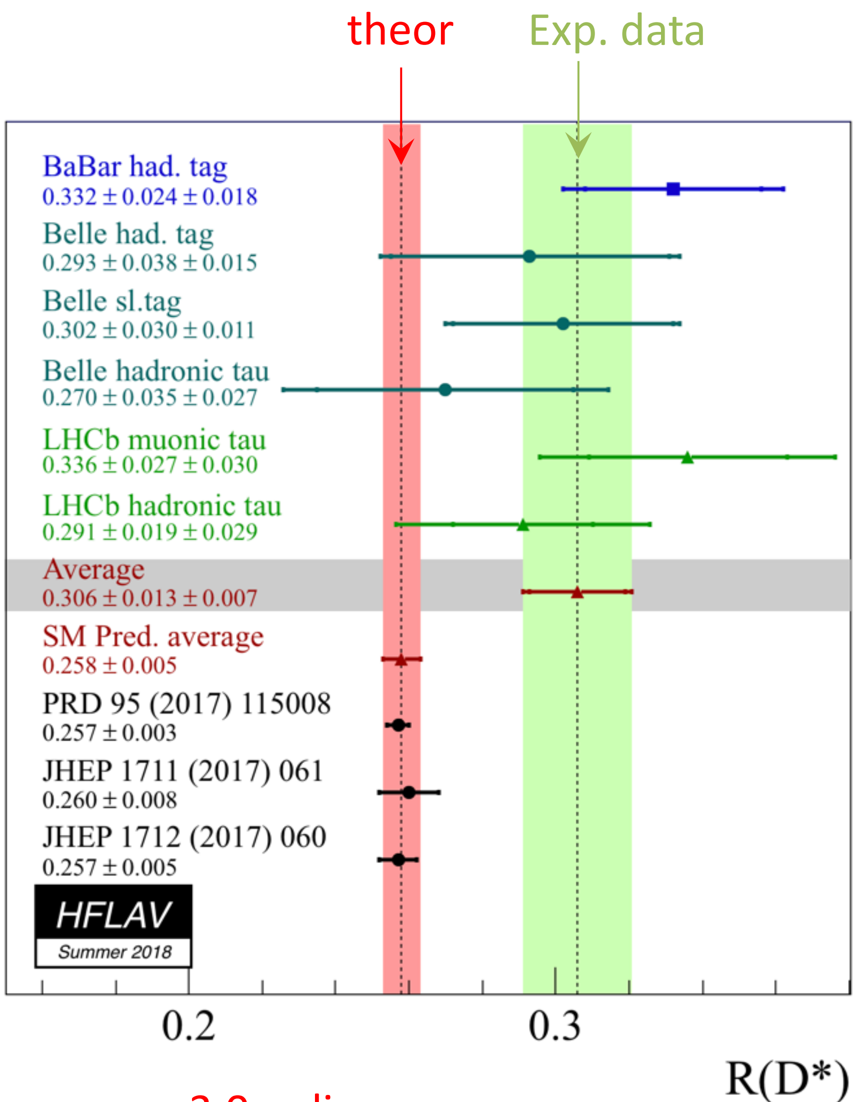
$$R(D^{(*)}) \equiv \mathcal{B}(\bar{B}^0 \to D^{(*)}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \to D^{(*)}\mu^-\bar{\nu}_\mu)$$



Semileptonic decays of B meson

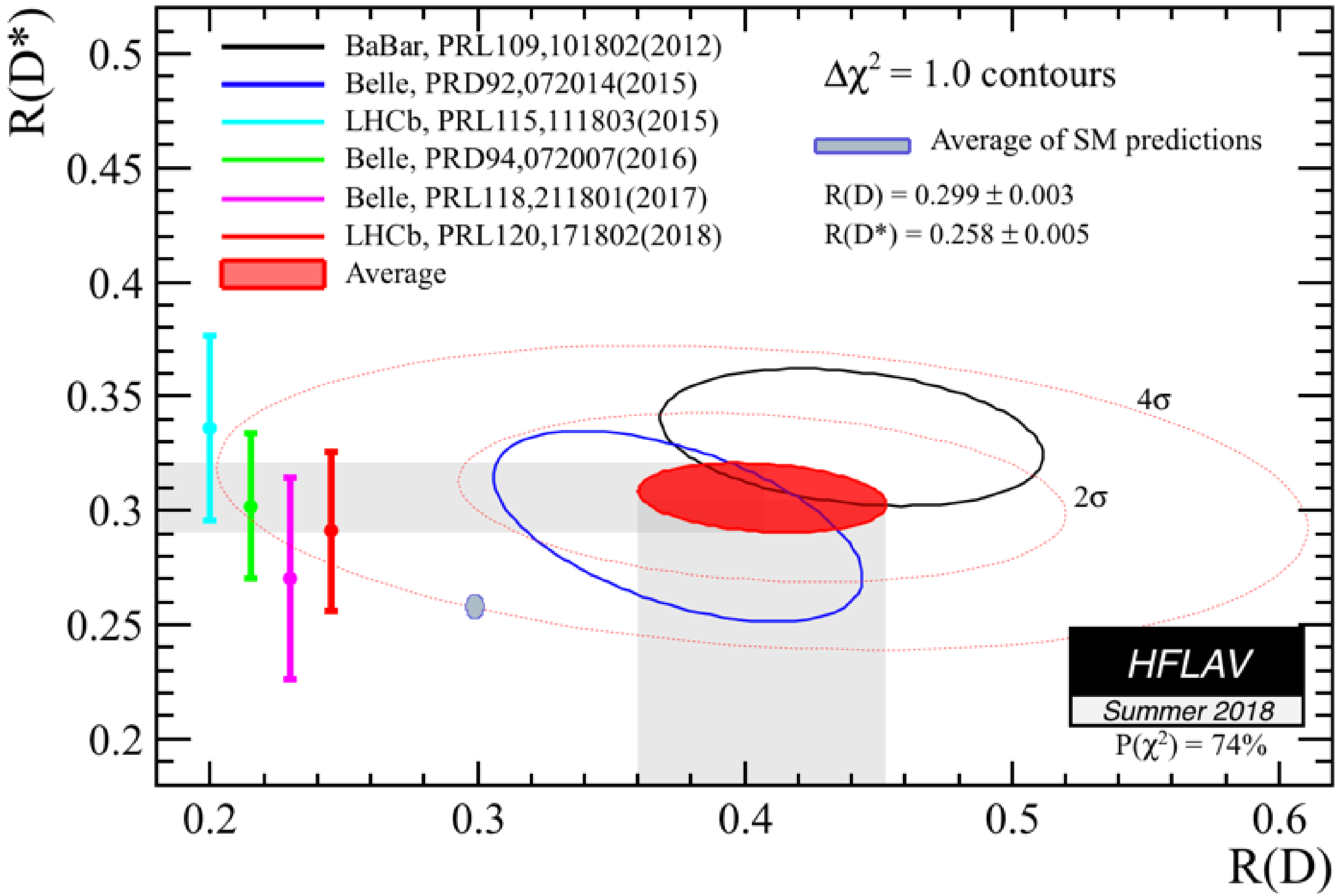


2.3 σ discreapancy



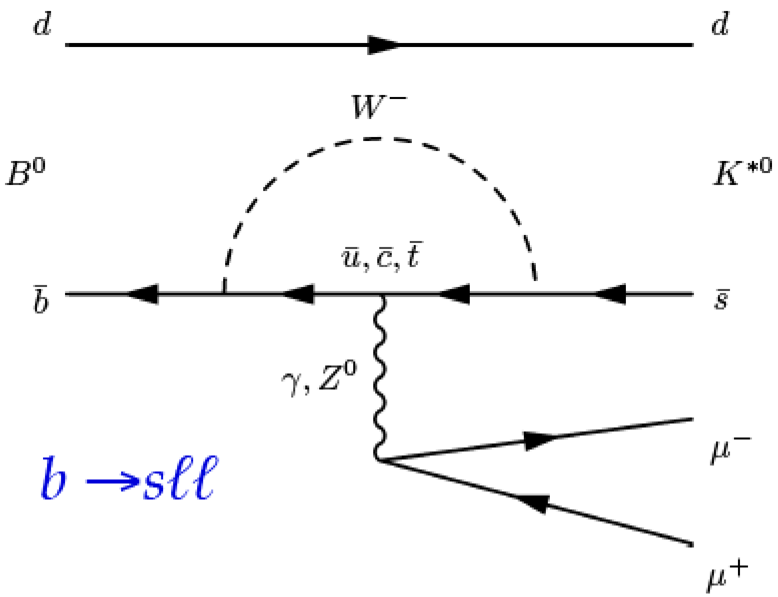
3.0 σ discreapancy

Combined data of R_D & R_{D*}



- (One of) the largest discrepancies between SM and measurement
- Up to 4.1 σ disagreement

Rare decays of B meson

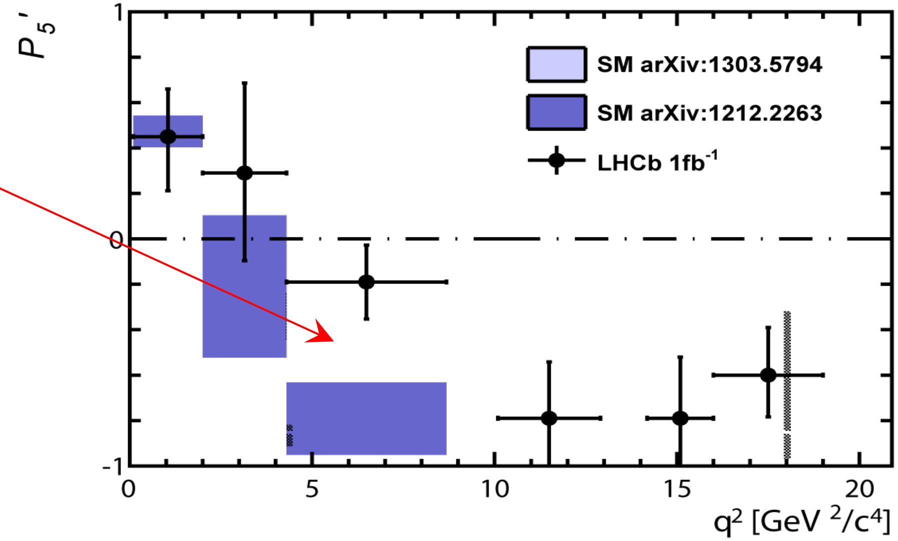


Loop-level decays $b \rightarrow s\ell^+\ell^-$:

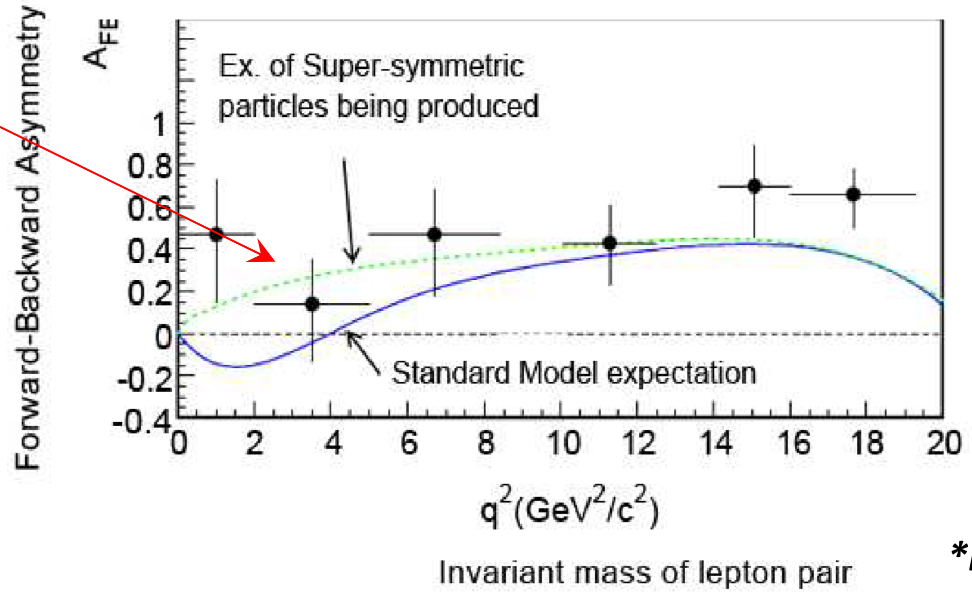
- forbidden at tree-level in SM
- sensitive to NP contributions in loops

Rare decays of B meson

3.4 σ

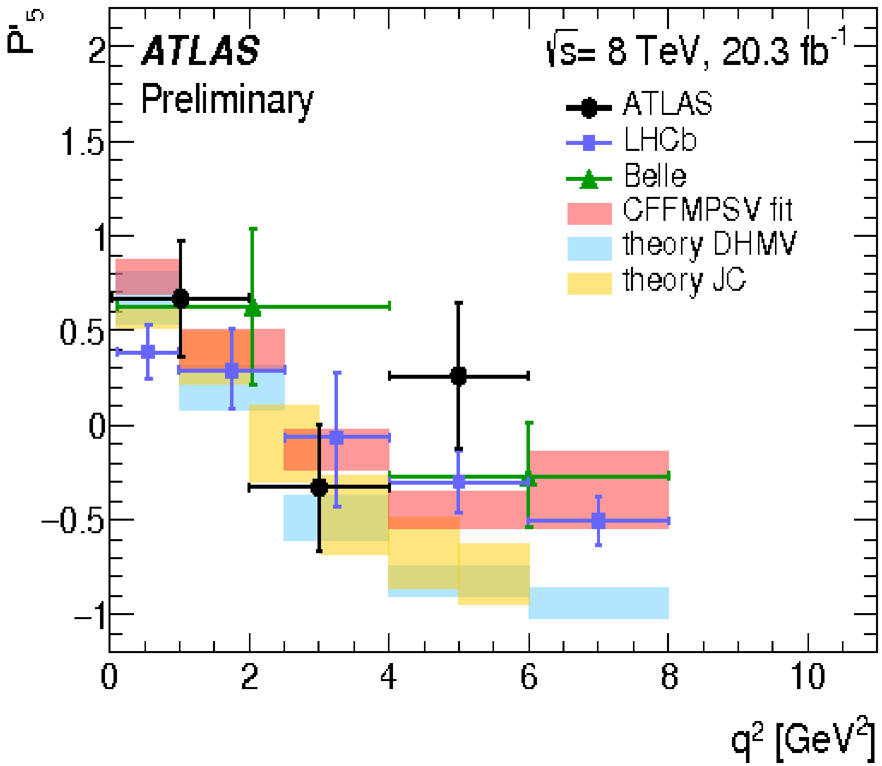


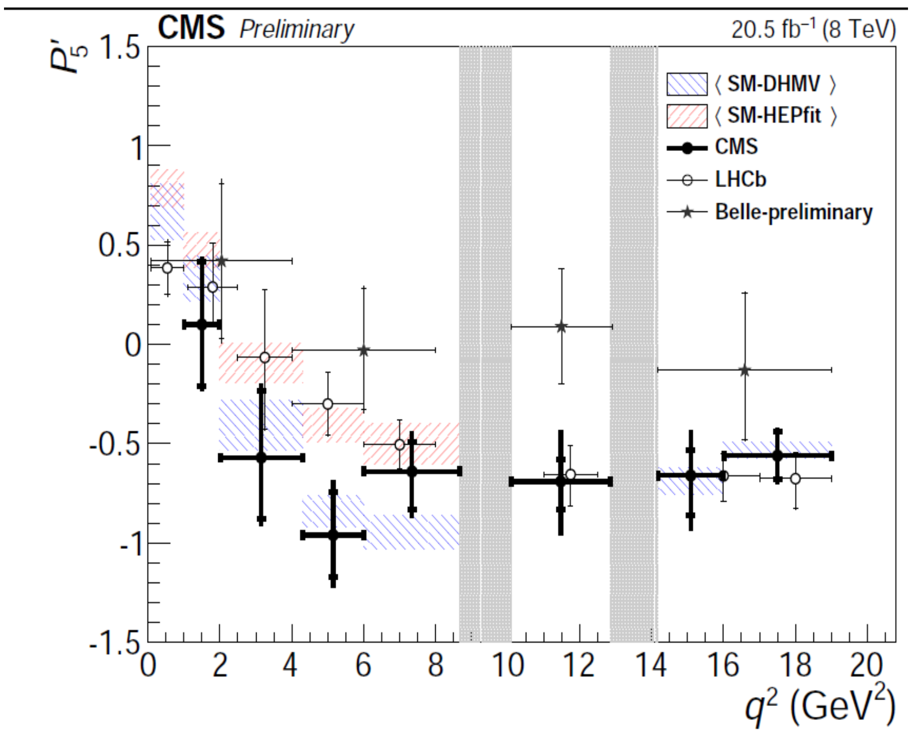
~ 3 σ



***Phys.Rev.Lett. 111 (2013) 191801**

Angular observables

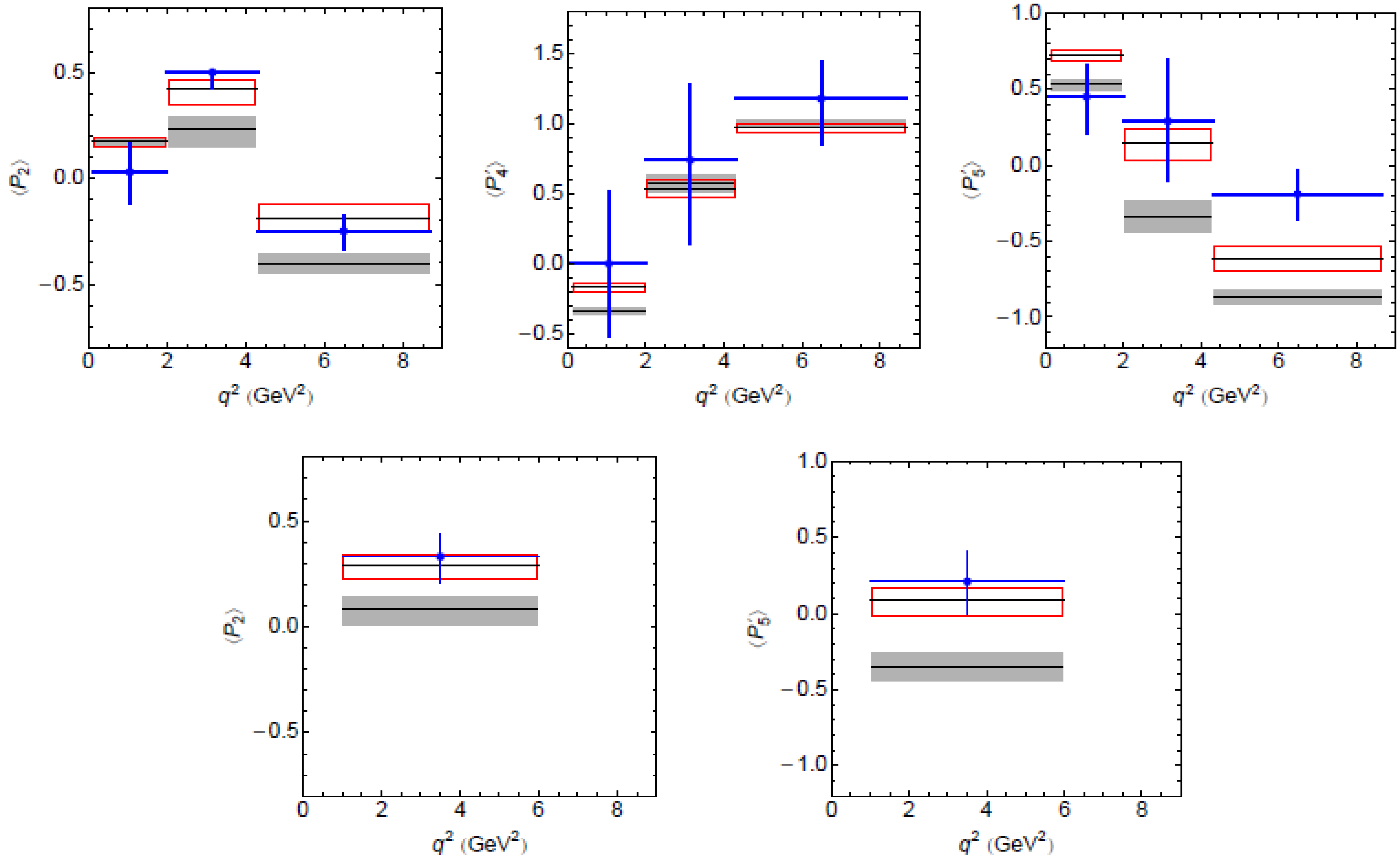




ATLAS measurement differs by 2.7σ from the SM prediction.

CMS results are consistent with SM prediction and other measurements

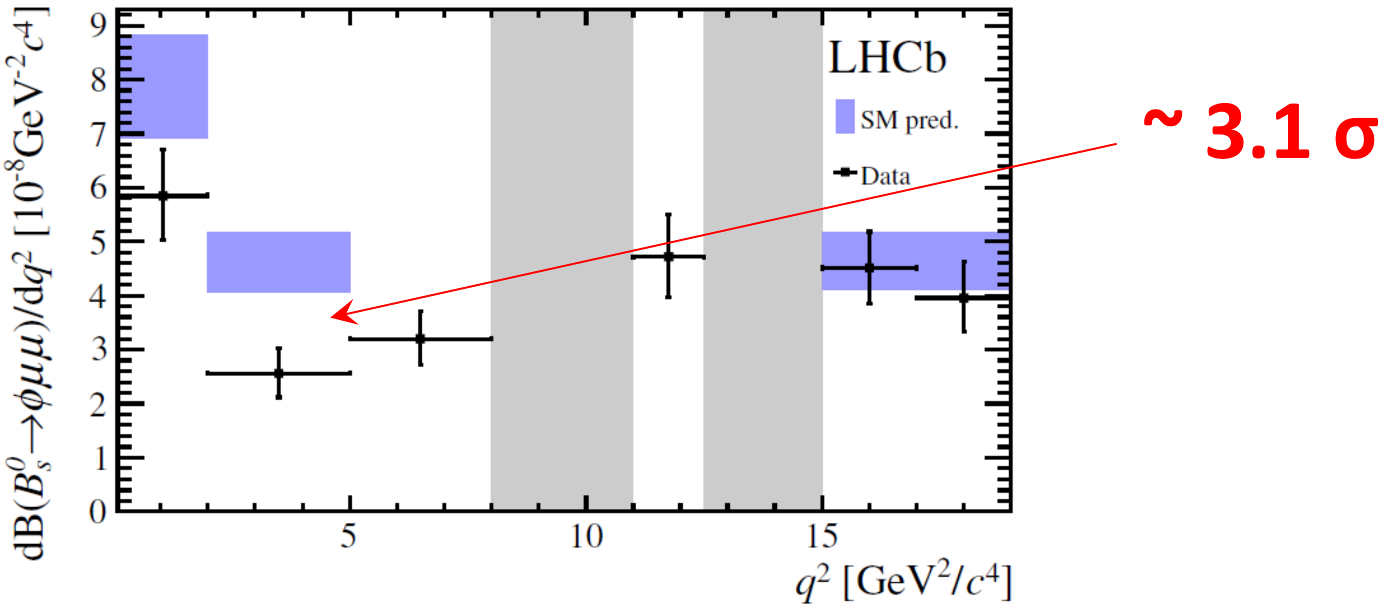
New physics in $b \to s$ transition



Comparison between the SM predictions (gray boxes), the experimental measurements (blue data points) and the predictions for the scenario with $\mathcal{C}_9^{\rm NP} = -1.5$ and other $\mathcal{C}_i^{\rm NP} = 0$ (red squares).

J. Matias, Phys. Rev. D 86 (2012) 094024

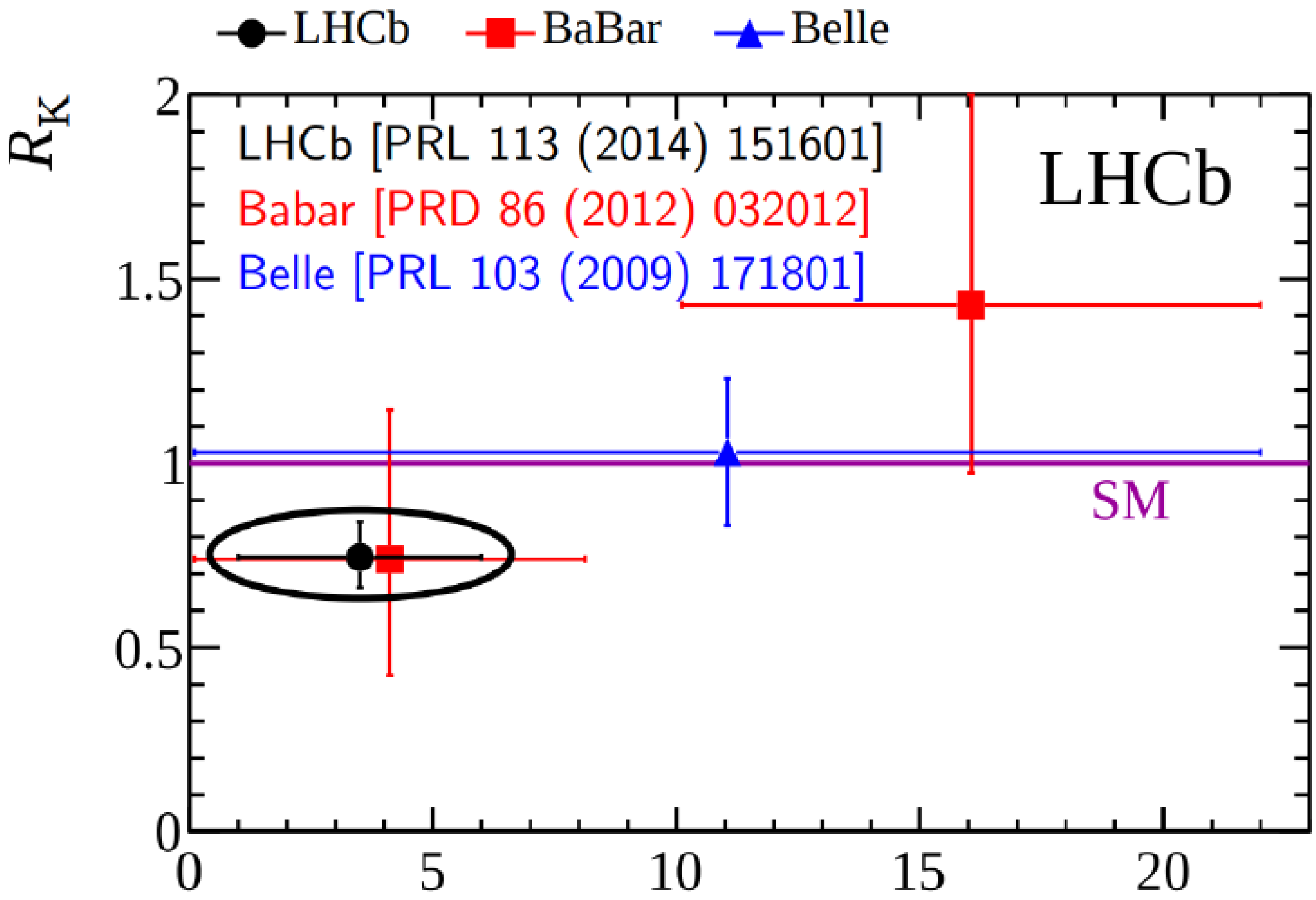
$B^0{}_s \rightarrow \varphi\ell^+\ell^-$



**R. Aaij et al. (LHCb Collaboration) JHEP 09 (2015) 179*

**SM ~ Eur.Phys.J. C75 (2015) 382 & JHEP 1608 (2016) 098*

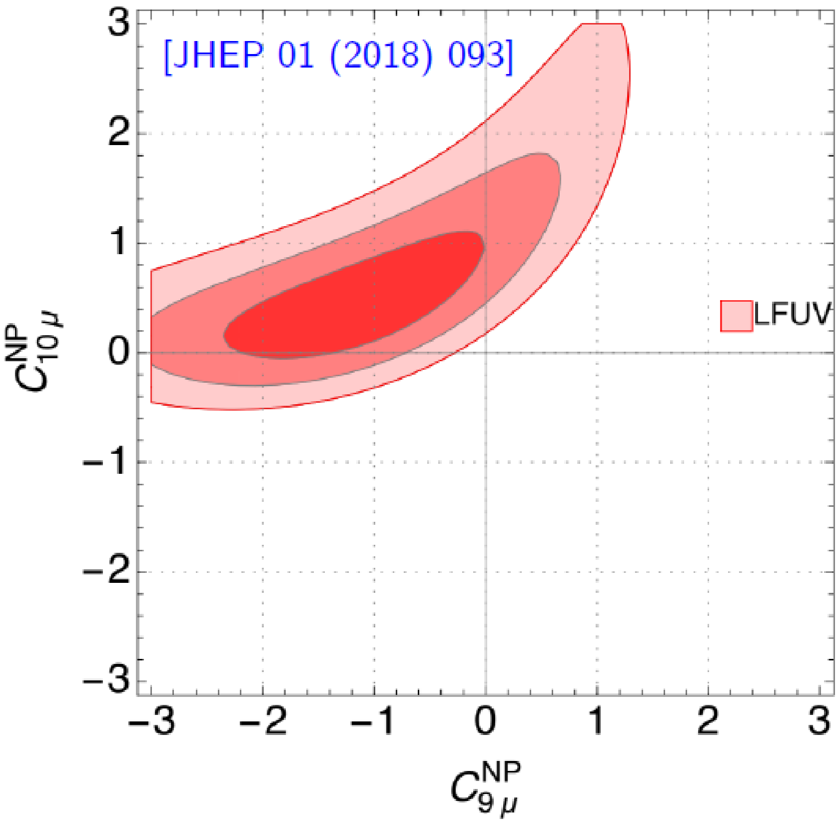
R_K

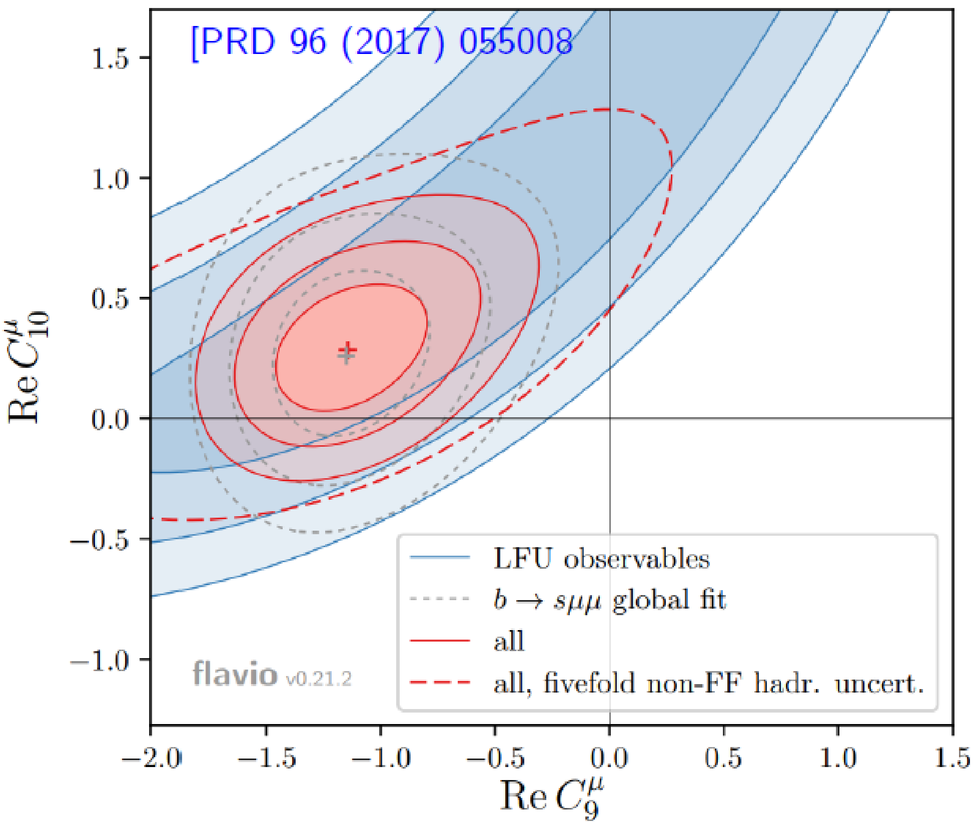


$$R_K \equiv \frac{\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \to K^+ e^+ e^-)}$$ in central q^2 region $[1,6]\mathrm{GeV}^2/c^4$

$$R_K = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst}) \sim 2.6\sigma \text{ below SM}$$

R_{K^*} and R_K





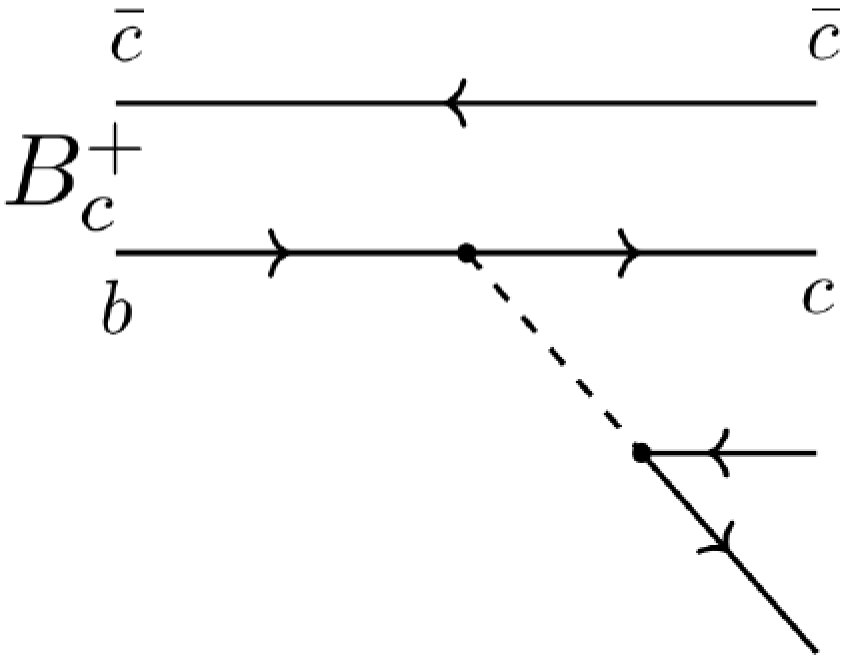
Combination of R_{K*} , R_K and [PRL 118 (2017) 111801] is ~4σ from SM

Experimental data

Relative decay rates of B_c meson.

Parameter	Measurements	Average
$\mathcal{B}(B_c^- \to J/\psi D_s^-)/\mathcal{B}(B_c^- \to J/\psi \pi^-)$	LHCb [5]: $2.90 \pm 0.57 \pm 0.24$ ATLAS [13]: $3.8 \pm 1.1 \pm 0.4$	3.09 ± 0.55
$\mathcal{B}(B_c^- \to J/\psi D_s^{*-}/\mathcal{B}(B_c^- \to J/\psi D_s^-)$	ATLAS [13]: $2.8^{+1.2}_{-0.8} \pm 0.3$ LHCb [5]: $2.37 \pm 0.56 \pm 0.10$	2.69 ± 0.78
$\mathcal{B}(B_c^- \to J/\psi D_s^{*-}/\mathcal{B}(B_c^- \to J/\psi \pi^-)$	ATLAS [13]: $10.4 \pm 3.1 \pm 1.6$	10.4 ± 3.5
$\mathcal{B}(B_c^- \to J/\psi K^-)/\mathcal{B}(B_c^- \to J/\psi \pi^-)$	LHCb [29]: $0.069 \pm 0.019 \pm 0.005$	0.069 ± 0.020
$\mathcal{B}(B_c^- \to J/\psi K^- K^+ \pi^-)/\mathcal{B}(B_c^- \to J/\psi \pi^-)$	LHCb [12]: $0.53 \pm 0.10 \pm 0.05$	0.53 ± 0.11
$\mathcal{B}(B_c^- \to J/\psi \pi^+ \pi^- \pi^-)/\mathcal{B}(B_c^- \to J/\psi \pi^-)$	LHCb [30]: $3.0 \pm 0.6 \pm 0.4$ LHCb [10]: $2.41 \pm 0.30 \pm 0.33$ CMS [31]: $2.55 \pm 0.80^{+0.33}_{-0.33}$	2.57 ± 0.35
$\mathcal{B}(B_c^- \to \psi(2S) \pi^-)/\mathcal{B}(B_c^- \to J/\psi \pi^-)$	LHCb [32]: $0.268 \pm 0.032 \pm 0.009$	0.268 ± 0.033

Semileptonic decays of B_c meson



$$\frac{\mathcal{B}(B_c^+ \to J/\psi \tau^+ \nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi \mu^+ \nu_\mu)} = 0.71 \pm 0.17(stat) \pm 0.18(syst).$$

***R. Aaij et al. Phys.Rev.Lett. 120 (2018) 121801**

Branching ratios of semileptonic B_c decays

Table 3. Branching ratios (in %) of semileptonic B_c decays into ground state charmonium states.

Mode	This work	[23]	[7]	[24, 25]	[26]	[27]	[28]
$B_c^- \to \eta_c \ell \nu$	0.95	0.81	0.98	0.75	0.97	0.59	0.44
$B_c^- \to \eta_c \tau \nu$	0.24	0.22	0.27	0.23		0.20	0.14
$B_c^- \to J/\psi \ell \nu$	1.67	2.07	2.30	1.9	2.35	1.20	1.01
$B_c^- \to J/\psi \tau \nu$	0.40	0.49	0.59	0.48		0.34	0.29
$B_c^- \to \overline{D}^- \ell \nu$	0.0033	0.0035	0.018		0.004	0.006	0.0032
$B_c^- \to \overline{D}^- \tau \nu$	0.0021	0.0021	0.0094	0.002			0.0022
$B_c^- \to \overline{D}^{*-} \ell \nu$	0.006	0.0038	0.034		0.018	0.018	0.011
$B_c^- \to \overline{D}^{*-} \tau \nu$	0.0034	0.0022	0.019	0.008			0.006

***** A. Issadykov, Mikhail A. Ivanov, G. Nurbakova, EPJ Web Conf. 158 (2017) 03002***

Branching ratios of semileptonic B_c decays

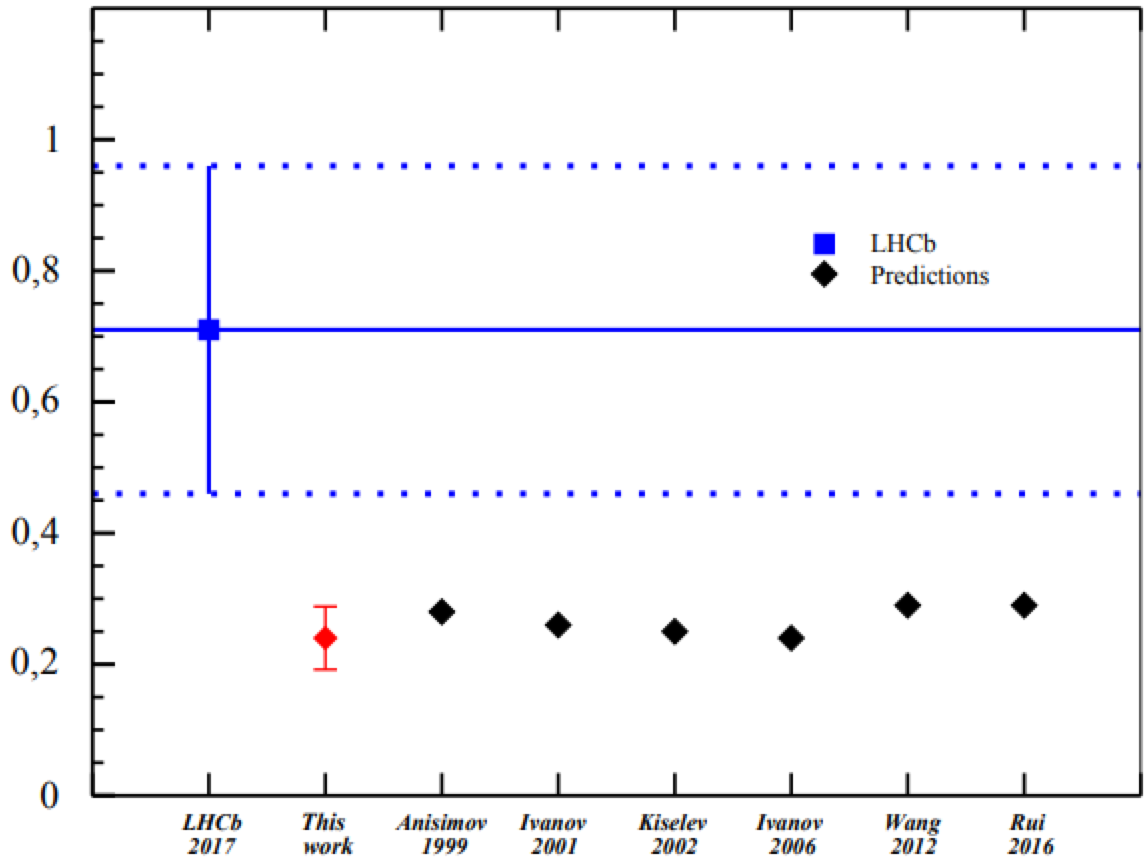
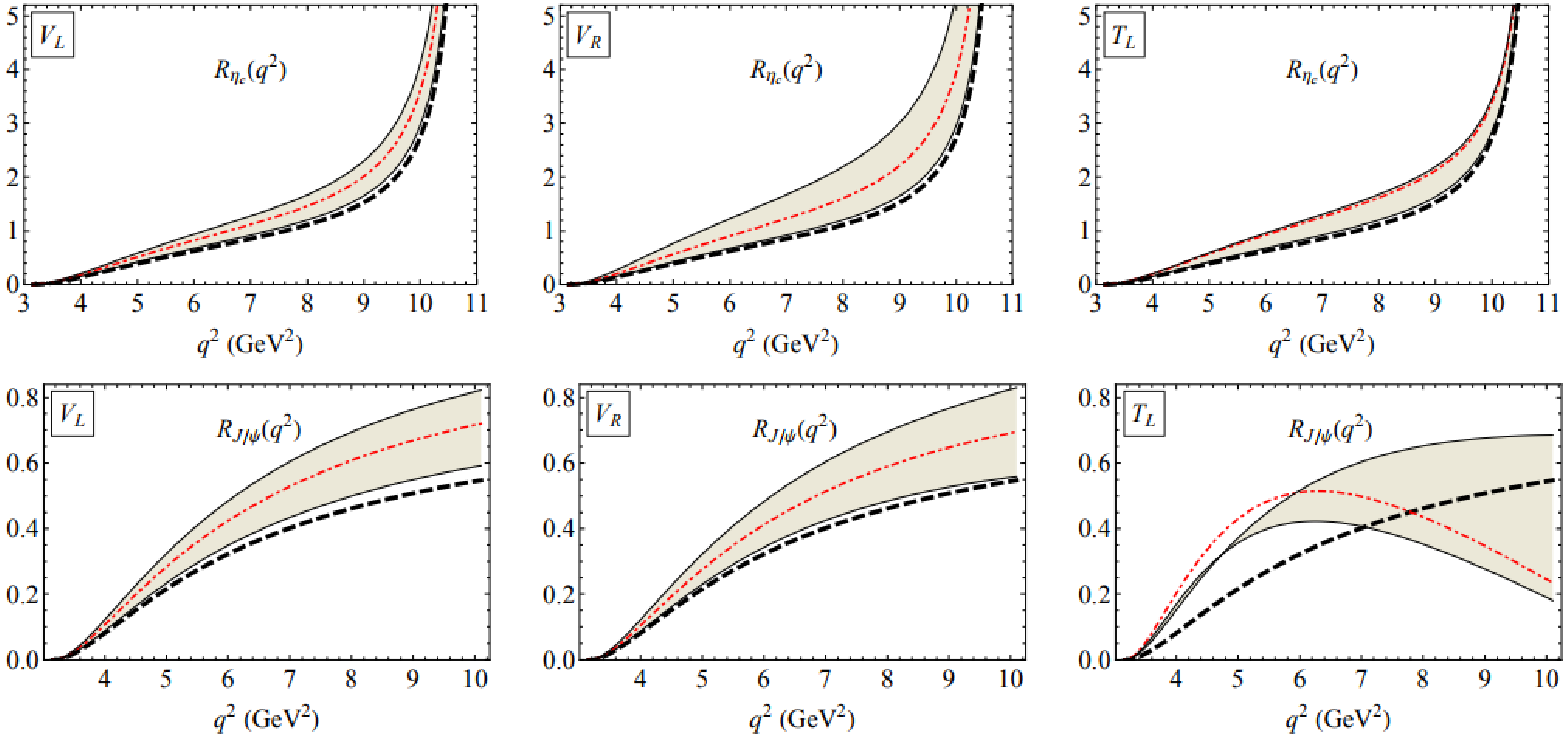


Figure 2: Theoretical predictions vs. LHCb data [15] for the ratio $\mathcal{R}_{\mathcal{J}/\psi}$. Solid line-central experimental value, dotted lines–experimental error bar.

***A. Issadykov, Mikhail A. Ivanov, Phys.Lett. B783 (2018) 178-182**

New Physics effects in semileptonic B_c decays



*C.T. Tran, Mikhail A. Ivanov, J. Körner, P. Santorelli, Phys.Rev. D97 (2018) 054014

Nonleptonic decays of B_c meson

LHCb collaboration(nonleptonic):

$$\frac{B(B_c^+ \to J/\psi K^+)}{B(B_c^+ \to J/\psi \pi^+)} = 0.069 \pm 0.0019(stat) \pm 0.005(syst).$$

***R. Aaij et al. [LHCb Collaboration], JHEP 1309 (2013) 075**

The predicted ratio of these branching fractions is proportional to

$$\frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} \approx \left| \frac{V_{us} f_{K^+}}{V_{ud} f_{\pi^+}} \right|^2 = 0.077$$

$$\frac{B(B_c^+ \to J/\psi K^+)}{B(B_c^+ \to J/\psi \pi^+)} = 0.079 \pm 0.007(stat) \pm 0.003(syst).$$

***R. Aaij et al. [LHCb Collaboration], JHEP 1609 (2016) 153**

Ratios of branching fractions

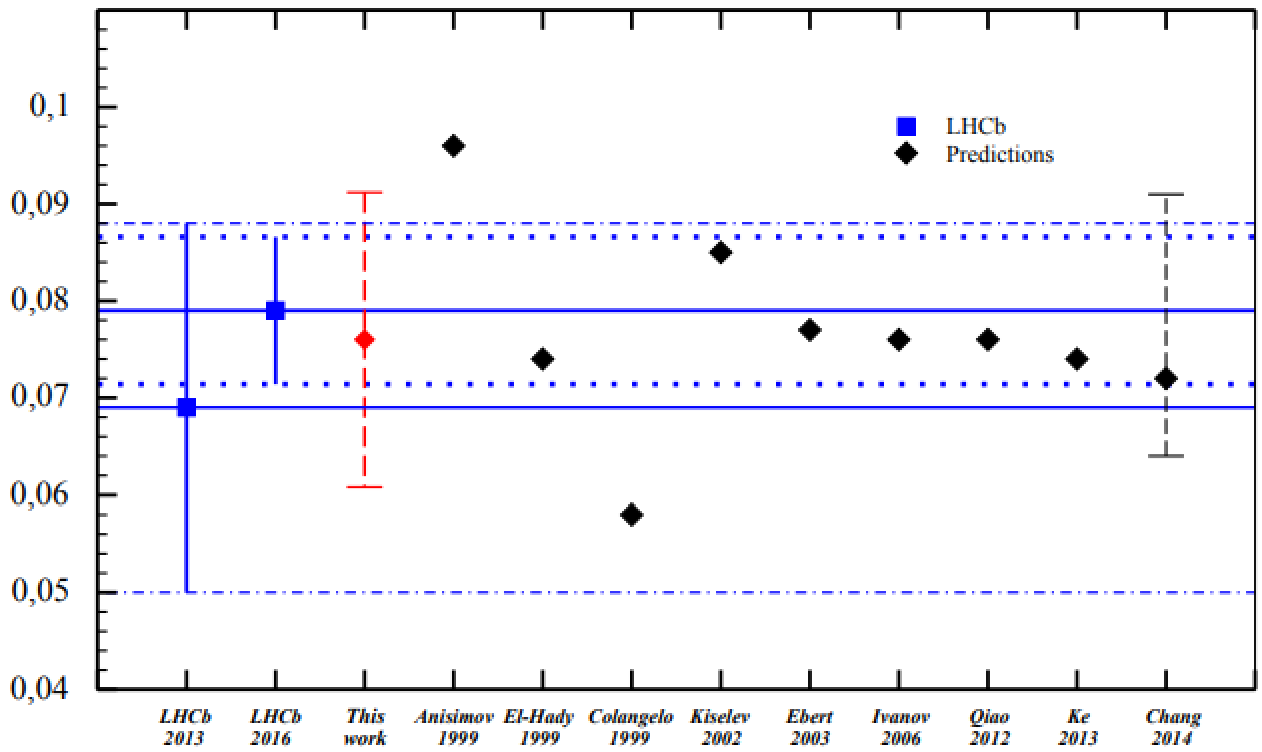


Figure 3: Theoretical predictions vs. LHCb data [10] and [11] for the ratio $\mathcal{R}_{K^+/\pi^+}$. Two solid lines- central experimental values, dash-dotted lines–experimental error bar from [10], dotted lines–experimental error bar from [11].

**A. Issadykov, Mikhail A. Ivanov, Phys.Lett. B783 (2018) 178-182*

Ratios of branching fractions

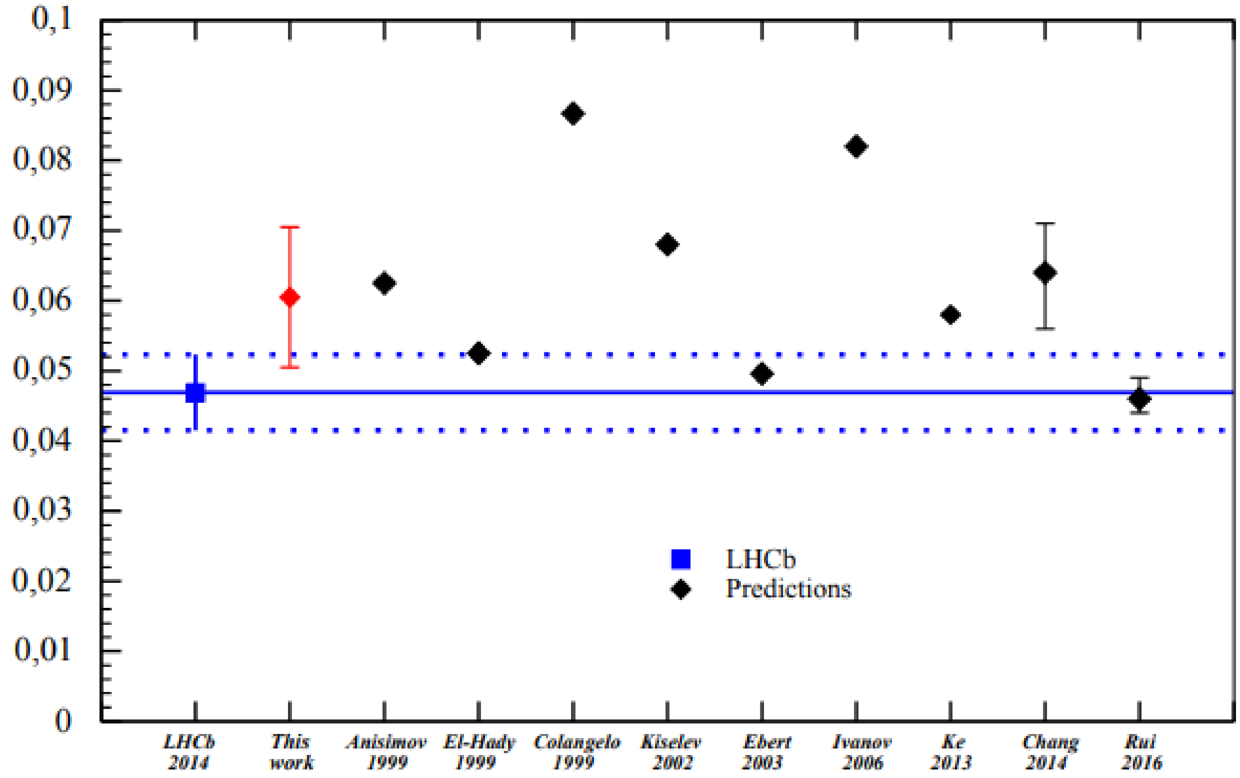


Figure 4: Theoretical predictions vs. LHCb data [1] for the ratio $\mathcal{R}_{\pi^+/\mu^+\nu_\mu}$. Solid line–central experimental value, dotted lines–experimental error bar.

A. Issadykov, Mikhail A. Ivanov, Phys.Lett. B783 (2018) 178-182

Nonleptonic decays of B_c meson

Ref.	$\mathcal{R}_{\pi^+/\mu^+\nu}$	$\mathcal{R}_{K^+/\pi^+}$	$\mathcal{R}_{\eta_c}$	$\mathcal{R}_{J/\psi}$
LHCb [1]	0.0469 ± 0.0054			
LHCb[10]		0.069 ± 0.019		
LHCb [11]		0.079 ± 0.0076		
LHCb[15]				0.71 ± 0.25
This work	0.0605 ± 0.012	0.076 ± 0.015	0.26 ± 0.05	0.24 ± 0.05
[3]	0.0525	0.074		
[4]	0.0866	0.058		
[5]	0.0625	0.096	0.34	0.28
[6]	0.058	0.075		
[7]	0.068	0.085	0.31	0.25
[8]	0.0496	0.077		
[9]	0.082	0.076	0.27	0.24
[14]		0.075		
[16]	$0.064^{+0.007}_{-0.008}$	$0.072^{+0.019}_{-0.008}$		
[18, 27]	$0.046^{+0.003}_{-0.002}$	0.082	0.63 ± 0.0	$0.29^{+0.01}_{-0.00}$
[19]			0.31	0.29
[22]			0.28	0.26

Effective Hamiltonian

In the SM, the effective Hamiltonian for the $b \to q\ell^+\ell^-$ decay can be written as

$$\mathcal{H}_{eff}^{SM} = -\frac{4G_F}{\sqrt{2}} V_{tq} V_{tb}^* \left\{ \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu) + \lambda_u^* \sum_{i=1}^{2} C_i(\mu) [\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu)] \right\},$$

where $\lambda_u^* \equiv \frac{V_{ub}^* V_{uq}}{V_{tb}^* V_{tq}}$, $\qquad q = s, d$

$b \to s$ *transition :*

$\lambda_{ts} = V_{tb}^* V_{ts} = 0.041$ $\qquad \lambda_{us} = V_{ub}^* V_{us} = 0.00088$

$b \to d$ *transition :*

$\lambda_{td} = V_{tb}^* V_{td} = 0.00825$ $\qquad \lambda_{ud} = V_{ub}^* V_{ud} = 0.00384$

Effective Hamiltonian

In the SM, the effective Hamiltonian for the $b \to q\ell^+\ell^-$ decay can be written as

$$\mathcal{H}_{eff}^{SM} = -\frac{4G_F}{\sqrt{2}} V_{tq} V_{tb}^* \left\{ \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu) + \lambda_u^* \sum_{i=1}^{2} C_i(\mu) [\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu)] \right\},$$

where $\lambda_u^* \equiv \frac{V_{ub}^* V_{uq}}{V_{tb}^* V_{tq}}, \qquad q = s, d$

$$\mathcal{O}_1^u = (\bar{q}_{a_1} \gamma^\mu P_L u_{a_2})(\bar{u}_{a_2} \gamma_\mu P_L b_{a_1}), \quad \mathcal{O}_2^u = (\bar{q} \gamma^\mu P_L u)(\bar{u} \gamma_\mu P_L b),$$

$$\mathcal{O}_1 = (\bar{q}_{a_1} \gamma^\mu P_L c_{a_2})(\bar{c}_{a_2} \gamma_\mu P_L b_{a_1}), \quad \mathcal{O}_2 = (\bar{q} \gamma^\mu P_L c)(\bar{c} \gamma_\mu P_L b),$$

$$\mathcal{O}_3 = (\bar{q} \gamma^\mu P_L b) \textstyle\sum_q (\bar{q} \gamma_\mu P_L q), \quad \mathcal{O}_4 = (\bar{q}_{a_1} \gamma^\mu P_L b_{a_2}) \textstyle\sum_q (\bar{q}_{a_2} \gamma_\mu P_L q_{a_1}),$$

$$\mathcal{O}_5 = (\bar{q} \gamma^\mu P_L b) \textstyle\sum_q (\bar{q} \gamma_\mu P_R q), \quad \mathcal{O}_6 = (\bar{q}_{a_1} \gamma^\mu P_L b_{a_2}) \textstyle\sum_q (\bar{q}_{a_2} \gamma_\mu P_R q_{a_1}),$$

$$\mathcal{O}_7 = \tfrac{e}{16\pi^2} \bar{m}_b (\bar{q} \sigma^{\mu\nu} P_R b) F_{\mu\nu}, \quad \mathcal{O}_8 = \tfrac{g}{16\pi^2} \bar{m}_b (\bar{q}_{a_1} \sigma^{\mu\nu} P_R \mathbf{T}_{a_1 a_2} b_{a_2}) \mathbf{G}_{\mu\nu},$$

$$\mathcal{O}_9 = \tfrac{e^2}{16\pi^2} (\bar{q} \gamma^\mu P_L b)(\bar{\ell} \gamma_\mu \ell), \quad \mathcal{O}_{10} = \tfrac{e^2}{16\pi^2} (\bar{q} \gamma^\mu P_L b)(\bar{\ell} \gamma_\mu \gamma_5 \ell),$$

Effective Hamiltonian

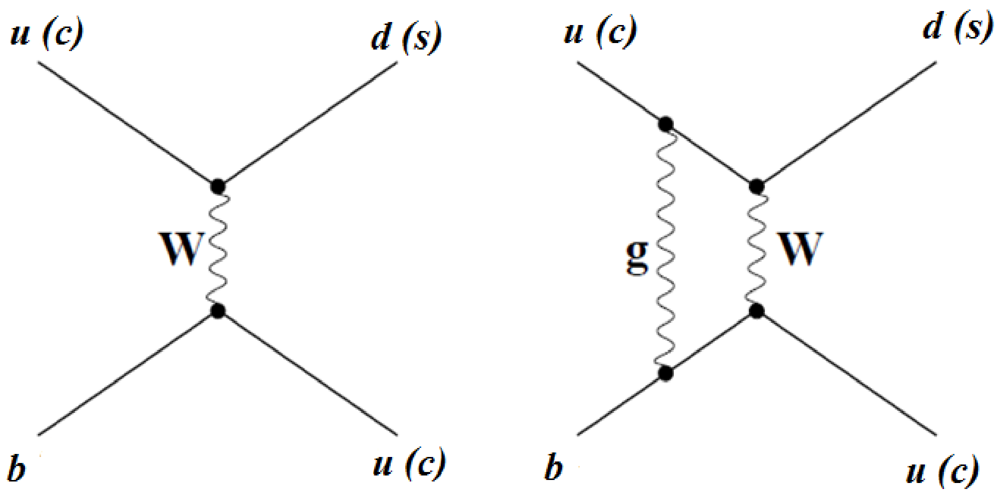
Effective Hamiltonian

Using the operator product expansion (OPE) formalism and renormalization group techniques, the effective Hamiltonian of the weak decays is derived.

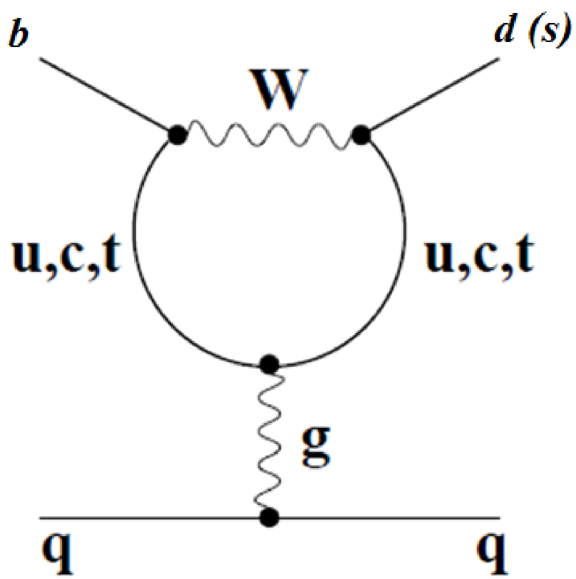
$$A(f \to i) = \langle f|\, \mathcal{H}_{\rm eff}\, |i\rangle = \frac{G_F}{\sqrt{2}} \lambda_{\rm CKM} \sum_k \underbrace{C_k(\mu)}_{\rm SD} \underbrace{\langle f|Q_k(\mu)|i\rangle}_{\rm LD}$$

- SD = Short-Distance contributions
- LD = Long-Distance contributions
- The Wilson coefficients $C_i(\mu)$ are calculated by using "matching"the full and effective theories, and the renormalization group.
- $Q_k(\mu)$ are the local operators generated by electroweak interactions and QCD
- The problem is to evaluate the matrix elements $\langle f|Q_k(\mu)|i\rangle$

Effective Hamiltonian

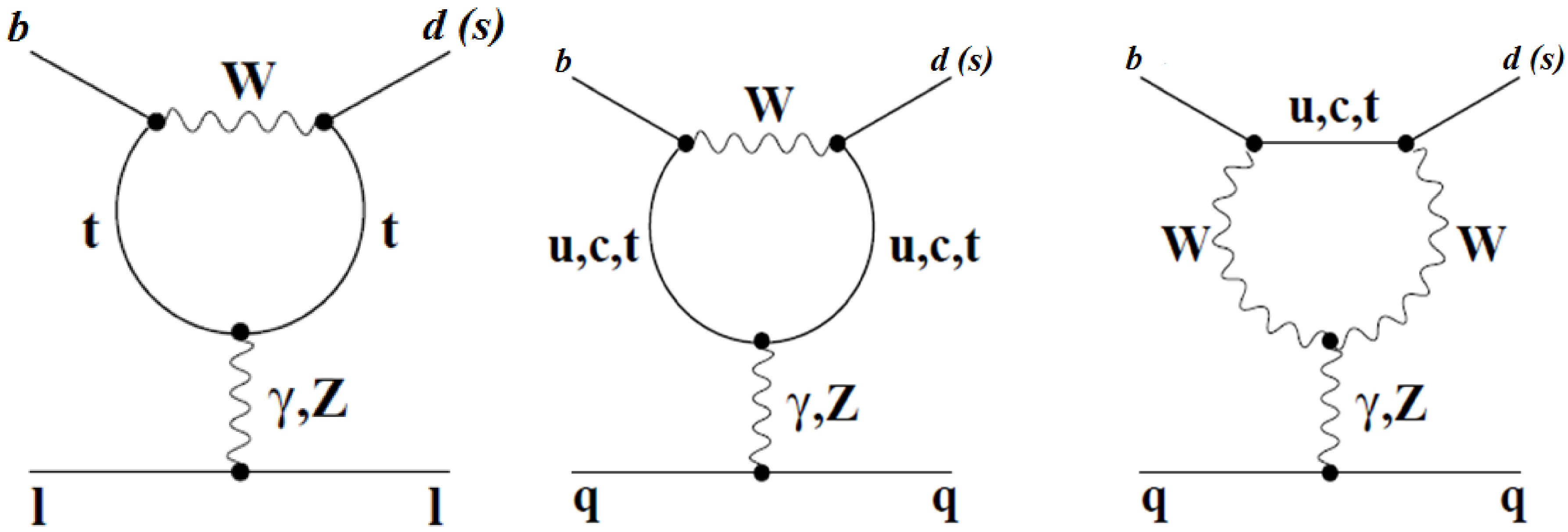


Current-current diagrams
Q_1 & Q_2 operators



QCD penguin diagram
$Q_3 - Q_6$ operators

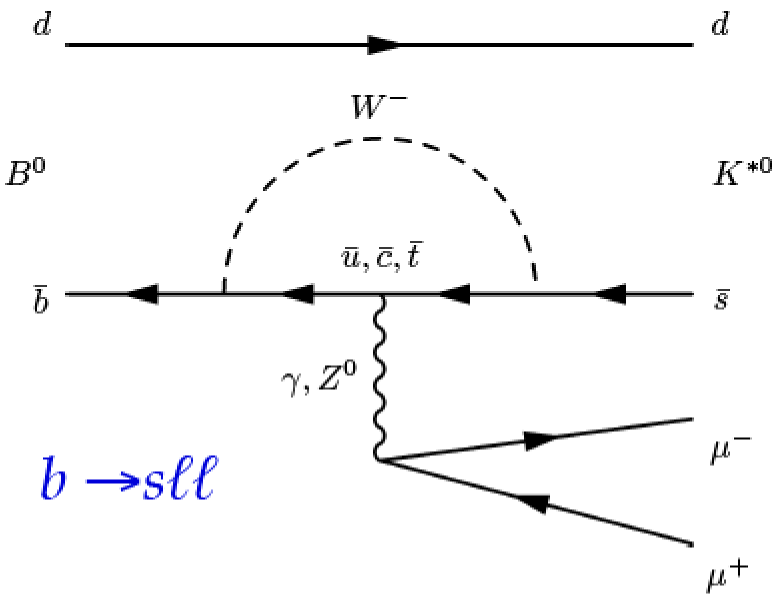
Effective Hamiltonian



Semileptonic electroweak penguin diagrams

$Q_7 - Q_{10}$ operators

$b \to s$ *transition*



Loop-level decays $b \to s\ell^+\ell^-$:

- forbidden at tree-level in SM
- sensitive to NP contributions in loops

Matrix element of $B \to K^* \ell^+ \ell^-$ transition

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha \lambda_t}{\pi} \cdot \Big\{ C_9^{\rm eff} < K^* \,|\, \bar{s}\, \gamma^\mu \, P_L \, b \,|\, B > (\bar{\ell}\gamma_\mu \ell)$$

$$- \frac{2\bar{m}_b}{q^2} C_7^{\rm eff} < K^* \,|\, \bar{s}\, i\sigma^{\mu\nu} q_\nu \, P_R \, b \,|\, B > (\bar{\ell}\gamma_\mu \ell)$$

$$+ \; C_{10} < K^* \,|\, \bar{s}\, \gamma^\mu P_L \, b \,|\, B > (\bar{\ell}\gamma_\mu \gamma_5 \ell) \Big\},$$

where $C_7^{\rm eff} = C_7 - C_5/3 - C_6$.

Wilson coefficients

$$
\begin{aligned}
C_9^{\text{eff}} &= C_9 + C_0 \left\{ h(\hat{m}_c, s) + \frac{3\pi}{\alpha^2} \kappa \sum_{V_i=\psi(1s),\psi(2s)} \frac{\Gamma(V_i \to l^+ l^-)\, m_{V_i}}{m_{V_i}{}^2 - q^2 - i m_{V_i} \Gamma_{V_i}} \right\} \\
&- \frac{1}{2} h(1, s) \left(4C_3 + 4C_4 + 3C_5 + C_6\right) \\
&- \frac{1}{2} h(0, s) \left(C_3 + 3C_4\right) + \frac{2}{9} \left(3C_3 + C_4 + 3C_5 + C_6\right) ,
\end{aligned}
$$

where $C_0 \equiv 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$. Here the charm-loop function is written as

$$
\begin{aligned}
h(\hat{m}_c, s) &= -\frac{8}{9} \ln \frac{\bar{m}_b}{\mu} - \frac{8}{9} \ln \hat{m}_c + \frac{8}{27} + \frac{4}{9} x \\
&- \frac{2}{9} (2 + x) |1 - x|^{1/2} \begin{cases} \left(\ln \left| \frac{\sqrt{1-x}+1}{\sqrt{1-x}-1} \right| - i\pi \right), & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} < 1, \\ 2 \arctan \frac{1}{\sqrt{x-1}}, & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} > 1, \end{cases} \\
h(0, s) &= \frac{8}{27} - \frac{8}{9} \ln \frac{\bar{m}_b}{\mu} - \frac{4}{9} \ln s + \frac{4}{9} i\pi,
\end{aligned}
$$

where $\hat{m}_c = \bar{m}_c / m_1$, $s = q^2 / m_1^2$ and $\kappa = 1/C_0$.

Matrix element of $B \to K^*\ell^+\ell^-$ transition

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha\lambda_t}{2\pi} \left\{T_1^\mu \left(\bar{\ell}\gamma_\mu\ell\right) + T_2^\mu \left(\bar{\ell}\gamma_\mu\gamma_5\ell\right)\right\},$$

$$T_i^\mu = T_i^{\mu\nu}\,\epsilon_{2\nu}^\dagger\,, \qquad (i = 1, 2)\,,$$

$$T_i^{\mu\nu} \;=\; \frac{1}{m_1 + m_2}\left\{-Pq\,g^{\mu\nu}\,A_0^{(i)} + P^\mu P^\nu\,A_+^{(i)} + q^\mu P^\nu\,A_-^{(i)} + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta\,V^{(i)}\right\}\,,$$

Matrix element of $B \to K^* \ell^+ \ell^-$ transition

$$T_i^{\mu\nu} = \frac{1}{m_1 + m_2} \left\{ -Pq\, g^{\mu\nu} A_0^{(i)} + P^\mu P^\nu A_+^{(i)} + q^\mu P^\nu A_-^{(i)} + i\varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta V^{(i)} \right\} ,$$

$$V^{(1)} = C_9^{\text{eff}} V + C_7^{\text{eff}} g \frac{2\bar{m}_b(m_1 + m_2)}{q^2} ,$$

$$A_0^{(1)} = C_9^{\text{eff}} A_0 + C_7^{\text{eff}} a_0 \frac{2\bar{m}_b(m_1 + m_2)}{q^2} ,$$

$$A_+^{(1)} = C_9^{\text{eff}} A_+ + C_7^{\text{eff}} a_+ \frac{2\bar{m}_b(m_1 + m_2)}{q^2} ,$$

$$A_-^{(1)} = C_9^{\text{eff}} A_- + C_7^{\text{eff}} (a_0 - a_+) \frac{2\bar{m}_b(m_1 + m_2)}{q^2} \frac{Pq}{q^2} ,$$

$$V^{(2)} = C_{10} V, \qquad A_0^{(2)} = C_{10} A_0, \qquad A_\pm^{(2)} = C_{10} A_\pm .$$

Form factors

$$\langle V(p_2,\epsilon_2)_{[\bar q_1 q_3]}\,|\,\bar q_2\, O^\mu\, q_1\,|\,P_{[\bar q_3 q_2]}(p_1)\rangle \;=$$

$$= \frac{\epsilon^\dagger_\nu}{m_1+m_2}\Big(-g^{\mu\nu}\,P\cdot q\,A_0(q^2)+P^\mu\,P^\nu\,A_+(q^2)+q^\mu\,P^\nu\,A_-(q^2)$$

$$+i\,\varepsilon^{\mu\nu\alpha\beta}\,P_\alpha\,q_\beta\,V(q^2)\Big),$$

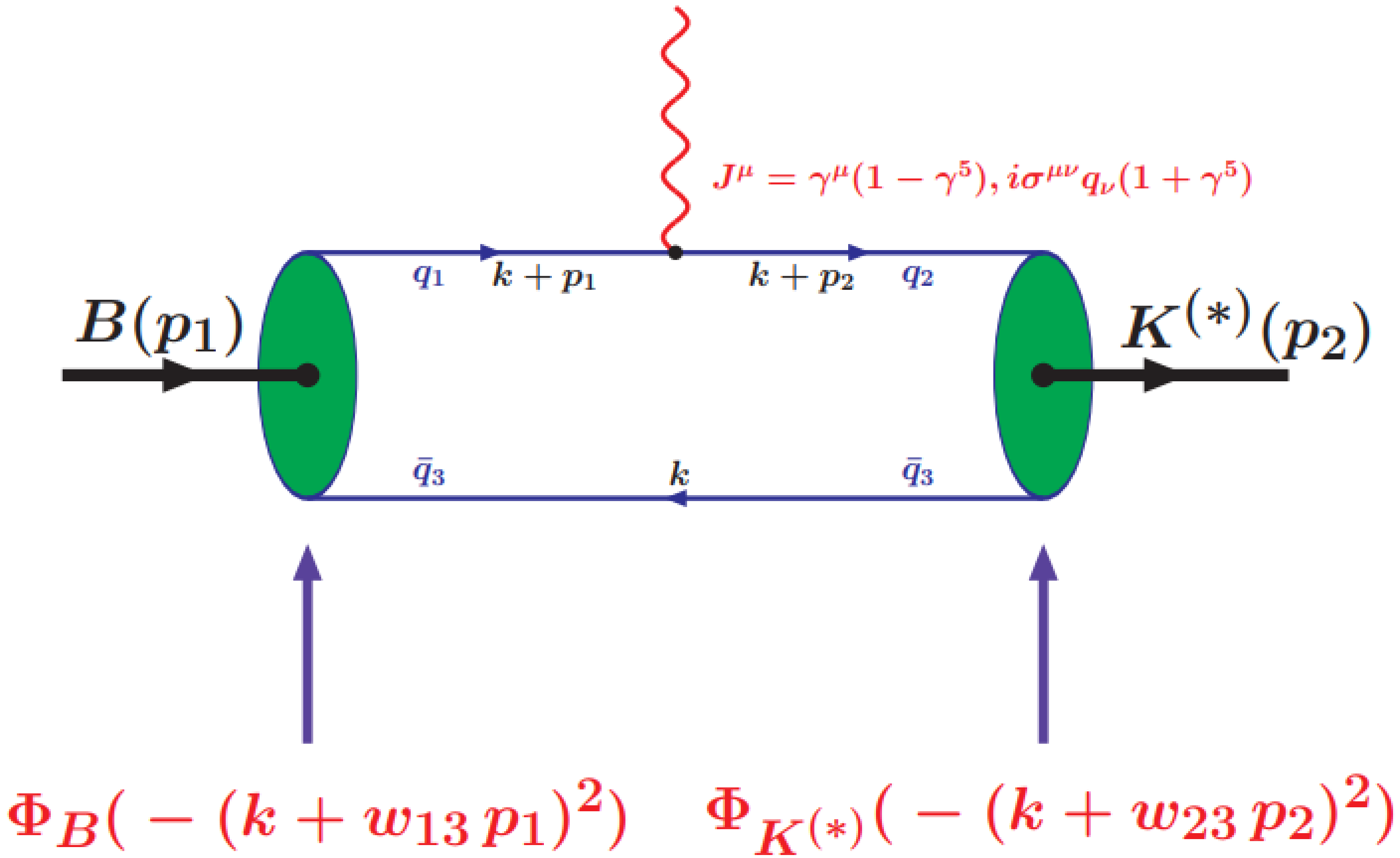
$$\langle V(p_2,\epsilon_2)_{[\bar q_1 q_3]}\,|\,\bar q_2\,(\sigma^{\mu\nu}q_\nu(1+\gamma^5))\,q_1\,|\,P_{[\bar q_3 q_2]}(p_1)\rangle \;=$$

$$= \epsilon^\dagger_\nu\Big(-(g^{\mu\nu}-q^\mu q^\nu/q^2)\,P\cdot q\,a_0(q^2)+(P^\mu\,P^\nu-q^\mu\,P^\nu\,P\cdot q/q^2)\,a_+(q^2)$$

$$+i\,\varepsilon^{\mu\nu\alpha\beta}\,P_\alpha\,q_\beta\,g(q^2)\Big).$$

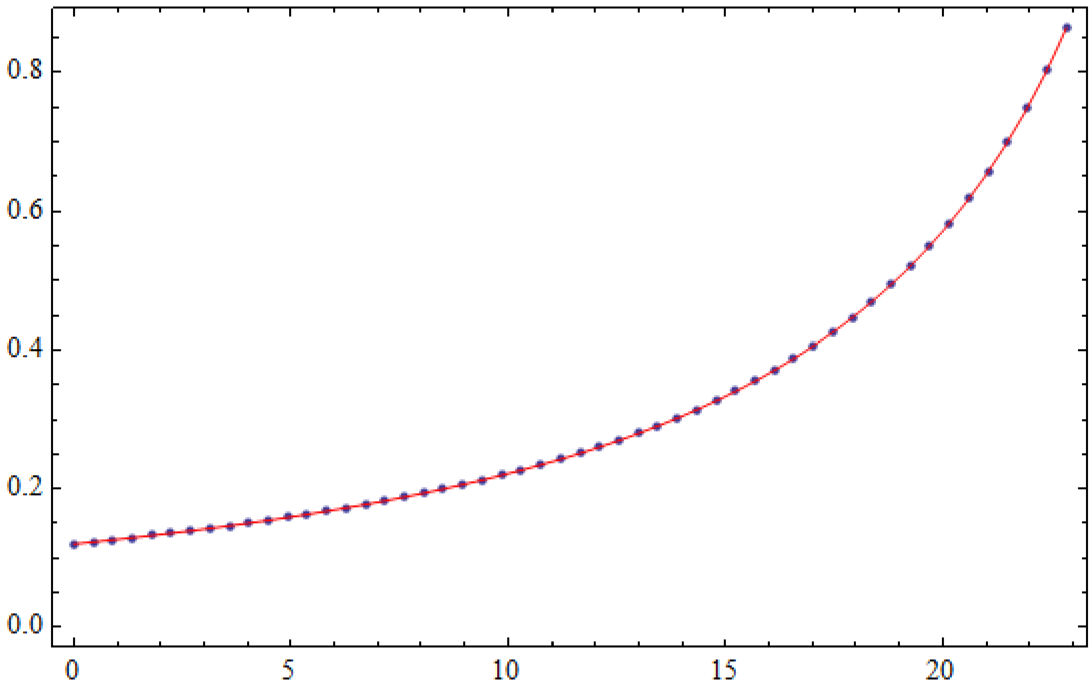
$$P\,=\,p_1+p_2,\;q\,=\,p_1-p_2,\;\epsilon_2^\dagger\cdot p_2\,=\,0,\;p_i^2\,=\,m_i^2.$$

Diagrammatic representation of the matrix elements



$$w_{ij} = m_{q_j}/(m_{q_i}+m_{q_j}) \; (i,j=1,2,3)$$

Form factors

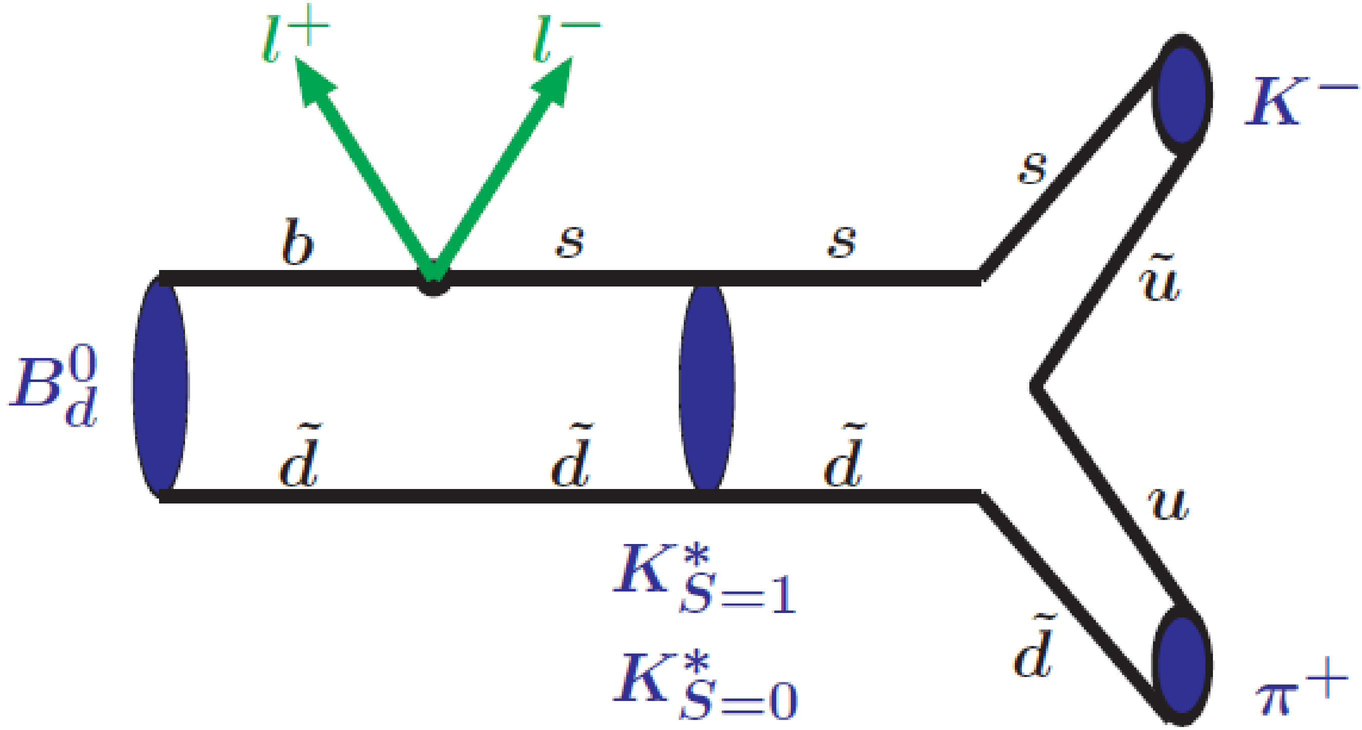


$$F(q^2) = \frac{F(0)}{1 - as + bs^2}\,, \qquad s = \frac{q^2}{m_1^2}\,.$$

$b \to s$ *transition*

Mode	Our	Others		Expt. [73–75]
$B \to K^* \mu^+ \mu^-$	12.7×10^{-7}	$(11.9 \pm 3.9) \times 10^{-7}$	[76]	$(9.24 \pm 0.93(\text{stat}) \pm 0.67(\text{sys})) \times 10^{-7}$
		19×10^{-7}	[77]	
		11.5×10^{-7}	[78]	
		14×10^{-7}	[79]	
$B \to K^* \tau^+ \tau^-$	1.35×10^{-7}	1.9×10^{-7}	[77]	–
		1.0×10^{-7}	[78]	
		2.2×10^{-7}	[79]	
$B \to K^* \gamma$	3.74×10^{-5}	11.4×10^{-5}	[80]	$(4.21 \pm 0.18) \times 10^{-5}$
		4.2×10^{-5}	[78]	
$B \to K^* \nu \bar{\nu}$	1.36×10^{-5}	1.5×10^{-5}	[78]	–
$B \to K \mu^+ \mu^-$	7.18×10^{-7}	5.7×10^{-7}	[77]	$(4.29 \pm 0.07(\text{stat}) \pm 0.21(\text{sys})) \times 10^{-7}$
		$(3.5 \pm 1.2) \times 10^{-7}$	[76]	
		4.4×10^{-7}	[78]	
		5×10^{-7}	[79]	
$B \to K \tau^+ \tau^-$	3.0×10^{-7}	1.3×10^{-7}	[77]	–
		1.0×10^{-7}	[78]	
		1.3×10^{-7}	[79]	
$B \to K \nu \bar{\nu}$	0.60×10^{-5}	0.56×10^{-5}	[78]	–

Schematic view of B→Kπ + ℓ⁺ℓ⁻ decay



** A. Issadykov, Mikhail A. Ivanov, S. Sakhiyev, Phys.Rev. D91 (2015) 074007*

b-s transition matrix elements

$$\langle S_{[\bar{q}_3 q_2]}(p_2)|\bar{q}_2 O^\mu q_1|B_{[\bar{q}_1 q_3]}(p_1)\rangle = F_+^{BS}(q^2)P^\mu + F_-^{BS}(q^2)q^\mu,$$

$$\langle S_{[\bar{q}_3 q_2]}(p_2)|\bar{q}_2(i\sigma^{\mu\nu}q_\nu(1+\gamma^5))q_1|B_{[\bar{q}_1 q_3]}(p_1)\rangle = -\frac{1}{m_1+m_2}(q^2 P^\mu - q\cdot P q^\mu)F_T^{BS}(q^2).$$

Branching fractions

Decay modes	Branching fractions			
	This work ($\Lambda_S = 1.5$ GeV)	[37]	[18]	[31]
$B_d^0 \to a_0^+(980)\mu^-\bar{\nu}_\mu$	0.52×10^{-4}	$(2.74 \pm 0.40) \times 10^{-4}$		1.84×10^{-4}
$B_d^0 \to a_0^+(980)\tau^-\bar{\nu}_\tau$	0.11×10^{-4}	$(1.31 \pm 0.23) \times 10^{-4}$		1.01×10^{-4}
$B_s^0 \to K_0^{*+}(800)\mu^-\bar{\nu}_\mu$	1.23×10^{-4}	$(2.06 \pm 0.31) \times 10^{-4}$		1.42×10^{-4}
$B_s^0 \to K_0^{*+}(800)\tau^-\bar{\nu}_\tau$	0.25×10^{-4}	$(1.07 \pm 0.19) \times 10^{-4}$		0.88×10^{-4}
$B_d^0 \to K_0^{*0}(800)\mu^+\mu^-$	3.47×10^{-7}	$(7.31 \pm 1.21) \times 10^{-7}$		
$B_d^0 \to K_0^{*0}(800)\tau^+\tau^-$	0.61×10^{-7}	$(1.33 \pm 0.36) \times 10^{-7}$		
$B_s^0 \to f_0(980)\mu^+\mu^-$	2.45×10^{-7}	$(5.14 \pm 0.78) \times 10^{-7}$	0.95×10^{-7}	5.21×10^{-7}
$B_s^0 \to f_0(980)\tau^+\tau^-$	0.42×10^{-7}	$(0.74 \pm 0.17) \times 10^{-7}$	1.1×10^{-7}	0.38×10^{-7}
$B_d^0 \to K_0^{*0}(800)\bar{\nu}\nu$	2.53×10^{-6}	$(6.30 \pm 0.97) \times 10^{-6}$		
$B_s^0 \to f_0(980)\bar{\nu}\nu$	1.79×10^{-6}	$(4.39 \pm 0.63) \times 10^{-6}$	0.87×10^{-6}	

[18] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Rev. D **53**, 3672 (1996); Phys. Rev. D **57**, 3186(E) (1998
[31] R. H. Li, C. D. Lu, W. Wang and X. X. Wang, Phys. Rev. D **79**, 014013 (2009) [arXiv:0811.2648 [hep-ph]].
[37] Z. G. Wang, Semi-leptonic $B \to S$ decays in the standard model and in the universal extra dimension mode

s-wave and p-wave contributions in the narrow width-limit

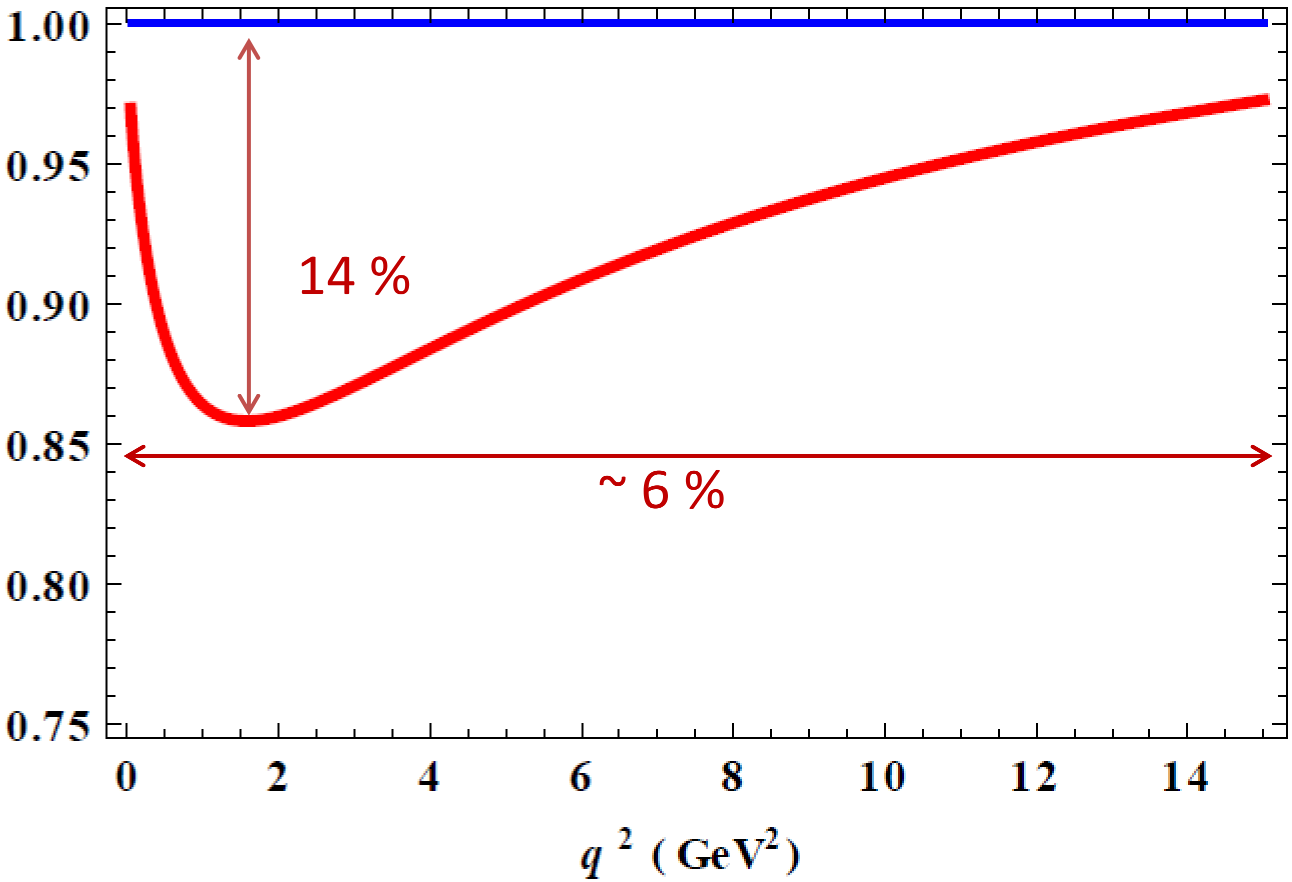
$$\int dm^2_{K\pi} |L_{K^*}(m^2_{K\pi})|^2 = \mathcal{B}(K^{*+} \to K^0 \pi^+) = \frac{2}{3} \, .$$

$$\int_{(m_{K^*}-\delta_m)^2}^{(m_{K^*}+\delta_m)^2} dm^2_{K\pi} |L_S(m^2_{K\pi})|^2 = 0.17$$

***U.G. Meissner, W. Wang, JHEP 01 (2014) 107**

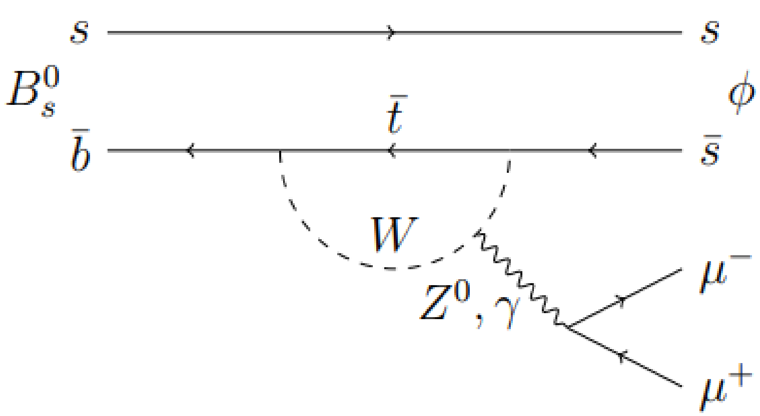
Impact of s wave contribution

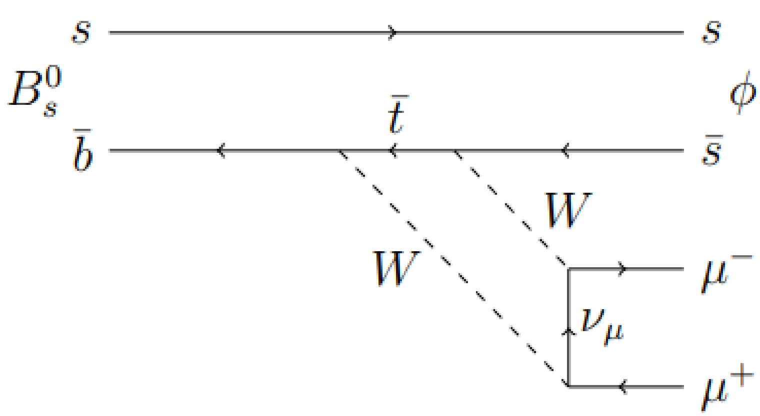
$$R(q^2) = \frac{2/3\, d\Gamma(B \to K^*(892)\mu^+\mu^-)}{2/3\, d\Gamma(B \to K^*(892)\mu^+\mu^-) + 0.17 d\Gamma(B \to K_0^*(800)\mu^+\mu^-)}$$



** A. Issadykov, Mikhail A. Ivanov, S. Sakhiyev ,Phys.Rev. D91 (2015) 074007*

$B^0{}_s \to \varphi \ell^+ \ell^-$





Examples of $b \to s$ loop diagrams contributing to the decay $B_s^0 \to \phi\mu^+\mu^-$ in the SM.

The values of branching fractions

TABLE IV. Total branching fractions.

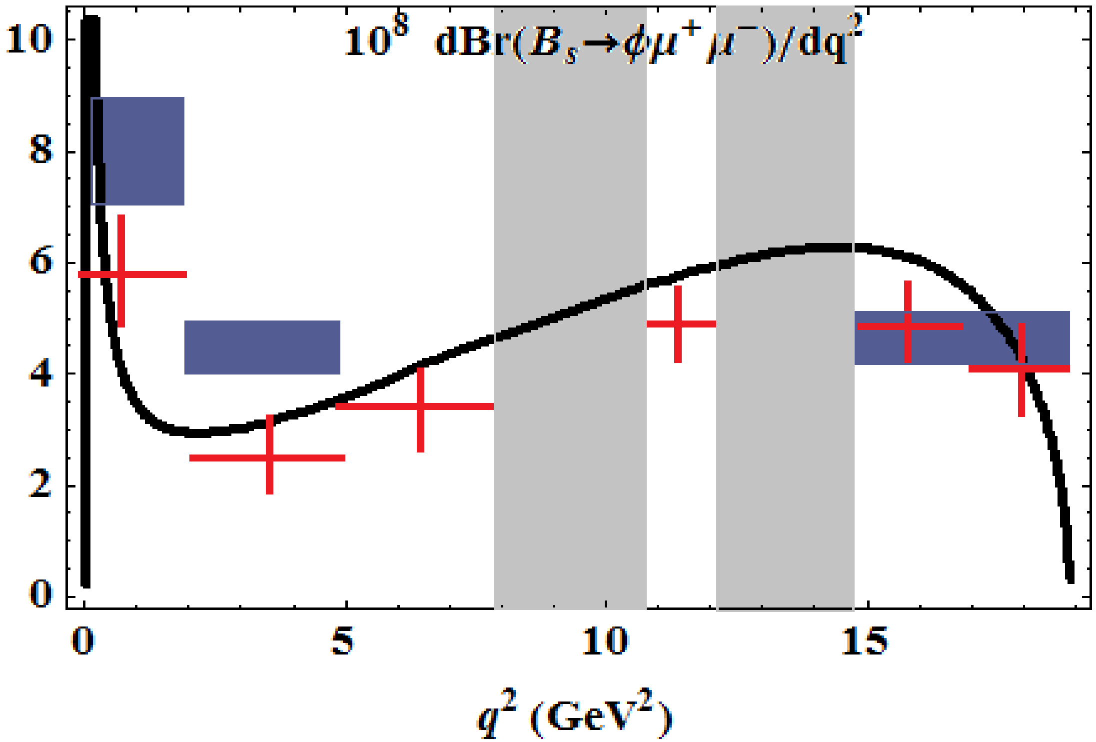
	This work	Ref. [32]	Ref. [33]	Ref. [38]	Ref. [43]	Ref. [3,44]
$10^7\mathcal{B}(B_s \to \phi\mu^+\mu^-)$	9.11 ± 1.82	11.1 ± 1.1	19.2	11.8 ± 1.1	16.4	7.97 ± 0.77
$10^7\mathcal{B}(B_s \to \phi\tau^+\tau^-)$	1.03 ± 0.20	1.5 ± 0.2	2.34	1.23 ± 0.11	1.51	
$10^5\mathcal{B}(B_s \to \phi\gamma)$	2.39 ± 0.48	3.8 ± 0.4				3.52 ± 0.34
$10^5\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	0.84 ± 0.16	0.796 ± 0.080			1.165	< 540
$10^2\mathcal{B}(B_s \to \phi J/\psi)$	0.16 ± 0.03	0.113 ± 0.016				0.108 ± 0.009

Our **Exp**

* [32] R. N. Faustov and V. O. Galkin, Eur. Phys. J. C **73**, 2593 (2013).
[33] U. O. Yilmaz, Eur. Phys. J. C **58**, 555 (2008).
[38] Y. L. Wu, M. Zhong, and Y. B. Zuo, Int. J. Mod. Phys. A **21**, 6125 (2006).
[43] C. Q. Geng and C. C. Liu, J. Phys. G **29**, 1103 (2003).
[44] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).
[3] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 09 (2015) 179.

S. Dubnicka et .al. Phys.Rev. D93 (2016), 094022

The branching ratio



***S. Dubnicka et .al. Phys.Rev. D93 (2016), 094022**

Two-loop corrections and cc-resonance contr.

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}}\lambda_t \sum_{i=1}^{10} C_i(\mu)Q_i(\mu)$$

Modify C_7^{eff} and C_9^{eff}

Condition:

$q^2/m_b^2 << 1$

$q^2/(4m_c)^2 << 1$

Areas of correction:

$1.1 < q^2 < 5.5$ ГэВ^2

$8.8 < q^2 < 22$ ГэВ^2

**C. Greub, V. Pilipp, C. Schupbach, JHEP 12 (2008) 040*

ANGULAR OBSERVABLES AND DIFFERENTIAL BRANCHING

$10^7\mathcal{B}(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	0.99 ± 0.1	0.86 ± 0.09	1.81 ± 0.36	1.11 ± 0.16
$[2, 5]$	0.90 ± 0.09	0.95 ± 0.1	1.88 ± 0.31	0.77 ± 0.14
$[5, 8]$	$--$	1.25 ± 0.13	2.25 ± 0.41	0.96 ± 0.15
$[15, 19]$	1.89 ± 0.19	1.95 ± 0.20	2.20 ± 0.16	1.62 ± 0.20
$F_L(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	0.37 ± 0.04	0.46 ± 0.05	0.46 ± 0.09	0.20 ± 0.09
$[2, 5]$	0.72 ± 0.07	0.74 ± 0.07	0.79 ± 0.03	0.68 ± 0.15
$[5, 8]$	$--$	0.57 ± 0.06	0.65 ± 0.05	0.54 ± 0.10
$[15, 19]$	0.34 ± 0.03	0.34 ± 0.03	0.36 ± 0.02	0.29 ± 0.07

S. Dubnicka et .al. Phys.Rev. D93 (2016), 094022

Effective Hamiltonian

In the SM, the effective Hamiltonian for the $b \to q\ell^+\ell^-$ decay can be written as

$$\mathcal{H}_{eff}^{SM} = -\frac{4G_F}{\sqrt{2}} V_{tq} V_{tb}^* \left\{ \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu) + \lambda_u^* \sum_{i=1}^{2} C_i(\mu)[\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu)] \right\},$$

where $\lambda_u^* \equiv \frac{V_{ub}^* V_{uq}}{V_{tb}^* V_{tq}}$, $\qquad q = s, d$

$b \to s$ *transition :*

$\lambda_{ts} = V_{tb}^* V_{ts} = 0.041$

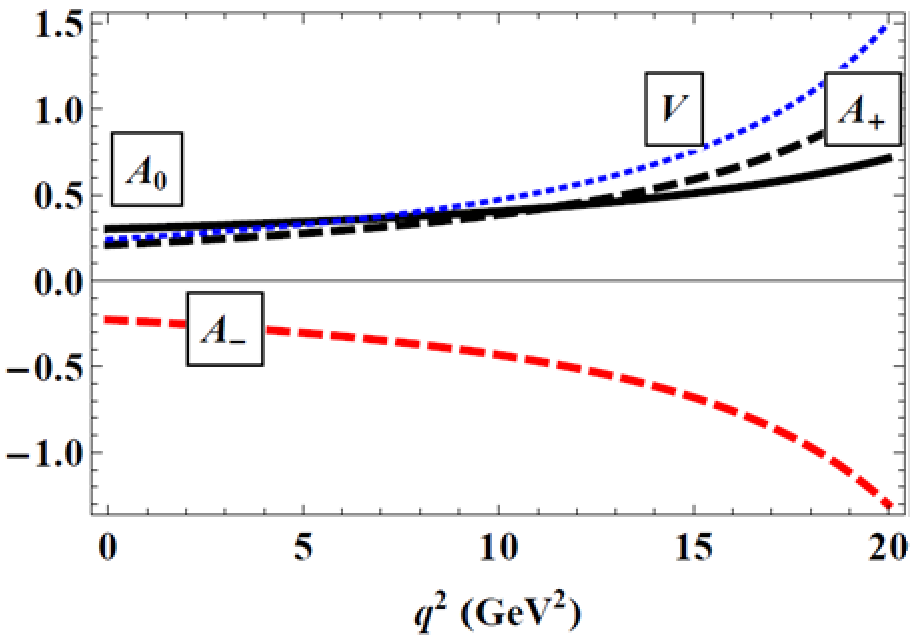
$\lambda_{us} = V_{ub}^* V_{us} = 0.00088$

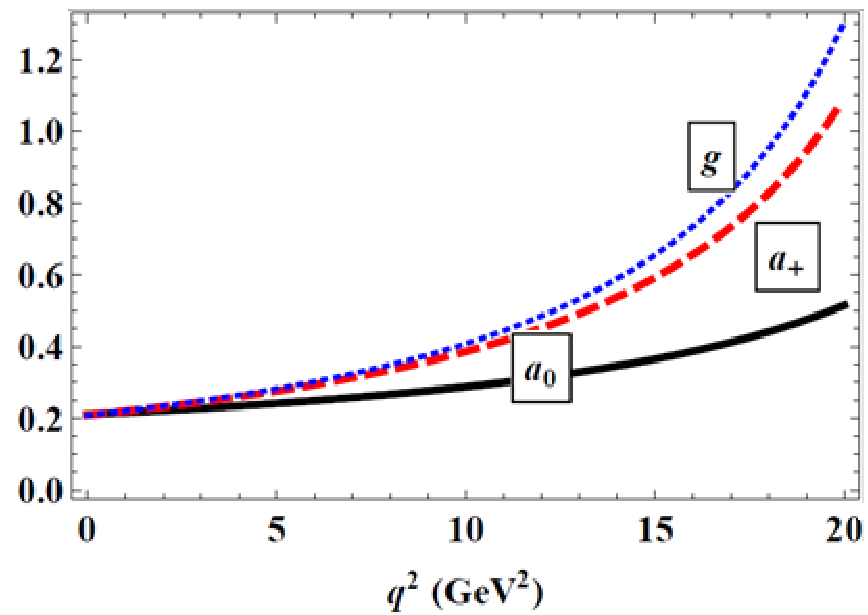
$b \to d$ *transition :*

$\lambda_{td} = V_{tb}^* V_{td} = 0.00825$

$\lambda_{ud} = V_{ub}^* V_{ud} = 0.00384$

Form factors of b-d transition





Branching ratios

TABLE VI: : Total branching fractions

	$\mathcal{B}(B_s \to K^{*0}\mu^+\mu^-)$	$\mathcal{B}(B_s \to K^{*0}J/\psi)$	$\frac{\mathcal{B}(B_s \to K^{*0}\mu^+\mu^-)}{\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-)}$	$\frac{\mathcal{B}(B_s \to K^{*0}J/\psi)}{\mathcal{B}(B_s \to \phi J/\psi)}$	$\mathcal{B}(B^+ \to \rho^+\mu^+\mu^-)$
Expt. [9, 10]	$(2.9 \pm 1)10^{-8}$	$(4.14 \pm +0.24)10^{-5}$	$(3.3 \pm 1.1) * 10^{-2}$	$4.05 * 10^{-2}$	
CCQM	$(2.73 \pm +0.55)10^{-8}$	$5.71 \pm +1.14)10^{-5}$	$(2.33 \pm 0.47) * 10^{-2}$	$3.57 * 10^{-2}$	$(3.46 \pm 0.69)10^{-8}$
[11]	$(2.89 \pm 0.73)10^{-8}$				$(4.16 \pm 0.68)10^{-8}$
[6]					$(4.33 \pm 1.14)10^{-8}$

[6] J. J. Wang, R. M. Wang, Y. G. Xu and Y. D. Yang, Phys. Rev. D **77** (2008) 014017 doi:10.1103/PhysRevD.77.014017 [arXiv:0711.0321 [hep-ph]].

[9] R. Aaij *et al.* [LHCb Collaboration], arXiv:1804.07167 [hep-ex].

[10] R. Aaij *et al.* [LHCb Collaboration], JHEP **1511** (2015) 082 doi:10.1007/JHEP11(2015)082

[11] B. Kindra and N. Mahajan, arXiv:1803.05876 [hep-ph].

Binned branching ratios

	$B_s \to K^{*0}\mu^+\mu^-$		$B^+ \to \rho^+\mu^+\mu^-$	
$10^8 * \mathcal{B}$	2 loop	1 loop	2 loop	1 loop
$[0.1, 1]$	0.17 ± 0.03	0.14 ± 0.03	0.23 ± 0.04	0.19 ± 0.04
$[2, 5]$	0.24 ± 0.05	0.25 ± 0.05	0.34 ± 0.06	0.36 ± 0.07
$[5, 8]$	$--$	0.34 ± 0.07	$--$	0.45 ± 0.09
$[11, 13]$	0.31 ± 0.06	0.33 ± 0.06	0.38 ± 0.04	0.40 ± 0.08
$[15, 17]$	0.35 ± 0.07	0.37 ± 0.07	0.41 ± 0.08	0.42 ± 0.08
$[17, 20]$	0.40 ± 0.08	0.41 ± 0.08	0.48 ± 0.10	0.49 ± 0.10
$[1, 6]$	0.41 ± 0.08	0.43 ± 0.08	0.58 ± 0.11	0.61 ± 0.12
$[15, 20]$	0.75 ± 0.15	0.78 ± 0.16	0.89 ± 0.18	0.92 ± 0.18

Binned angular observables

	$B_s \to K^{*0}\mu^+\mu^-$		$B^+ \to \rho^+\mu^+\mu^-$	
A_{FB}	2 loop	1 loop	2 loop	1 loop
$[0.1, 1]$	0.11 ± 0.01	0.12 ± 0.01	0.11 ± 0.01	0.11 ± 0.01
$[2, 5]$	0.036 ± 0.004	-0.034 ± 0.003	0.036 ± 0.004	-0.027 ± 0.003
$[5, 8]$	$--$	-0.249 ± 0.025	$--$	-0.227 ± 0.023
$[11, 13]$	-0.38 ± 0.04	-0.40 ± 0.04	-0.37 ± 0.04	-0.38 ± 0.04
$[15, 17]$	-0.39 ± 0.04	-0.40 ± 0.04	-0.39 ± 0.04	-0.39 ± 0.04
$[17, 20]$	-0.31 ± 0.03	-0.31 ± 0.03	-0.32 ± 0.03	-0.33 ± 0.03
$[1, 6]$	0.034 ± 0.003	-0.035 ± 0.004	0.035 ± 0.004	-0.027 ± 0.003
$[15, 20]$	-0.35 ± 0.04	-0.35 ± 0.04	-0.35 ± 0.04	-0.36 ± 0.04

	$B_s \to K^{*0}\mu^+\mu^-$		$B^+ \to \rho^+\mu^+\mu^-$	
F_L	2 loop	1 loop	2 loop	1 loop
$[0.1, 1]$	0.23 ± 0.02	0.30 ± 0.03	0.25 ± 0.03	0.32 ± 0.03
$[2, 5]$	0.72 ± 0.07	0.74 ± 0.07	0.75 ± 0.08	0.77 ± 0.08
$[5, 8]$	$--$	0.57 ± 0.06	$--$	0.61 ± 0.06
$[11, 13]$	0.40 ± 0.04	0.39 ± 0.04	0.42 ± 0.04	0.42 ± 0.04
$[15, 17]$	0.34 ± 0.03	0.33 ± 0.03	0.35 ± 0.04	0.35 ± 0.04

Conclusion

We have found that the theoretical predictions for the ratio $\mathcal{R}_{J/\psi}$ are more than 2 σ less than the experimental data. This may indicate on the possibility of New physics effects in this decay.

At the same time the ratios of the branching fractions $\mathcal{R}_{\pi^+/\mu^+\nu}$ and $\mathcal{R}_{K^+/\pi^+}$ are in good agreement with the LHCb data and other theoretical approaches.

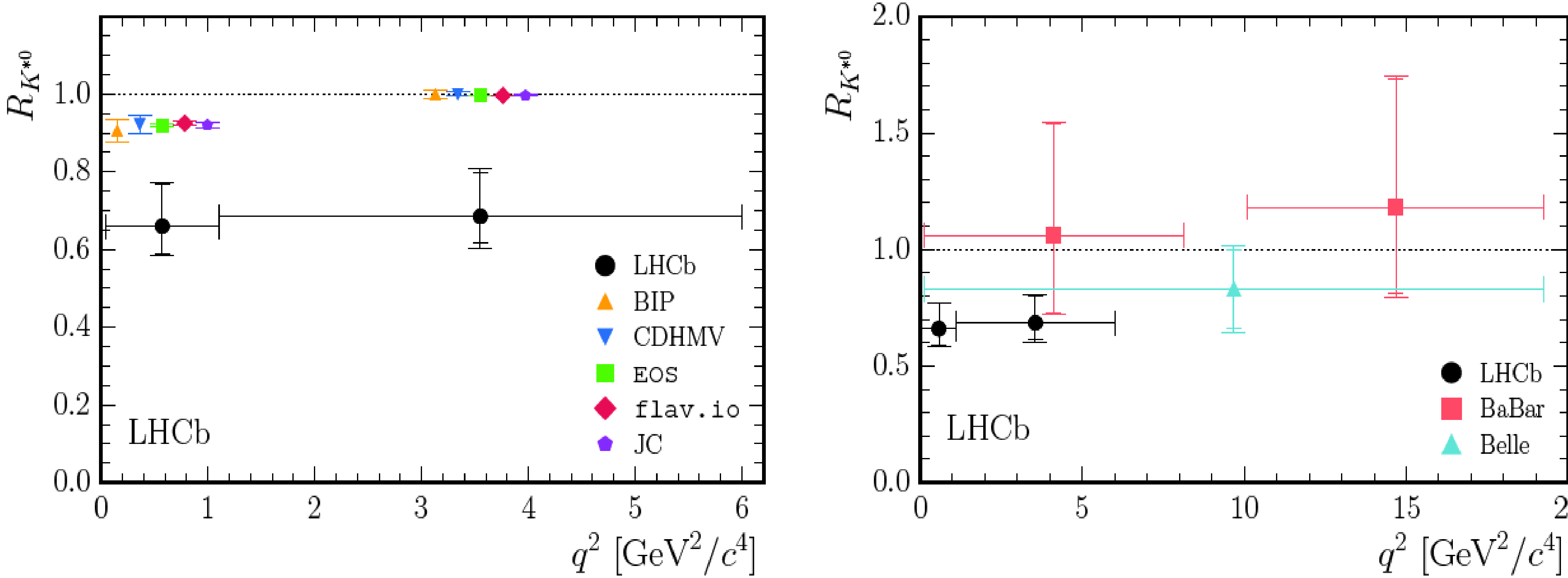
Since our result for $\mathcal{R}_{J/\psi}$ is different from the data at the level of 2 σ, we can urge to more precise measurement of the $B_c \to J/\psi\, \ell \bar{\nu}_\ell$ channel which currently has quite large uncertainties.This might be very important since it may imply that the new physics (if there is any) has strong couplings to the leptons but not hadrons.

- **Phys.Lett. B783 (2018) 178-182** (arXiv:1804.00472)
- **Phys.Rev. D96 (2017) no.7, 076017** (arXiv:1708.09607)
- **EPJ Web Conf. 158 (2017) 03002**

Thank you for your attention

Backup slides

R_{K^*}



$$R_{K^*} = \begin{cases} 0.66^{+0.11}_{-0.07}(\text{stat}) \pm 0.03(\text{syst}), & \text{at low } q^2 \ (\sim 2.2\sigma \text{ below SM}) \\ 0.69^{+0.11}_{-0.07}(\text{stat}) \pm 0.05(\text{syst}), & \text{at central } q^2 \ (\sim 2.4\sigma \text{ below SM}) \end{cases}$$

Angular observables

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta^*\, d\cos\theta\, d\chi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta^* + J_{1c}\cos^2\theta^* + (J_{2s}\sin^2\theta^* + J_{2c}\cos^2\theta^*)\cos 2\theta$$

$$+J_3\sin^2\theta^*\sin^2\theta\cos 2\chi + J_4\sin 2\theta^*\sin 2\theta\cos\chi + J_5\sin 2\theta^*\sin\theta\cos\chi$$

$$+(J_{6s}\sin^2\theta^* + J_{6c}\cos^2\theta^*)\cos\theta + J_7\sin 2\theta^*\sin\theta\sin\chi + J_8\sin 2\theta^*\sin 2\theta\sin\chi$$

$$+J_9\sin^2\theta^*\sin^2\theta\sin 2\chi\Big]$$

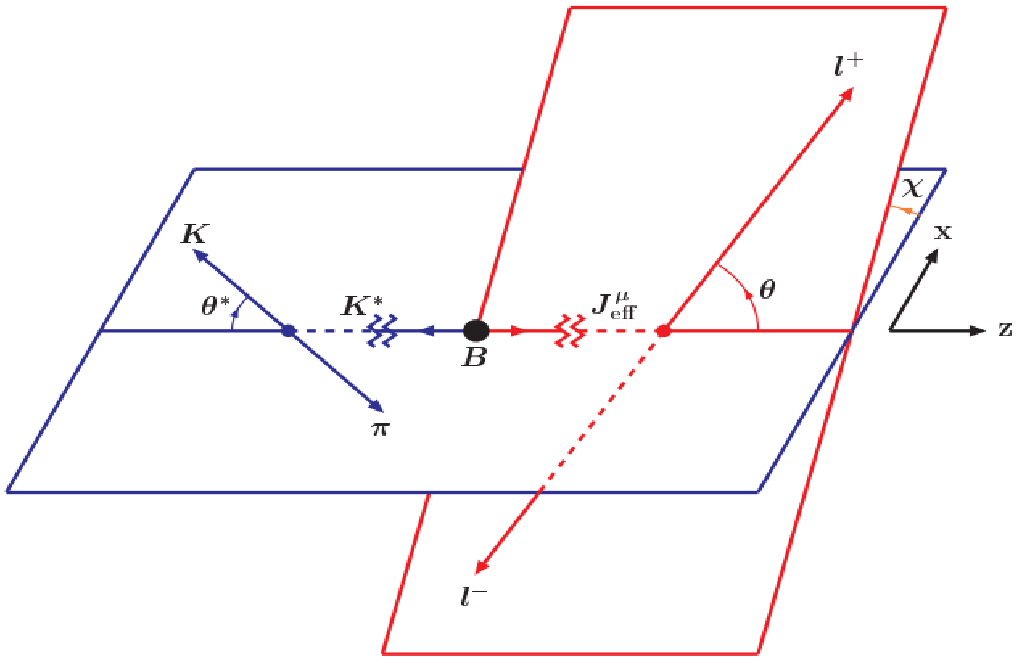


FIG. 6: Definition of the angles θ, θ^* and χ in the cascade decay $B \to K^*(\to K\pi)\bar{\ell}\ell$.

Angular observables

$$\frac{d\Gamma}{dq^2} = \int d\cos\theta\, d\cos\theta^* d\chi \,\frac{d^4\Gamma}{dq^2\, d\cos\theta^*\, d\cos\theta\, d\chi} = \frac{1}{4}\left(3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s}\right)$$

$$= \frac{G_F^2}{(2\pi)^3}\left(\frac{\alpha|\lambda_t|}{2\pi}\right)^2 \frac{|\mathbf{p_2}|\, q^2\, \beta_\ell}{12\, m_1^2}\mathcal{H}_{\rm tot}\,, \qquad \frac{d\mathcal{B}}{dq^2} = \frac{1}{\Gamma_B}\frac{d\Gamma}{dq^2}\,,$$

$$A_{\rm FB} = \frac{1}{d\Gamma/dq^2}\left[\int_0^1 - \int_{-1}^0\right] d\cos\theta\, \frac{d^2\Gamma}{dq^2 d\cos\theta} = -\frac{3}{4}\frac{J_{6s}}{d\Gamma/dq^2} = -\frac{3}{4}\beta_\ell \frac{\mathcal{H}_P^{12}}{\mathcal{H}_{\rm tot}}\,,$$

$$F_L = -\frac{J_{2c}}{d\Gamma/dq^2} = \frac{1}{2}\beta_\ell^2 \frac{\mathcal{H}_L^{11} + \mathcal{H}_L^{22}}{\mathcal{H}_{\rm tot}}.$$

Angular observables

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R^*} + A_\parallel^L A_\parallel^{R^*}\right) ,$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R^*})\right] + \beta_\ell^2\,|A_S|^2 ,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] , \qquad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + |A_0^R|^2\right] ,$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + |A_\perp^R|^2 - |A_\parallel^R|^2\right] , \qquad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L^*} + A_0^R A_\parallel^{R^*})\right]$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L^*} - A_0^R A_\perp^{R^*}) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^{R^*} A_S)\right] ,$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L^*} - A_\parallel^R A_\perp^{R^*})\right] , \qquad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_0^L A_S^* + A_0^{R^*} A_S) ;$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L^*} - A_0^R A_\parallel^{R^*}) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* - A_\perp^{R^*} A_S))\right] ,$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L^*} + A_0^R A_\perp^{R^*})\right] , \qquad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L^*} A_\perp^L + A_\parallel^{R^*} A_\perp^R)\right] .$$

$$
\begin{aligned}
A_{\perp}^{L,R} &= N\,\frac{1}{\sqrt{2}}\left[(H^{(1)}_{+1+1} - H^{(1)}_{-1-1}) \mp (H^{(2)}_{+1+1} - H^{(2)}_{-1-1})\right], \\
A_{\parallel}^{L,R} &= N\,\frac{1}{\sqrt{2}}\left[(H^{(1)}_{+1+1} + H^{(1)}_{-1-1}) \mp (H^{(2)}_{+1+1} + H^{(2)}_{-1-1})\right], \\
A_{0}^{L,R} &= N\,\left(H^{(1)}_{00} \mp H^{(2)}_{00}\right), \\
A_t &= -2\,N\,H^{(2)}_{0t},
\end{aligned}
$$

where the overall factor is given by

$$
N = \left[\frac{1}{4}\frac{G_F^2}{(2\pi)^3}\left(\frac{\alpha|\lambda_t|}{2\pi}\right)^2 \frac{|\mathbf{p_2}|q^2\beta_\ell}{12m_1^2}\right]^{\frac{1}{2}}.
$$

$$
\begin{aligned}
H^i_{t0} &= \epsilon^{\dagger\mu}(t)\epsilon_2^{\dagger\alpha}(0)T^i_{\mu\alpha} = \frac{1}{m_1+m_2}\frac{m_1\,|\mathbf{p_2}|}{m_2\sqrt{q^2}}\left(Pq\,(-A^i_0+A^i_+)+q^2 A^i_-\right), \\
H^i_{\pm1\pm1} &= \epsilon^{\dagger\mu}(\pm)\epsilon_2^{\dagger\alpha}(\pm)T^i_{\mu\alpha} = \frac{1}{m_1+m_2}\left(-Pq\,A^i_0 \pm 2\,m_1\,|\mathbf{p_2}|\,V^i\right), \\
H^i_{00} &= \epsilon^{\dagger\mu}(0)\epsilon_2^{\dagger\alpha}(0)T^i_{\mu\alpha} = \\
&= \frac{1}{m_1+m_2}\frac{1}{2\,m_2\sqrt{q^2}}\left(-Pq\,(m_1^2-m_2^2-q^2)\,A^i_0 + 4\,m_1^2\,|\mathbf{p_2}|^2\,A^i_+\right).
\end{aligned}
$$

$$\langle P_1\rangle_{\rm bin} = \frac{1}{2}\frac{\int_{\rm bin} dq^2 J_3}{\int_{\rm bin} dq^2 J_{2s}} = -2\,\frac{\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_T^{11}+\mathcal{H}_T^{22}]}{\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_U^{11}+\mathcal{H}_U^{22}]},$$

$$\langle P_2\rangle_{\rm bin} = \frac{1}{8}\frac{\int_{\rm bin} dq^2 J_{6s}}{\int_{\rm bin} dq^2 J_{2s}} = -\,\frac{\int_{\rm bin} dq^2\, \beta_\ell\, f(q^2)\mathcal{H}_P^{12}}{\int_{\rm bin} dq^2\, \beta_\ell^2\, [\mathcal{H}_U^{11}+\mathcal{H}_U^{22}]},$$

$$\langle P_3\rangle_{\rm bin} = -\frac{1}{4}\frac{\int_{\rm bin} dq^2 J_9}{\int_{\rm bin} dq^2 J_{2s}} = -\,\frac{\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_{IT}^{11}+\mathcal{H}_{IT}^{22}]}{\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_U^{11}+\mathcal{H}_U^{22}]},$$

$$\langle P_4'\rangle_{\rm bin} = \frac{1}{\mathcal{N}_{\rm bin}}\int_{\rm bin} dq^2 J_4 = 2\,\frac{\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_I^{11}+\mathcal{H}_I^{22}]}{N_{\rm bin}},$$

$$\langle P_5'\rangle_{\rm bin} = \frac{1}{2\mathcal{N}_{\rm bin}}\int_{\rm bin} dq^2 J_5 = -2\,\frac{\int_{\rm bin} dq^2\, \beta_\ell\, f(q^2)[\mathcal{H}_A^{12}+\mathcal{H}_A^{21}]}{N_{\rm bin}},$$

$$\langle P_6'\rangle_{\rm bin} = \frac{-1}{2\mathcal{N}_{\rm bin}}\int_{\rm bin} dq^2 J_7 = -2\,\frac{\int_{\rm bin} dq^2\, \beta_\ell\, f(q^2)[\mathcal{H}_{II}^{12}+\mathcal{H}_{II}^{21}]}{N_{\rm bin}},$$

$$\langle P_8'\rangle_{\rm bin} = \frac{-1}{\mathcal{N}_{\rm bin}}\int_{\rm bin} dq^2 J_8 = +2\,\frac{\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_{IA}^{11}+\mathcal{H}_{IA}^{22}]}{N_{\rm bin}},$$

where the normalization $\mathcal{N}_{\rm bin}$ is defined as

$$\mathcal{N}_{\rm bin} = \sqrt{-\int_{\rm bin} dq^2[J_{2s}]\cdot\int_{\rm bin} dq^2[J_{2c}]}$$

$$= \sqrt{\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_U^{11}+\mathcal{H}_U^{22}]\cdot\int_{\rm bin} dq^2\, \beta_\ell^2\, f(q^2)[\mathcal{H}_L^{11}+\mathcal{H}_L^{22}]}.$$

TABLE III: Definition of helicity structure functions and their parity properties.

parity-conserving (p.c.)	parity-violating (p.v.)
$\mathcal{H}_U^{ij} = \mathrm{Re}\left(H^i_{+1+1}H^{\dagger j}_{+1+1}\right) + \mathrm{Re}\left(H^i_{-1-1}H^{\dagger j}_{-1-1}\right)$	$\mathcal{H}_P^{ij} = \mathrm{Re}\left(H^i_{+1+1}H^{\dagger j}_{+1+1}\right) - \mathrm{Re}\left(H^i_{-1-1}H^{\dagger j}_{-1-1}\right)$
$\mathcal{H}_{IU}^{ij} = \mathrm{Im}\left(H^i_{+1+1}H^{\dagger j}_{+1+1}\right) + \mathrm{Im}\left(H^i_{-1-1}H^{\dagger j}_{-1-1}\right)$	$\mathcal{H}_{IP}^{ij} = \mathrm{Im}\left(H^i_{+1+1}H^{\dagger j}_{+1+1}\right) - \mathrm{Im}\left(H^i_{-1-1}H^{\dagger j}_{-1-1}\right)$
$\mathcal{H}_L^{ij} = \mathrm{Re}\left(H^i_{00}H^{\dagger j}_{00}\right)$	$\mathcal{H}_A^{ij} = \frac{1}{2}\left[\mathrm{Re}\left(H^i_{+1+1}H^{\dagger j}_{00}\right) - \mathrm{Re}\left(H^i_{-1-1}H^{\dagger j}_{00}\right)\right]$
$\mathcal{H}_{IL}^{ij} = \mathrm{Im}\left(H^i_{00}H^{\dagger j}_{00}\right)$	$\mathcal{H}_{IA}^{ij} = \frac{1}{2}\left[\mathrm{Im}\left(H^i_{+1+1}H^{\dagger j}_{00}\right) - \mathrm{Im}\left(H^i_{-1-1}H^{\dagger j}_{00}\right)\right]$
$\mathcal{H}_T^{ij} = \mathrm{Re}\left(H^i_{+1+1}H^{\dagger j}_{-1-1}\right)$	$\mathcal{H}_{SA}^{ij} = \frac{1}{2}\left[\mathrm{Re}\left(H^i_{+1+1}H^{\dagger j}_{0t}\right) - \mathrm{Re}\left(H^i_{-1-1}H^{\dagger j}_{0t}\right)\right]$
$\mathcal{H}_{IT}^{ij} = \mathrm{Im}\left(H^i_{+1+1}H^{\dagger j}_{-1-1}\right)$	$\mathcal{H}_{ISA}^{ij} = \frac{1}{2}\left[\mathrm{Im}\left(H^i_{+1+1}H^{\dagger j}_{0t}\right) - \mathrm{Im}\left(H^i_{-1-1}H^{\dagger j}_{0t}\right)\right]$
$\mathcal{H}_I^{ij} = \frac{1}{2}\left[\mathrm{Re}\left(H^i_{+1+1}H^{\dagger j}_{00}\right) + \mathrm{Re}\left(H^i_{-1-1}H^{\dagger j}_{00}\right)\right]$	
$\mathcal{H}_{II}^{ij} = \frac{1}{2}\left[\mathrm{Im}\left(H^i_{+1+1}H^{\dagger j}_{00}\right) + \mathrm{Im}\left(H^i_{-1-1}H^{\dagger j}_{00}\right)\right]$	
$\mathcal{H}_S^{ij} = \mathrm{Re}\left(H^i_{0t}H^{\dagger j}_{0t}\right)$	
$\mathcal{H}_{IS}^{ij} = \mathrm{Im}\left(H^i_{0t}H^{\dagger j}_{0t}\right)$	
$\mathcal{H}_{ST}^{ij} = \frac{1}{2}\left[\mathrm{Re}\left(H^i_{+1+1}H^{\dagger j}_{0t}\right) + \mathrm{Re}\left(H^i_{-1-1}H^{\dagger j}_{0t}\right)\right]$	
$\mathcal{H}_{IST}^{ij} = \frac{1}{2}\left[\mathrm{Im}\left(H^i_{+1+1}H^{\dagger j}_{0t}\right) + \mathrm{Im}\left(H^i_{-1-1}H^{\dagger j}_{0t}\right)\right]$	
$\mathcal{H}_{SL}^{ij} = \mathrm{Re}\left(H^i_{00}H^{\dagger j}_{0t}\right)$	
$\mathcal{H}_{ISL}^{ij} = \mathrm{Im}\left(H^i_{00}H^{\dagger j}_{0t}\right)$	