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Tracks and primary vertex fits for SPD experiment

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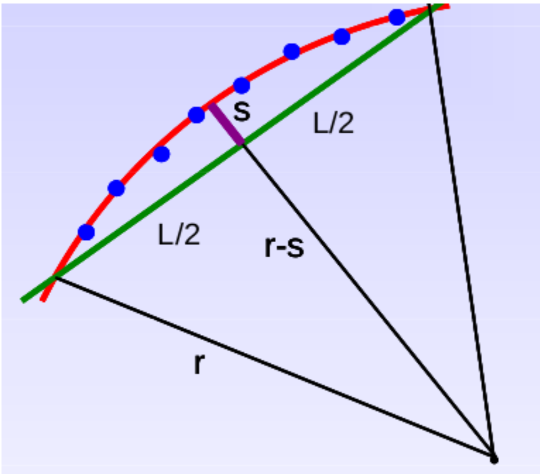
Introduction

- Track reconstruction is an essential part of most HEP experiments
- Track reconstruction is traditionally divided into separate sub-tasks:
  - track finding
  - track fitting
- Track finding:
  - division of set of measurements in a tracking detectors into subsets
  - each subset contains measurements believed to originate from the same particle
- Track fitting:
  - starts with the measurements inside one subset as provided by the track finder
  - aims to estimate a track parameters using the information from the measurements
  - evaluates the quality and final acceptance of the track candidate
- Several experiments in high-energy-physics implement their own track fitters, however, they use similar algorithms, such as well known Kalman filter
- We propose to use Kalman filter algorithm in a track fitter program

Track's momentum measurement



In uniform magnetic field B alone z-axis,
p_t – transverse momentum is estimated

$$p_t\,[\mathrm{GeV/c}] = 0.3 \cdot B\,[\mathrm{T}] \cdot r\,[\mathrm{m}]$$

$$r = \frac{L^2}{8s} + \frac{s}{2} \qquad \text{if } s << L \qquad r \approx \frac{L^2}{8s}$$

sagitta (s) / radius (r) is obtained by a circle fit through measurement points along the track with point resolution $\sigma_{r\varphi}$ for each point

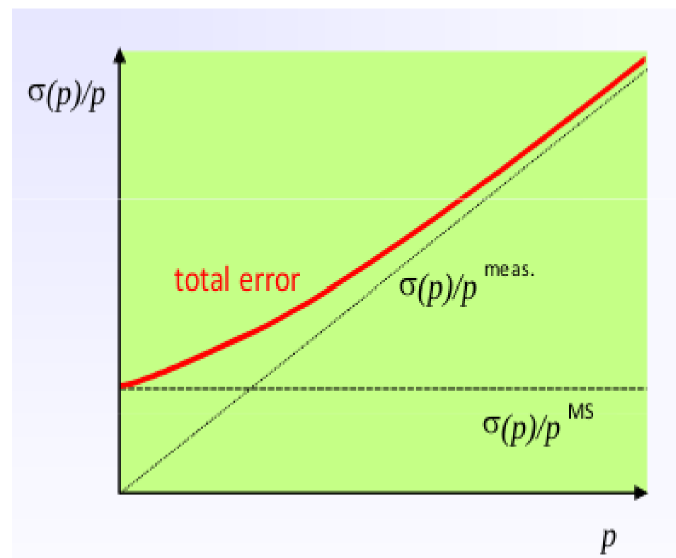
$$\boxed{\frac{\sigma_{p_T}}{p_T} = \frac{8\,p_T}{0.3BL^2} \cdot \sigma_S}$$

$$\sigma_S = \sqrt{\frac{A_N}{N+4}} \cdot \frac{\sigma_{r\phi}}{8}$$

A_N – statistical factor
(R.L.Gluckstern, NIM 24 (1963) 381)

Remember contribution from multiple scattering

$$\sigma_{r\varphi}\Big|^{MS} \propto \theta_0 \qquad \text{where} \qquad \theta_0 \propto \frac{1}{p_T}\sqrt{\frac{L}{X_0}}$$



Relative track's momentum resolution:
- degrades linearly with momentum
- improves linearly with B field
- improves quadratically with radial extension of detector
- improves with number of track point measurements

Kalman filter

Kalman filter was developed by R.E.Kalman during 1950's, it is a method of estimation the state of dynamic systems

a) applied by the NASA in the rocket trajectory control for the Apollo program

b) military applications: compute plane (or rocket) trajectory by radar tracking

Assumption:

- trajectory of a particle between two adjacent surface (or two system states) is described by a deterministic function (dynamic model) plus random disturbances (material effects, etc)
- the system equation: propagates the state (τ_κ) in one surface to the next (and covariance matrix)

$$\tau_k = F_k(\tau_{k-1}) + \delta_k\,,\ \langle\delta_k\rangle = 0,\ Cov(\delta_k) = Q_k\,,\quad C_{k|k-1} = F_k C_{k|k-1} F^T{}_k + Q_k$$

F_k – state transition model, Q_k - covariance of the process noise

- measurement equation: mapping the track on the surface and considers some measurement error

$$m_k = H_k\,(\tau_k) + \varepsilon_k\,,\quad \langle\varepsilon_k\rangle = 0\,,\ Cov(\varepsilon_k) = V_k$$

H_k – measurement model, V_k – measurement noise (or precision)

- filtering (or updating) based on $\tau_{k/k-1}$ and m_k

$$\tau_{k|k} = \tau_{k|k-1} + [(H^T{}_k\, V^{-1}\, H_k) + C_{k|k-1}]^{-1}\, [H^T{}_k\, V^{-1}\, (m_k - H_k\ \tau_k)],\quad C_{k|k} = [(H^T{}_k\, V^{-1}\, H_k) + C_{k|k-1}]^{-1}$$

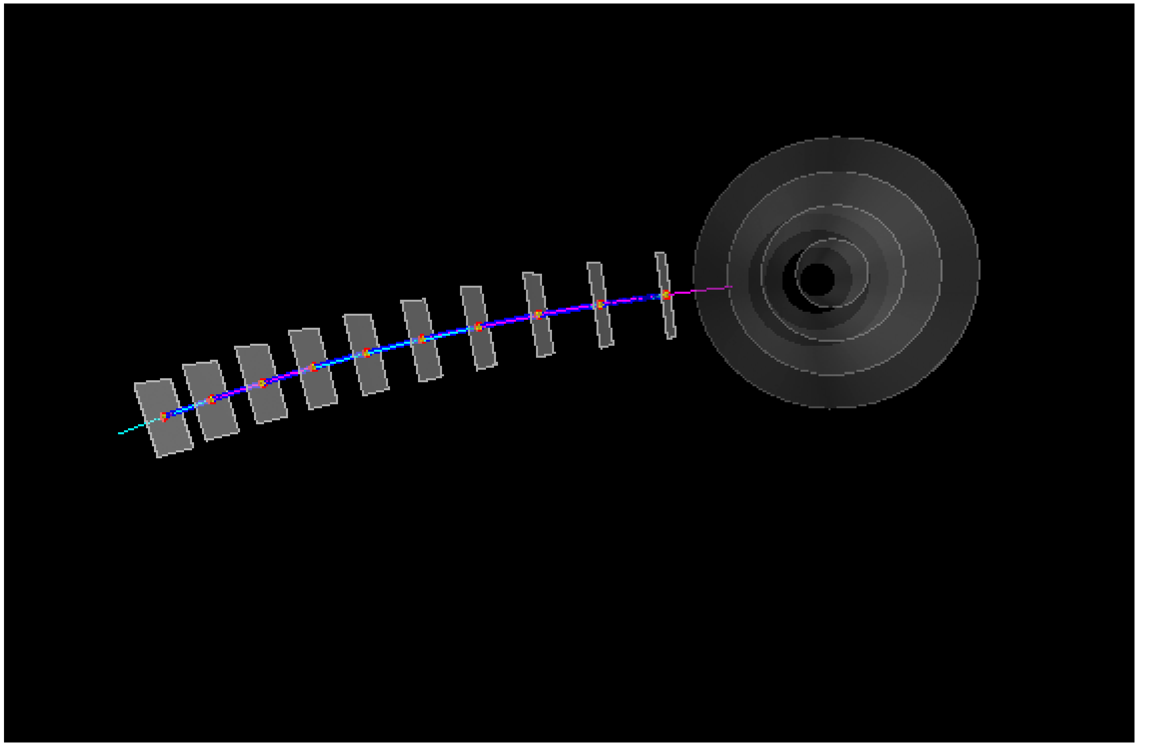
- and residual is $r_{k|k} = m_k - H_k\, \tau_{k|k}$
- which allows to compute a χ^2 to test the goodness of the fit

$$\chi^2{}_{k|k} = r^T{}_k\, V^{-1}\, r_k\quad,\ \chi^2 = \sum \chi^2{}_k$$

- in our case of KF extrapolation uses the standard Runge-Kutta method with some modification (such method is used in ATLAS experiment)
- track can be fitted in forward and backward direction:
  - a) forward direction gives the best estimation of track parameters at the end
  - b) backward direction gives the best estimation of the track parameters at the interaction vertex
- using both these results “smoothing track” state can be calculated

Kalman fitter (stand-alone program)

1) at this moment we use the special standalone program in which Kalman fitter is realized (not yet a part of SPDroot framework)
2) this program was developed on the base of GenFit2 package (arXiv:1902.04405, 19.02.2019) with our modification
3) program gives possibility to construct the set of tracking detectors with different parameters
4) user can define the number of plane for each tracking detector and add detector's resolution for measurements
4) program produces hit points in “virtual” detectors and apply resolution for each point
5) simple example of track after Kalman fitter is shown bottom with hit points on virtual planes



Ideal geometry of SPD tracking

1) following steps were done for tests of Kalman fitter in SPD geometry

2) introduce hybrid magnetic field in stand-alone program

3) add 94 cylinders (or 47, 31, 23) with radius started from R = 65.5 cm and step size = 1.0 cm (or 2.0 cm, 3.0 cm and 4.09 cm) as the prototype of SPD tracking geometry

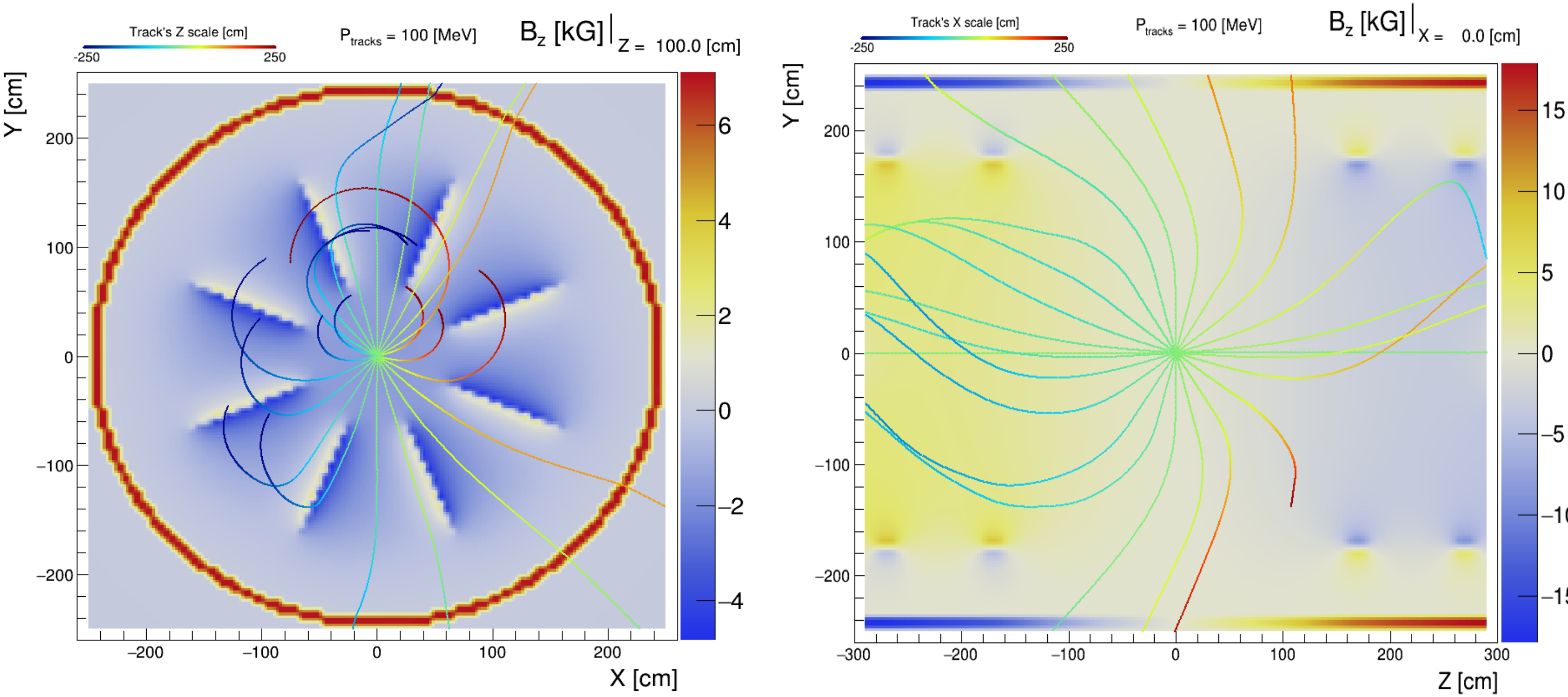
4) propagate tracks in hybrid field

5) produce the “virtual” hit points in position of crossing track with the detector's cylinder

6) add detector resolution effects to each x,y,z – coordinates (150 μm, Gaussian)

7) do Kalman fitter procedure

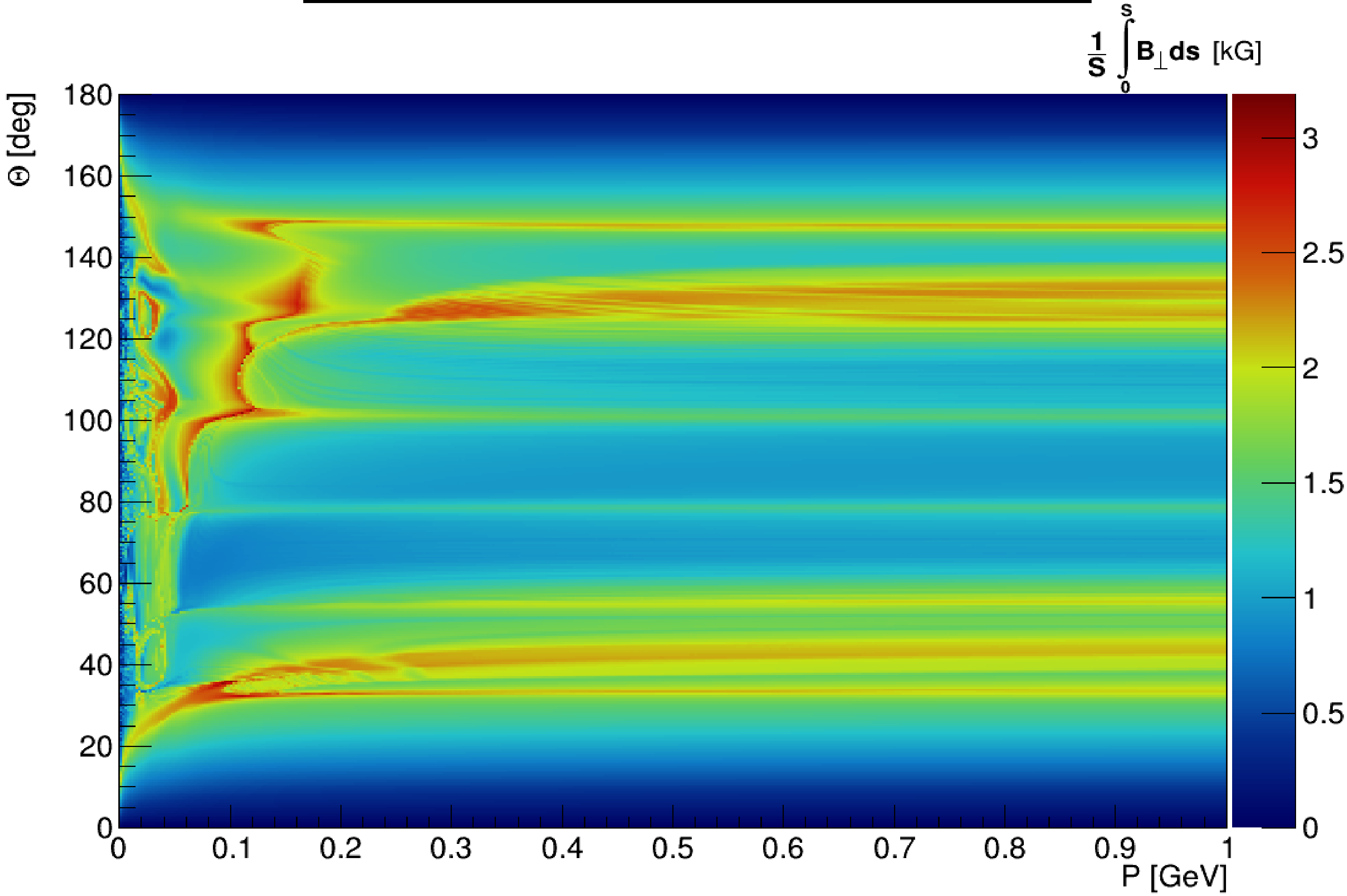
Field integrals calculation



Hybrid magnetic field had been "probed" by extrapolation of positively charged particles:

- in momentum range: 0 – 1 GeV (step 25 MeV)
- in θ angle range: $0^0 \leq \theta \leq 180^0$ (step 0.5 degree)
- in φ angle range: $0^0 \leq \varphi \leq 360^0$ (step 5 degree)

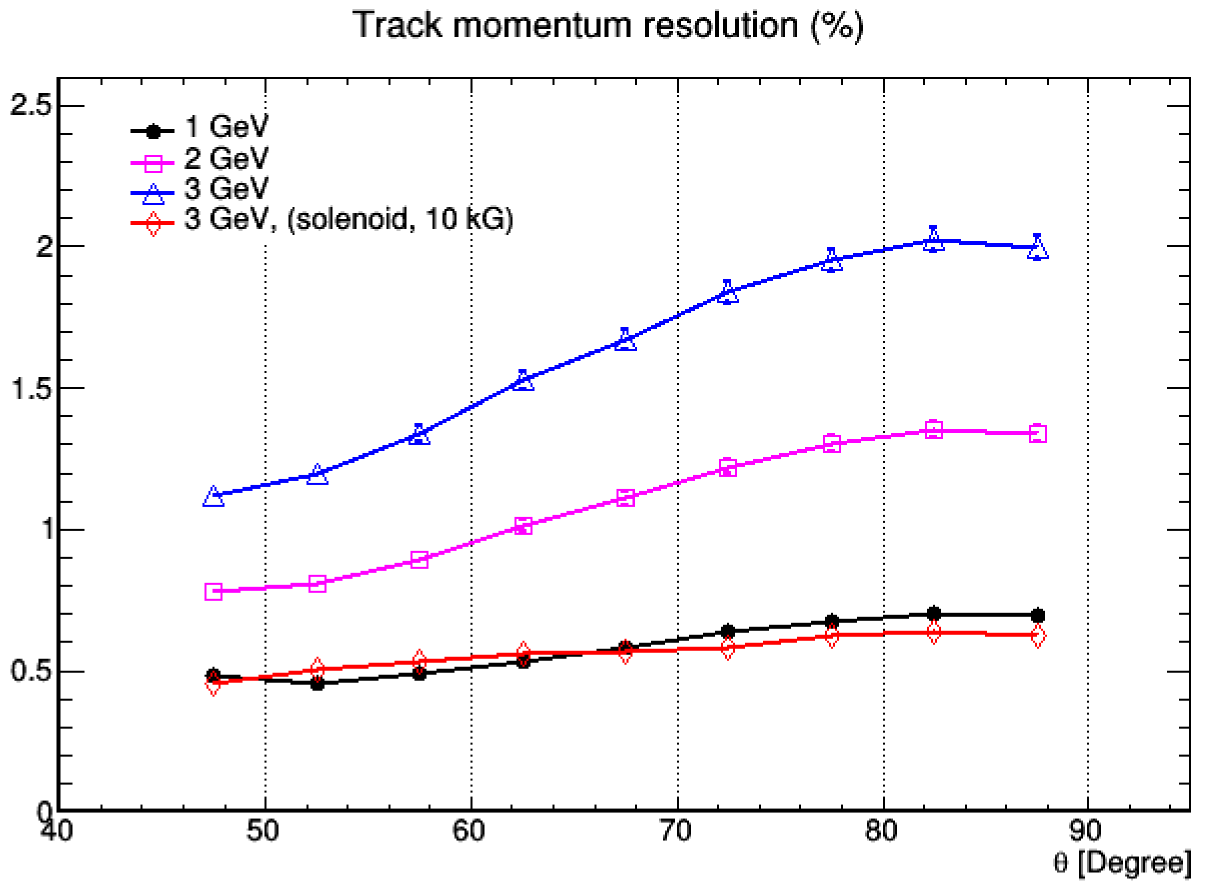
Field integrals calculation



Integral of transverse field (field component perpendicular to particle motion direction, normalized on the path length of the particle in the field, as function of *P* and θ angle (integrated over all ϕ angle)

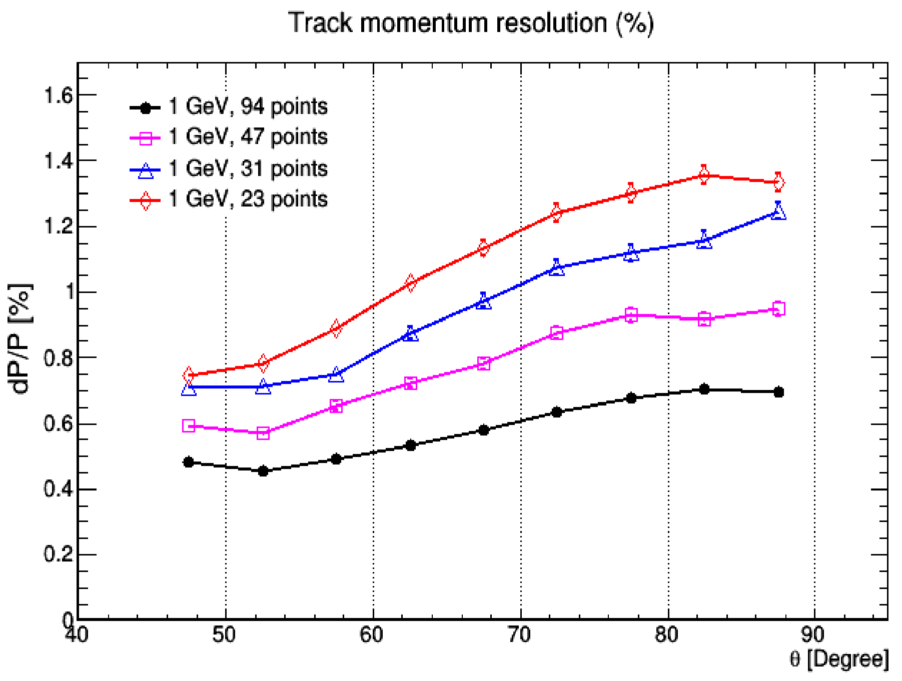
Track's momentum resolution (ideal stand-alone MC)

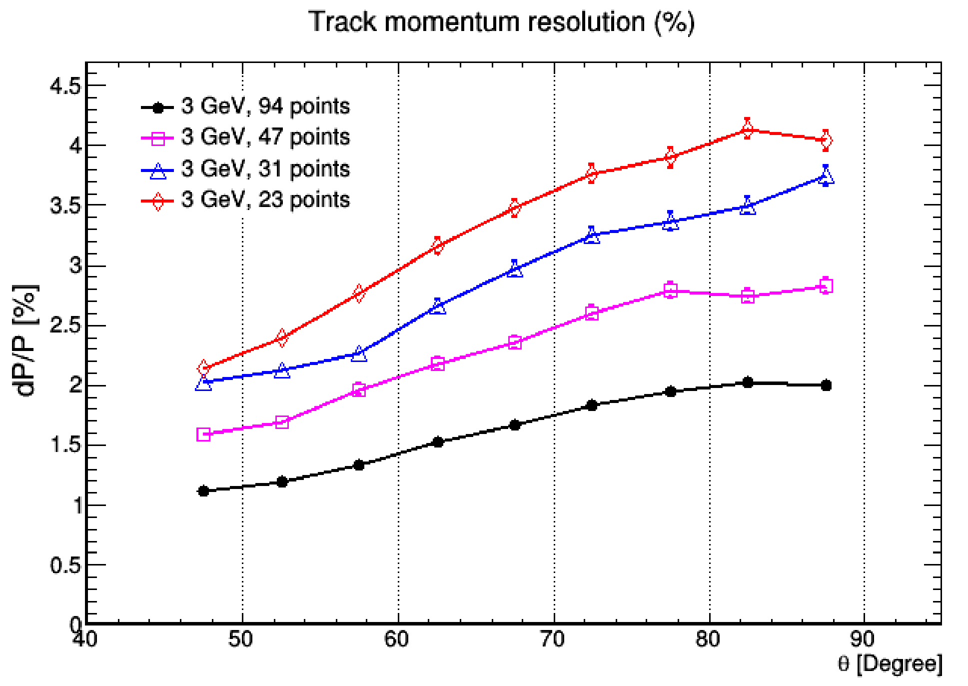
a) 94 tracker's cylinders start from R = 65.5 cm, with step = 1.0 cm
b) generated tracks with 1 GeV, 2 GeV and 3 GeV momentum and inside range $45^0 \le \theta \le 90^0$ and $0^0 \le \varphi \le 360^0$
c) add 150 μm space resolution for each 94 points



Hybrid magnetic field shows more worse track's momentum resolution comparing to solenoid type field (10 kG)

Track's momentum resolution vs number of measured points (ideal stand-alone MC)

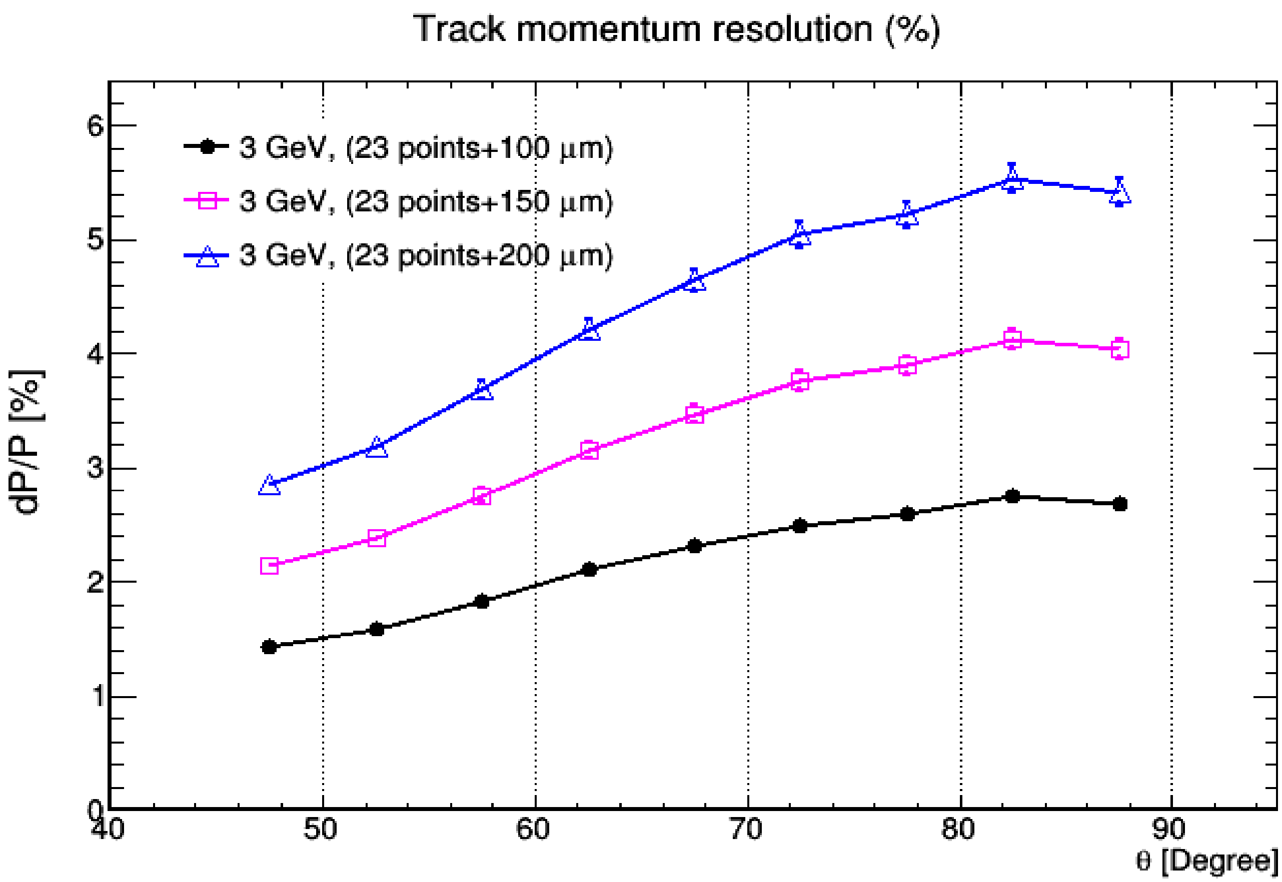




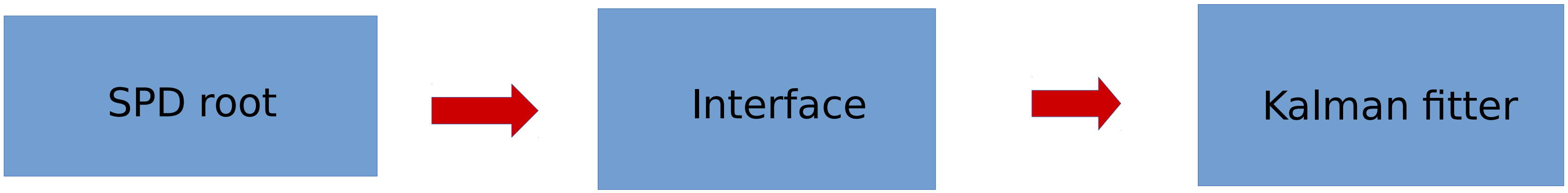
Clear and obvious conclusion:

decrease number of measured points => do worse track's momentum resolution

Track's momentum resolution vs detector resolution (ideal stand-alone MC)



«Real» geometry for SPD tracker (SPD MC)



1) use last version of hybrid magnetic field map in SPDroot (map_hyb_1T5cm.dat)

2) use SPDroot built-in generator with muons (1 GeV and 3 GeV, barrel part) and run SPDroot simulation to produce root-file output with simulated hits

3) we can run different tracker configuration in SPDroot program (change number of layers in straw tracker and add vertex detector (5 additional silicon layers) in simulation)

4) use tools of SPDroot package to produce an output with x,y,z – coordinate of points on particle's trajectories (MC hits)

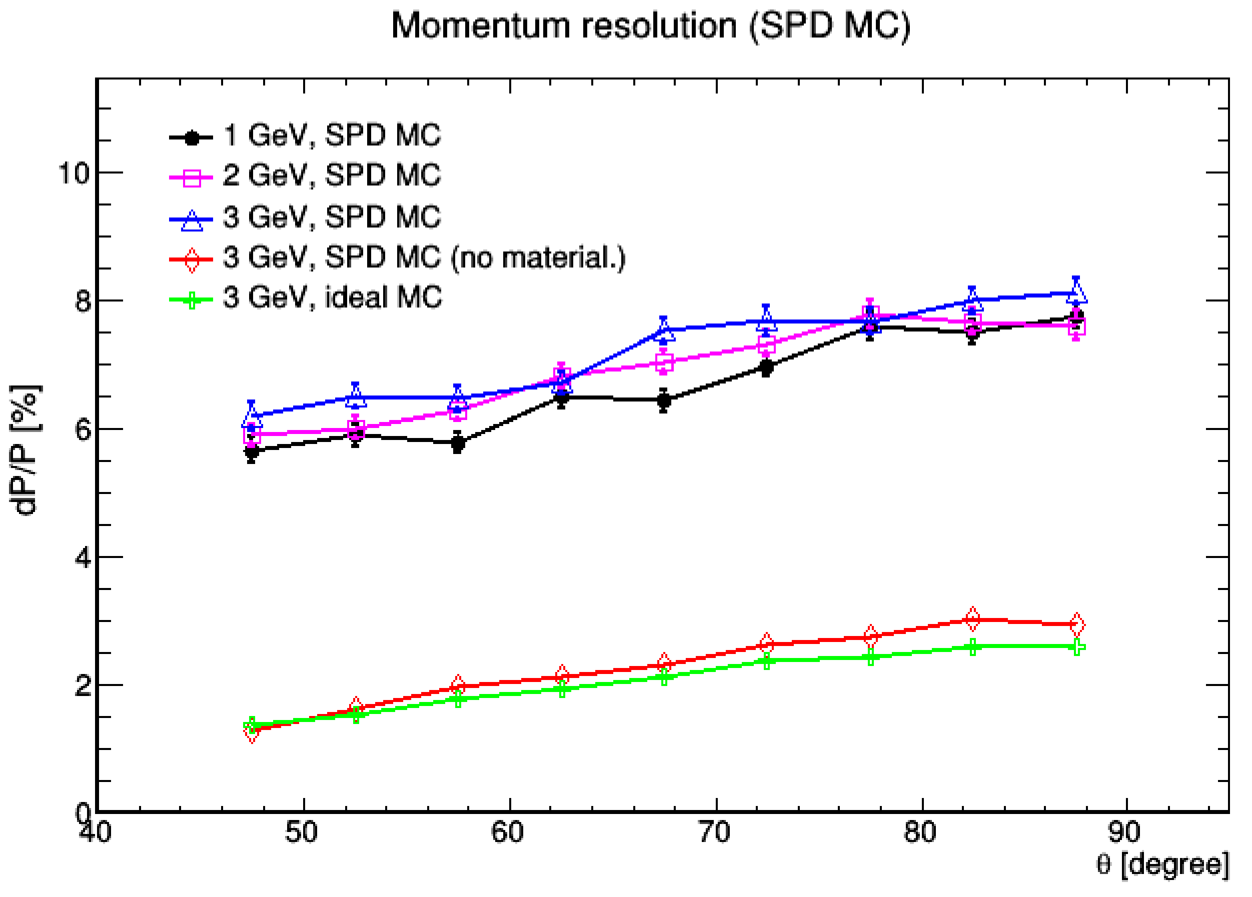
5) add hybrid magnetic field in Kalman fitter stand-alone program

6) read simulated MC hits in stand-alone program

7) add detector resolution effect for each hit point x,y,z – coordinate (Gaussian, 150 μm for straw tracker and 50 μm (25 μm) for silicon tracker)

8) do Kalman fitter procedure

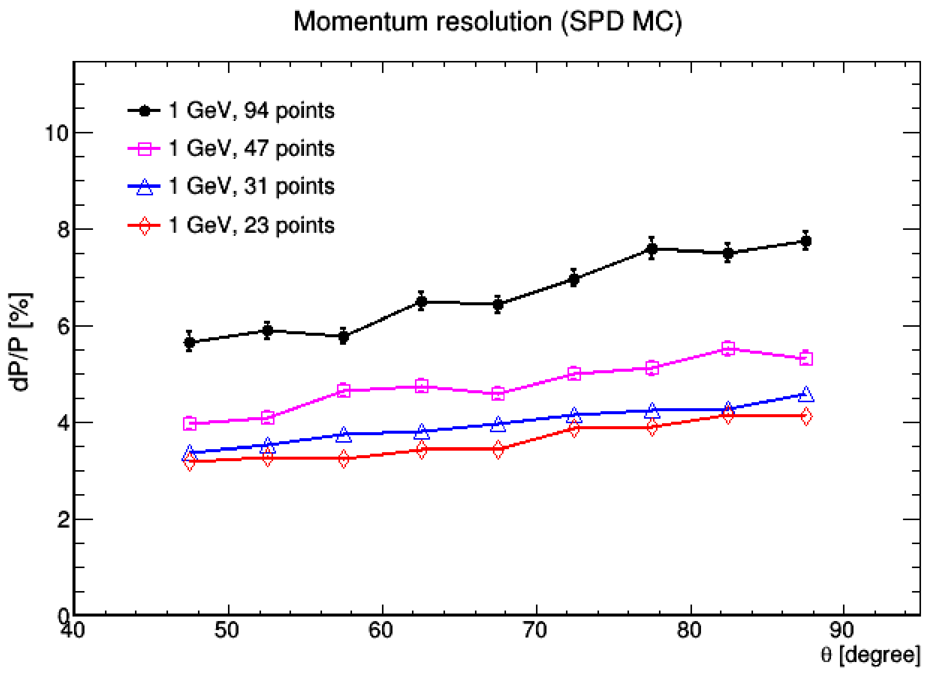
Track's momentum resolution (94 points, SPD MC)

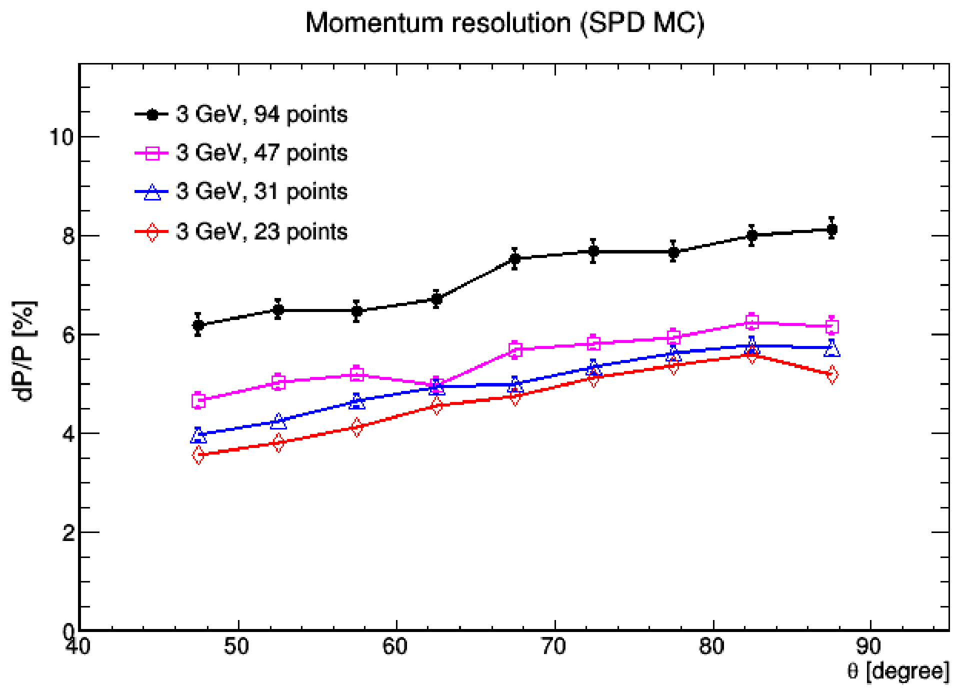


In most up-to-date SPD geometry of trackers material (multiple scattering) do track's momentum resolution worse.

SPD MC and “ideal stand-alone MC” in the case of absence of materials give very similar results (just a cross check)

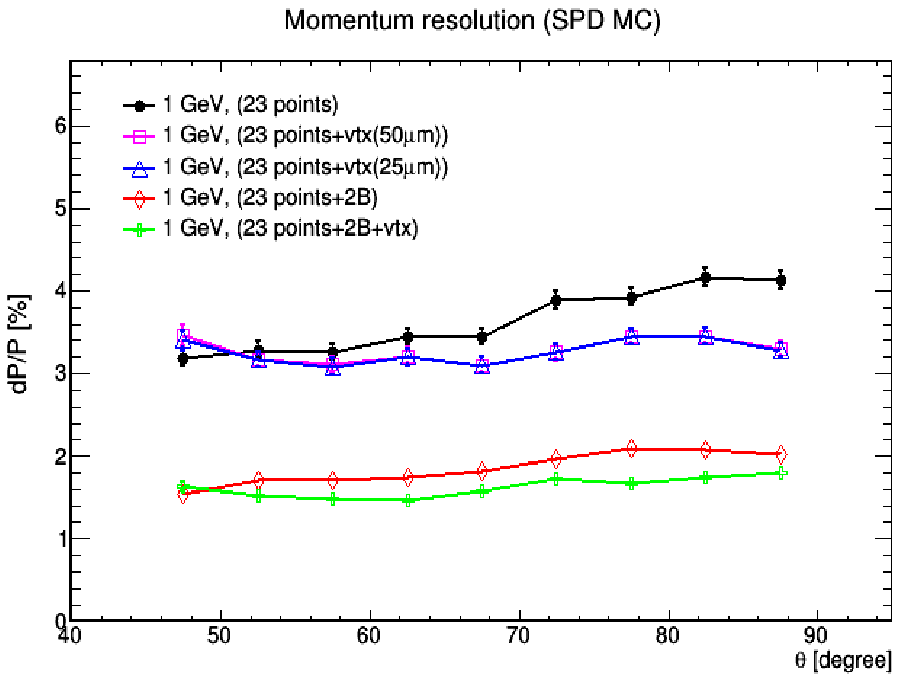
Momentum resolution vs segmentation (SPD MC)

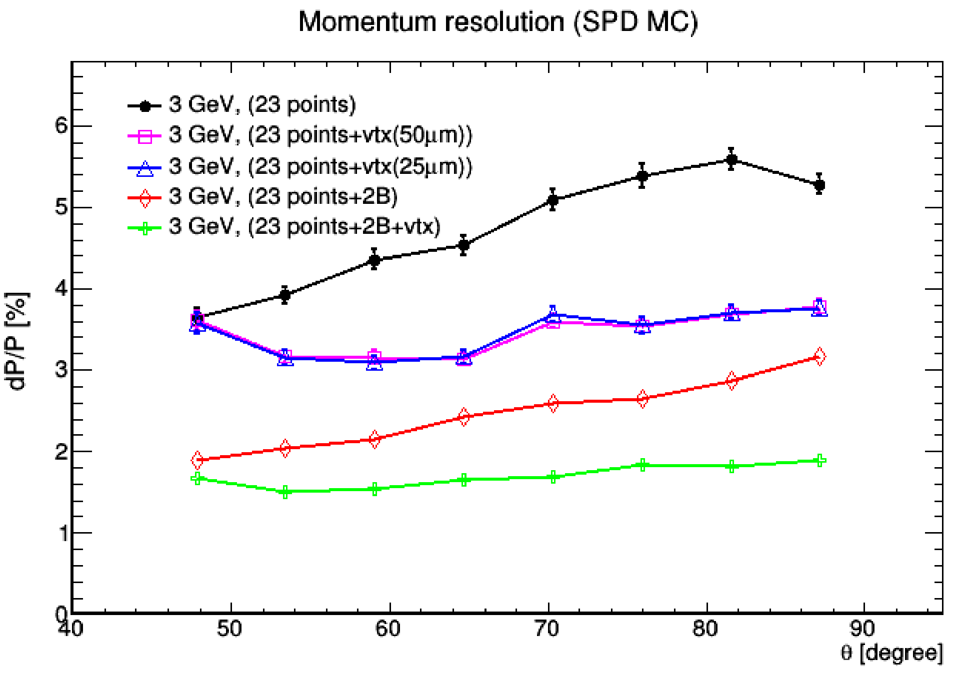




Decrease number points => decrease multiple scattering =>
improve track's momentum resolution

Track momentum resolution + vertex detector (SPD MC)





a) 23 points on track (straw tracker) without vertex detector (black)
b) 23 points on track (straw tracker) with vertex detector (resolution 50 μm) (magenta)
c) 23 points on track (straw tracker) with vertex detector (resolution 25 μm) (blue)
d) 23 points on track (straw tracker) without vertex detector with 2 times stronger field (red)
f) 23 points on track (straw tracker) with vertex detector with 2 times stronger field (green)

Conclusion:
- add vertex detector => improve track's momentum resolution
- increase field value => improve track's momentum resolution
- increase field and add vertex detector => best way to improve track's momentum resolution

Primary vertex fit

1) we have checked 2 different programs for primary vertex fit in our study

2) RAVE - detector-independent toolkit to reconstruct vertices (was developed in CMS experiment https://rave.hepforge.org/). This program was used only for cross check (as need to apply additional efforts for implementation of hybrid field and non-helix track extrapolation)

3) at the present moment for primary vertex fit we use program on the base of Kalman fit (similar to program in CBM experiments, CBM-SOFT-note-2006-002)

4) this program can work with hybrid magnetic field from SPDroot and track's extrapolation from ideal stand-alone MC program

5) we combine stand-alone program for tracks fit with the primary vertex fit program

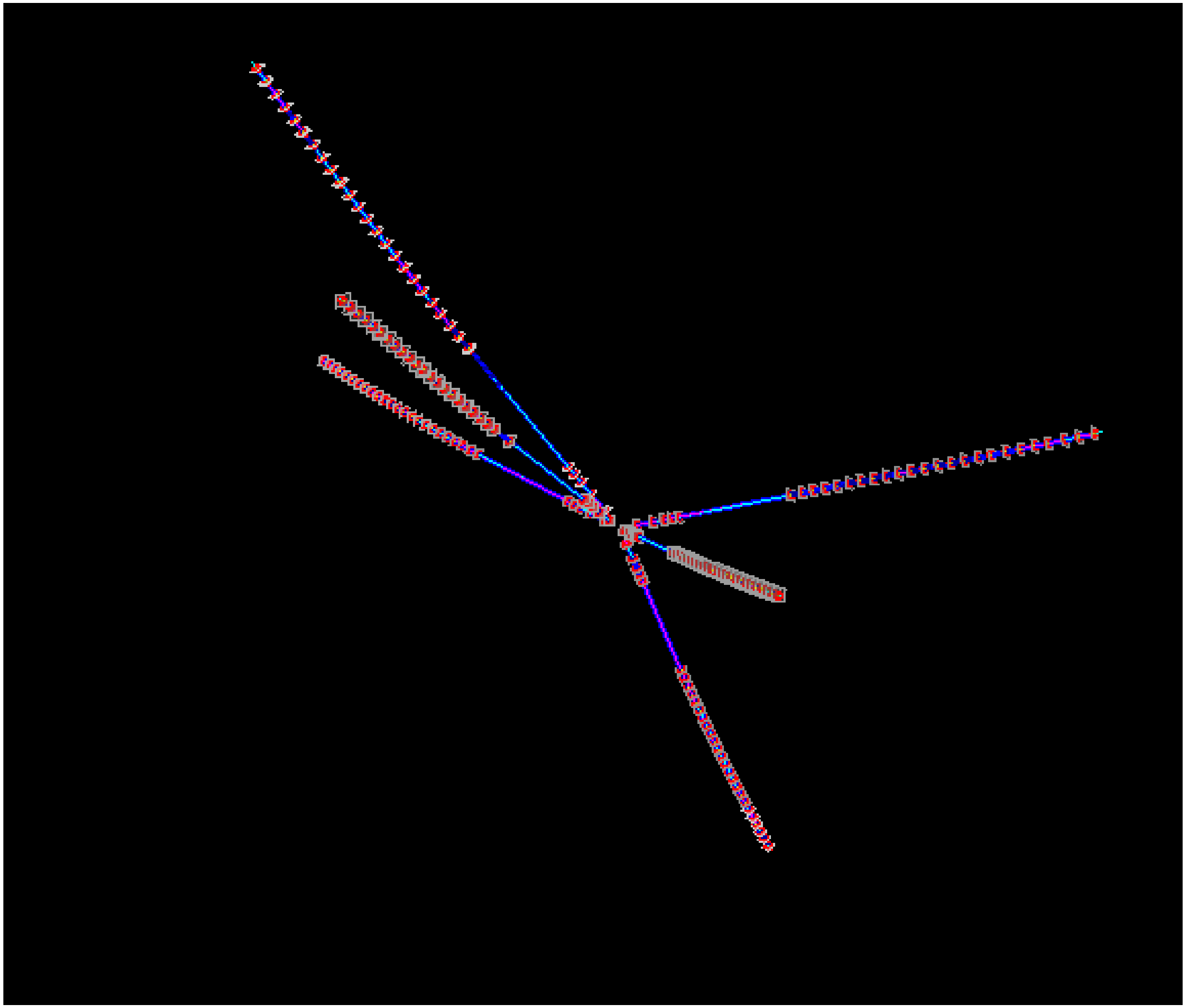
6) this primary vertex fit program had been checked with SPD geometry and with hit points from SPDroot program (23 points in straw and 5 points in silicon tracking detectors)

7) 1 step => simulate 6 tracks with 1 GeV momentum (or 3 GeV) in angle range $45^0 \leq \theta \leq 135^0$ and $0^0 \leq \varphi \leq 360^0$ with SPDroot program

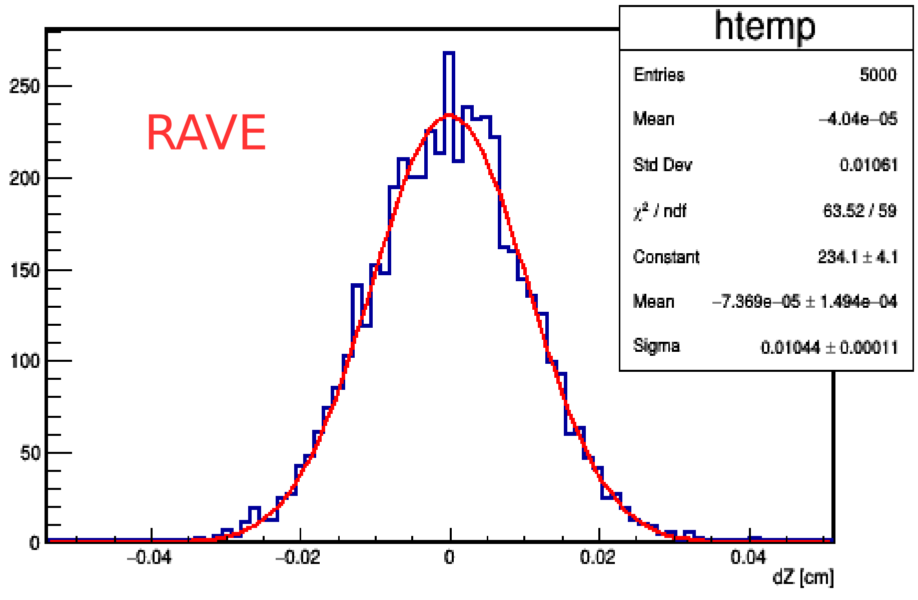
8) 2 step => use tools of SPDroot package to produce an output with x,y,z – coordinate of points on trajectories (MC hits)

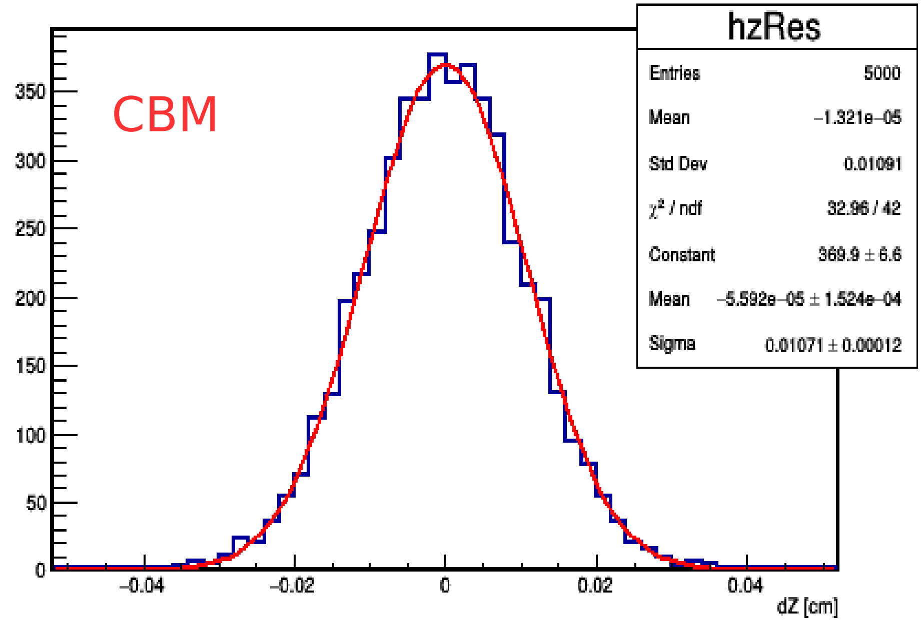
9) 3 step => fit tracks with Kalman fit procedure (use stand-alone program with 150 μm point resolution for straw tracker and 50 μm (or 25 μm) point resolution for silicon tracker) and then use these tracks for primary vertex fit

Primary vertex fit



Vertex position resolution (in z, RAVE vs CBM)





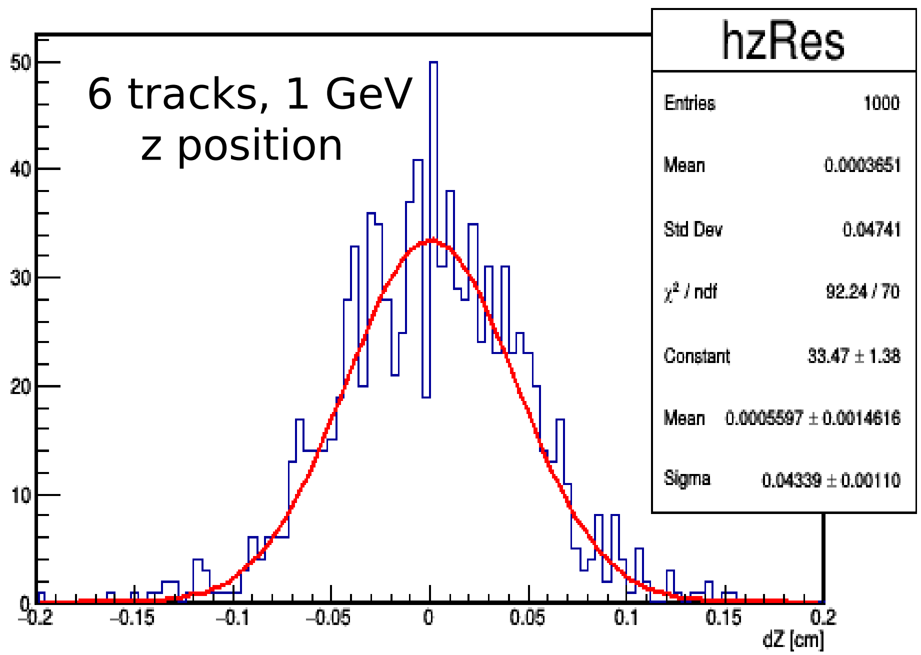
We did comparison of RAVE and CBM primary vertex fitter programs with constant field along z-axis and ideal stand-alone geometry of SPD tracker (just for cross check):

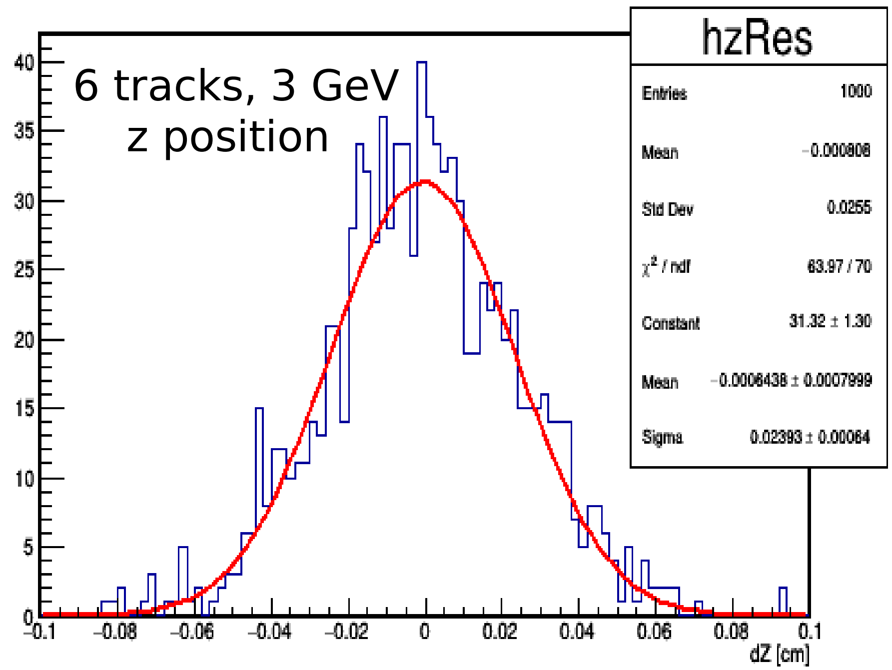
- simulated 6 tracks of 3 GeV momentum inside range $45^0 \le \theta \le 135^0$ and $0^0 \le \varphi \le 360^0$
- did primary vertex fit using these 2 programs (RAVE and CBM)

We can conclude that RAVE and CBM give the same result

CBM program had been used in our study of primary vertex fit procedure

Vertex position resolution (in z) without vertex detector



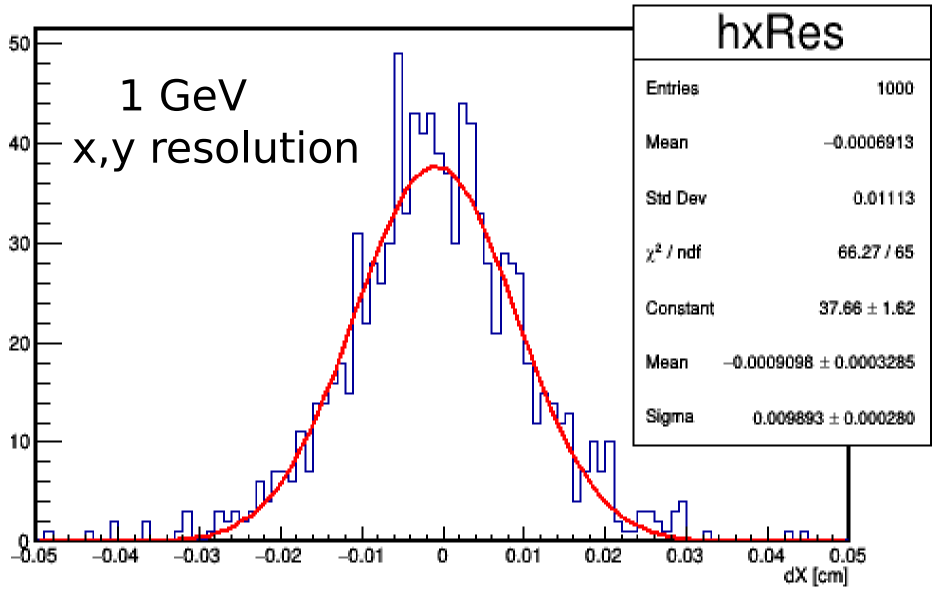


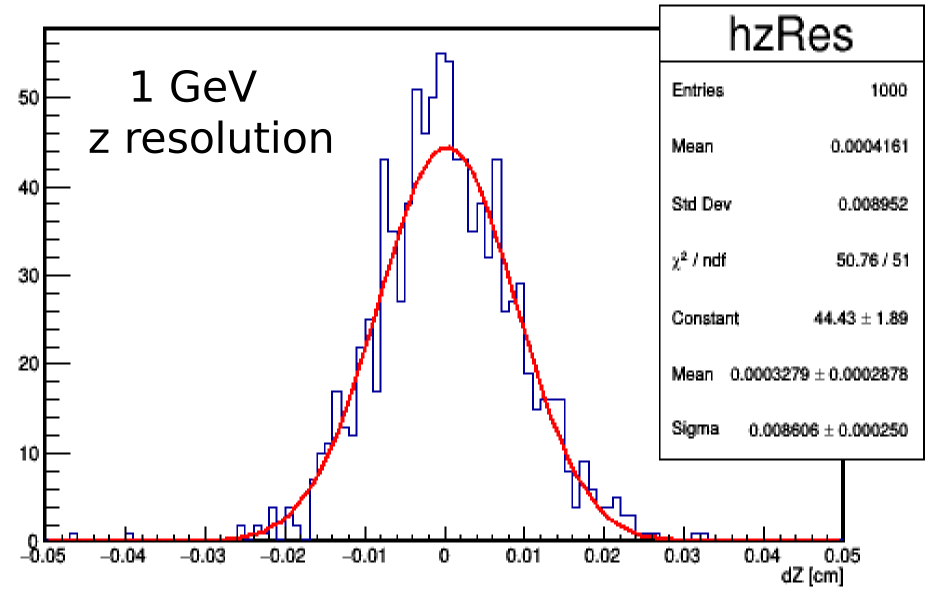
simulated 6 tracks of 1 GeV and 3 GeV momentum inside range $45^0 \leq \theta \leq 135^0$ and $0^0 \leq \varphi \leq 360^0$. Used only MC points in straw tracker:

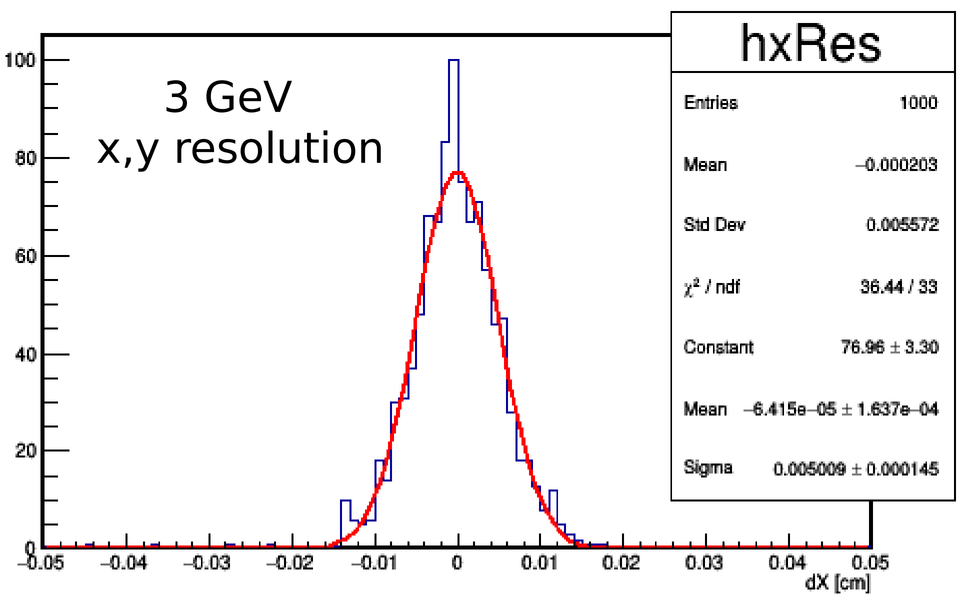
- vertex position resolution depended on track's momentum used for fit

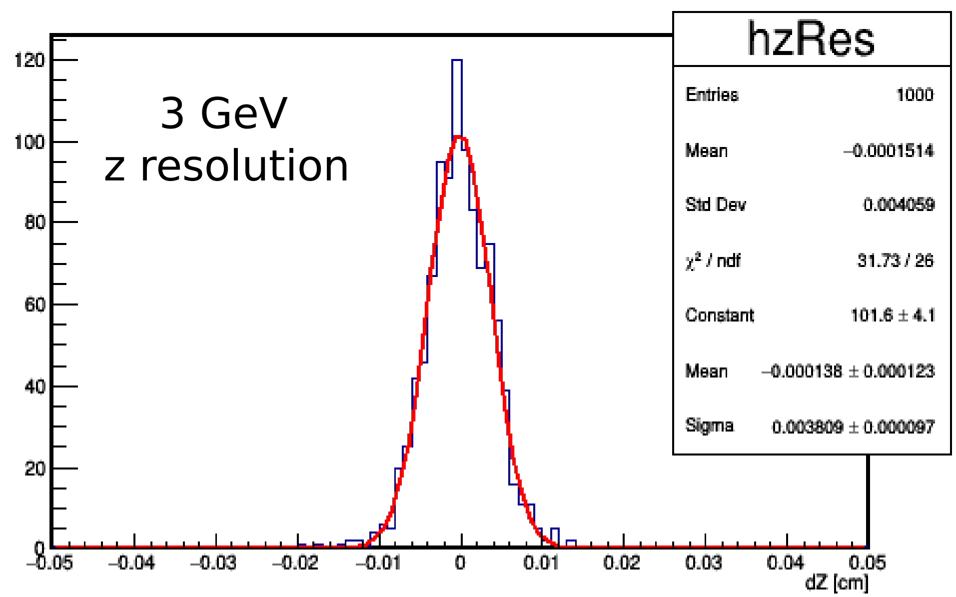
- worse position resolution only on the base of straw tracker hits (~430 μm for 1 GeV and ~240 μm for 3 GeV particles)

Vertex position resolution with vertex detector



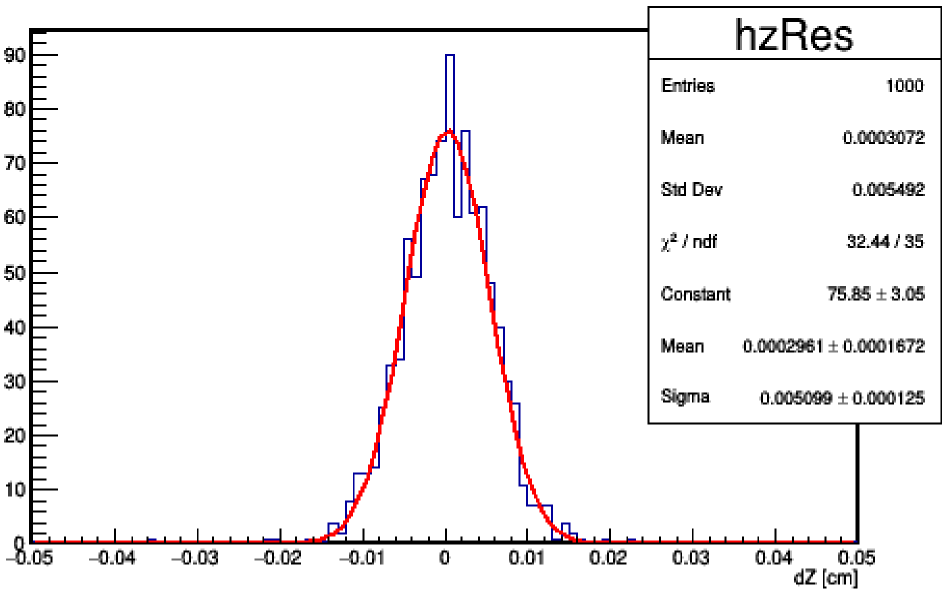


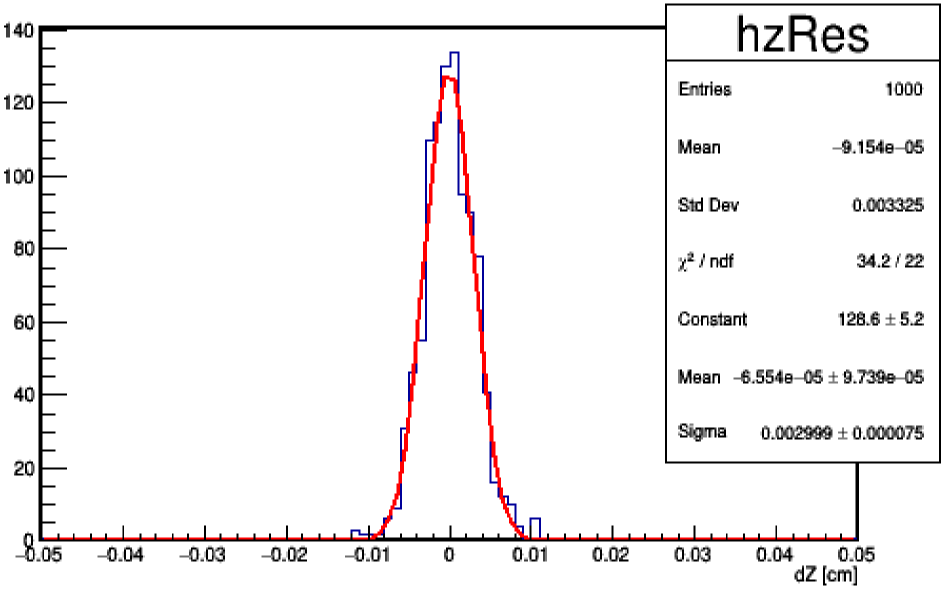




x,y resolution ~100 μm for 1 GeV and ~50 μm for 3 GeV particles and
z resolution ~86 μm for 1 GeV and ~40 μm for 3 GeV particles

Vertex position resolution (in z) with vertex detector (25 μm precision)





6 tracks with 150 μm space point resolution (in straw tracker) and vertex detector (silicon tracker) with 25 μm space resolution

Conclusion => primary vertex position resolution strongly depends on vertex detector resolution:

- from 86 μm => 50 μm for 1 GeV track's momentum and
- from 40 μm => 30 μm for 3 GeV track's momentum if vertex detector resolution will change from 50 μm to 25 μm

Summary

1) we have prepared the special stand-alone program on base of Kalman filter for tracks and primary vertex fits

2) simple interface gives possibility to use MC hits produced by SPDroot simulation

3) using this stand-alone program we can study different setups of SPD trackers