Quantum phase transitions: NEGF and hydrodynamic.

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Let us consider the equilibrium system. How the statistical physics can contributes to the problem of hadron matter equation of state?

Then the temperature Green function formalism must be used. Action S in Minkowski space transforms to the Euclidian one $S_e = S(t = i\tau)$, $\tau \in [0, \beta]$.

$$Z = \int D\psi^+ D\psi D A e^{-S_e}.$$

QCD. For example the temperature propagator for fermions is:

$$G = -\gamma_0 \left\{ \text{ch}(\varepsilon(t-t')) - \frac{(\gamma_i p^i + m)\gamma_0}{\varepsilon} \text{sh}(\varepsilon(t-t')) \right\}$$

$$\left\{ \frac{(\gamma_i p^i + m)\gamma_0}{2\varepsilon} \text{th}\left(\frac{\varepsilon\beta}{2}\right) - \frac{1}{2} + \theta(t-t') \right\}$$

where $\epsilon = \sqrt{p^2 + m^2} - \mu$. The difficult model.

But some predictions could be made.

Three statements:

If the critical point exists the critical behavior is known.

Hydrodynamic equations near the equilibrium are similar to the stochastic equations of critical dynamics.

Non-equilibrium Green functions (NEGF) is the most reliable approach.

The common statement is that the critical point theory demonstrates universality.
It is true for the quantum field models too. Namely:
Effective theories are used in infra-red (IR) region. Then non-relativistic (IR) description

$$H_0 = \sqrt{m_0^2 c^4 + \mathbf{p}^2 \mathbf{c}^2} \to \frac{\mathbf{p}^2}{2m_0}$$

The order parameter has the bosonic nature:
Let us suppose that $\Psi = F(\psi)$ is an microscopic presentation of the oder parameter for the phase transition in the difficult initial model (QCD can be considered as an example) with action $S(\psi, A)$. Then one can construct the effective action using

$$e^{iS_{eff}(\Psi)} = \int D\psi DA \delta(\Psi - F(\psi)) e^{iS(\psi,A)} =$$

$$\int D\psi DAD\Psi' e^{iS(\psi,A) - \Psi'(\Psi - F(\psi))}$$

The propagator in the temperature Green functions formalism is

$$< \Psi\Psi >= \frac{1}{i\omega_n + \mathbf{p^2/(2m)} - \mu} \quad \text{or}$$

$$e^{-t(\mathbf{p^2/(2m)}-\mu)}(\Theta(t-t') + \frac{1}{e^{\beta(\mathbf{p^2/(2m)}-\mu)} - 1}) \to \frac{1}{\mathbf{p^2/(2m)} - \mu}$$

t is imaginary "time".

And a quantum field model near the critical point is equivalent to some classical one.

The tensor structure of the order parameter is essential only.

What is $F(\Phi)$? It is really the question. QCD.

Then it is possible that the quark-gluon plasma critical point was described in our articles devoted to the classical superconnective phase transition:

Temperature Green's functions in Fermi systems: The superconducting phase transition *M. V. Komarova, M. Yu. Nalimov, J. Honkonen* Theor. Math. Phys., 2013, Volume 176, Number 1, Pages 906-912

Renormalization-group study of a superconducting phase transition: asymptotic behavior of higher expansion orders and results of three-loop calculations *G. A. Kalagov, M. V. Kompanietz, M.Yu. Nalimov* Theor. Math. Phys., 2014, Volume 181, Number 2, Pages 1448-1458

Renormalization-group investigation of a superconducting $U(r)$-phase transition using five loops calculations, *G.A. Kalagov, M.V. Kompaniets, M.Yu. Nalimov*, Nuclear Physics, Section B (2016), pp. 16-44, NUPHB13611, DOI information: 10.1016/j.nuclphysb.2016.02.004

The initial action for the non relativistic fermions is

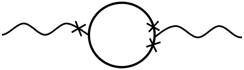
$$S_\psi = \psi^\dagger_\alpha(\partial_t - \frac{\Delta}{2m} - \mu)\psi_\alpha - \frac{\lambda}{2}\psi^\dagger_\alpha\psi^\dagger_\gamma\psi_\gamma\psi_\alpha, \tag{1.1}$$

The microscopic analogs of order parameters are $\chi_{\alpha\gamma} = \psi_\alpha \psi_\gamma$ and $\chi^\dagger_{\alpha\gamma} = \psi^\dagger_\alpha \psi^\dagger_\gamma$. Then the effective action is

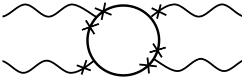
$$S_\chi = \frac{1}{2\lambda}\mathrm{tr}\chi\chi^\dagger - \mathrm{tr}\ln \begin{pmatrix} -\chi^\dagger & -i\omega_s - \frac{\Delta}{2m} - \mu \\ -i\omega_s + \frac{\Delta}{2m} + \mu & -\chi \end{pmatrix}, \quad (1.2)$$

here $\omega_s = \pi T(2s+1)$ are Matsubara frequencies, $s \in \mathbb{Z}$. Using the Taylor expansions for $\ln(1+\dots)$ we can rewrite the action as

$$S_\chi = \frac{1}{2\lambda}\mathrm{tr}\chi\chi^\dagger + \frac{1}{2}$$



$$+ \frac{1}{4}$$



$$+ \dots (1.3)$$

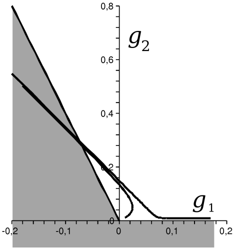
here wave lines denote the field χ, $\chi^\dagger$, the plain lines denote free $\langle \psi\psi^\dagger \rangle$ propagators, cross corresponds to $\psi^\dagger$, $\chi^\dagger$ fields. In the form of a Ginzburg-Landau functional it tends to the model of ϕ^4 type.

RG. Usual fermions with $s = 1/2$ then second order phase transition. If fermions obey some additional symmetry or one have more complex order parameter:

$$\beta_{(g1)} = -\epsilon g_1 + \frac{1}{2}(r^2 - r + 8)g_1^2 + 2(r-1)g_1 g_2 + \frac{3}{2}g_2^2 + \text{...up to 5 loops;}$$

$$\beta_{(g2)} = -\epsilon g_2 + 6 g_1 g_2 + \frac{1}{2}(2r-5)g_2^2 + \text{...up to 5 loops;}$$

Borel transform using large order asymptotes obtained leads to



First-order phase transition near-by the continues one was found.

The critical dynamics description can be based on two different approaches:

- The microscopic description of interacting quantum particles system based on time depended Green functions at finite temperature (NEGF)
- Hydrodynamics: some stochastic model.

To write the hydrodynamics equations one have to regard the conservation lows and the obvious relations.
According the Vasiljev's green book the stochastic equations must have the form

$$\partial_t \phi_a = (\alpha_{ab} + \beta_{ab}) \frac{\delta S_{stat}}{\delta \phi_b} + \eta_a$$

$$\alpha_{ab} = \alpha_{ba}, \quad \beta_{ab} = -\beta_{ba}, \quad \frac{\delta \beta_{ab}}{\delta \phi_a} = 0.$$

These equations leads to equilibrium. There is Onzager reciprocal relations (extended).

The stochastic equations then are

$$(\partial_t + v_i\partial_i)\psi = \lambda(1+ib)\Big[\partial^2\psi - \frac{g_1\psi^+\psi^2}{3} + g_2 m\psi\Big] + i\lambda g_3\psi\Big[g_2\psi^+\psi - m - \frac{v^2}{2}\Big] + f_{\psi^+},$$

$$(\partial_t + v_i\partial_i)\psi^+ = \lambda(1-ib)\Big[\partial^2\psi^+ - \frac{g_1(\psi^+)^2\psi}{3} + g_2 m\psi^+\Big]$$

$$-i\lambda g_3\psi^+\Big[g_2\psi^+\psi - m - \frac{v^2}{2}\Big] + f_\psi,$$

$$\partial_t m + \partial_i(\rho v_i) = -\lambda u\partial^2\Big[g_2\psi^+\psi - m - \frac{v^2}{2}\Big] + i\lambda g_3\left[\psi^+\partial^2\psi - \psi\partial^2\psi^+\right] - c^2\partial_i v_i + f_m,$$

$$\rho(\partial_t v_i + v_j\partial_j v_i) = \gamma\partial^2 v_i + \frac{(3\zeta+\gamma)}{3}\partial_j\partial_i v_i - (\partial_i\psi^+)\Big[\partial^2\psi - \frac{g_1\psi^+\psi^2}{3} + g_2 m\psi\Big]$$

$$+\rho\partial_i\Big[\frac{v^2}{2}\Big] - (\partial_i\psi)\Big[\partial^2\psi^+ - \frac{g_1(\psi^+)^2\psi}{3} + g_2 m\psi^+\Big] + (c^2+\rho)\partial_i\Big[g_2\psi^+\psi - m - \frac{v^2}{2}\Big]\Big] + f_v,$$

ψ, ψ^+ – microscopic analogs of order parameter, γ, ζ — viscosities, λ –kinetic coefficient, c – sound velocity, ρ – density ($\rho = \rho_0 + m$).

It is rather difficult equations system. ($\partial_i v_i = 0$ – Fh -model, $v = 0$ – F-model, $b = g_2 = 0$ – E-model).
Hydrodynamics is a description in the limit $kr_c \to 0$, k is a wave number. In this limit $c^2 \partial v$ term is more essential. But it is in the free part of action (linear part of equations). Then the propagators were derived. Detailed dimensional analysis shows that the true hydrodynamic equations are:

$$\partial_t \psi = \lambda\big[\partial^2\psi - \frac{g\psi^+\psi^2}{3}\big] + f_{\psi^+},$$
$$c.c.$$

The details can be found in : Yu. A. Zhavoronkov, M. V. Komarova, Yu. G. Molotkov, M. Yu. Nalimov, and J. Honkonen CRITICAL DYNAMICS OF THE PHASE TRANSITION TO THE SUPERFLUID STATE Theoretical and Mathematical Physics, 200(2): 1237(2019)

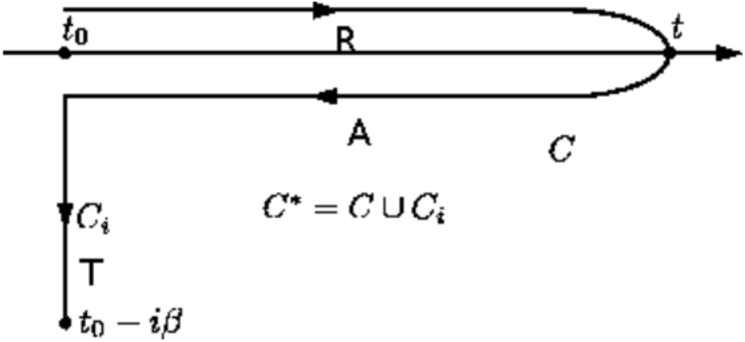
The same problem can be considered using the time-dependent Green functions at finite temperature. The model with the action

$$S(\psi,\psi^+) = \int d\mathbf{x} \int_C dt[\psi^+(\partial_t + i\frac{\Delta}{2m} + i\mu)\psi - i\frac{g}{4}(\psi^+\psi)^2],$$

now t is at the contour C which depends from the time and temperature. $\mathrm{Re}(t) \in (t_0, t_f)$, where t_0 and t_f are initial and final times,

$$\mathrm{Im} t \in (0, -\beta), \qquad \beta = \frac{1}{kT}.$$

Keldysh-Schwinger contour



Propagators of the theory $R, A. T$ indicate the branches of C contour for times of fields

$$G_{RR} = e^{-i\epsilon(t-t')}(\Theta(t-t') + n(\epsilon)), \qquad G_{AA} = e^{-i\epsilon(t-t')}(\Theta(t'-t) + n),$$

$$G_{TT} = e^{-i\epsilon(\tau-\tau')}(\Theta(\tau-\tau') + n), \qquad G_{RT} = e^{-i\epsilon(t-t_0+i\tau')}n,$$

$$G_{RA} = e^{-i\epsilon(t-t')}n, \quad G_{AT} = e^{-i\epsilon(t-t_0+i\tau')}n, \quad G_{AR} = e^{-i\epsilon(t-t')}(n+1),$$

$$G_{TR} = e^{-i\epsilon(t_0-i\tau-t')}(n+1), \qquad G_{TA} = e^{-i\epsilon(t_0-i\tau-t')}(n+1).$$

$$\langle\psi_i(t)\psi_j^+(t')\rangle \equiv G_{ij}, \quad n(\epsilon) = 1/(e^{\beta\epsilon} - 1), \quad \epsilon \equiv p^2/(2m) - \mu.$$

p is a momentum, $\tau \equiv -\mathrm{Im}t$.
Using the Dyson equation in two-loops approximations **the dissipation was found**

$$G^{-1} = G_0^{-1} - \Sigma,$$

G is the matrix of dressed propagators, G_0 – initial propagators, Σ – self energy.
The dissipation origin is similar to Landau dissipation

IR effective theory can be constructed for the investigation of critical behaviour, one obtain the IR effective theory with the action

$$S = 4\eta\alpha\eta^+ + \eta^+(\partial_t - iu\alpha\Delta - \alpha\Delta)\xi + \xi^+(\partial_t - iu\alpha\Delta + \alpha\Delta)\eta + \frac{ig_1}{2}\eta^+\xi\xi^+\xi + + \frac{ig_2}{2}\eta\xi^+\xi\xi^+,$$

in the new variables

$$\xi = \frac{1}{\sqrt{2}}(\psi_R + \psi_A) \quad \xi^+ = \frac{1}{\sqrt{2}}(\psi_R^+ + \psi_A^+)$$
$$\eta = \frac{1}{\sqrt{2}}(\psi_R - \psi_A) \quad \eta^+ = \frac{1}{\sqrt{2}}(\psi_R^+ - \psi_A^+)$$

The renormalization group analysis in $4 - \varepsilon$ space dimension. **Due to the diagram structure the theory is renormalized multiplicatively.** The results of RG analysis coincide the hydrodynamics one mentioned above.

Conclusions:
If the critical point exists the critical behavior is known or can be simply calculated.

Hydrodynamic equations near the equilibrium are similar to the stochastic equations of critical dynamics.

Non-equilibrium Green functions (NEGF) is the most reliable approach.

Thank you very much.