SEMILEPTONIC DECAYS OF HEAVY BARYONS

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INTRODUCTION

• Semileptonic decays of heavy baryons (FCCC): $B_Q \to B_{Q'}\ell\nu_\ell$ ($Q = b, c$, $Q' = c, s, u, d$, $\ell = e, \mu, \tau$)
• Rare semileptonic decays (FCNC): $B_Q \to B_{Q'}\ell^+\ell^-$
Additional source for the determination of $V_{QQ'}$.
Main assumption: heavy-quark–light-diquark picture of heavy baryons B_Q (Qqq), ($q = u, d, s$)
Three-body calculation $\longrightarrow$ two-step two-body calculation:

- diquark d as qq' bound state
- baryon as the Qd bound state

Diquarks

• Diquark is a composite (qq') system:
– diquark is not point-like object: Its interaction with gluons is smeared by the form factor expressed through the overlap integral of diquark wave functions

• Pauli principle for ground state diquarks:
– (qq') diquark can have $S = 0, 1$ (scalar $[q, q']$, axial vector $\{q, q'\}$)
– (qq) diquarks can have only $S = 1$ (axial vector $\{q, q\}$)

• Both light and heavy quarks and diquarks are considered fully relativistically without nonrelativistic (v/c) expansion

Semileptonic decay:

Active heavy quark and spectator light diquark.

Two types of heavy baryons:

- Λ_Q type – light scalar diquark (Λ_b, Λ_c, Ξ_b, Ξ_c – spin 1/2)
- $\Omega_Q(\Sigma_Q)$ type – light axial vector diquark (Ω_b, Ω_c, Σ_b, Σ_c, Ξ'_b, Ξ'_c– spin 1/2; Ω^*_b, Ω^*_c, Σ^*_b, Σ^*_c, Ξ^*_b, Ξ^*_c– spin 3/2)

Heavy Quark Symmetry (HQS) for heavy-to-heavy baryon decays:

$m_Q \to \infty$:

heavy quark spin and mass decouple $\to$ heavy baryon properties are determined by light quarks $\to$

- masses of ground state baryons with spin 1/2 and 3/2 containing the axial vector diquark are degenerate
- for $\Lambda_b \to \Lambda_c$ one universal form factor $\zeta(w)$ (Isgur-Wise function)
- for $\Omega_b \to \Omega_c$ two universal form factors $\zeta_1(w)$ and $\zeta_2(w)$
- isospin violating decay amplitudes, e.g. $\Lambda_b \to \Sigma_c$, vanish

$1/m_Q$ order:

- for $\Lambda_Q \to \Lambda_{Q'}$ one additional mass parameter $\bar{\Lambda}$ and one additional function $\chi(w)$
- for $\Omega_Q \to \Omega_{Q'}$ one additional mass parameter $\bar{\Lambda}$ and five additional functions

- Σ_Q, Ξ'_Q baryons with axial vector diquark can decay strongly or radiatively to Λ_Q and Ξ_Q, respectively

$\Longrightarrow$ their weak branching ratios will be very small ($\sim 10^{-7} - 10^{-12}$)

$\Longrightarrow$ we mostly consider Λ_Q and Ξ_Q decays

RELATIVISTIC QUARK MODEL

Relativistic quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - center-of-mass relative momentum of quarks (diquarks)
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center-of-mass energies:

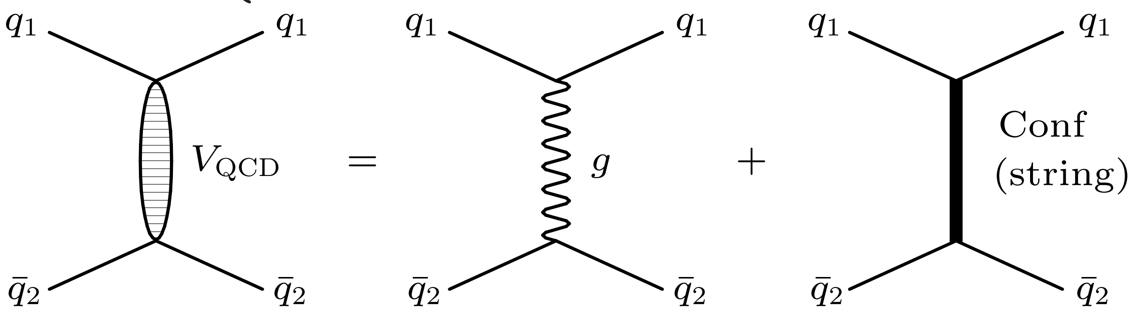
$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- **Baryons in quark-diquark picture**

(qq)-interaction:

$$V_{qq} = \frac{1}{2} V_{q\bar{q}}$$

$$V_{q\bar{q}}(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V_{\rm conf}^V(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V_{\rm conf}^S(\mathbf{k})\right\}u_1(q)u_2(-q)$$



$\mathbf{k} = \mathbf{p} - \mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \qquad \epsilon(p) = \sqrt{\mathbf{p}^2+m^2}$$

.

- Lorentz structure of the quark potential

$$V_{\text{conf}} = V_{\text{conf}}^V + V_{\text{conf}}^S$$

In nonrelativistic limit

$$\left.\begin{aligned} V_{\text{conf}}^V(r) &= (1-\varepsilon)(Ar+B) \\ V_{\text{conf}}^S(r) &= \varepsilon(Ar+B) \end{aligned}\right\} \quad \text{Sum :} \quad (Ar+B)$$

ε - mixing parameter

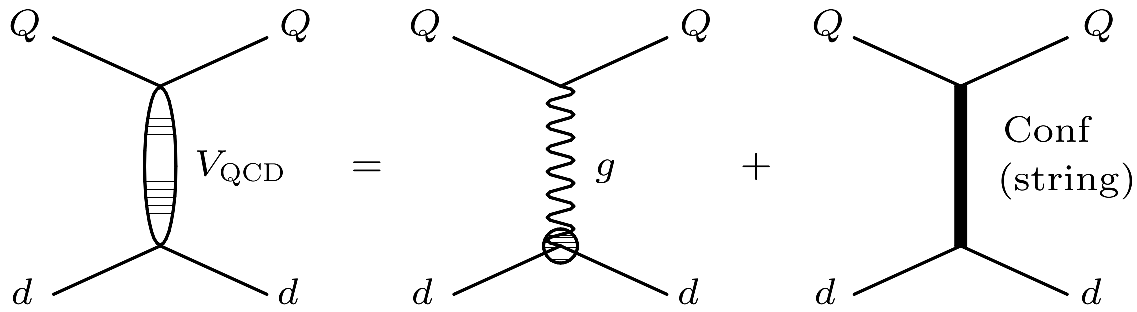
$$V_{\text{NR}}(r) = V_{\text{Coul}}(r) + V_{\text{conf}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar + B$$

$$V_{\text{Coul}}(r) = -\frac{4}{3}\frac{\alpha_s}{r}$$

(dQ)-interaction: $d=(qq')$

$$V(\mathbf{p},\mathbf{q};M)=\frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p)\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma^\nu u_Q(q)$$

$$+\psi_d^*(P)\bar{u}_Q(p)J_{d;\mu}\Gamma_Q^\mu V_{\rm conf}^V(\mathbf{k})u_Q(q)\psi_d(Q)+\psi_d^*(P)\bar{u}_Q(p)V_{\rm conf}^S(\mathbf{k})u_Q(q)\psi_d(Q)$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu}=\begin{cases}\dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark}\\[2ex] -\dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}}+\dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark } (\mu_{\rm d}=0)\end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark

diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu=-i(g_{\mu\rho}\delta_\sigma^\nu-g_{\mu\sigma}\delta_\rho^\nu)$

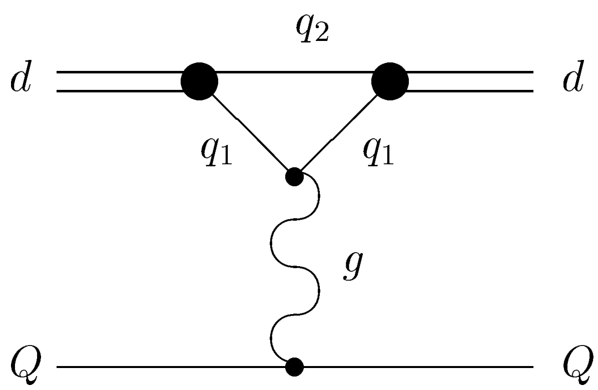
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il}=-i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

Vertex of diquark-gluon interaction:



$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction

The diquark-gluon interaction form factor can be parameterized by

$$F(r) = 1 - e^{-\xi r - \zeta r^2}$$

- Parameters of the model

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays $(J/\psi \to \eta_c + \gamma)$ and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ${}^3P_J$ states and HQET

$(1+\kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction ! (flux tube model)

Freezing of α_s

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2+M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad \mu = \frac{2m_1m_2}{m_1+m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV (from M_ρ)

Quark masses:

$m_b = 4.88$ GeV $\quad$ $m_s = 0.50$ GeV

$m_c = 1.55$ GeV $\quad$ $m_{u,d} = 0.33$ GeV

Light diquarks

Table 1: Masses of light ground state diquarks (in MeV)

Quark content	Diquark type	Mass our	Mass NJL	Mass BSE	Mass BSE	Mass Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

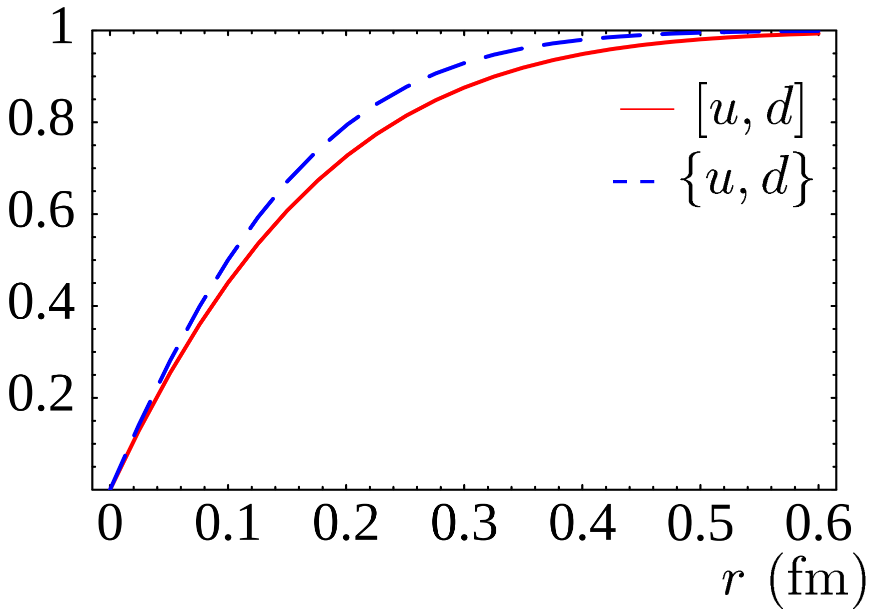
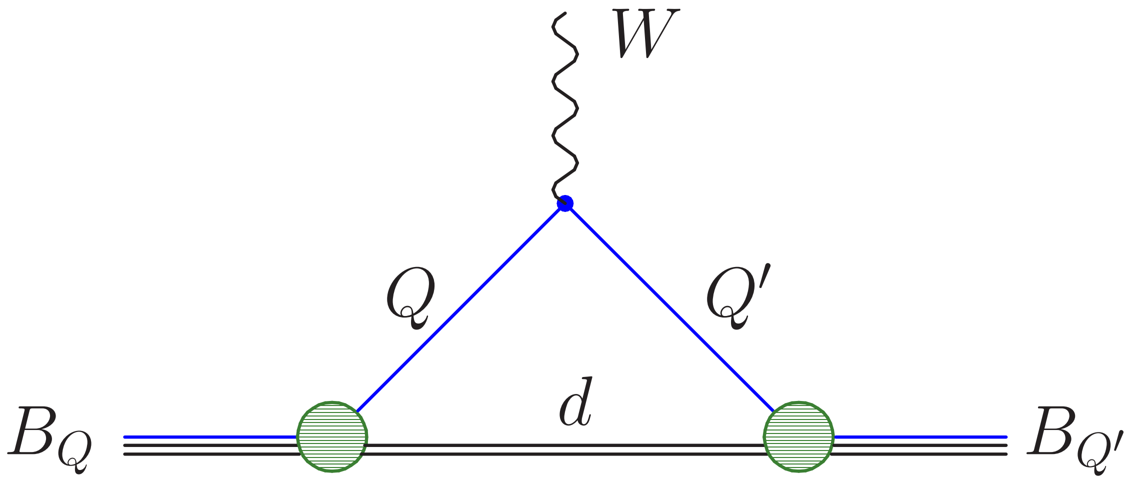


Figure 1: Form factors $F(r)$ for scalar $[u, d]$ (solid line) and axial vector $\{u, d\}$ (dashed line) diquarks.

SEMILEPTONIC DECAYS

- Heavy-to-heavy and heavy-to-light semileptonic decays of baryons: $B_Q \to B_{Q'} e\nu$ ($Q = b, c$, $Q' = c, u$)

Additional source for the determination of V_{cb} and V_{ub}.



Active heavy quark and spectator light diquark.

Matrix elements of weak current

Matrix element of weak current $J_\mu^W = \bar{Q}'\gamma_\mu(1-\gamma_5)Q$:

$$\langle B_{Q'}(P')|J_\mu^W|B_Q(P)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6}\bar{\Psi}_{B_{Q'}\,\mathbf{P}'}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B_Q\,\mathbf{P}}(\mathbf{q}),$$

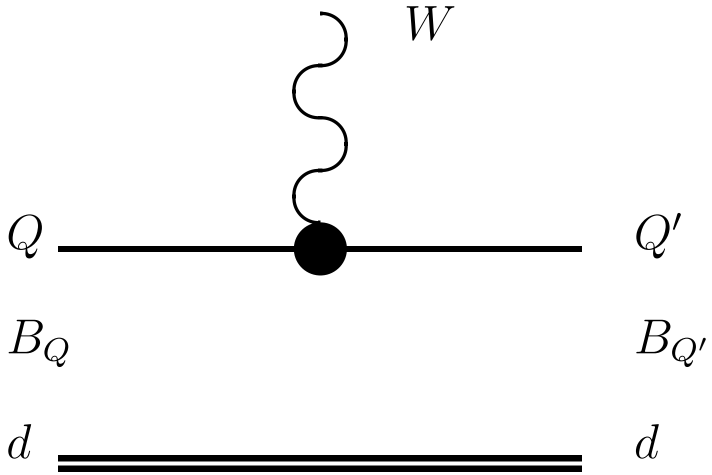


Figure 2: Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element.

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q}) = \psi_d^*(p_d)\bar{u}_{Q'}(p_{Q'})\gamma_\mu(1-\gamma^5)u_Q(q_Q)\psi_d(q_d)(2\pi)^3\delta(\mathbf{p}_d-\mathbf{q}_d)$$

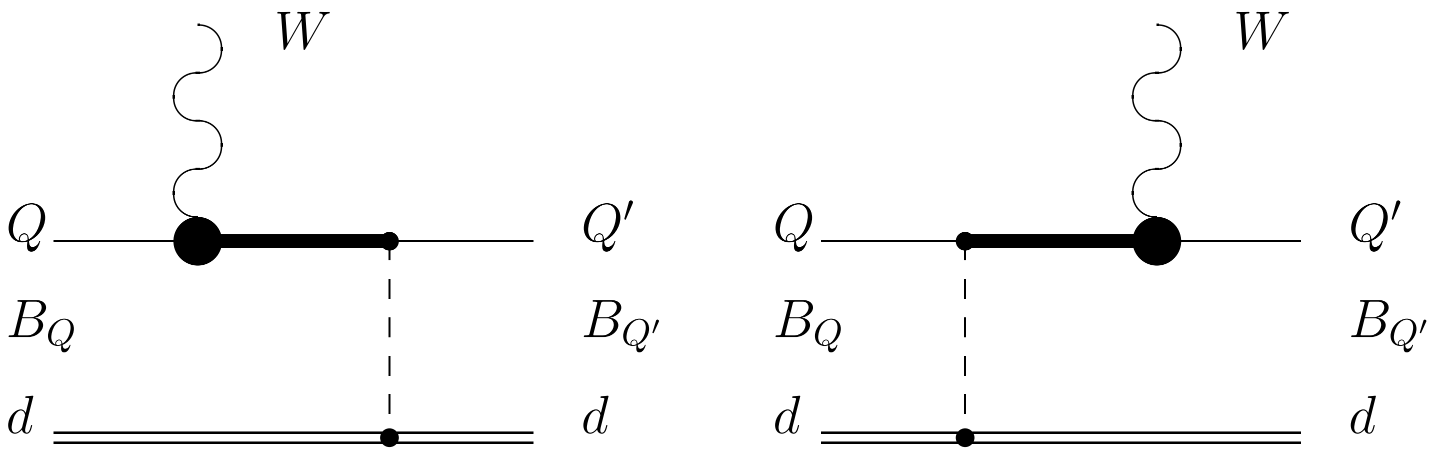


Figure 3: Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}_{Qd}$. Bold lines denote the negative-energy part of the quark propagator.

$$
\begin{aligned}
\Gamma^{(2)}_\mu(\mathbf{p},\mathbf{q}) \quad = \quad & \psi^*_d(p_d)\bar{u}_{Q'}(p_{Q'})\Big\{\gamma_\mu(1-\gamma^5)\frac{\Lambda^{(-)}_Q(k)}{\epsilon_Q(k)+\epsilon_Q(p_{Q'})}\gamma^0\mathcal{V}_{Qd}(\mathbf{p}_d-\mathbf{q}_d) \\
& +\mathcal{V}_{Qd}(\mathbf{p}_d-\mathbf{q}_d)\frac{\Lambda^{(-)}_{Q'}(k')}{\epsilon_{Q'}(k')+\epsilon_{Q'}(q_Q)}\gamma^0\gamma_\mu(1-\gamma^5)\Big\}u_Q(q_Q)\psi_d(q_d),
\end{aligned}
$$

where

$$
\Lambda^{(-)}(p) = \frac{\epsilon(p) - \big(m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p})\big)}{2\epsilon(p)}
$$

$\mathbf{k} = \mathbf{p}_{Q'} - \boldsymbol{\Delta};\ \mathbf{k}' = \mathbf{q}_Q + \boldsymbol{\Delta};\ \boldsymbol{\Delta} = \mathbf{P}' - \mathbf{P};\ \epsilon(p) = \sqrt{m^2 + \mathbf{p}^2}$

Wave function $\Psi_{B_{Q'}\mathbf{P}'}$ of the moving baryon is connected with the rest-frame wave function $\Psi_{B_{Q'}\mathbf{0}} \equiv \Psi_{B_{Q'}}$ by the transformation

$$\Psi_{B_{Q'}\mathbf{P}'}(\mathbf{p}) = D_{Q'}^{1/2}(R_{L_{\mathbf{P}'}}^{W}) D_{d}^{\mathcal{I}}(R_{L_{\mathbf{P}'}}^{W}) \Psi_{B_{Q'}\mathbf{0}}(\mathbf{p}), \qquad \mathcal{I} = 0, 1,$$

where R^W – Wigner rotation, $L_{\mathbf{P}'}$ – Lorentz boost from the baryon rest frame to a moving one.

- Rotation matrix $D_Q^{1/2}(R)$ of heavy quark spin:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_{Q'}^{1/2}(R_{L_{\mathbf{P}'}}^{W}) = S^{-1}(\mathbf{p}_{Q'}) S(\mathbf{P}') S(\mathbf{p}),$$

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p) + m}\right).$$

- Rotation matrix $D_d^{\mathcal{I}}(R)$ of diquark with spin $\mathcal{I} = 0, 1$:
  $D_d^0(R^W) = 1$ for scalar diquark
  $D_d^1(R^W) = R^W$ for axial vector diquark.

- heavy-to-heavy decays $\Lambda_b \to \Lambda_c l \nu_l$ $b \to c$ transition CKM favored V_{cb} $Br \sim 10^{-2}$
- heavy-to-light decays $\Lambda_b \to p l \nu_l$ $b \to u$ transition CKM suppressed V_{ub} $Br \sim 10^{-4}$

Broad kinematical range:
the square of momentum transfer to the lepton pair q^2 varies
from 0 to $q^2_{\rm max} \approx 10$ GeV^2 for decays to Λ_c
from 0 to $q^2_{\rm max} \approx 20$ GeV^2 for decays to p
$\Longrightarrow$ the explicit determination of the q^2 dependence of the decay form factors in the whole kinematical range is needed

Large recoil of the final baryon requires consistent relativistic treatment (e.g. boost of the baryon wave functions from the rest to the moving reference frame)

Presence of heavy quarks in Λ_b and Λ_c baryons allows one to use expansions in the inverse powers of heavy quark masses $1/m_{b,c} \Longrightarrow$ significant simplifications, heavy quark symmetry relations can be used

Light u, d, s quarks should be treated relativistically

Large recoils allow one to neglect small relative momentum ($|\mathbf{p}|$) with respect to recoil ($|\mathbf{\Delta}|$) in the energies of light quarks in energetic light baryons $\epsilon_q(p+\Delta) \equiv \sqrt{m_q^2 + (\mathbf{p}+\mathbf{\Delta})^2} \longrightarrow \epsilon_q(\Delta) \equiv \sqrt{m_q^2 + \mathbf{\Delta}^2}$.
Such replacement is made in subleading contribution $\Gamma^{(2)}_\mu(\mathbf{p}, \mathbf{q})$ and permits to perform one of the integrations using the quasipotential equation. As a result, the weak decay matrix element is expressed through the usual overlap integral of initial and final meson wave functions

Heavy-to-heavy semileptonic $\Lambda_b \to \Lambda_c$ decays

HQS ($m_Q \to \infty$):

heavy quark spin and mass decouple $\to$ heavy baryon properties are determined by light diquarks $\to$

- masses of ground state baryons with spin 1/2 and 3/2 containing the axial vector diquark are degenerate
- for $\Lambda_b \to \Lambda_c$ one universal form factor $\zeta(w)$ (Isgur-Wise function)
- for $\Omega_b \to \Omega_c$ two universal form factors $\zeta_1(w)$ and $\zeta_2(w)$
- isospin violating decay amplitudes, e.g. $\Lambda_b \to \Sigma_c$, vanish

$1/m_Q$ order:

- for $\Lambda_Q \to \Lambda_{Q'}$ one additional mass parameter $\bar{\Lambda}$ and one additional function $\chi(w)$
- for $\Omega_Q \to \Omega_{Q'}$ one additional mass parameter $\bar{\Lambda}$ and five additional functions

- **Form factors of heavy baryons with scalar diquark**

Hadronic matrix elements for $\Lambda_Q \to \Lambda_q e\nu$:

$$\langle \Lambda_q(p',s')|V^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')\Big[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{p^\mu}{M_{\Lambda_Q}} + F_3(q^2)\frac{p'^\mu}{M_{\Lambda_q}}\Big]u_{\Lambda_Q}(p,s),$$
$$\langle \Lambda_q(p',s')|A^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')\Big[G_1(q^2)\gamma^\mu + G_2(q^2)\frac{p^\mu}{M_{\Lambda_Q}} + G_3(q^2)\frac{p'^\mu}{M_{\Lambda_q}}\Big]\gamma_5 u_{\Lambda_Q}(p,s),$$

An other popular parametrisation

$$\langle \Lambda_q(p',s')|V^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')\Big[f_1^V(q^2)\gamma^\mu - f_2^V(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{M_{\Lambda_Q}} + f_3^V(q^2)\frac{q^\mu}{M_{\Lambda_Q}}\Big]u_{\Lambda_Q}(p,s),$$
$$\langle \Lambda_q(p',s')|A^\mu|\Lambda_Q(p,s)\rangle = \bar{u}_{\Lambda_q}(p',s')[f_1^A(q^2)\gamma^\mu - f_2^A(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{M_{\Lambda_Q}} + f_3^A(q^2)\frac{q^\mu}{M_{\Lambda_Q}}\Big]\gamma_5 u_{\Lambda_Q}(p,s)$$

Relations

$$f_1^V(q^2) = F_1(q^2) + (M_{\Lambda_Q} + M_{\Lambda_q})\left[\frac{F_2(q^2)}{2M_{\Lambda_Q}} + \frac{F_3(q^2)}{2M_{\Lambda_q}}\right],$$
$$f_2^V(q^2) = -\frac{1}{2}\left[F_2(q^2) + \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}F_3(q^2)\right], \qquad f_3^V(q^2) = \frac{1}{2}\left[F_2(q^2) - \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}F_3(q^2)\right],$$
$$f_1^A(q^2) = G_1(q^2) - (M_{\Lambda_Q} - M_{\Lambda_q})\left[\frac{G_2(q^2)}{2M_{\Lambda_Q}} + \frac{G_3(q^2)}{2M_{\Lambda_q}}\right],$$
$$f_2^A(q^2) = -\frac{1}{2}\left[G_2(q^2) + \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}G_3(q^2)\right], \qquad f_3^A(q^2) = \frac{1}{2}\left[G_2(q^2) - \frac{M_{\Lambda_Q}}{M_{\Lambda_q}}G_3(q^2)\right].$$

- **Heavy quark expansion**

- In heavy quark limit $m_Q \to \infty$

$$F_1(w) = G_1(w) = \zeta(w), \qquad F_2(w) = F_3(w) = G_2(w) = G_3(w) = 0,$$

$$w = v \cdot v' = \frac{M_{\Lambda_Q}^2 + M_{\Lambda_{Q'}}^2 - q^2}{2M_{\Lambda_Q}M_{\Lambda_{Q'}}}.$$

- At $1/m_Q$ order in HQET

$$F_1(w) = \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)\left[2\chi(w) + \zeta(w)\right],$$

$$G_1(w) = \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right],$$

$$F_2(w) = G_2(w) = -\frac{\bar{\Lambda}}{2m_{Q'}}\frac{2}{w+1}\zeta(w), \qquad F_3(w) = -G_3(w) = -\frac{\bar{\Lambda}}{2m_Q}\frac{2}{w+1}\zeta(w).$$

In our model up to $1/m_Q$ order

$$F_1(w) = \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)[2\chi(w) + \zeta(w)]$$
$$+4(1-\varepsilon)(1+\kappa)\left[\frac{\bar{\Lambda}}{2m_{Q'}}\frac{1}{w-1} - \frac{\bar{\Lambda}}{2m_Q}(w+1)\right]\chi(w),$$
$$G_1(w) = \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right] - 4(1-\varepsilon)(1+\kappa)\frac{\bar{\Lambda}}{2m_Q}w\chi(w),$$
$$F_2(w) = -\frac{\bar{\Lambda}}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\left[\frac{\bar{\Lambda}}{2m_{Q'}}\frac{1}{w-1} + \frac{\bar{\Lambda}}{2m_Q}w\right]\chi(w),$$
$$G_2(w) = -\frac{\bar{\Lambda}}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\frac{\bar{\Lambda}}{2m_{Q'}}\frac{1}{w-1}\chi(w),$$
$$F_3(w) = -G_3(w) = -\frac{\bar{\Lambda}}{2m_Q}\frac{2}{w+1}\zeta(w) + 4(1-\varepsilon)(1+\kappa)\frac{\bar{\Lambda}}{2m_Q}\chi(w).$$

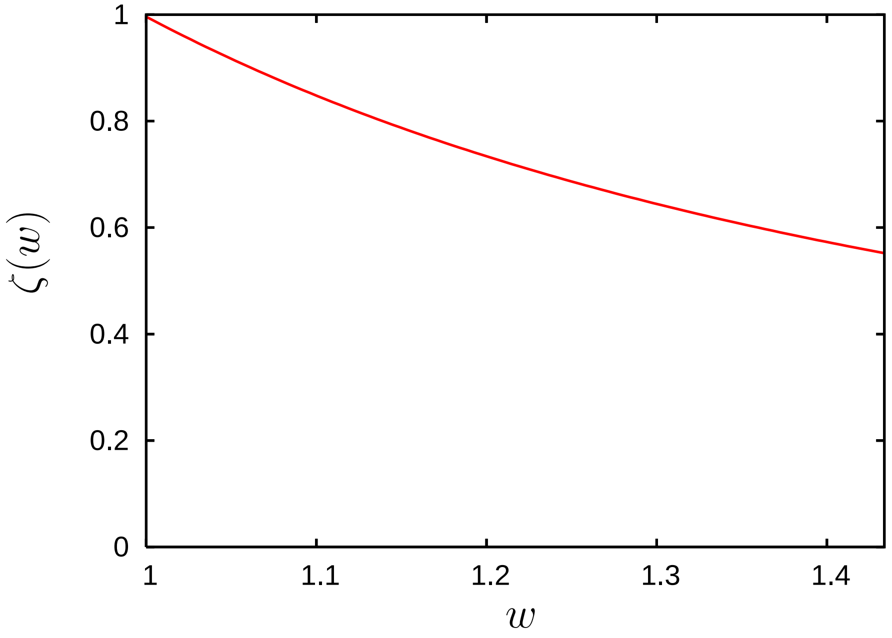
⋆ For $(1-\varepsilon)(1+\kappa) = 0$ HQET results are reproduced ($\kappa = -1$ in our model)!

- Leading order Isgur-Wise function ($\mathbf{e}_{\mathbf{\Delta}} = \mathbf{\Delta}/\sqrt{\mathbf{\Delta^2}}$)

$$\zeta(w) = \lim_{m_Q \to \infty} \int \frac{d^3p}{(2\pi)^3} \Psi_{\Lambda_{Q'}}\left(\mathbf{p} + 2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\,\mathbf{e}_{\mathbf{\Delta}}\right) \Psi_{\Lambda_Q}(\mathbf{p}).$$

- Subleading function (very small)

$$\chi(w) = -\frac{w-1}{w+1} \lim_{m_Q \to \infty} \int \frac{d^3p}{(2\pi)^3} \Psi_{\Lambda_{Q'}}\left(\mathbf{p} + 2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\,\mathbf{e}_{\mathbf{\Delta}}\right) \frac{\bar{\Lambda} - \epsilon_d(p)}{2\bar{\Lambda}} \Psi_{\Lambda_Q}(\mathbf{p}).$$



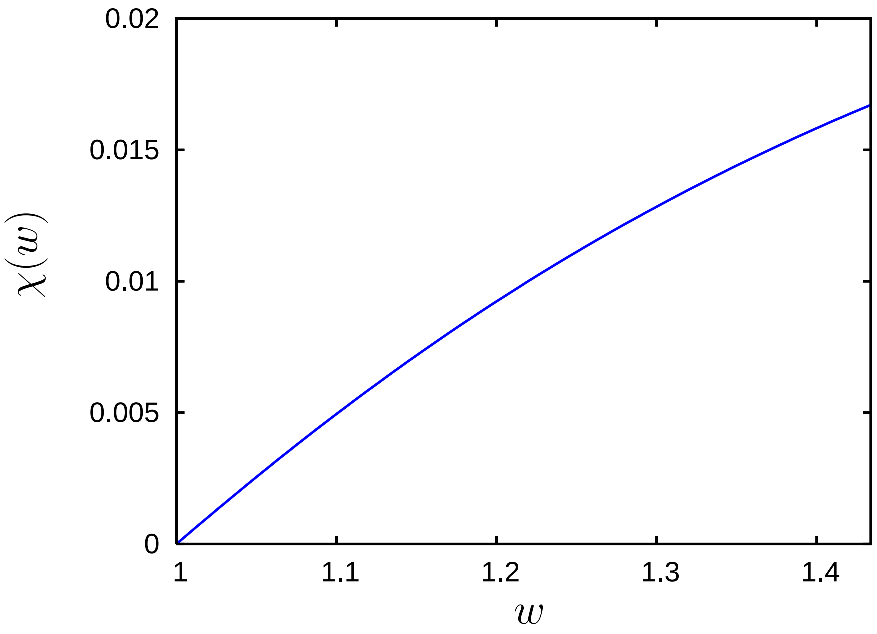


Table 2: Comparison of different theoretical predictions for semileptonic decay rates Γ (in $10^{10}\mathrm{s}^{-1}$) of bottom baryons.

Decay	Our RQM	Singleton NRQM	Cheng NRQM	Körner NRQM	Ivanov RTQM	Ivanov BS	Cardarelli LF	Albertus NRQM	Huang sum rule
$\Lambda_b \to \Lambda_c e\nu$	5.10	5.9	5.1	5.14	5.39	6.09	5.08 ± 1.3	5.82	5.4 ± 0.4
$\Xi_b \to \Xi_c e\nu$	5.03	7.2	5.3	5.21	5.27	6.42	5.68 ± 1.5	4.98	
$\Sigma_b \to \Sigma_c e\nu$	1.44	4.3			2.23	1.65			
$\Xi_b' \to \Xi_c' e\nu$	1.34								
$\Omega_b \to \Omega_c e\nu$	1.29	5.4	2.3	1.52	1.87	1.81			
$\Sigma_b \to \Sigma_c^* e\nu$	3.23				4.56	3.75			
$\Xi_b' \to \Xi_c^* e\nu$	3.09								
$\Omega_b \to \Omega_c^* e\nu$	3.03			3.41	4.01	4.13			

Our prediction for the branching ratio ($|V_{cb}| = 0.039$, $\tau_{\Lambda_b} = 1.466 \times 10^{-12}$s)

$$Br^{\rm theor}(\Lambda_b \to \Lambda_c l\nu) = 7.2\%$$

Experiment

$$Br^{\rm exp}(\Lambda_b \to \Lambda_c l\nu) = (6.2^{+1.4}_{-1.3})\%$$

Heavy-to-light semileptonic $\Lambda_b \to p$ decays

- **Without heavy quark expansion**

Large recoil:

$$|\mathbf{p}| \ll |\mathbf{\Delta}| \Longrightarrow \epsilon_q(p+\Delta) \equiv \sqrt{m_q^2 + (\mathbf{p}+\mathbf{\Delta})^2} \longrightarrow \epsilon_q(\Delta) \equiv \sqrt{m_q^2 + \mathbf{\Delta}^2}$$

in subleading contribution $\Gamma_\mu^{(2)}(\mathbf{p},\mathbf{q})$ and perform one of the integrations using the quasipotential equation.

$$F_i(q^2) = F_i^{(1)}(q^2) + \varepsilon F_i^{(2)S}(q^2) + (1-\varepsilon)F_i^{(2)V}(q^2)$$
$$G_i(q^2) = F_i^{(1)}(q^2) + \varepsilon G_i^{(2)S}(q^2) + (1-\varepsilon)G_i^{(2)V}(q^2) \qquad (i=1,2,3)$$

$$F_1^{(1)}(q^2) = \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_F\left(\mathbf{p} + \frac{2\epsilon_d}{E_F+M_F}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_Q(p)+m_Q}{2\epsilon_Q(p)}}\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}$$
$$\times\left\{1 + \frac{\epsilon_d}{\epsilon_q(p+\Delta)+m_q}\left[1+\frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\frac{E_F-M_F}{E_F+M_F}\right] + \frac{\epsilon_d}{\epsilon_Q(p)+m_Q}\right.$$
$$-\frac{1}{3}\frac{\mathbf{p}^2}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)} - \frac{\mathbf{p}\mathbf{\Delta}}{E_F+M_F}\left[\frac{1}{\epsilon_q(p+\Delta)+m_q}\right.$$
$$\left.\left.-\frac{1}{\epsilon_Q(p)+m_Q} + \frac{2M_F}{E_F+M_F}\frac{\epsilon_d}{(\epsilon_q(p+\Delta)+m_q)(\epsilon_Q(p)+m_Q)}\right]\right\}\Psi_I(\mathbf{p})$$

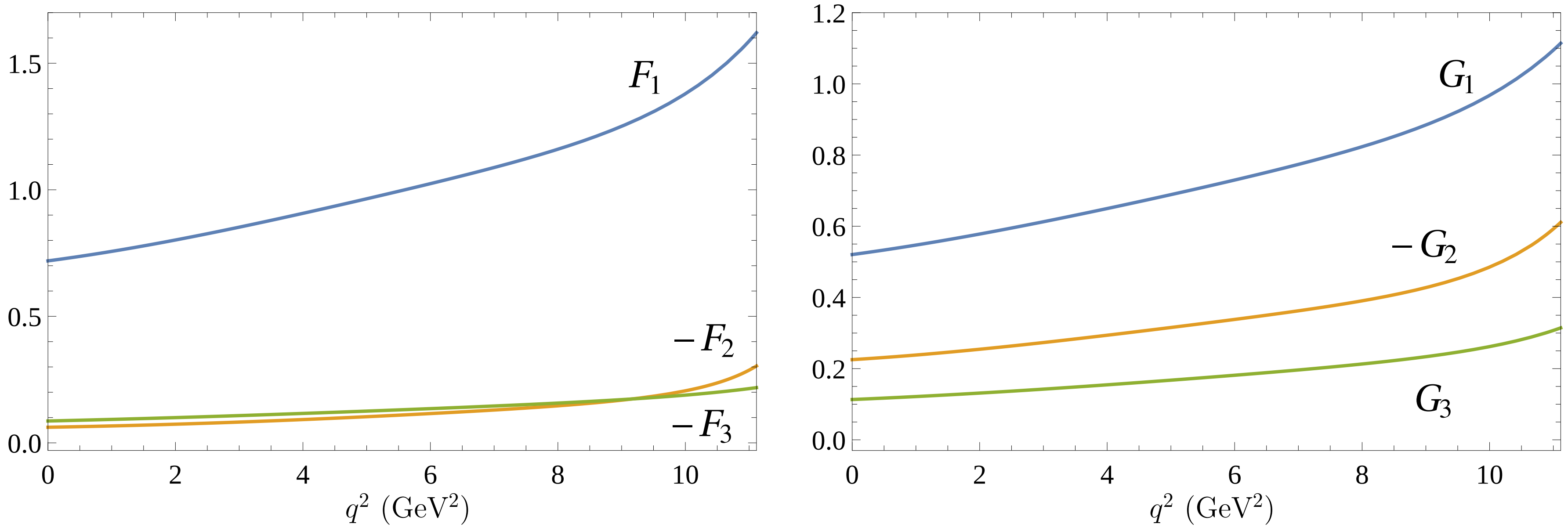


Figure 4: Form factors of the weak $\Lambda_b \to \Lambda_c$ transition.

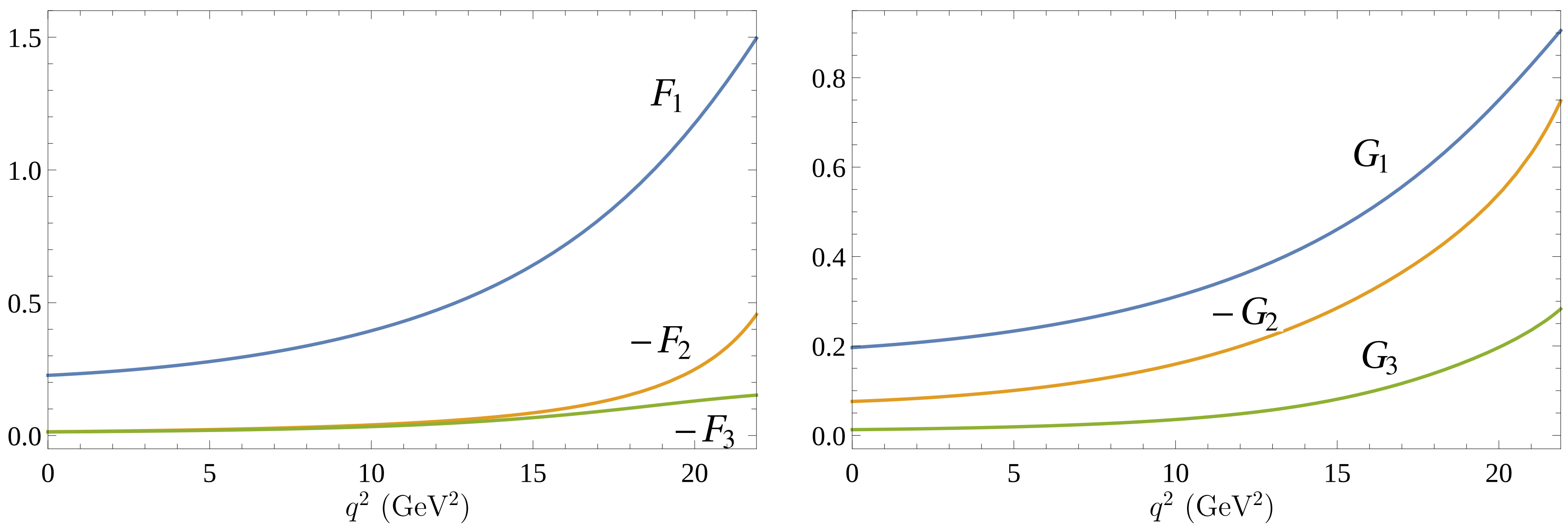


Figure 5: Form factors of the weak $\Lambda_b \to p$ transition.

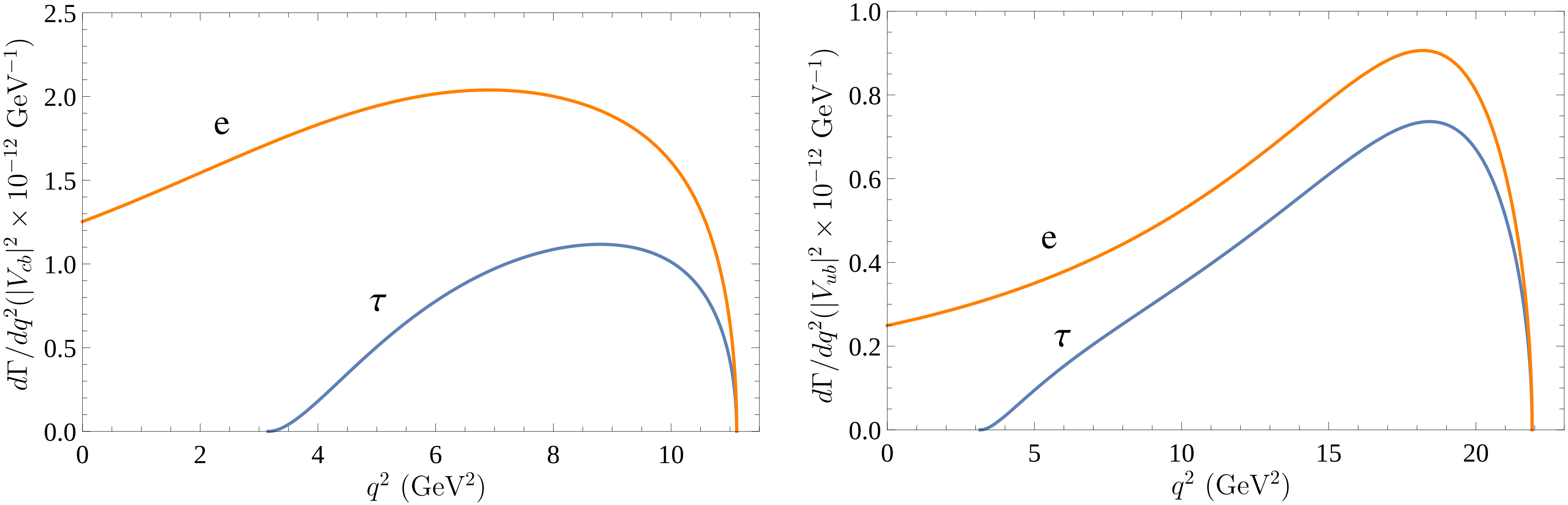


Figure 6: Predictions for the differential decay rates of the $\Lambda_b \to \Lambda_c l\nu$ (left) and $\Lambda_b \to pl\nu$ (right) semileptonic decays.

Table 3: Comparison of theoretical predictions for baryon semileptonic decays with experimental data.

Parameter	our RQM	Ivanov CCQM	Pervin SRQM	Dutta ELA	Ke LFQM	Detmold Lattice	Experiment PDG
$\Lambda_b \to \Lambda_c l\nu$							
Γ (ns^{-1})	44.2		53.9				
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	29.1					$21.5 \pm 0.8 \pm 1.1$	
Br (%)	6.48	6.9		4.83	6.3		$6.2^{+1.4}_{-1.3}$
$\Lambda_b \to \Lambda_c \tau\nu$							
Γ (ns^{-1})	13.9		20.9				
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	9.11					$7.15 \pm 0.15 \pm 0.27$	
Br (%)	2.03	2.0		1.63			
$\Lambda_b \to p l\nu$							
$\Gamma/\lvert V_{ub}\rvert^2$ (ps^{-1})	18.7	13.3	7.55			$25.7 \pm 2.6 \pm 4.6$	
Br (%)	0.045	0.029		0.0389	0.0254		
$\Lambda_b \to p \tau\nu$							
$\Gamma/\lvert V_{ub}\rvert^2$ (ps^{-1})	12.1	9.6	6.55			$17.7 \pm 1.3 \pm 1.6$	
Br (%)	0.029	0.021		0.0275			
$\Xi_b \to \Xi_c l\nu$							
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	25.7						
Br (%)	6.15			9.22			
$\Xi_b \to \Xi_c \tau\nu$							
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	8.4						
Br (%)	2.00			2.35			
$\Xi_b \to \Lambda l\nu$							
$\Gamma/\lvert V_{ub}\rvert^2$ (ps^{-1})	10.0						
Br (%)	0.026						

Ratios of branching fractions

$$R_{\Lambda_c} = \frac{Br(\Lambda_b \to \Lambda_c \tau\nu)}{Br(\Lambda_b \to \Lambda_c l\nu)}, \qquad R_p = \frac{Br(\Lambda_b \to p\tau\nu)}{Br(\Lambda_b \to pl\nu)}, \qquad R_{\Lambda cp} = \frac{\int_{15\,\mathrm{GeV}^2}^{q^2_{max}} \frac{d\Gamma(\Lambda_b \to p\mu\nu)}{dq^2} dq^2}{\int_{7\,\mathrm{GeV}^2}^{q^2_{max}} \frac{d\Gamma(\Lambda_b \to \Lambda_c\mu\nu)}{dq^2} dq^2}.$$

Table 4: Predictions for the ratios of Λ_b baryon decay rates.

Ratio	our	Dutta	Lattice	Experiment (LHCb)
R_{Λ_c}	0.313	0.3379	$0.3318 \pm 0.0074 \pm 0.0070$	
R_p	0.649	0.7071		
$R_{\Lambda cp}$	$(0.78 \pm 0.08)\frac{\lvert V_{ub}\rvert^2}{\lvert V_{cb}\rvert^2}$	0.0101	$(1.471 \pm 0.095 \pm 0.109)\frac{\lvert V_{ub}\rvert^2}{\lvert V_{cb}\rvert^2}$	$(1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$

Comparing our result for $R_{\Lambda cp}$ with experimental data we find

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.113 \pm 0.011|_{\mathrm{theor}} \pm 0.006|_{\mathrm{exp}},$$

in good agreement with the experimental ratio of these matrix elements extracted from inclusive decays

$$\frac{|V_{ub}|_{\mathrm{incl}}}{|V_{cb}|_{\mathrm{incl}}} = 0.105 \pm 0.006,$$

and with the corresponding ratio found in our previous analysis of exclusive semileptonic B and B_s meson decays [$|V_{cb}| = (3.90 \pm 0.15) \times 10^{-2}$, $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$]

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.104 \pm 0.012.$$

Charm baryon Λ_c and Ξ_c semileptonic decays

- $\Lambda_c \to \Lambda l \nu_l$
- $\Lambda_c \to n l \nu_l$
- $\Xi_c \to \Xi l \nu_l$
- $\Xi_c \to \Lambda l \nu_l$ $\qquad l = e, \mu$

Decays with τ lepton are not kinematically allowed

CKM matrix elements $|V_{cs}|$ and $|V_{cd}|$

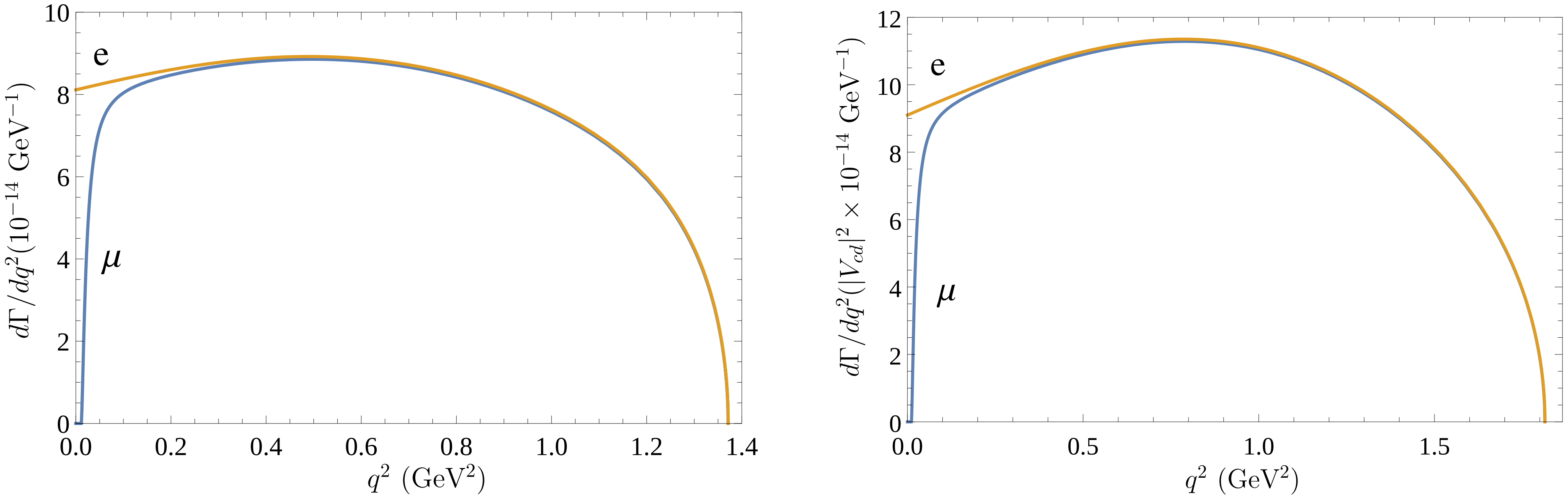


Figure 7: Differential decay rates of the $\Lambda_c^+ \to \Lambda l^+ \nu_l$ (left) and $\Lambda_c^+ \to n l^+ \nu_l$ (right) semileptonic decays.

Table 5: Theoretical predictions for the Λ_c baryon semileptonic decays and experimental data.

Parameter	our RQM	Ivanov CCQM	Pervin SRQM	Liu LCSR	Azizi LCSR	Experiment PDG
$\Lambda_c^+ \to \Lambda e^+ \nu_e$						
Γ (ns^{-1})	162	139	236			
Br (%)	3.25	2.78	4.72	3.0 ± 0.3		3.6 ± 0.4
$\Lambda_c^+ \to \Lambda \mu^+ \nu_\mu$						
Γ (ns^{-1})	157	135	236			
Br (%)	3.14	2.69	4.72	3.0 ± 0.3		3.5 ± 0.5
$\Lambda_c^+ \to n e^+ \nu_e$						
Γ (ns^{-1})	13.4		13.5			
$\Gamma/\lvert V_{cd}\rvert^2$ (ps^{-1})	0.265	0.20			8.21 ± 2.80	
Br (%)	0.268	0.207	0.27		8.69 ± 2.89	
$\Lambda_c^+ \to n \mu^+ \nu_\mu$						
$\Gamma/\lvert V_{cd}\rvert^2$ (ps^{-1})	0.260	0.19			8.3 ± 2.85	
Br (%)	0.262	0.202			8.78 ± 2.89	

Semileptonic Ξ_c baryon decays

- Until 2019 the absolute branching fractions of both neutral Ξ_c^0 and charged Ξ_c^+ baryons were not measured. All decay modes were only measured relative to $\Xi_c^0 \to \Xi^- \pi^+$ and $\Xi_c^+ \to \Xi^- \pi^+ \pi^+$ modes.
- Belle Collaboration [Phys. Rev. Lett. **122**, no. 8, 082001 (2019)] presented the first measurement of absolute branching fractions of the neutral Ξ_c^0 baryon

$$Br(\Xi_c^0 \to \Xi^- \pi^+) = (1.80 \pm 0.50 \pm 0.14)\%$$

- Then the absolute branching fractions of its charged partner Ξ_c was also reported by Belle [arXiv:1904.12093]

$$Br(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) = (2.86 \pm 1.21 \pm 0.38)\%$$

- Multiplying the CLEO II values for the ratios

$$\Gamma(\Xi_c^0 \to \Xi^- e^+ \nu_e)/\Gamma(\Xi_c^0 \to \Xi^- \pi^+) = 3.1 \pm 1.1$$

$$\Gamma(\Xi_c^+ \to \Xi^0 e^+ \nu_e)/\Gamma(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) = 2.3^{+0.7}_{-0.8}$$

by the recently measured by the Belle Collaboration branching fractions we get

$$Br(\Xi_c^0 \to \Xi^- e^+ \nu_e) = 5.58 \pm 2.62$$

$$Br(\Xi_c^+ \to \Xi^0 e^+ \nu_e) = 6.58 \pm 3.85$$

- Using ARGUS Collaboration ratio

$$\Gamma(\Xi_c^0 \to \Xi^- e^+ \text{anything})/\Gamma(\Xi_c^0 \to \Xi^- \pi^+) = 1.0 \pm 0.5$$

we get for the semi-inclusive branching fraction

$$Br(\Xi_c^0 \to \Xi^- e^+ \text{anything}) = (1.80 \pm 1.07)\%$$

Table 6: Comparison of theoretical predictions for the Ξ_c semileptonic decay branching fractions (in %) with available experimental data.

Decay	our RQM	Zhao LFQM	Geng $SU(3)$	Geng $SU(3)$	Azizi LCSR	Experiment
$Br(\Xi_c^0 \to \Xi^- e^+ \nu_e)$	2.38	1.35	4.87 ± 1.74	2.4 ± 0.3	7.26 ± 2.54	5.58 ± 2.62
$Br(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu)$	2.31			2.4 ± 0.3	7.15 ± 2.50	
$Br(\Xi_c^+ \to \Xi^0 e^+ \nu_e)$	9.40	5.39	$3.38^{+2.19}_{-2.26}$	9.8 ± 1.1	28.6 ± 10.0	6.58 ± 3.85
$Br(\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu)$	9.11			9.8 ± 1.1	28.2 ± 9.9	
$Br(\Xi_c^+ \to \Lambda e^+ \nu_e)$	0.127	0.082		0.166 ± 0.018		
$Br(\Xi_c^+ \to \Lambda \mu^+ \nu_\mu)$	0.124					

- Under the exact $SU(3)$ limit

$$\frac{\Gamma(\Lambda_c \to n e \nu_e)}{|V_{cd}|^2} = \frac{3\Gamma(\Lambda_c \to \Lambda e \nu_e)}{2|V_{cs}|^2} = \frac{6\Gamma(\Xi_c \to \Lambda e \nu_e)}{|V_{cd}|^2} = \frac{\Gamma(\Xi_c \to \Xi e \nu_e)}{|V_{cs}|^2}.$$

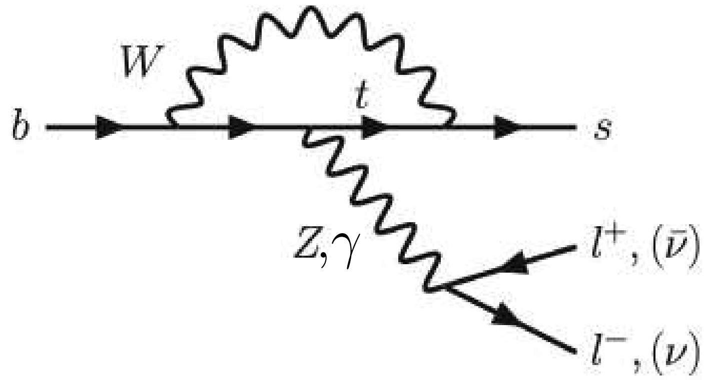
Table 7: Predictions for the ratios $\Gamma/|V_{cq}|^2$ in ps^{-1} $(q = s, d)$

Decay	our result	exact $SU(3)$	difference
$\Lambda_c \to n e \nu_e$	0.265	0.265	
$\Lambda_c \to \Lambda e \nu_e$	0.167	0.177	6%
$\Xi_c \to \Lambda e \nu_e$	0.059	0.044	34%
$\Xi_c \to \Xi e \nu_e$	0.215	0.265	19%

Rare semileptonic $\Lambda_b \to \Lambda l^+ l^-$ baryon decays

The effective Hamiltonian for the rare $b \to s$ transitions (FCNC)

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} c_i \mathcal{O}_i,$$



G_F — Fermi constant
V_{tj} — CKM matrix elements
c_i — Wilson coefficients

$\mathcal{O}_i$ — standard model operators:

$$\mathcal{O}_i \sim (\bar{s}b)(\bar{c}c), \text{ for } i = 1 \ldots 6$$
$$\mathcal{O}_8 \sim m_b \bar{s}(\sigma \cdot G) b$$

The only operators with a tree-level non-vanishing matrix element in $b \to s\ell^+\ell^-$ are given by:

$$\mathcal{O}_7 = \frac{e}{4\pi^2} \bar{s}_L \sigma_{\mu\nu} m_b b_R F^{\mu\nu} ,$$

$$\mathcal{O}_9 = \frac{e^2}{4\pi^2} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \ell ,$$

$$\mathcal{O}_{10} = \frac{e^2}{4\pi^2} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \gamma_5 \ell .$$

Then the matrix element of the $b \to s l^+ l^-$ transition amplitude between baryon states is given by

$$\mathcal{M}(\Lambda_b \to \Lambda l^+ l^-) = \frac{G_F \alpha}{2\sqrt{2}\pi} |V_{ts}^* V_{tb}| \left[T_\mu^{(1)} (\bar{l}\gamma^\mu l) + T_\mu^{(2)} (\bar{l}\gamma^\mu \gamma_5 l) \right], \quad (1)$$

where

$$\begin{aligned} T_\mu^{(1)} &= c_9^{eff} \langle \Lambda | \bar{s}\gamma^\mu (1-\gamma^5) b | \Lambda_b \rangle - \frac{2m_b}{q^2} c_7^{eff} \langle \Lambda | \bar{s} i \sigma^{\mu\nu} q_\nu (1+\gamma^5) b | \Lambda_b \rangle, \\ T_\mu^{(2)} &= c_{10} \langle \Lambda | \bar{s}\gamma^\mu (1-\gamma^5) b | \Lambda_b \rangle \end{aligned} \quad (2)$$

$T^{(m)}$ $(m = 1, 2)$ are expressed through the form factors and the Wilson coefficients.

The matrix element of the flavour changing neutral current for the rare $\Lambda_b \to \Lambda l^+ l^-$ baryon

$$\langle \Lambda(p', s') | \bar{s}\gamma^\mu b | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') \left[f_1^V(q^2)\gamma^\mu - f_2^V(q^2) i\sigma^{\mu\nu} \frac{q_\nu}{M_{\Lambda_b}} + f_3^V(q^2) \frac{q^\mu}{M_{\Lambda_b}} \right] u_{\Lambda_b}(p, s),$$

$$\langle \Lambda(p', s') | \bar{s}\gamma^\mu \gamma_5 b | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') [f_1^A(q^2)\gamma^\mu - f_2^A(q^2) i\sigma^{\mu\nu} \frac{q_\nu}{M_{\Lambda_b}} + f_3^A(q^2) \frac{q^\mu}{M_{\Lambda_b}}] \gamma_5 u_{\Lambda_b}(p, s),$$

$$\langle \Lambda(p', s') | \bar{s} i\sigma^{\mu\nu} q_\nu b | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') \left[\frac{f_1^{TV}(q^2)}{M_{\Lambda_b}} \left(\gamma^\mu q^2 - q^\mu \not{q} \right) - f_2^{TV}(q^2) i\sigma^{\mu\nu} q_\nu \right] u_{\Lambda_b}(p, s),$$

$$\langle \Lambda(p', s') | \bar{s} i\sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') \left[\frac{f_1^{TA}(q^2)}{M_{\Lambda_b}} \left(\gamma^\mu q^2 - q^\mu \not{q} \right) - f_2^{TA}(q^2) i\sigma^{\mu\nu} q_\nu \right] \gamma_5 u_{\Lambda_b}(p, s),$$

$u_{\Lambda_b}(p, s)$ and $u_\Lambda(p', s')$ — Dirac spinors, $\qquad q = p' - p.$

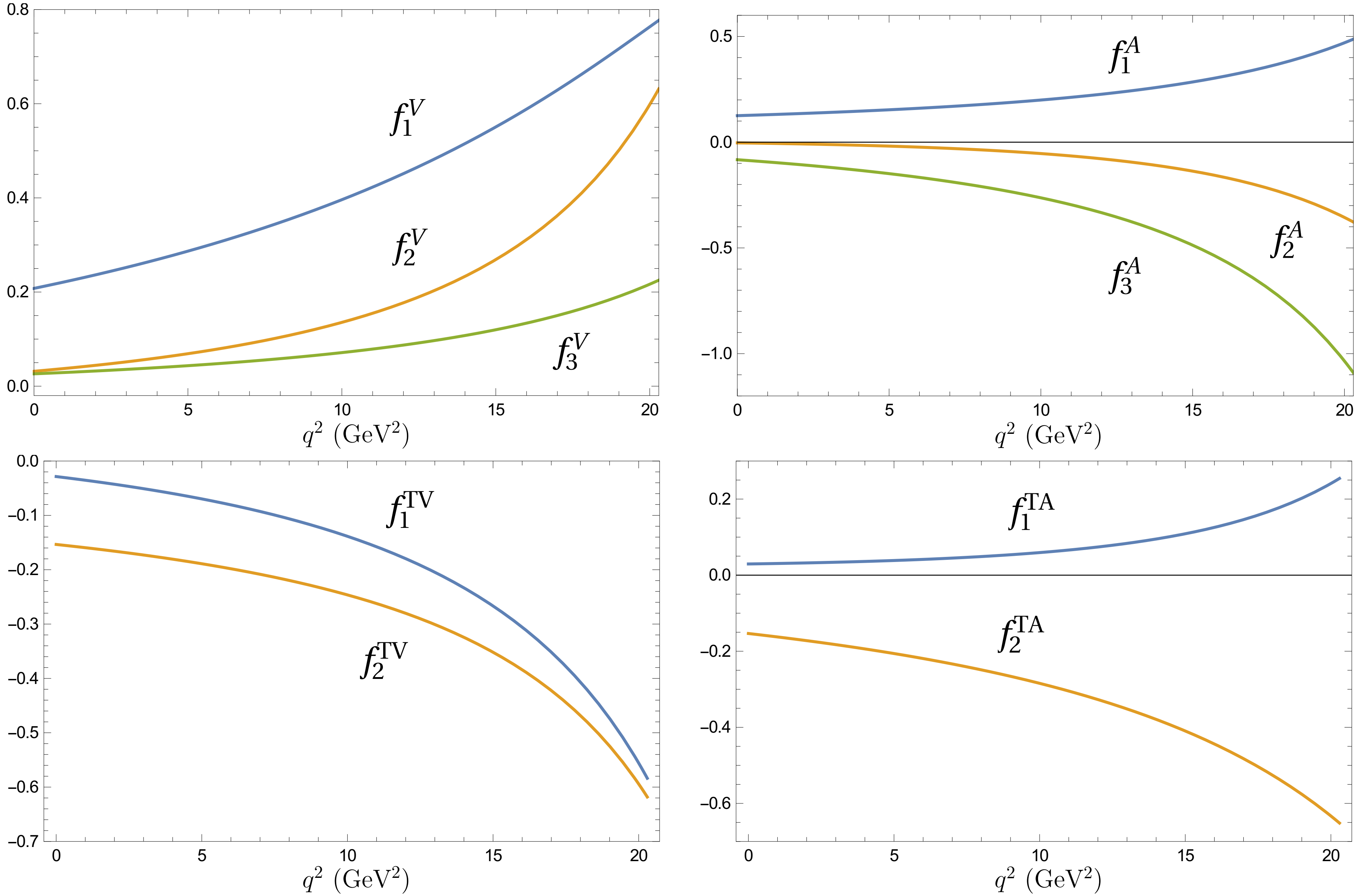


Figure 8: Form factors of the rare weak $\Lambda_b \to \Lambda$ transition.

The effective Wilson coefficient c_9^{eff} contains additional pertubative and long-distance contributions

$$c_9^{\text{eff}} = c_9 + \mathcal{Y}_{\text{pert}}(q^2) + \mathcal{Y}_{\text{BW}}(q^2).$$

- perturbative part

$$\begin{aligned}
\mathcal{Y}_{\text{pert}}(q^2) \quad = \quad & h\left(\frac{m_c}{m_b}, \frac{q^2}{m_b^2}\right)(3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6) \\
& -\frac{1}{2}h\left(1, \frac{q^2}{m_b^2}\right)(4c_3 + 4c_4 + 3c_5 + c_6) \\
& -\frac{1}{2}h\left(0, \frac{q^2}{m_b^2}\right)(c_3 + 3c_4) + \frac{2}{9}(3c_3 + c_4 + 3c_5 + c_6),
\end{aligned}$$

where

$$h\left(\frac{m_c}{m_b}, \frac{q^2}{m_b}\right) = -\frac{8}{9}\ln\frac{m_c}{m_b} + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x)|1-x|^{1/2}\begin{cases} \ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi, & x \equiv \frac{4m_c^2}{q^2} < 1, \\ 2\arctan\frac{1}{\sqrt{x-1}}, & x \equiv \frac{4m_c^2}{q^2} > 1, \end{cases}$$

$$h\left(0, \frac{q^2}{m_b}\right) = \frac{8}{27} - \frac{4}{9}\ln\frac{q^2}{m_b} + \frac{4}{9}i\pi.$$

- long-distance (nonperturbative) contributions are assumed to originate from the $c\bar{c}$ resonances $(J/\psi, \psi' \ldots)$ and have a usual Breit-Wigner structure:

$$\mathcal{Y}_{\rm BW}(q^2) = \frac{3\pi}{\alpha^2} \sum_{V_i = J/\psi, \psi(2S) \ldots} \frac{\Gamma(V_i \to l^+ l^-) M_{V_i}}{M_{V_i}^2 - q^2 - i M_{V_i} \Gamma_{V_i}}.$$

Contributions of the vector $V_i(1^{--})$ charmonium states: J/ψ, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$

The lepton angle differential decay distribution

$$\frac{d^2\Gamma(\Lambda_b \to \Lambda l^+ l^-)}{dq^2 d\cos\theta} = \frac{d\Gamma(\Lambda_b \to \Lambda l^+ l^-)}{dq^2} \left[\frac{3}{8}(1+\cos^2\theta)(1-F_L) + A_{FB}^{\ell}\cos\theta + \frac{3}{4} F_L \sin^2\theta\right],$$

θ — angle between the Λ_b baryon and the positively charged lepton in the dilepton rest frame.

- Lepton forward-backward asymmetry

$$A_{FB}^{\ell}(q^2) = \frac{\frac{d\Gamma}{dq^2}(\text{forward}) - \frac{d\Gamma}{dq^2}(\text{backward})}{\frac{d\Gamma}{dq^2}}$$

- F_L — fraction of longitudinally polarized dileptons

Hadron angle differential distribution of the decay $\Lambda_b \to \Lambda(\to p\pi^-)l^+l^-$

$$\frac{d^2\Gamma(\Lambda_b \to \Lambda l^+ l^-)}{dq^2 d\cos\theta_h} = Br(\Lambda \to p\pi^-)\frac{d\Gamma(\Lambda_b \to \Lambda l^+ l^-)}{dq^2}\frac{1}{2}\left(1 + 2A^h_{FB}\cos\theta_h\right),$$

θ_h — angle between the proton and the Λ baryon in the Λ_b rest frame.

- A^h_{FB} — hadron forward-backward asymmetry

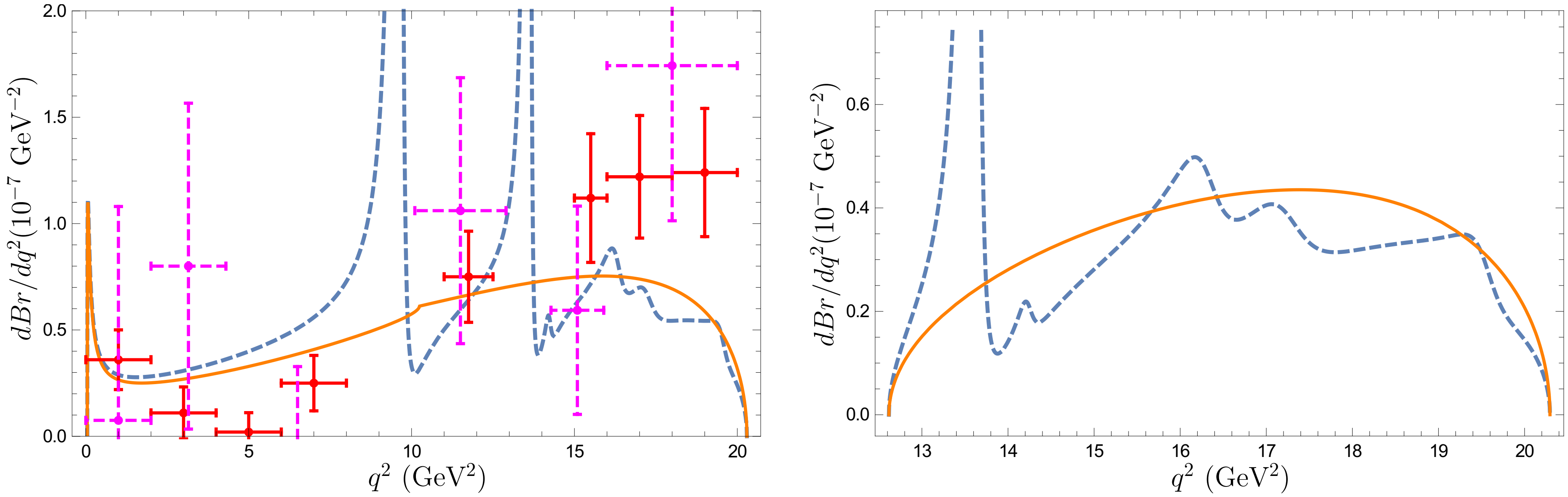


Figure 9: Predictions for the differential branching ratios for the $\Lambda_b \to \Lambda\mu^+\mu^-$ (left) and $\Lambda_b \to \Lambda\tau^+\tau^-$ (right) rare decays. Available experimental data from LHCb are given by dots with solid error bars, CDF data are given by dots with dashed error bars.

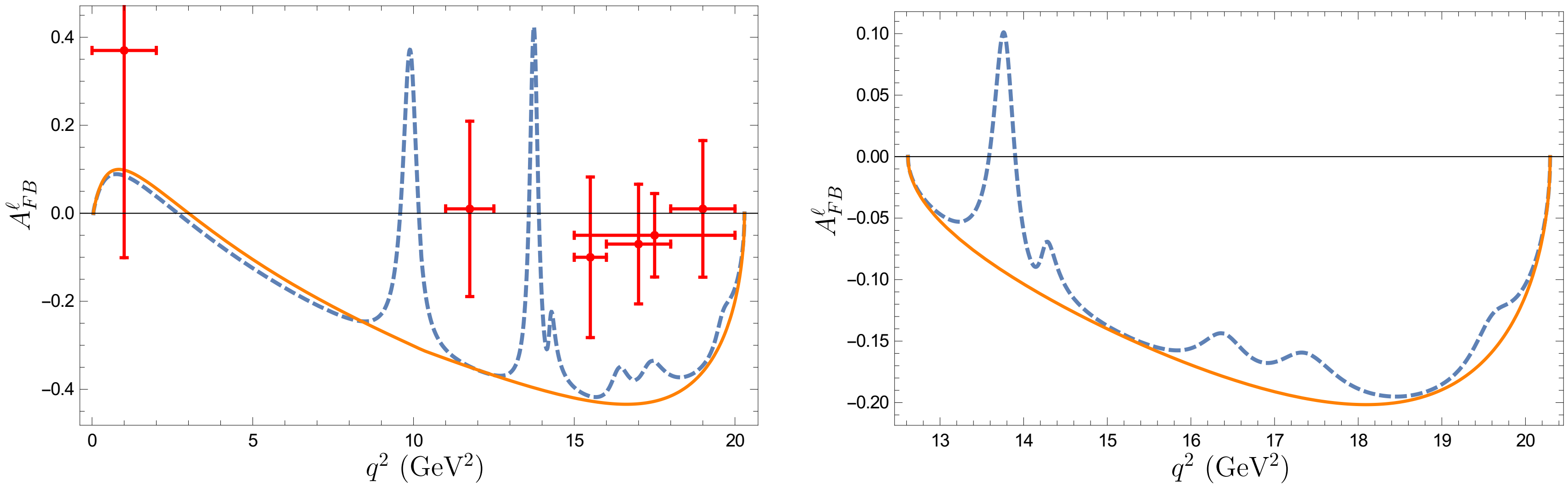


Figure 10: Predictions for the lepton forward-backward asymmetries $A^{\ell}_{FB}(q^2)$ in the $\Lambda_b \to \Lambda\mu^+\mu^-$ (left) and $\Lambda_b \to \Lambda\tau^+\tau^-$ (right) rare decays. Data from LHCb are given by dots with solid error bars.

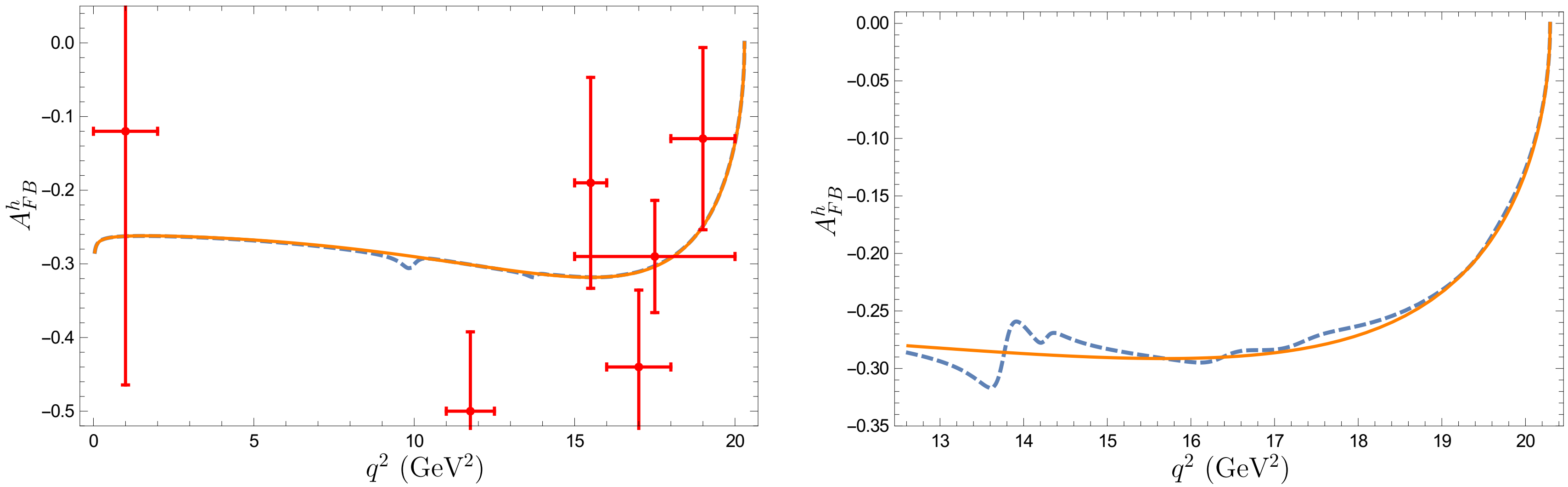


Figure 11: Predictions for the hadron forward-backward asymmetries $A^{h}_{FB}(q^2)$ in the $\Lambda_b \to \Lambda\mu^+\mu^-$ (left) and $\Lambda_b \to \Lambda\tau^+\tau^-$ (right) rare decays. Data from LHCb are given by dots with solid error bars.

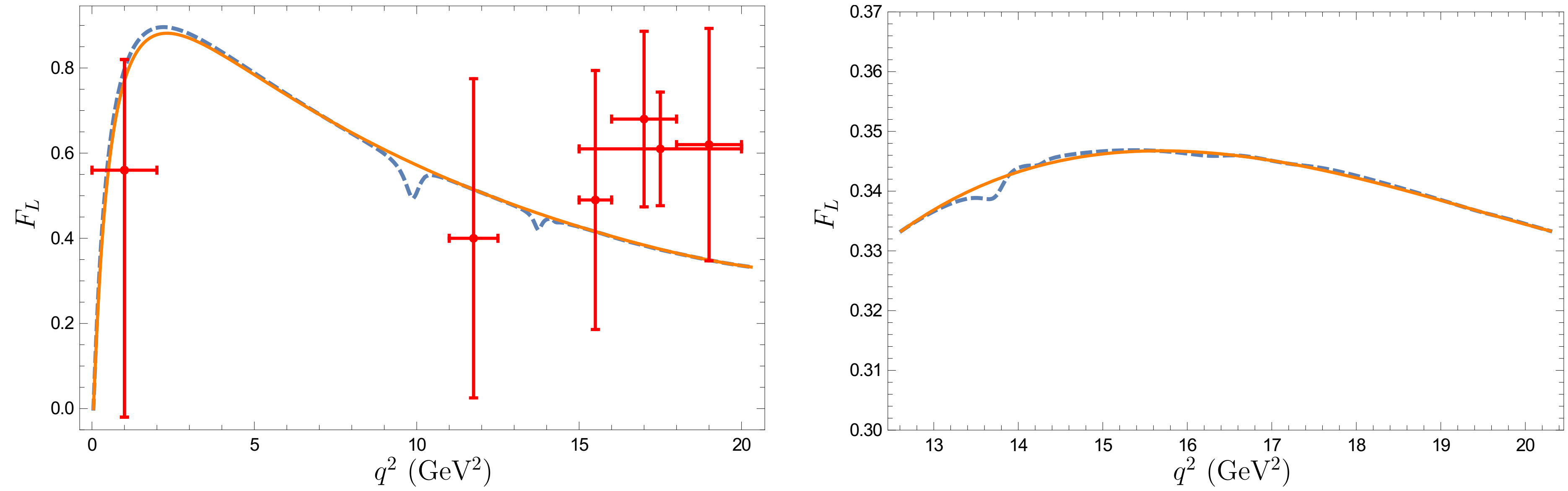


Figure 12: Prediction for the fraction of longitudinally polarized dileptons $F_L(q^2)$ in the $\Lambda_b \to \Lambda\mu^+\mu^-$ (left) and $\Lambda_b \to \Lambda\tau^+\tau^-$ (right) rare decays. Data from LHCb are given by dots with solid error bars.

Table 8: Comparison of theoretical predictions for baryon rare decay branching fractions $(\times 10^{-6})$ with experimental data.

Decay	our RQM	Ivanov CCQM	Aliev LCSR	Wang LCSR	Liu RQM	Mott NRQM	Gan LCSR	Experiment
$\Lambda_b \to \Lambda e^+e^-$	1.07	1.0	4.6(1.6)		$1.21 \sim 2.32$		$2.03\,\binom{26}{9}$	
$\Lambda_b \to \Lambda\mu^+\mu^-$	1.05	1.0	4.0(1.2)	$6.1\,\binom{5.8}{1.7}$	$0.53 \sim 0.89$	0.70		$1.08(28)$
$\Lambda_b \to \Lambda\tau^+\tau^-$	0.26	0.2	0.8(3)	$2.1\,\binom{2.3}{0.6}$	$0.037 \sim 0.083$	0.22		
$\Lambda_b \to n e^+e^-$	0.0381							
$\Lambda_b \to n\mu^+\mu^-$	0.0375							
$\Lambda_b \to n\tau^+\tau^-$	0.0121							

Rare radiative Λ_b baryon decays

The exclusive rare radiative decay rate $\Lambda_b \to \Lambda\gamma$ for the emission of a real photon ($k^2 = 0$)

$$\Gamma(\Lambda_b \to \Lambda\gamma) = \frac{\alpha}{64\pi^4} G_F^2 m_b^2 M_{\Lambda_b}^3 |V_{tb}V_{ts}|^2 |c_7^{\rm eff}(m_b)|^2 (|f_2^{TV}(0)|^2 + |f_2^{TA}(0)|^2) \left(1 - \frac{M_\Lambda^2}{M_{\Lambda_b}^2}\right)^3 .$$

Table 9: Comparison of theoretical predictions for the rare radiative decay branching fraction ($\times 10^{-5}$) with available experimental data.

Decay	our RQM	Mannel HQS	Ivanov CCQM	Wang LCSR	Gan LCSR	Colangelo QCDSR	Experiment
$\Lambda_b \to \Lambda\gamma$	1.0	$0.77(^{22}_{19})$	0.4	$0.73(15)$	$0.061(^{14}_{13})$	3.1(6)	< 130
$\Lambda_b \to n\gamma$	0.037						

CONCLUSIONS

- Baryons were considered in the framework of the relativistic quark–diquark picture.
- The wave functions of heavy and light baryons were calculated in the relativistic quark model with the set of model parameters, fixed from previous considerations of meson properties.
- Light quarks, light diquarks and heavy quarks were treated fully relativistically without application of the nonrelativistic v/c and heavy quark $1/m_Q$ expansions.
- Internal structure of the diquark was taken into account by calculating the form factor of the diquark–gluon interaction.
- Diquark and baryon wave functions were used for the calculation of semileptonic decays.
- Structure of weak decay matrix elements agrees with model independent predictions of HQET both at leading and subleading orders of heavy quark expansion.
- Leading and subleading Isgur-Wise functions for heavy baryon decays were explicitly expressed through the overlap integrals of wave functions in the whole accessible kinematic range.
- Heavy-to-light and heavy-to-heavy baryon semileptonic decays were considered without using heavy quark expansion.
- Semileptonic decays of charm baryons were studied
- Rare semileptonic and radiative decays were investigated
- Calculated decay rates and differential distributions agree with available experimental data.