Back reaction in graphene excited by a strong laser field

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I. What is graphene ?

- Graphene is a monolayer carbon crystalline material with the hexagonal honeycomb lattice with two interpenetrating triangular sublattices A and B sites.

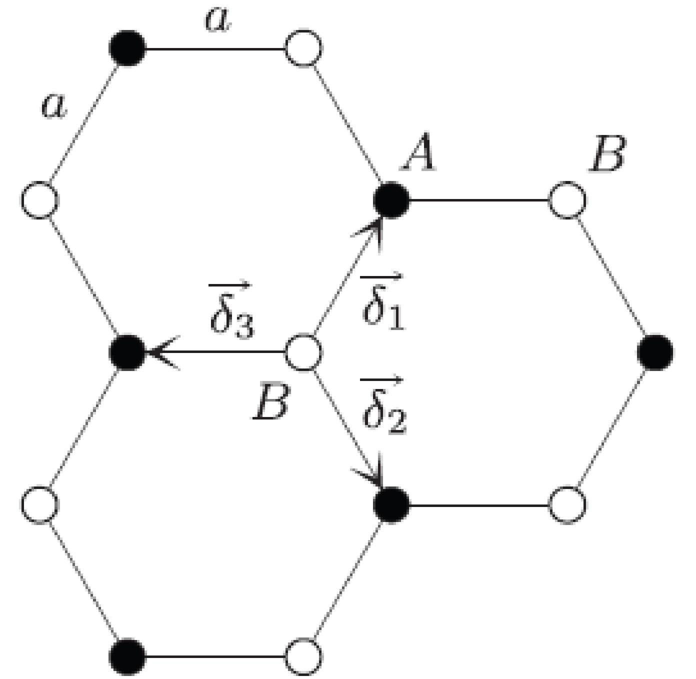
  Here lattice vectors

  $\vec{\delta}_i \quad (i = 1,2,3)$ are

  $\vec{\delta}_1 = \frac{a}{2}(1, \sqrt{3})$, $\vec{\delta}_2 = \frac{a}{2}(1, -\sqrt{3})$,

  $\vec{\delta}_3 = -a(1,0)$;

  $a = 1.42$ Å is the distance between nearest carbon atoms.



II. Why is graphene ?

Graphene is an actual perspective material for microelectronics: minimal thickness (monoatomic layer), dynamical strength, high thermal and electro-conductivity [Geim, Castro Neto, Guinea, Peres, Novoselov].

However, it is important an others thing: graphene is rather simple model of the standard QED and allows experimental verification of theoretical predictions.

This circumstance increases confidence in correctness of the standard QED in one or other extremal conditions, that where experimental proof meets with large difficulties.

As an example, it can point up to the strong field QED, where the vacuum creation of $e^{-}e^{+}$ plasma was predicted in the region of critical electric fields with strength $E \sim E_c = m^2 c^2 / e\hbar \sim 10^{18}$ V/m (for electrons) [Sauter; Heisenberg, Euler; Schwinger].

III. In which respects are simplifications ?

If to consider graphene as D=2+1 QFT, the spin degrees of freedom can not be realized and number of equation of motion decreases greatly. All other specific features of the standard QED survive.

The new element is demand of massless of the electrons in the low-momentum approximation around two inequivalent “Dirac points” corresponding to valleys of the first Brillouin zone.

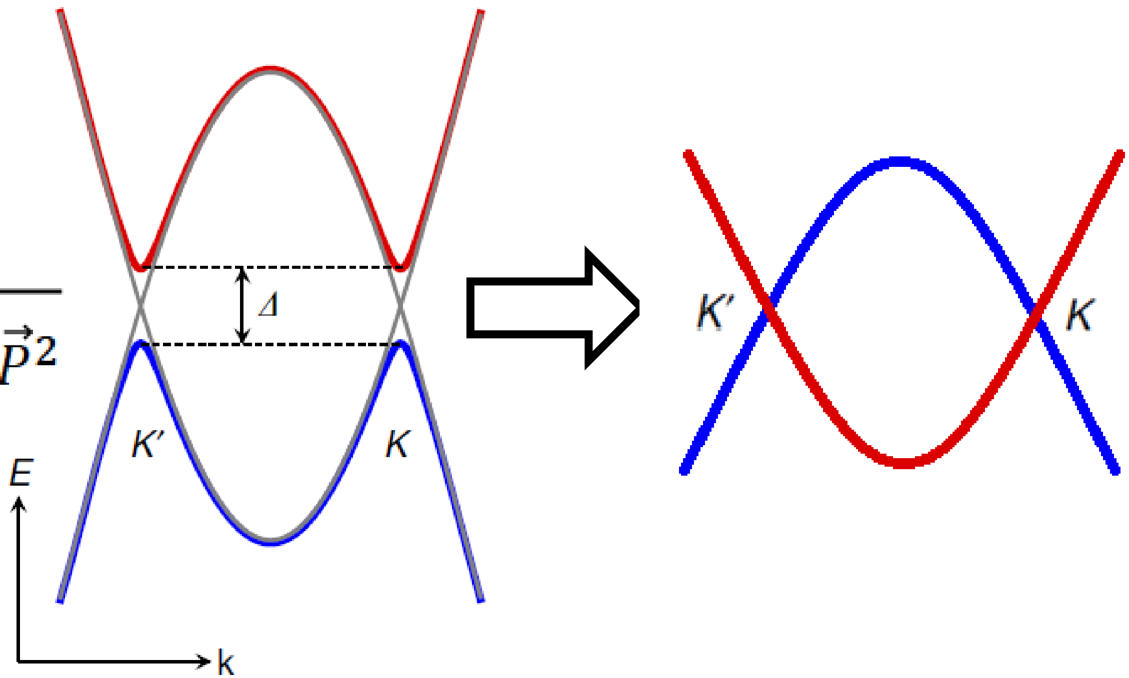
So graphene as QFT is massless D=2+1 QED.

Such picture is well-founded [Neto, Guinea, Peres, Novoselov, Geim; Gusynin, Sharapov, Carbotte; Vozmediano, Katsnelson, Guinea]

Dirac points

$m \neq 0$

$\varepsilon = v_F\sqrt{m_*^2 + \vec{P}^2}$



$m = 0$

$\varepsilon = v_F\sqrt{\vec{P}^2}$

Structure of the Brillouin zone in the materials with "relativistic" dispersion laws in the low-momentum regions; v_F is the Fermi velocity which plays a part of the light velocity; points K′, K in the momentum space are the Dirac points; m_* is effective mass.

Dirac points

Absence of the carries masses in graphene brings to strong response of graphene under action of any weak electric field. It reflects on nulling of the critical field $E_c = 0$. So, any electric field is strong.

IV. Statement of problem

We will consider excitation of eh-plasma in graphene under action of a semiclassical spatially homogeneous external electric field with arbitrary polarization and time dependence. The corresponding vector potential $\vec{A}_{ex}(t)$ is given in the Hamiltonian gauge $A^0(t) = 0$, i.e. $A^k_{ex}(t) = \ (0, A^1_{ex}(t), A^2_{ex}(t))$.

IV. Statement of problem

The aim is construction of kinetic theory of eh-plasma excitation in such fields with taken into account of the internal (plasma) electric fields , generated by eh-currents (back reaction).

It results to some new effects: partial suppression of the external field and creation of eh-plasma as result of back action of the plasma field and also to radiation of electromagnetic field from specimen of graphene on the frequencies of eh-plasma oscillations. The corresponding currents and radiation are accessible to experiment verification

V. Equation of motion and Hamiltonian

Bellow it will be consider the low-momentum model (the formalism allows generalization on the case of the tight bindings model of nearest neighbour interaction of the carbon atoms [Neto, Guinea, Peres, Novoselov, Geim; Gusynin, Sharapov, Carbotte; Kao, Lewkowicz, Rosenstein].

In the continuum limit the Dirac-type equation for the low-energy excitations in graphene in a time dependent electric field described above is

$$i\hbar\dot{\Psi}(\vec{x},t) = v_F\hat{\vec{P}}\vec{\sigma}\Psi(\vec{x},t), \text{ (\$)}$$

where $\hat{P}_k = -i\hbar\nabla_k - \left(\frac{e}{c}\right)A_k(t)$ is the quasi-momentum ($k = 1, 2$) and σ_k are the Pauli matrices corresponding to the pseudospin structure of graphene.

V. Equation of motion and Hamiltonian

The Hamiltonian of the theory,

$$H(t) = \frac{i\hbar}{2}\int d^2x\left[\Psi^+(\vec{x},t)\dot{\Psi}(\vec{x},t) - \dot{\Psi}^+(\vec{x},t)\Psi(\vec{x},t)\right],$$

is the 00 component of the corresponding energy-momentum tensor and it can be transformed with help of the equation of motion to the form

$$H(t) = v_F\int d^2x\,\Psi^+(\vec{x},t)\hat{\vec{P}}\vec{\sigma}\Psi(\vec{x},t).$$

Here we dropped the spin indices.

V. Equation of motion and Hamiltonian

The wave function here is a two-component spinor permitting the decomposition

$$\Psi^T(\vec{x},t) = \frac{1}{(2\pi\hbar)^2}\int d^2p \left(\Psi_{\vec{p}}^{(1)}(t), \Psi_{-\vec{p}}^{(2)}(t)\right) e^{i\vec{p}\vec{x}/\hbar},$$

which translates the Hamiltonian function to the momentum representation.

VI. Quasiparticle representation (QPR)

Now we use analogy with the standard QED and pass to the QPR [Grib, Mamaev, Mostepanenko] that allows to describe the problem of vacuum generation of charged particles in vivid physical terms.

Idea of the QPR is based on natural writing of the energy density on of excited eh-pairs in presence of a semiclassical field

$$\varepsilon(t) = 2N_f \int d^2p\, \varepsilon(\vec{p}, t) f(\vec{p}, t),$$

where $\varepsilon(\vec{p}, t) = \sqrt{m^2 + \vec{P}^2}$ is quasienergy of the carries with the quasimomentum $\vec{P} = \vec{p} - \frac{e}{c}\vec{A}$, and

VI. Quasiparticle representation (QPR)

$$f(\vec{p},t) = <\mathrm{in}|a^+(\vec{p},t)a(\vec{p},t)|\mathrm{in}>$$

is the distribution function of quasiparticles, |in> is in-vacuum state at $t \to -\infty$, $a^+(\vec{p},t)$ and $a(\vec{p},t)$ are creation and annihilation operators of quasielectrons. Finally, N_f is the number of flavors of carries: in the case of a single-layer graphene $N_f = 2 \cdot 2 = 4$ (two Dirac points in the Brillouin zone, two “pseudospin” number, corresponding to a fermion with two spin projections). Factor 2 corresponds to equal contributions of electrons and holes.

Transition to the QPR can be realized with help of an unitary transformation $\Psi(t) = U(t)\Phi(t)$, $U(t)U^+(t)$ =1, conserving norm of the vector, $|\Psi(t)|^2 = |\Phi(t)|^2$.

VI. Quasiparticle representation (QPR)

As it was shown in the work [Dora, Moessner], this is achieved with the unitary transformation

$$U^{+}(t) v_F \vec{P} \vec{\sigma} U(t) = \varepsilon(\vec{p}, t) \sigma_3 = H_{\vec{p}}(t), (\$\$)$$

and $\Phi = U^{+}\Psi$ with unitary matrix

$$U(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} \exp(-i\kappa/2) & \exp(-i\kappa/2) \\ \exp(i\kappa/2) & -\exp(i\kappa/2) \end{pmatrix}.$$

The function κ is defined by the condition (\$\$), corresponding to $\tan\kappa = P^2/P^1$, where $P^k = p^k - \frac{e}{c} A^k(t)$. The quasienergy $\varepsilon(\vec{p}, t)$ in (\$\$) is determined by the dispersion relation in the vicinity of the Dirac points.

$$\varepsilon(\vec{p}, t) = v_F \sqrt{P^2} = v_F \sqrt{(P^1)^2 + (P^2)^2}.$$

VI. Quasiparticle representation (QPR)

Equation ($) transforms then to the form

$$i\hbar\dot{\Phi} = H_{\vec{p}}(t)\Phi + \frac{1}{2}\lambda\hbar\sigma_1\Phi, \quad (\$*)$$

where $H_{\vec{p}}(t)$ is defined by Equation ($$) and

$$\lambda(\vec{p},t) = \dot{\kappa} = \frac{ev_F^2[E_1P_2 - E_2P_1]}{\varepsilon^2(\vec{p},t)}. \quad (*)$$

Introducing the notation

$$\Phi(\vec{p},t) = \begin{bmatrix} a(\vec{p},t) \\ b^+(-\vec{p},t) \end{bmatrix}, \quad (\$\$\$)$$

the Hamiltonian function can be rewritten in the quasiparticle form $H(t) = \int [dp]\, \varepsilon(\vec{p},t)\Phi^+(\vec{p},t)\sigma_3\Phi(\vec{p},t)$

$$= \int [dp]\, \varepsilon(\vec{p},t)[a^+(\vec{p},t)a(\vec{p},t) - b(-\vec{p},t)\, b^+(-\vec{p},t)],$$

VI. Quasiparticle representation (QPR)

where the abbreviation $[dp] = \int d^2p$ has been used.

Apparently, the realization of the unitary transformation in the explicit form in both the low-energy and the tight-binding models is a result of the fact that these models belong to the class of massless field theories.

At this stage one can go over to the occupation number representation and replace the amplitudes $a^+(t), a(t)$ and $b^+(t), b(t)$ by the corresponding creation and annihilation operators for electrons and holes considered as quasiparticles. These operators are defined on the in-vacuum state |in> with vector potential $\vec{A}_{in}$

VI. Quasiparticle representation (QPR)

and satisfy the canonical anti-commutation relation $\{a(\vec{p},t), a^{+}(\vec{p}',t)\}_{+} = \{b(\vec{p},t), b^{+}(\vec{p}',t)\}_{+} = (2\pi)^{2}\delta(\vec{p}-\vec{p}')$.
Other elementary anti-commutators are equal to zero.

From Equations ($), ($$) and ($$$) it follows the equations of motion of the Heisenberg type for the description of the unitary evolution of the creation and annihilation operators, e.g.,

$$\dot{a}(\vec{p},t) = \frac{i}{\hbar}[H(t), a(\vec{p},t)] - \frac{i}{2}\lambda(\vec{p},t)b^{+}(-\vec{p},t) = \frac{i}{\hbar}[H_{tot}(t), a(\vec{p},t)], \quad (**)$$

$$\dot{b}(\vec{p},t) = \frac{i}{\hbar}[H(t), b(-\vec{p},t)] + \frac{i}{2}\lambda(\vec{p},t)a^{+}(\vec{p},t) = \frac{i}{\hbar}[H_{tot}(t), b(-\vec{p},t)], \quad (***)$$

where the amplitude of the transitions between states with the positive and negative energies of the quasiparticles is defined by Equation (*).

VI. Quasiparticle representation (QPR)

From Equations ($*), (**) and (***) it follows that evolution of the system is unitary. The Fock space is constructed on the time dependent vacuum state. In Equations (**) and (***) $H_{tot} = H + H_{pol}$, where

$$H_{pol} = \frac{\hbar}{2}\int [dp]\lambda(\vec{p},t)[a^{+}(\vec{p},t)b^{+}(-\vec{p},t) - b(-\vec{p},t)a(\vec{p},t)]$$

describes the dynamics of vacuum polarization.

VI. Quasiparticle representation (QPR)

Let us mark, hat this unitary transformation in graphene can be realized in an explicit form, in contrast to the massive D=3+1 QED [Grib, Mamaev, Mostepanenko].

The QPR is not "uniquely true theory" in strong field QED. It connects with the fact that the notion of the quasiparticle energy in the point of time is relative. This shortcoming of QPR was noted in literature (e.g., [Birrell, Davies]). However, other formalism with habitual physical meaning is absent (e.g., [Unger, Dong, Flores, Su, Grobe]).

VII. Kinetic equations (KE)

The distribution function $f(\vec{p},t) = <\mathrm{in}|a^{+}(\vec{p},t)a(\vec{p},t)|\mathrm{in}>$ is the main object of kinetic theory. This function satisfies the kinetic equation , that can be obtained with help of the equations of motion for the operators $a^{+}(\vec{p},t), a(\vec{p},t)$.

Such kinetics in graphene was constructed in [Smolyansky, Panferov, Blaschke, Gevorgyan] by analogy with the standard QED [Marinov, Trunov; Bialynicky-Birula, Gornicki, Rafelski; Schmidt, Blaschke, Smolyansky, Toneev].

The corresponding KE can be written in QPR in the integro-differential form

$$\dot{f}(\vec{p},t) = \frac{1}{2}\lambda(\vec{p},t)\int_{t_0}^{t} dt'\,\lambda(\vec{p},t')[1-2f(\vec{p},t')]\cos\theta(t,t')$$

VII. Kinetic equations (KE)

or in the form of equivalent system of ODE's

$$\dot{f} = \frac{1}{2}\lambda u, \qquad \dot{u} = \lambda(1-2f) - \frac{2\varepsilon}{\hbar}v, \qquad \dot{v} = \frac{2\varepsilon}{\hbar}u.$$

Here

$$\theta(t,t') = \frac{2}{\hbar}\int_{t'}^{t} dt''\,\varepsilon(\vec{p},t'')$$

is the dynamical phase.

The same KE's can obtain from the twelve system of KE's in the standard QED for arbitrary polarization of an external electric field [Alexandrov, Dmitriev, Smolyansky].

VIII. Currents

The nuclear problem is experimental verification of the theory.

One possibility is related with measurement of currents.

According to definition [Schwinger], the current densities

$$j_k(t) = -e\,\frac{\delta H(t)}{\delta A_k(t)}.$$

Results of calculations can be represented in the form

$$j_k(t) = j_k^{cond}(t) + j_k^{pol}(t)$$

where $$j_i^{cond}(t) = 8\int [dp]\, v_q^i(\vec{p},t) f(\vec{p},t),$$

$$j_i^{pol}(t) = 4\int [dp]\, \varepsilon(\vec{p},t) l_i(\vec{p},t) u(\vec{p},t),$$

where $v_q^i(\vec{p},t) = P^i/\varepsilon(\vec{p},t)$

VIII. Currents

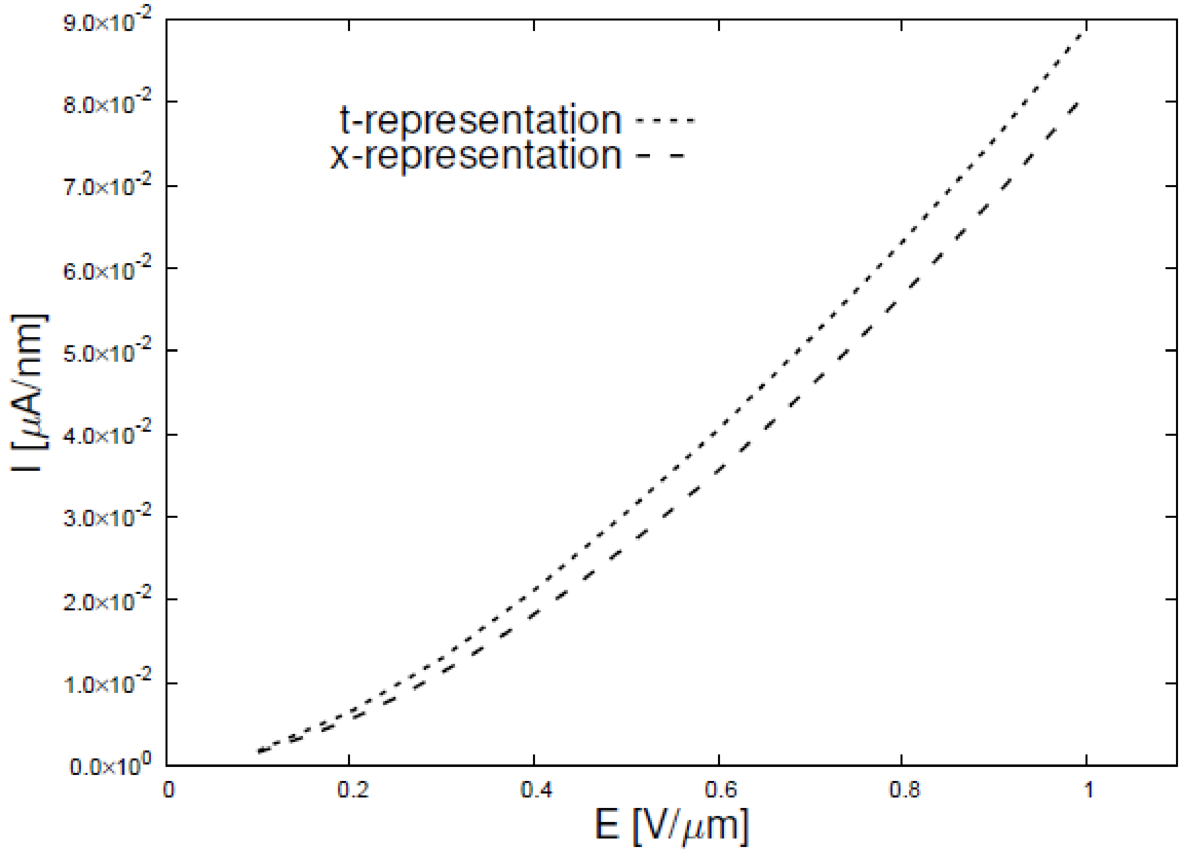
and $l_i(\vec{p},t) = \delta\lambda(\vec{p},t)/\delta E^i$ is defined by the components

$$l_1(\vec{p},t) = \frac{ev_F^2 P_2}{\varepsilon^2},\ l_2(\vec{p},t) = \frac{ev_F^2 P_1}{\varepsilon^2}.$$

At present, direct comparison of experimental currents with theory was curried only for constant electric field, for example, $E^1 = E_0 = const$, $E^2 = 0$, that corresponds to two models of electric field: $A^1 = -E_0 t$ (t-representation) or $A^2 = -E_0 x$ (x-representation).

In the last field model the I-V-characteristic can be calculated also in the different realizations of the tunnel Zener-Klein approach in the x-representation. (see [Smolyansky, Panferov, Blaschke, Gevorgyan]) or in the framework of the exact solution of the task [Gavrilov, Gitman].

VIII. Currents



Comparison of result of calculations

of I-V – characteristics on the bases of the considered exact kinetic approach (t-representation) and approximate Zener-Klein method (x-representation) [Smolyansky, Panferov, Blaschke, Gevorgyan].

VIII. Currents

This picture demonstrate satisfactory agreement of two approaches between them and experiment [Vandecasteele, Barreiro, Lazzeri, Bachtold, Mauri] in the region of sufficiently small constant electric fields. Difference increases with growth of the field strength. However, the kinetic approach pretends on more exact prediction. Besides that, kinetic approach is unique instrument in the case of fast oscillating fields.

Represented I-V curves show the law $I \sim V^{3/2} \sim E^{3/2}$, i.e. dependence $I(E)$ is nonanalytic in region $E=0$ and hence the standard perturbation theory here is not applicable.

IX. Back reaction problem

Hard core of the problem is that the inner currents of eh-plasma in turn generates corresponding internal plasma field $E_{in}^{k}(t)$, which is found in antiphase to external field and hence it relaxes the action of an external field and intensity of the eh-production.

In the standard QED this problem was investigated in [Cooper, Kluger; Mottola, Eisenberg, Svetitsky, Cooper, Mottola; Brout, Massar, Parentani, Popescu; Bloch, Miserny, Prozorkevich, Roberts, Schmidt, Smolyansky, Vinnik].

In the some other approach this problem was analyzed in the framework of the theory of cascade process [Narozhny, Fedotov].

IX. Back reaction problem

Internal field can be found from Maxwell equation

$$\dot{E}_{in}^{k}(t) = -j^{k}(t).$$

This Maxwell equation is written in the model D=2+1 space-time, $k = 1,2$.

Effectiveness of the back reaction can estimate, for example, with help of the exhaustion coefficient,

$$R_{exh}(t) = \frac{\vec{E}_{in}^{2}(t)}{\vec{E}_{ex}^{2}(t)}.$$

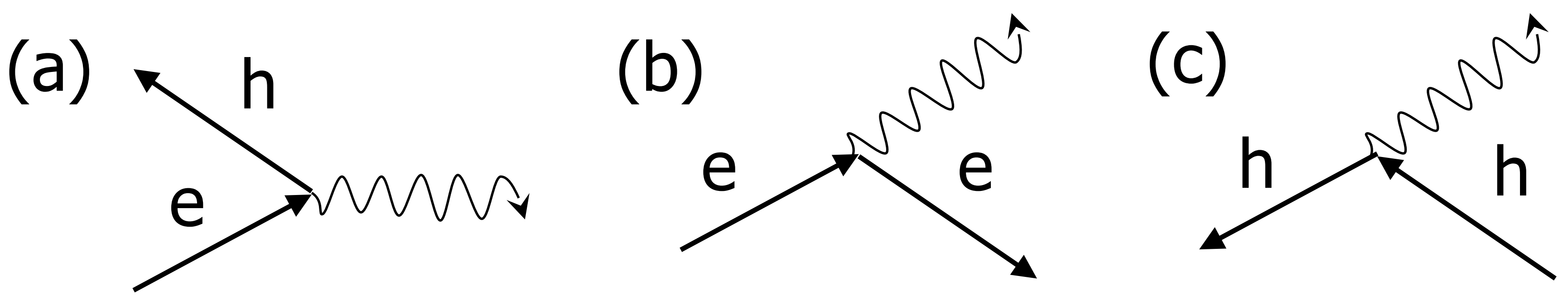
X. Radiation

There are two groups of processes, where D=2+1 massless QED is insufficient for description of real existent phenomena in graphene and where necessity arises in widening of the theory up to D=3 space-time. It is the spin phenomena (in D=2+1 graphene QED the real existing spin degrees of freedom reflects on the degeneracy flavor factor 2 only) [Werner, Trauzettel, Kashuba] and radiation from graphene [Yokomizo, Baudish at al]. The back-reaction problem belongs to this enumeration also.

So, the question is about the charge confinement problem of D=2 graphene plane in the real D=3+1 space-time.

X. Radiation

There are two sources of radiation in graphene: quasiclassical radiation of plasma motions (collective mechanism, Sect. 9) and quantum radiation as result of annihilation of eh-pairs (a) and spontaneous radiation of electrons (b) and holes (c):



X. Radiation

In the case of radiation of plasma field, it can use the well know approach [Abbott, Griffiths] for description of radiation of the electrically neutral infinite plane currents induced by linearly polarized electric field,

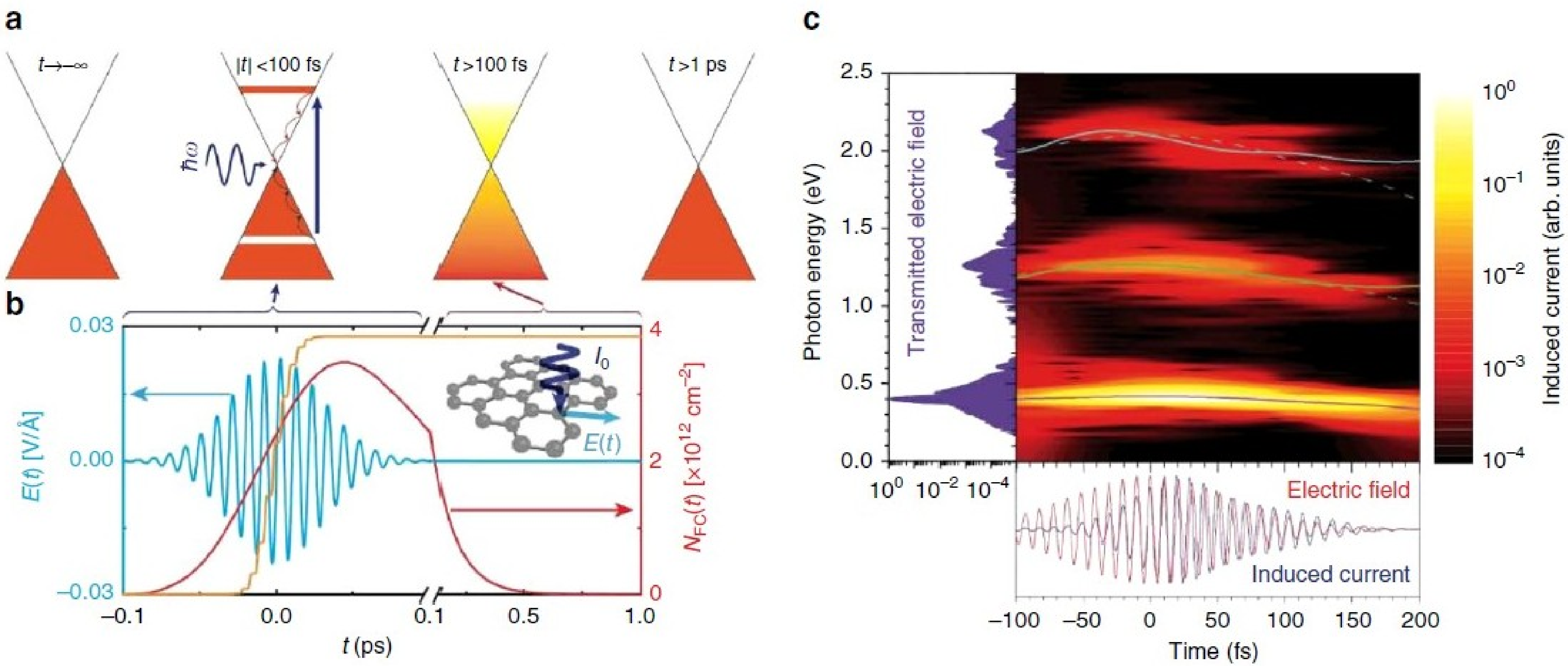
$$E_{act}(t) = E_{ex}(t - x/c) - \frac{1}{2} c\mu_0 j(t - x/c),$$

where $E_{act}(t)$ is acting field and x is distance from graphene plane. Neglecting by retardation, one obtain

$$E_{act}(t) = E_{ex}(t) - \frac{1}{2} c\mu_0 j(t).$$

X. Radiation

Results of theoretical simulation and experiments in the optical diapason are presented on the figure taken from the work [Baudisch et al].



X. Radiation

Main conclusions:

a) strong nonlinear response

b) large widening of each harmonic

c) presence of the odd harmonics only

d) visible role of polarization phenomena (residual currents [Panferov, Smolyansky, Blaschke])

e) excellent qualitative agreement theoretical simulations and experiment

Theory of quantum radiation from vacuum plasma takes now the first steps [Brout, Massar, Parentani, Popescu; Smolyansky, Panferov, Fedotov; Yokomizu].

XI. Conclusion

We showed that the QED model of graphene is rather simple in comparison with the D=3+1 standard QED, where verification is laboured over very large value of critical field strength E_c, and, in the same time, this model allows direct experimental examination. Specific of graphene is reflected in absence of critical field that leads to noticeable generation of the eh-plasma already in rather small electric fields. On the other hand, graphene QED is nonanalitical in the neighborhood $E=0$, as the standard QED. It forces to use nonperturbative methods of calculations of observed values.

XI. Conclusion

Such kind approach was presented was presented above. But it is nonunique. For example, in the graphene QED, Ishikawa method is known. This approach is formulated without QPR. In the present review we showed in the graphene QED an alternative method, which is effective in other regions of physics: strong field QED, heavy ion physics and cosmology.

List of actual problems, connected with confinement of the graphene plane in D=3 space:

- spin physics
- back reaction problem
- radiation of plasma oscillations
- quantum radiation

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