The II International Workshop on Theory of Hadronic Matter Under Extreme Conditions
Dubna (Russia)
18 September 2019

Joint Institute for Nuclear Research
SCIENCE BRINGING NATIONS TOGETHER

Collectivity and electromagnetic fields in proton-nucleus collisions

Lucia Oliva

Collaborators:
Elena Bratkovskaya, Pierre Moreau, Vadim Voronyuk

PHSD

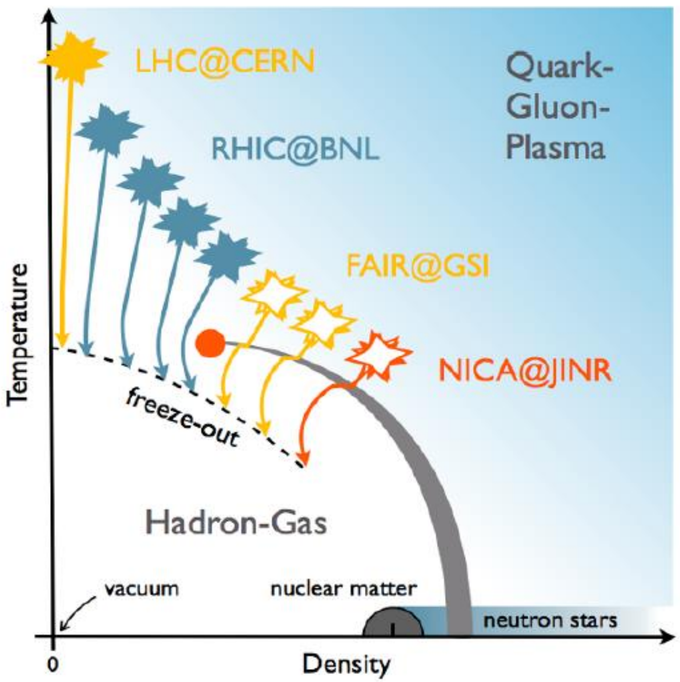
Helmholtzzentrum für Schwerionenforschung GmbH

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211
Strong-interaction matter under extreme conditions

Alexander von Humboldt Stiftung/Foundation

QCD PHASE DIAGRAM

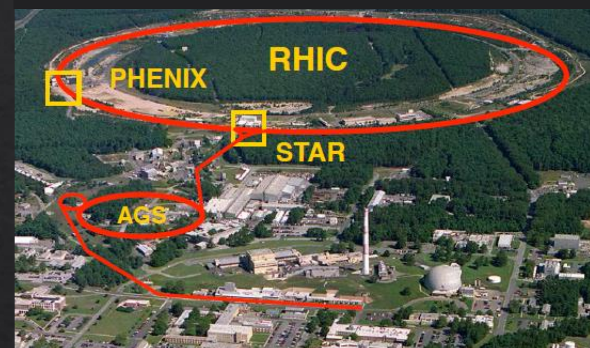


High energy heavy ion collisions

- ✓ allow to experimentally investigate the QCD phase diagram
- ✓ recreate the extreme condition of temperature and density required to form the **QUARK-GLUON PLASMA**

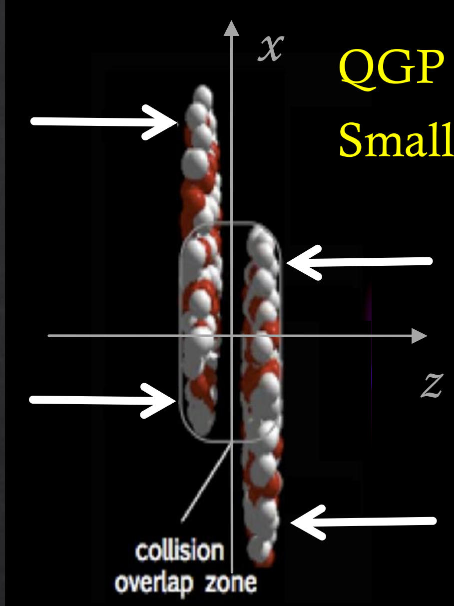
Large Hadron Collider (LHC)

Relativistic Heavy Ion Collider (RHIC)



Facility for Antiproton and Ion Research (FAIR)

Nuclotron-based Ion Collider fAcility (NICA)

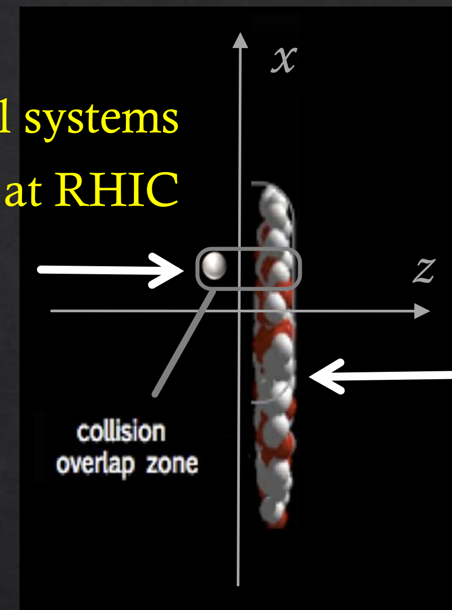


QGP initially expected only in high energy collisions of two heavy ions

Small colliding systems initially regarded as control measurements

Signatures of collective flow found in small systems

p+Pb collisions at LHC, p/d/^{3}He+Au at RHIC



PHENIX Coll., Nature Phys. 15 (2019) 214

nature physics

LETTERS

https://doi.org/10.1038/s41567-018-0360-0

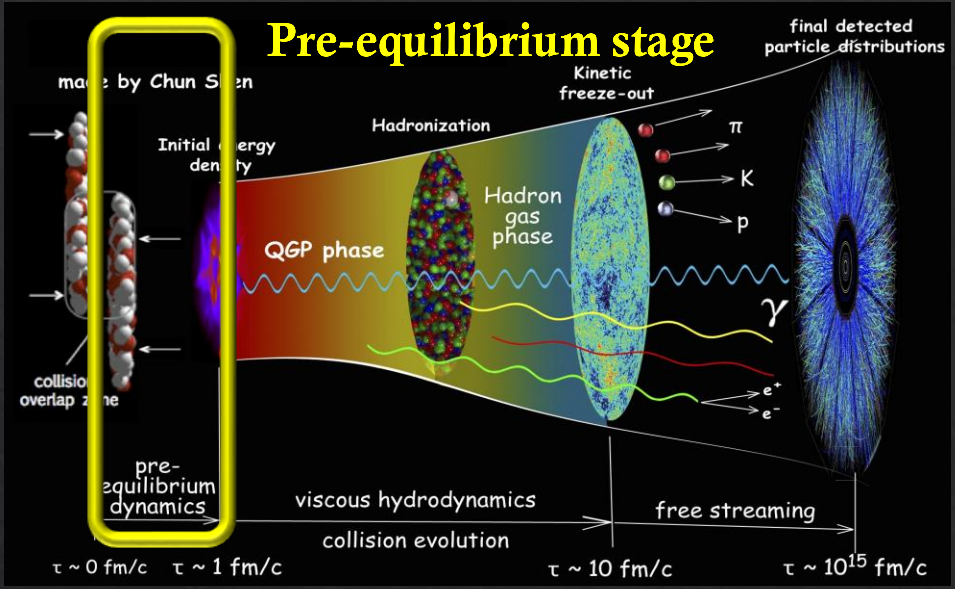
Creation of quark–gluon plasma droplets with three distinct geometries

PHENIX Collaboration*

Experimental studies of the collisions of heavy nuclei at relativistic energies have established the properties of the quark–gluon plasma (QGP), a state of hot, dense nuclear matter in which quarks and gluons are not bound into hadrons[1–4]. In this state, matter behaves as a nearly inviscid fluid[5] that efficiently translates initial spatial anisotropies into correlated momentum anisotropies among the particles produced, creating a common velocity field pattern known as collective flow. In recent years, comparable momentum anisotropies have been measured in small-system proton–proton (p+p) and proton–nucleus (p+A) collisions, despite expectations that the volume and lifetime of the medium produced would be too small to form a QGP. Here we report on the observation of elliptic and triangular flow patterns of charged particles produced in proton–gold (p+Au), deuteron–gold (d+Au) and helium–gold (^{3}He+Au) collisions at a nucleon–nucleon centre-of-mass energy $\sqrt{s_{NN}} = 200$ GeV. The unique combination of three distinct initial geometries and two flow patterns provides unprecedented model discrimination. Hydrodynamical models, which include the formation of a short-lived QGP droplet, provide the best simultaneous description of these measurements.

COLLECTIVITY IN SMALL SYSTEMS AS SIGN OF QGP DROPLETS?

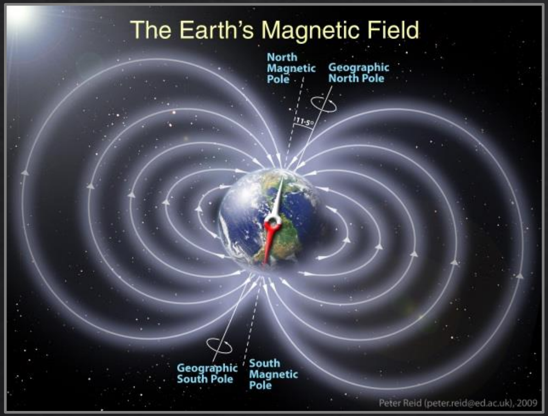
proton-induced collisions at top RHIC energy



Intense magnetic field

$eB_y \sim 5\text{-}50\ m_\pi^2 \sim 10^{18}\text{-}10^{19}$ G

Kharzeev, McLerran and Warringa, NPA 803 (2008) 227
Skokov, Illarionov and Toneev, IJMPA 24 (2009) 5925



Earth's magnetic field

~ 1 G

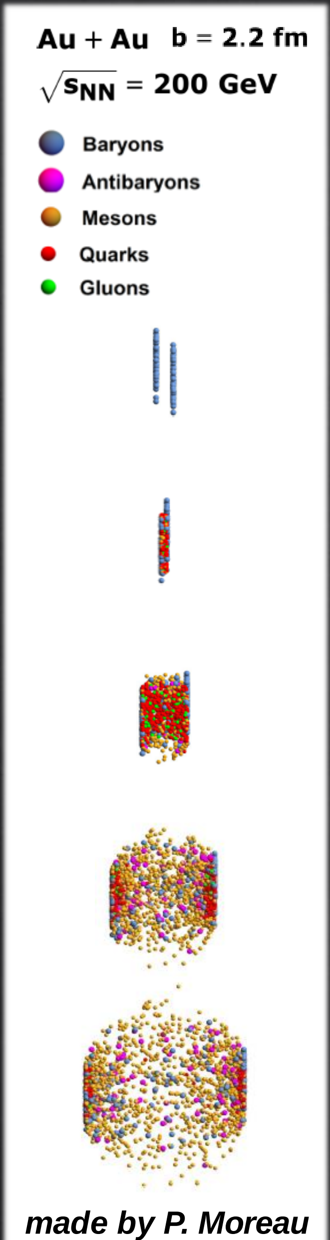
laboratory

~ 10^6 G

magnetar

~ 10^{14}-10^{15} G

PHSD: Parton-Hadron-String Dynamics



A consistent non-equilibrium transport approach to describe large and small colliding systems

To study the phase transition from hadronic to partonic matter and QGP properties from a microscopic origin

talk of Elena Bratkovskaya on Monday

- INITIAL A+A COLLISIONS: nucleon-nucleon collisions lead to the formation of strings that decay to pre-hadrons
- FORMATION OF QGP: if the energy density is above ε_C pre-hadrons dissolve in massive quarks and gluons + mean-field potential
- QGP STAGE: evolution based on off-shell transport eqs. derived by Kadanoff-Baym eqs. with the Dynamical Quasi-Particle Model (DQPM) defining parton spectral functions, i.e. masses and widths
- HADRONIZATION: massive off-shell partons with broad spectral functions hadronize to off-shell baryon and mesons
- HADRONIC PHASE: evolution based on the off-shell transport equations with hadron-hadron interactions

Cassing and Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215
Cassing, EPJ ST 168 (2009) 3; NPA856 (2011) 162

PHSD + electromagnetic fields

PHSD includes the dynamical formation and evolution of the retarded electomagnetic field (EMF) and its influence on quasi-particle dynamics

$$\left\{\frac{\partial}{\partial t} + \left(\frac{\mathbf{p}}{p_0} + \nabla_{\mathbf{p}} U\right)\nabla_{\mathbf{r}} + (-\nabla_{\mathbf{r}} U + e\mathbf{E} + e\mathbf{v}\times\mathbf{B})\nabla_{\mathbf{p}}\right\} f = C_{\text{coll}}(f, f_1, \ldots, f_N)$$

Lorentz force

TRANSPORT EQUATIONS

consistent solution of particle and field evolution equations

charge distribution

electric current

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\cdot\mathbf{E} = 4\pi\rho \qquad \nabla\times\mathbf{B} = \frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}$$

MAXWELL EQUATIONS

Voronyuk *et al*. (HSD), PRC 83 (2011) 054911

PHSD + electromagnetic fields

PHSD includes the dynamical formation and evolution of the retarded electomagnetic field (EMF) and its influence on quasi-particle dynamics

$$\left\{\frac{\partial}{\partial t} + \left(\frac{\mathbf{p}}{p_0} + \nabla_{\mathbf{p}} U\right) \nabla_{\mathbf{r}} + (-\nabla_{\mathbf{r}} U + e\mathbf{E} + e\mathbf{v} \times \mathbf{B}) \nabla_{\mathbf{p}} \right\} f = C_{\mathrm{coll}}(f, f_1, \ldots, f_N)$$

Lorentz force

TRANSPORT EQUATIONS

consistent solution of particle and field evolution equations

charge distribution

electric current

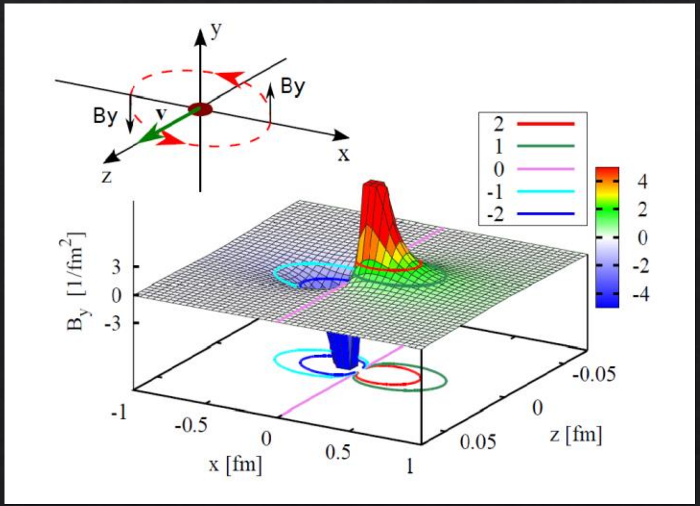
$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{E} = 4\pi\rho \qquad \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}$$

MAXWELL EQUATIONS

$$e\mathbf{E}(t, \mathbf{r}) = \alpha_{em} \frac{1 - \beta^2}{\left[(\mathbf{R} \cdot \boldsymbol{\beta})^2 + R^2 (1 - \beta^2)\right]^{3/2}} \mathbf{R}$$

$$e\mathbf{B}(t, \mathbf{r}) = \boldsymbol{\beta} \times e\mathbf{E}(t, \mathbf{r})$$

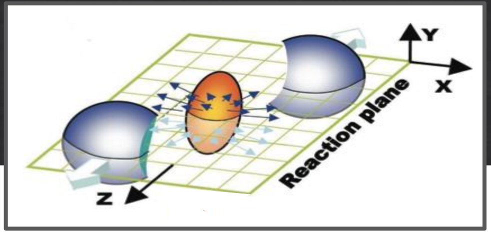
single freely moving charge



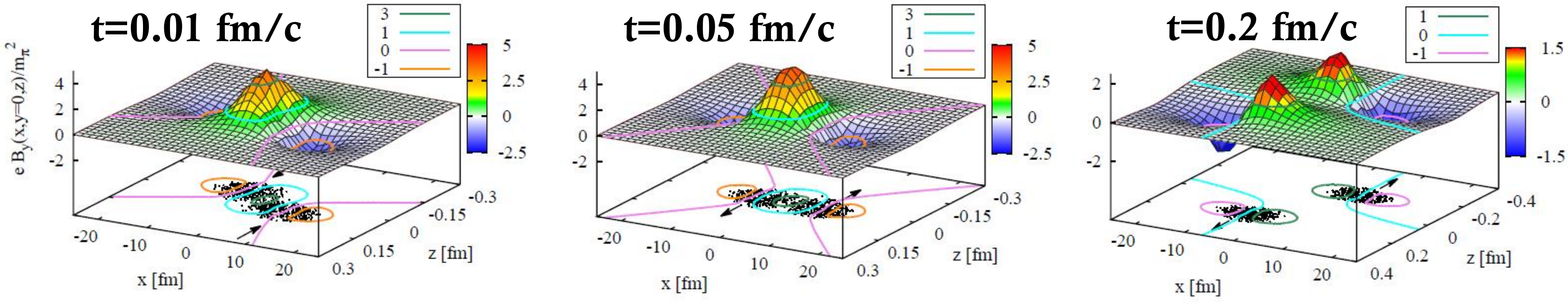
Voronyuk *et al*. (HSD), PRC 83 (2011) 054911

electromagnetic fields in HICs

in a nuclear collision the magnetic field is a superposition of solenoidal fields from different moving charges



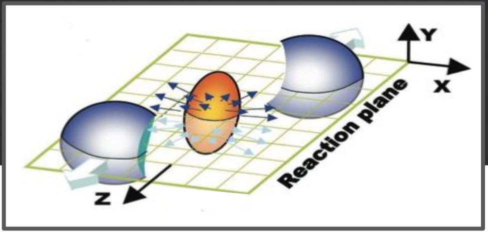
Au+Au @RHIC 200 GeV – b = 10 fm



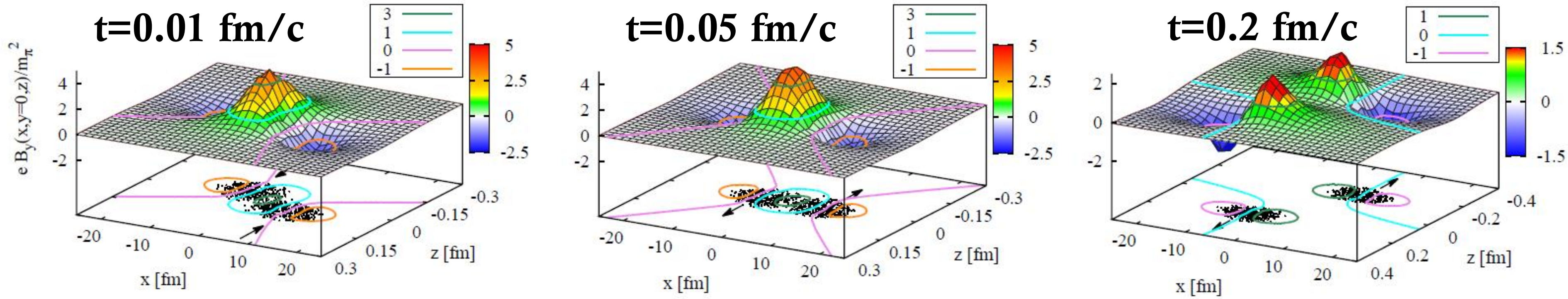
Voronyuk *et al*. (HSD), PRC 83 (2011) 054911

electromagnetic fields in HICs

in a nuclear collision the magnetic field is a superposition of solenoidal fields from different moving charges

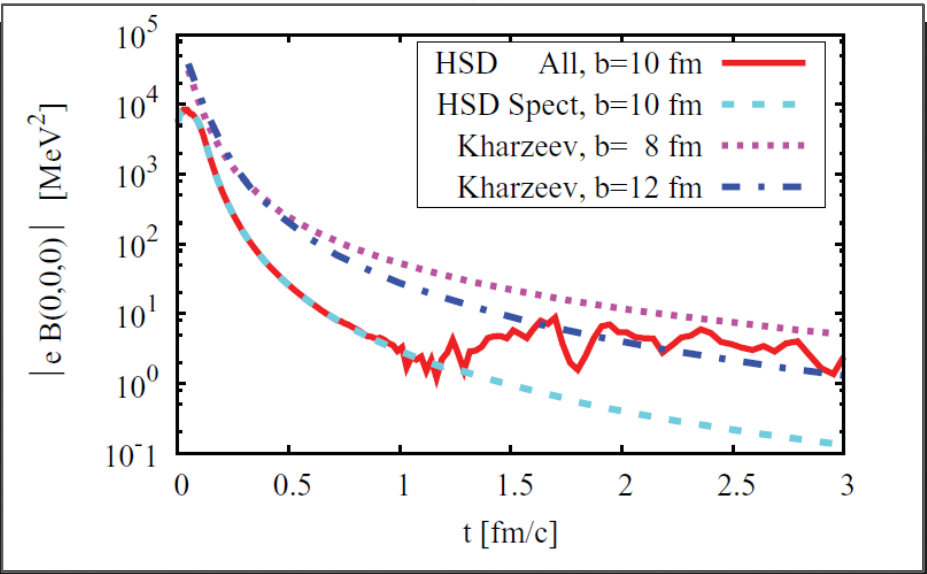


Au+Au @RHIC 200 GeV – b = 10 fm



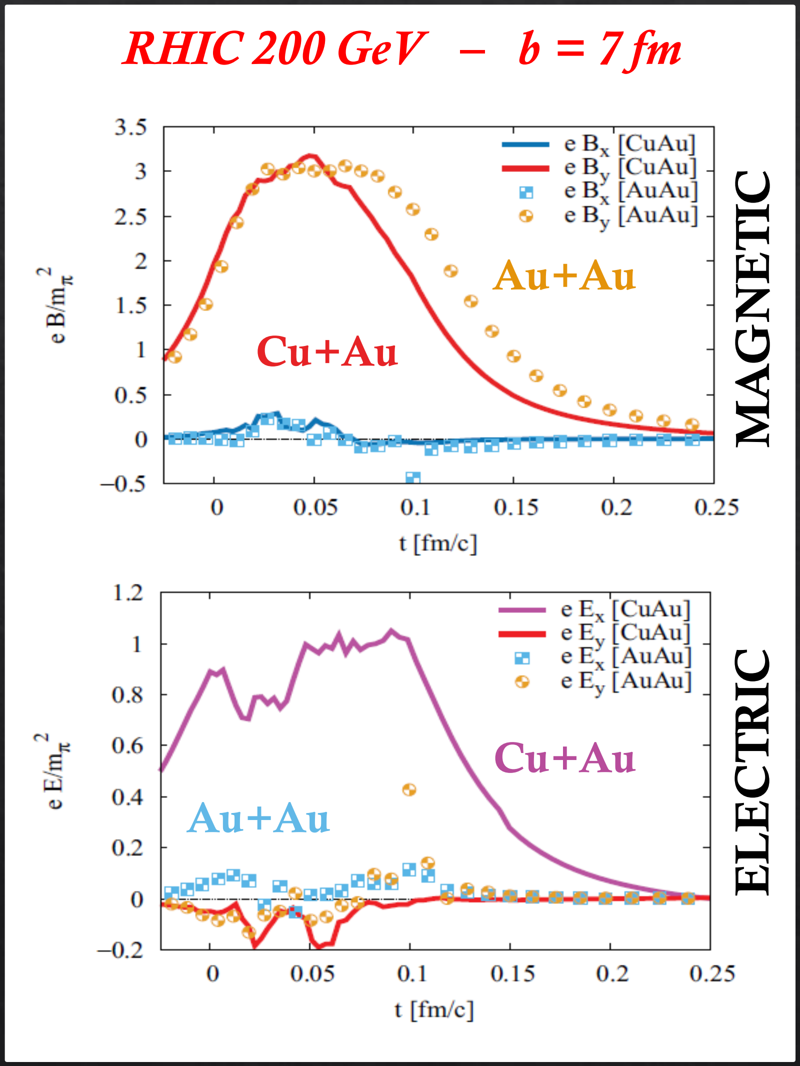
MAGNETIC FIELD

- ❖ dominated by the y-component
- ❖ maximal strength reached during nuclear overlapping time
- ❖ only due to spectators up to $t \sim 1$ fm/c
- ❖ drops down by three orders of magnitude and become comparable with that from participants



Voronyuk *et al*. (HSD), PRC 83 (2011) 054911

electromagnetic fields in HICs

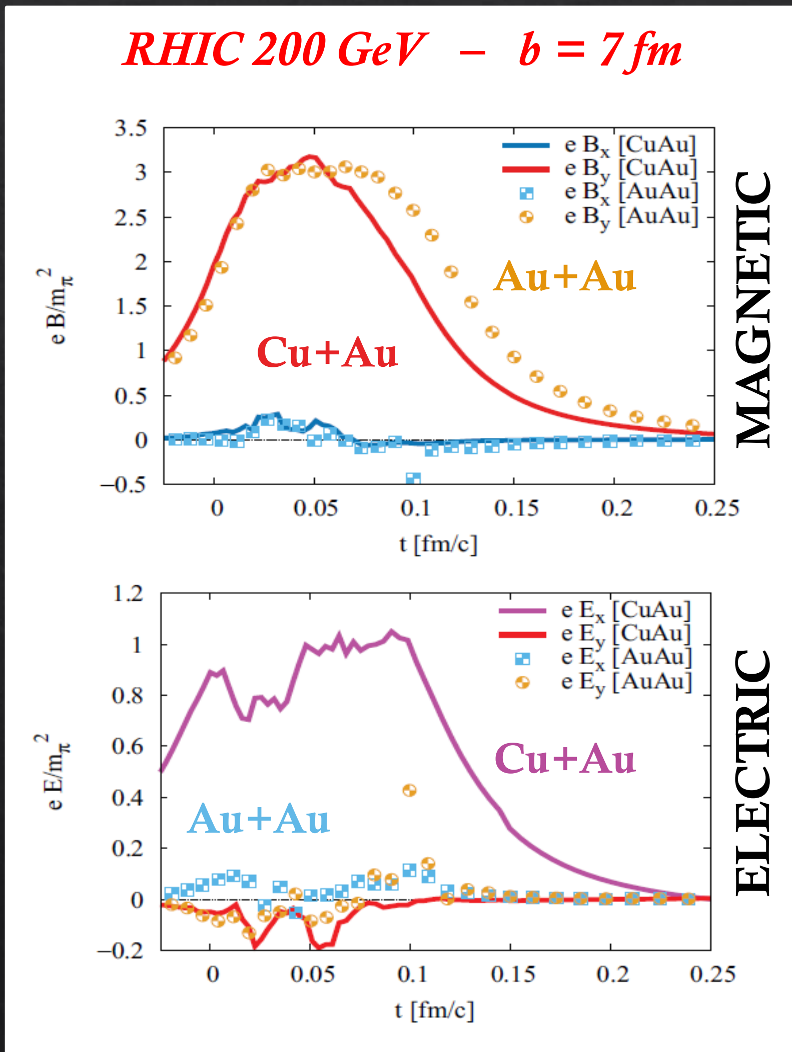


✓ SYMMETRIC SYSTEMS (e.g. Au+Au)
transverse momentum increments due to electric and magnetic fields compensate each other

✓ ASYMMETRIC SYSTEMS (e.g. Cu+Au)
an intense electric fields directed from the heavy nuclei to light one appears in the overlap region

Voronyuk *et al*. (PHSD), PRC 90 (2014) 064903
Toneev *et al*. (PHSD), PRC 95 (2017) 034911

electromagnetic fields in HICs

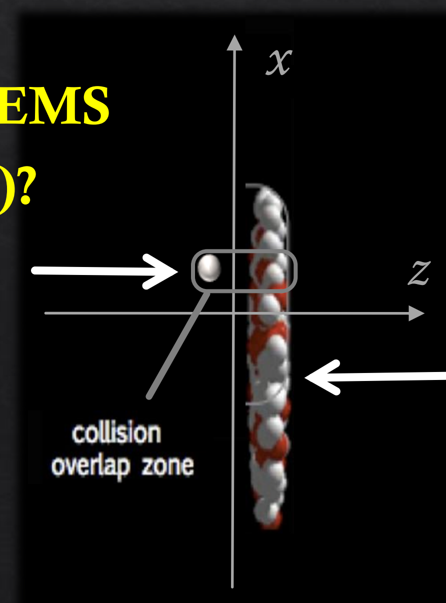


Voronyuk *et al*. (PHSD), PRC 90 (2014) 064903
Toneev *et al*. (PHSD), PRC 95 (2017) 034911

✓ SYMMETRIC SYSTEMS (e.g. Au+Au)
transverse momentum increments due to electric and magnetic fields compensate each other

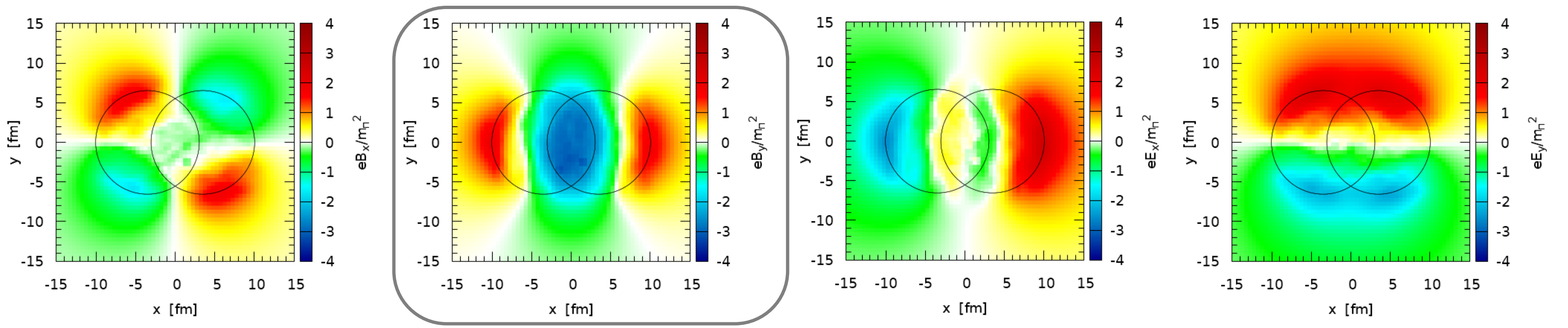
✓ ASYMMETRIC SYSTEMS (e.g. Cu+Au)
an intense electric fields directed from the heavy nuclei to light one appears in the overlap region

SMALL SYSTEMS (e.g. p+Au)?

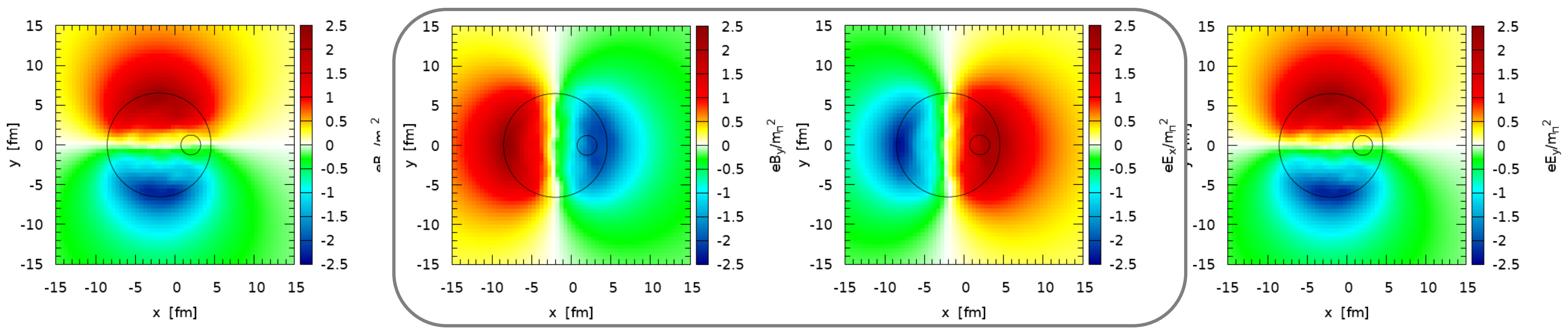


p+Au: electromagnetic fields

Au+Au @ RHIC 200 GeV b=7 fm



B_x B_y E_x E_y

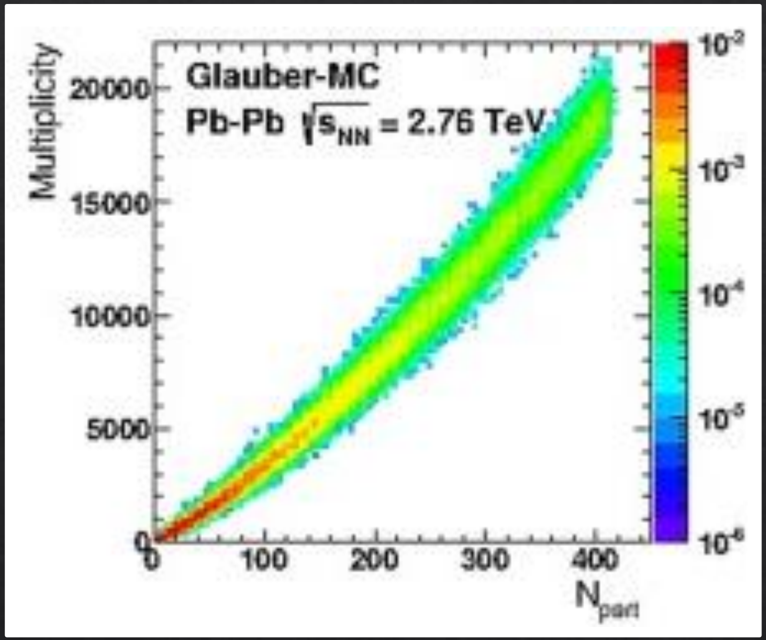


p+Au @ RHIC 200 GeV b=4 fm

LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770

Centrality in small systems

In heavy ion collisions centrality characterizes the amount of overlap or size of the fireball in the collision region

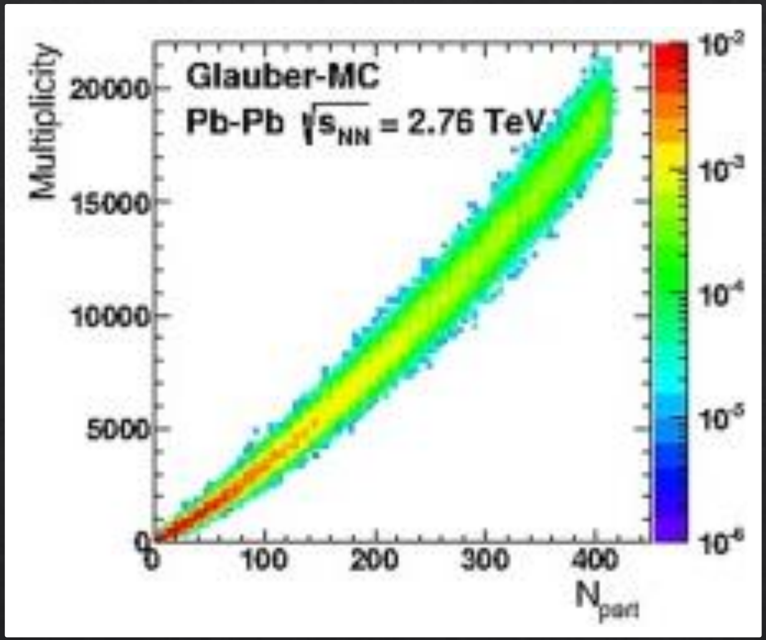


ALICE, NPA 932 (2014) 399

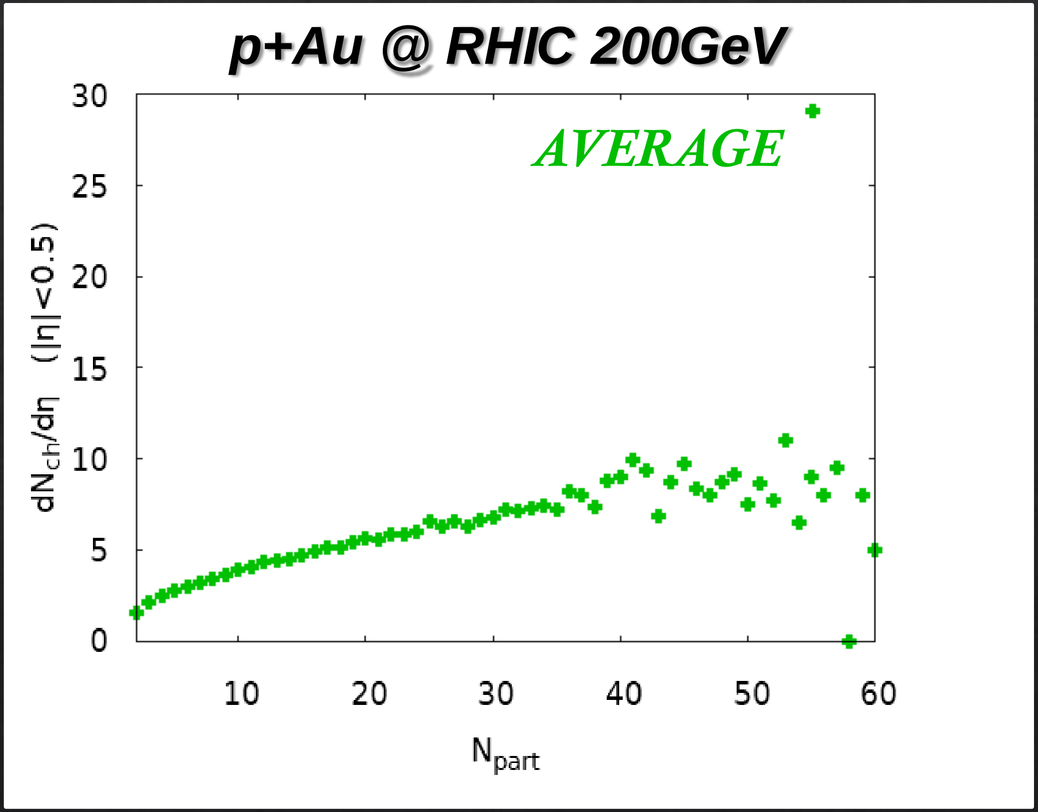
Correlation between participant number and charged particle multiplicity at midrapidity

p+Au: centrality determination

In heavy ion collisions centrality characterizes the amount of overlap or size of the fireball in the collision region



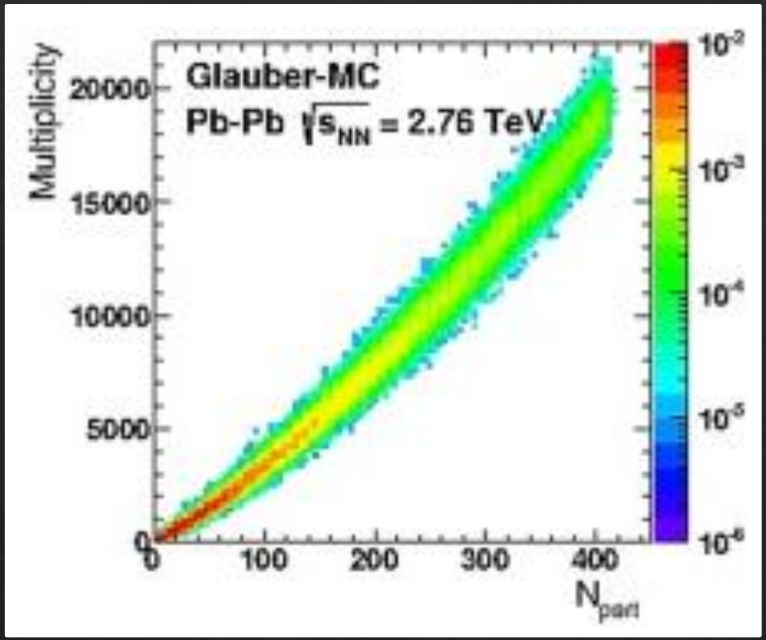
ALICE, NPA 932 (2014) 399



Correlation between participant number and charged particle multiplicity at midrapidity

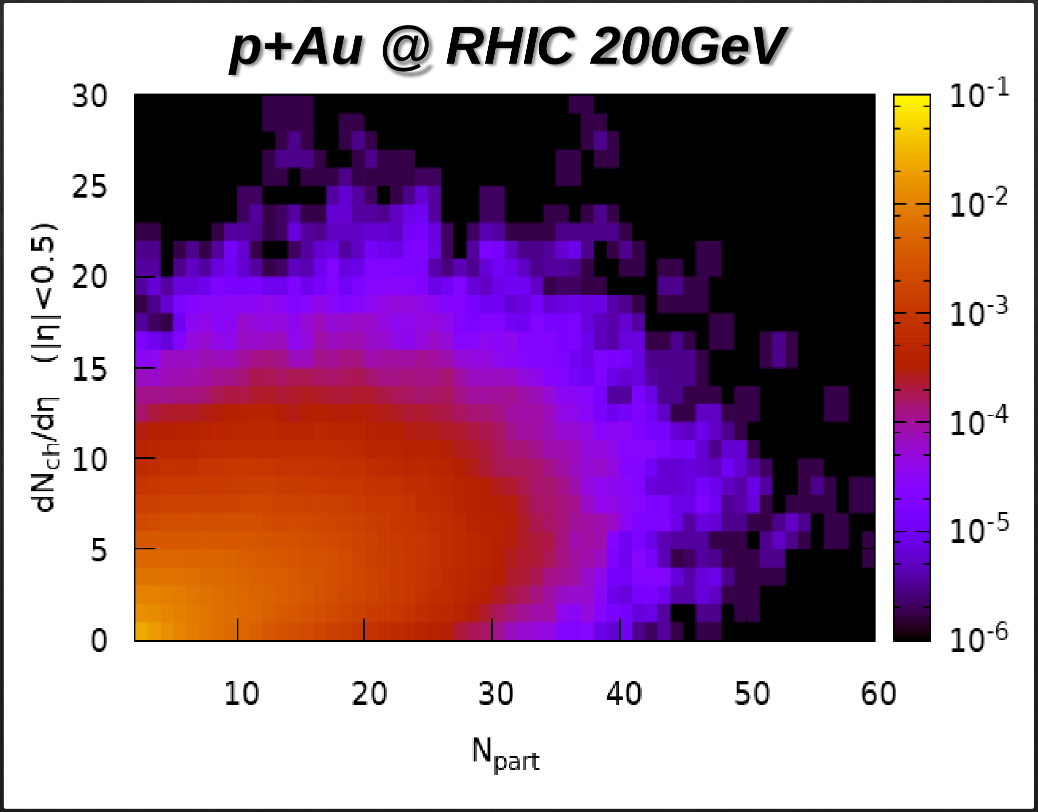
p+Au: centrality determination

In heavy ion collisions centrality characterizes the amount of overlap or size of the fireball in the collision region



ALICE, NPA 932 (2014) 399

LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770



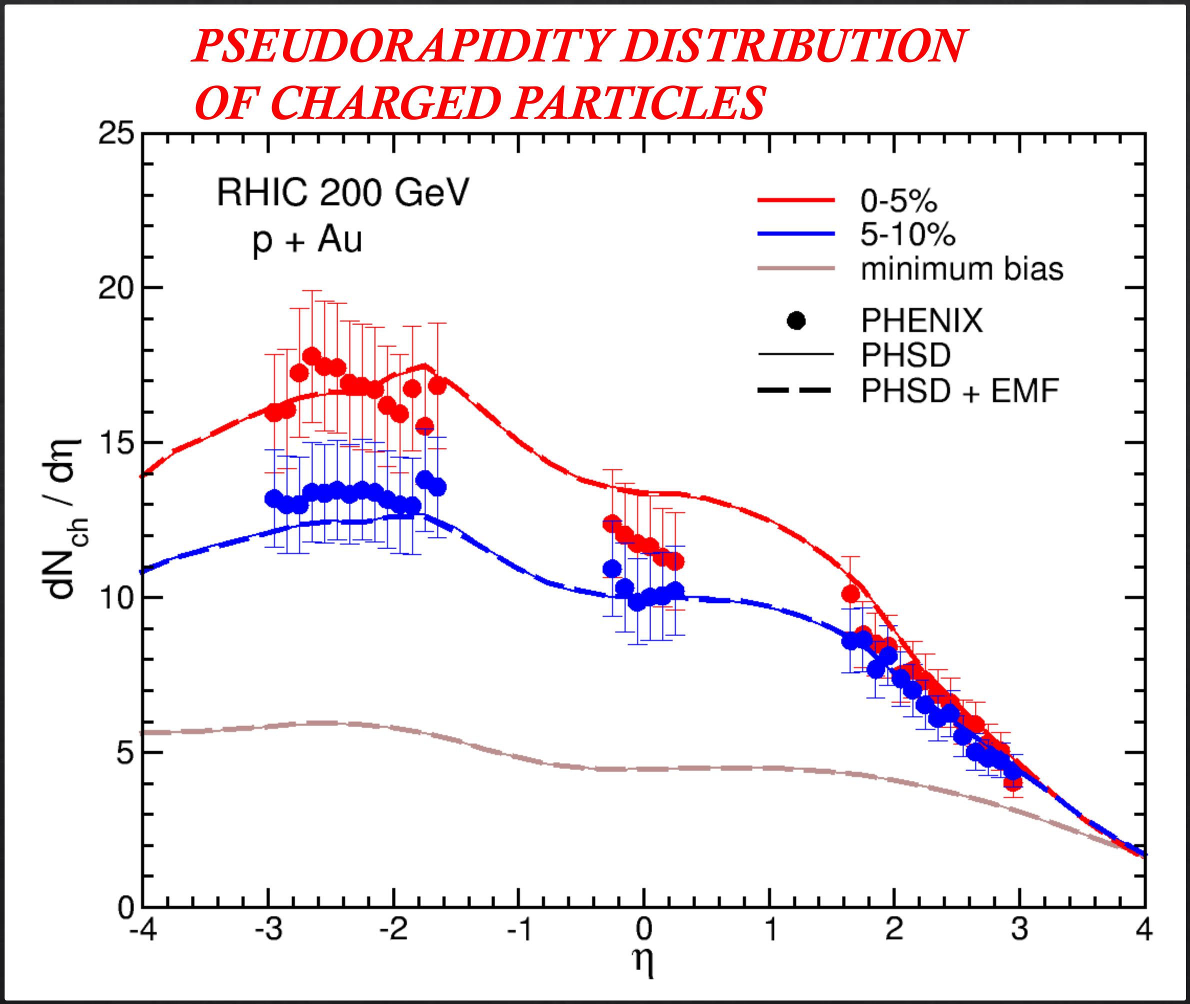
Correlation between participant number and charged particle multiplicity at midrapidity

BUT

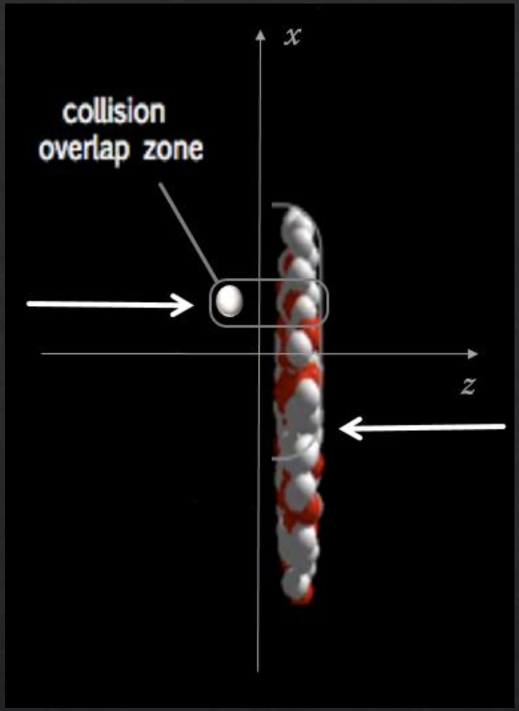
large dispersion in both quantities in p+A respect to A+A collisions

p+Au: rapidity distributions

Exp. Data: PHENIX Collaboration, PRL 121 (2018) 222301



LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770

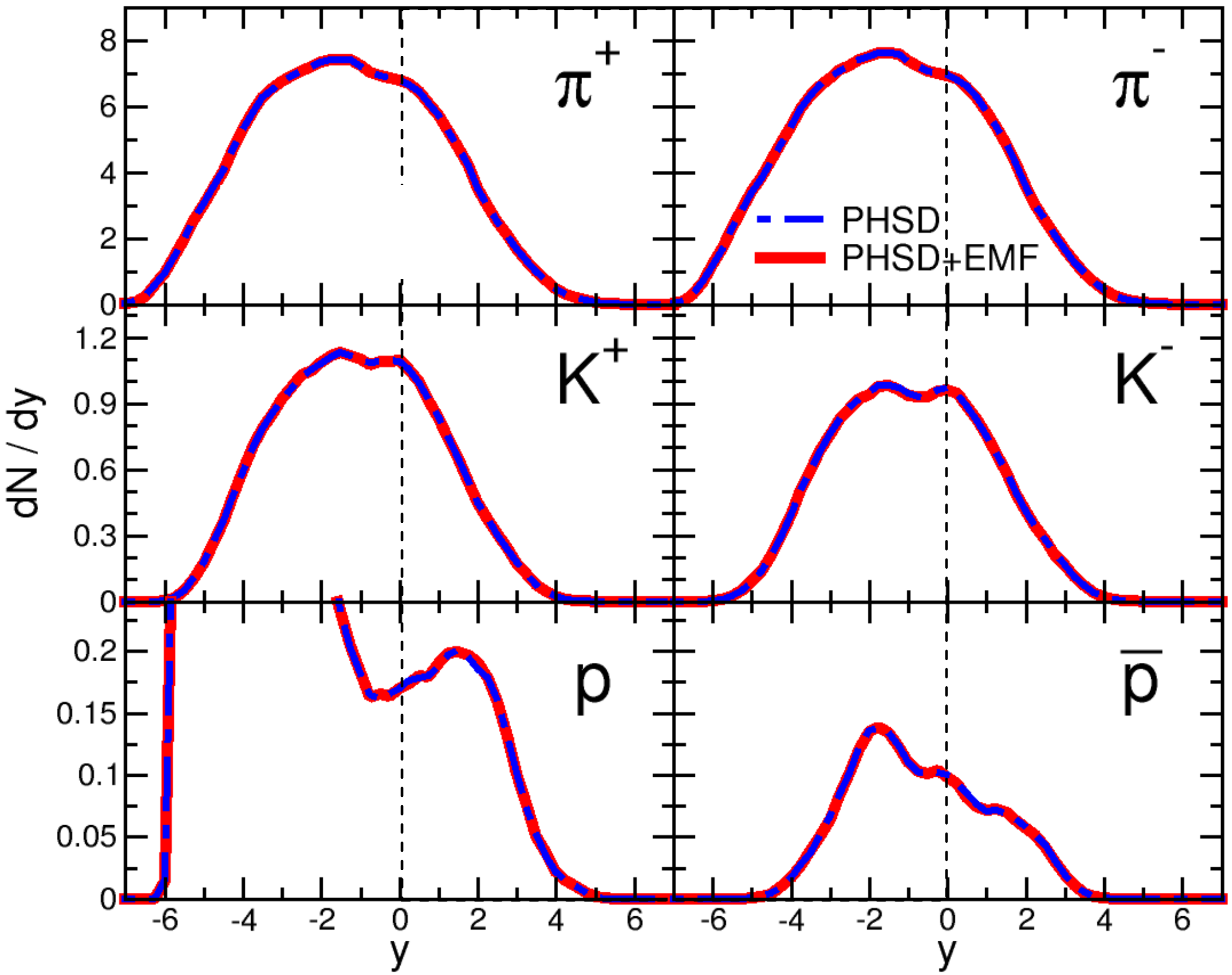


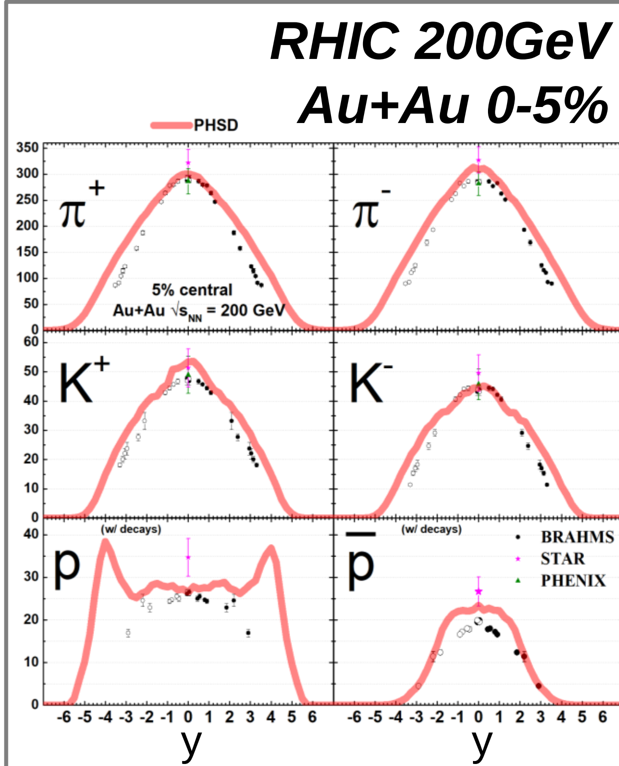
- enhanced particle production in the Au-going directions
- asymmetry increases with centrality of the collision

p+Au: rapidity distributions

RAPIDITY DISTRIBUTION OF IDENTIFIED PARTICLES

RHIC 200GeV
p+Au 0-5%



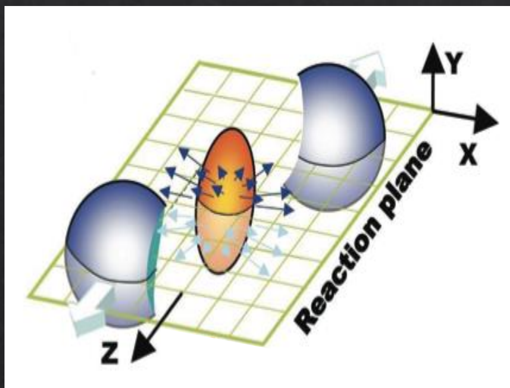


symmetric colliding system

Anisotropic radial flow

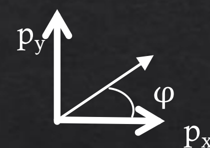
A DEEPER INSIGHT…INITIAL-STATE FLUCTUATIONS

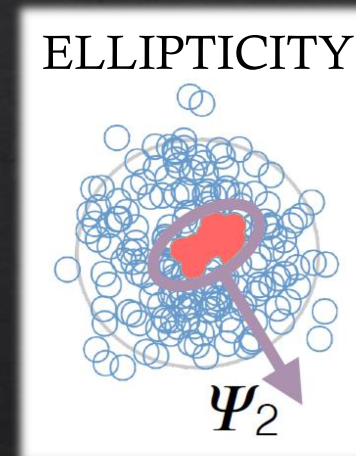
Not a simple **almond shape**
- ➢ odd harmonics = 0



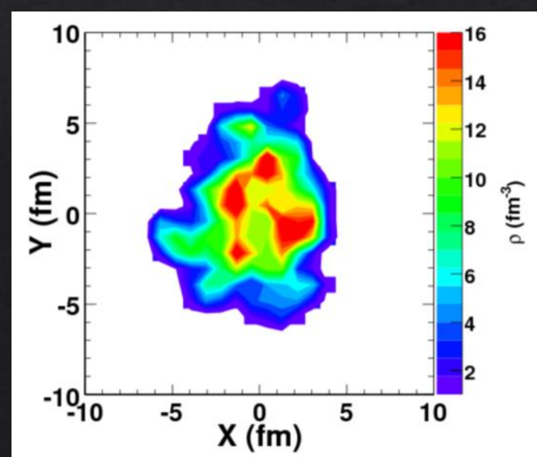
$$E\frac{d^3N}{d^3p}=\frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(1+2\sum_{n=1}^{\infty}v_n(p_T,y)\cos\left(n\left(\phi-\Psi_r\right)\right)\right)$$

$$v_n=\left\langle\cos\left(n\left(\phi-\Psi_r\right)\right)\right\rangle$$

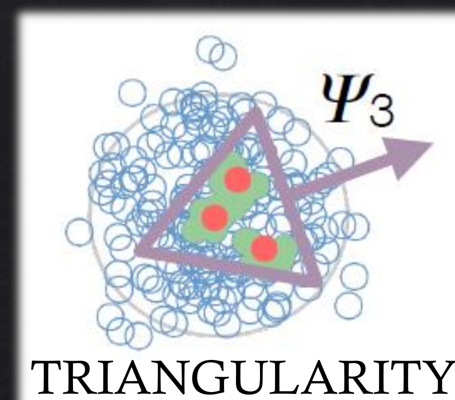




But a **"lumpy" profile due to fluctuations** of nucleon position in the overlap region
- ➢ odd harmonics ≠ 0



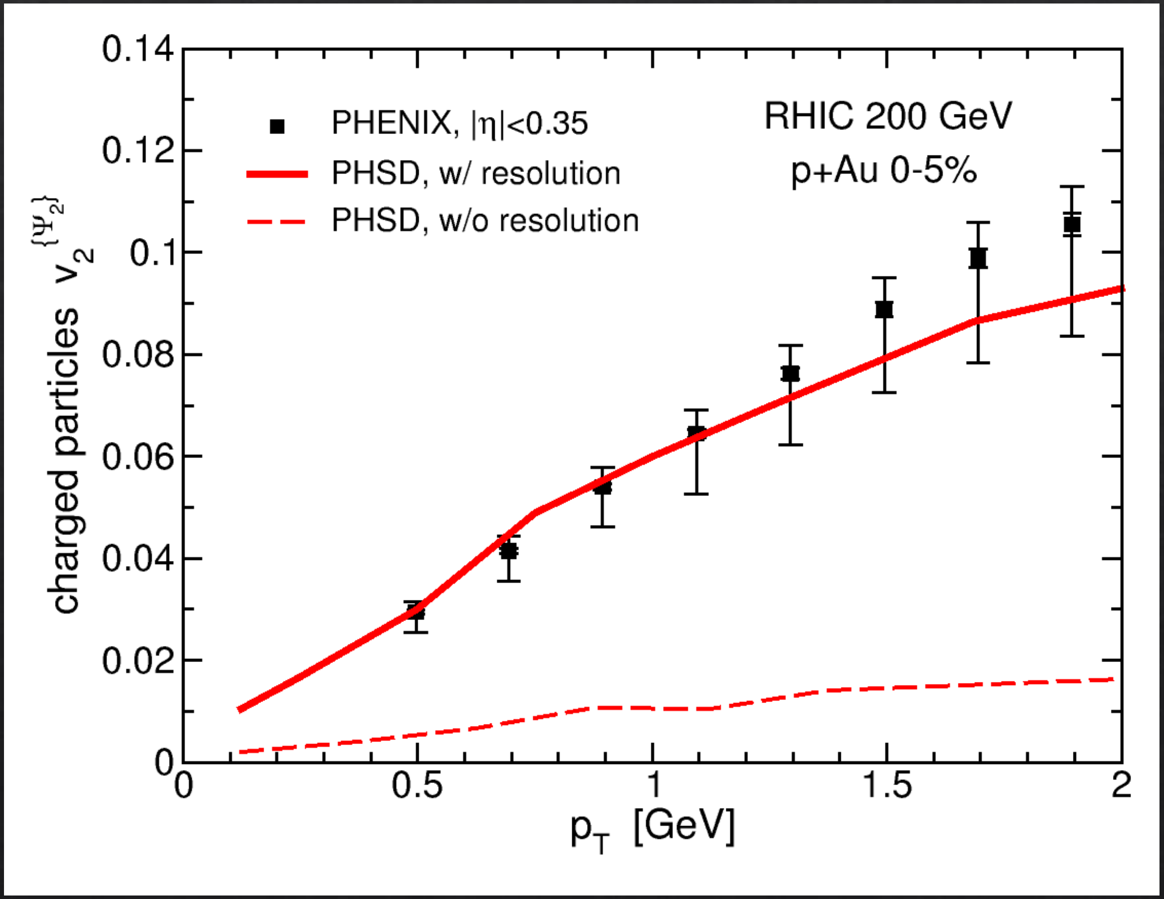
Plumari et al., PRC 92 (2015) 054902



p+Au: elliptic flow

ELLIPTIC FLOW OF CHARGED PARTICLES

$$v_2(p_T) = \frac{\langle\cos[2(\varphi(p_T) - \Psi_2)]\rangle}{Res(\Psi_2)}$$



Event-plane angle in $-3 < \eta < -1$:
$Res(\Psi_2^{PHSD}) = 0.175$
$Res(\Psi_2^{PHENIX}) = 0.171$

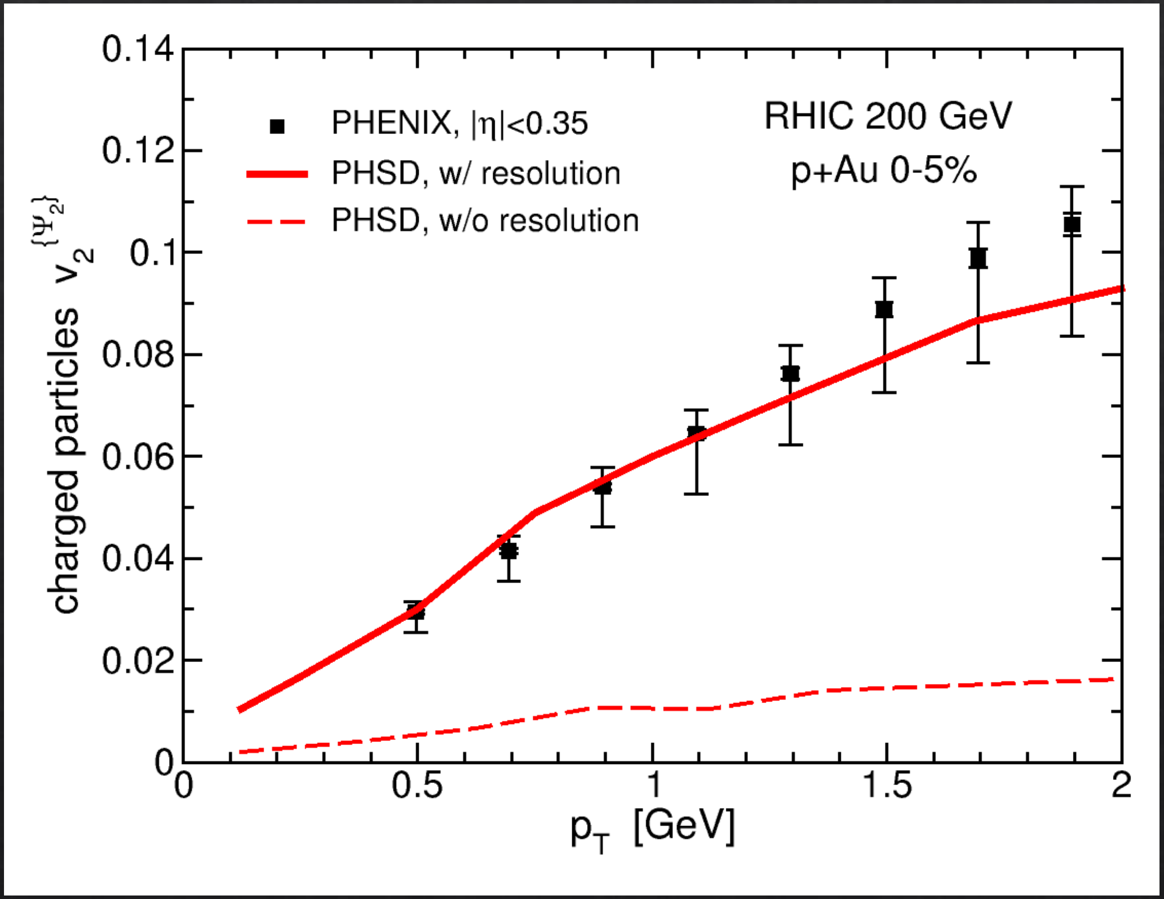
magnitude correlated with the determination of the reaction plane

LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770
Exp. data: Aidala et al. (PHENIX Collaboration), PRC 95 (2017) 034910

p+Au: elliptic flow

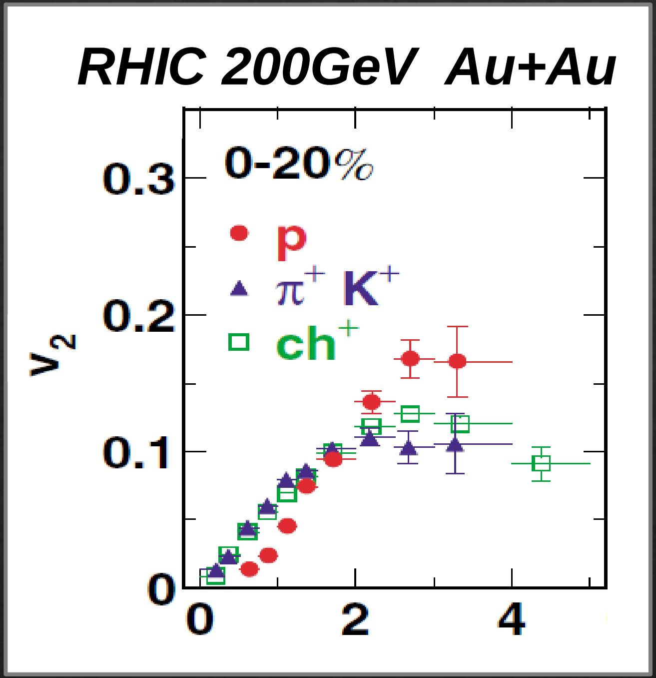
ELLIPTIC FLOW OF CHARGED PARTICLES

$$v_2(p_T) = \frac{\langle \cos[2(\varphi(p_T) - \Psi_2)] \rangle}{Res(\Psi_2)}$$



LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770

Exp. data: Aidala et al. (PHENIX Collaboration), PRC 95 (2017) 034910

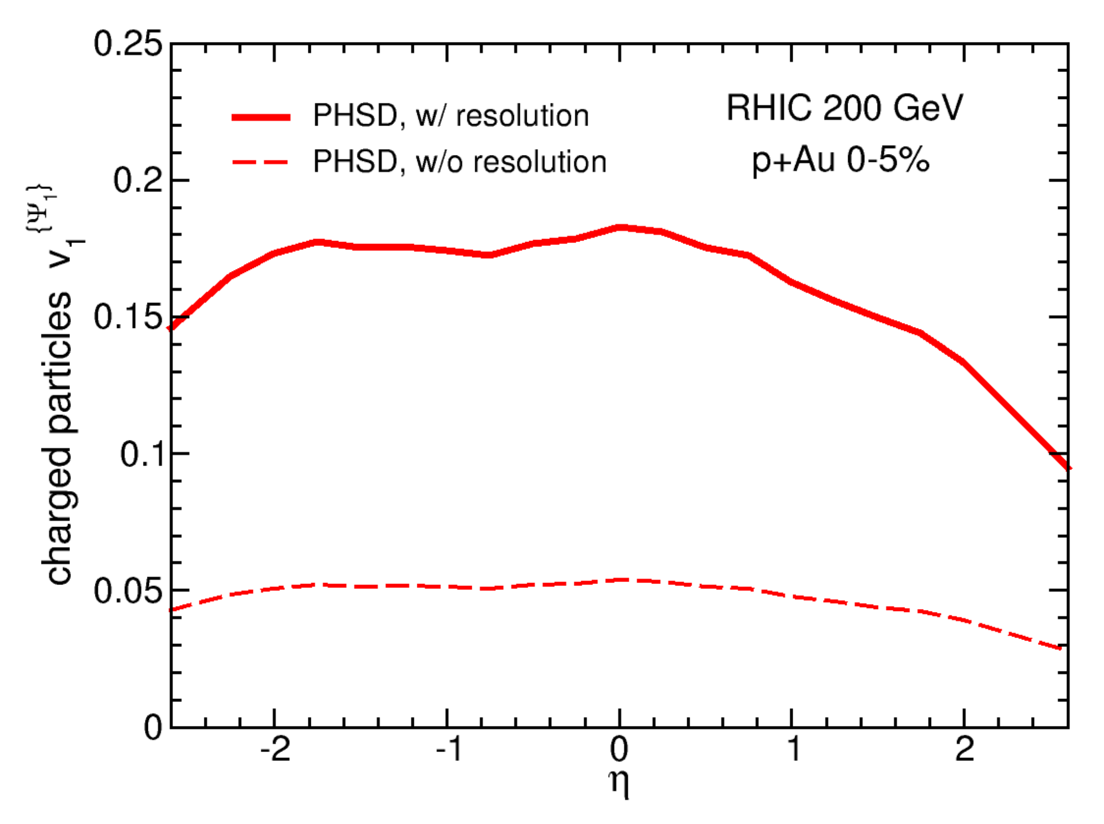


PHENIX, PRL 91 (2003) 182301

comparable to that found in large colliding systems

p+Au: directed flow

pseudorapidity dependence of the
DIRECTED FLOW OF CHARGED PARTICLES



LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770

$$v_1(\eta) = \frac{\langle \cos[\varphi(\eta) - \Psi_1] \rangle}{Res(\Psi_1)}$$

Event-plane angle in $-4 < \eta < -3$: $Res\left(\Psi_1^{PHSD}\right) = 0.397$

magnitude correlated with the determination of the reaction plane

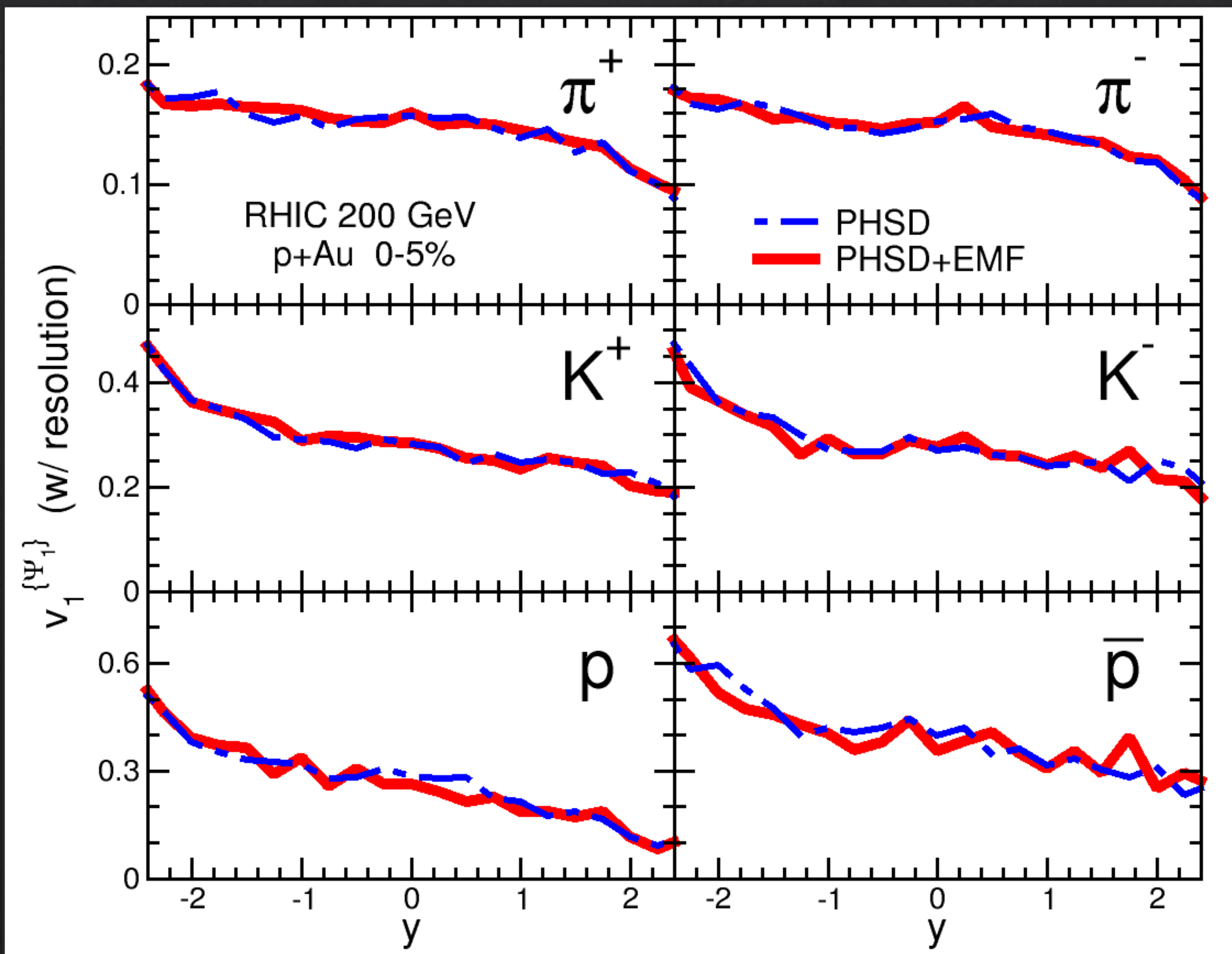
SPLITTING OF POSITIVELY AND NEGATIVELY CHARGED PARTICLES INDUCED BY THE ELECTROMAGNETIC FIELD?

p+Au: directed flow

rapidity dependence of the
DIRECTED FLOW OF IDENTIFIED PARTICLES

$$v_1(\eta) = \frac{\langle \cos[\varphi(\eta) - \Psi_1] \rangle}{Res(\Psi_1)}$$

LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770



SPLITTING INDUCED BY THE EM FIELD?

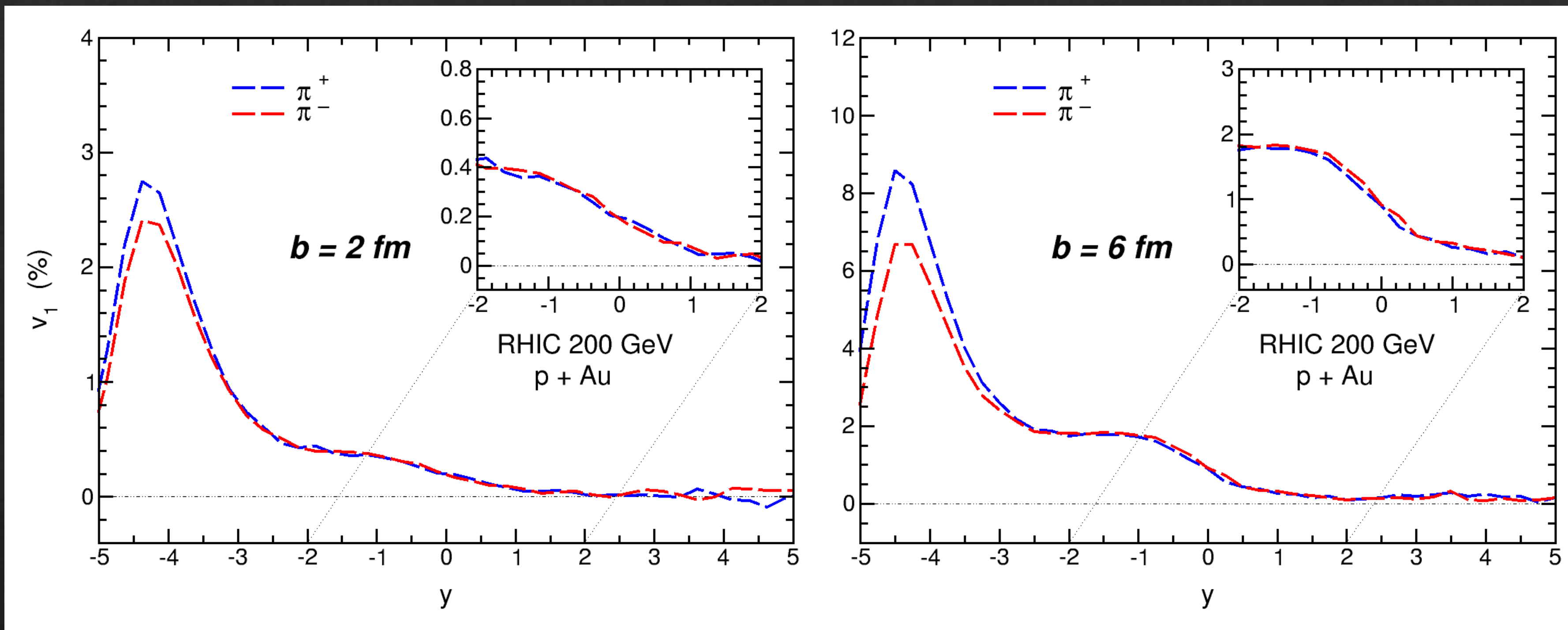
no visible changes
with and without
electromagnetic fields
for 5% central collisions

BUT…

p+Au: directed flow

rapidity dependence of the
DIRECTED FLOW OF PIONS

$$v_1(y) = \langle \cos[\varphi(y)] \rangle = \langle p_x / p_T \rangle$$



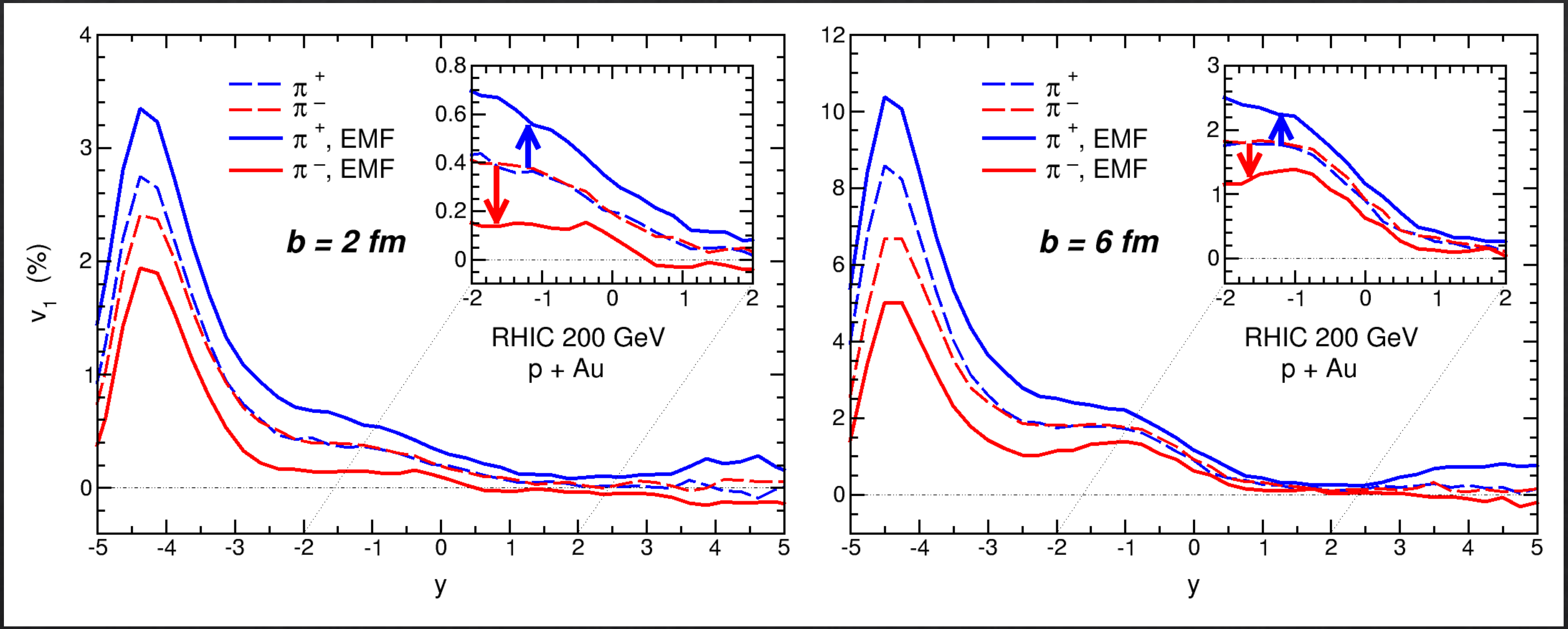
LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770

SPLITTING INDUCED BY THE EM FIELD?

p+Au: directed flow

rapidity dependence of the
DIRECTED FLOW OF PIONS

$$v_1(y) = \langle \cos[\varphi(y)] \rangle = \langle p_x/p_T \rangle$$



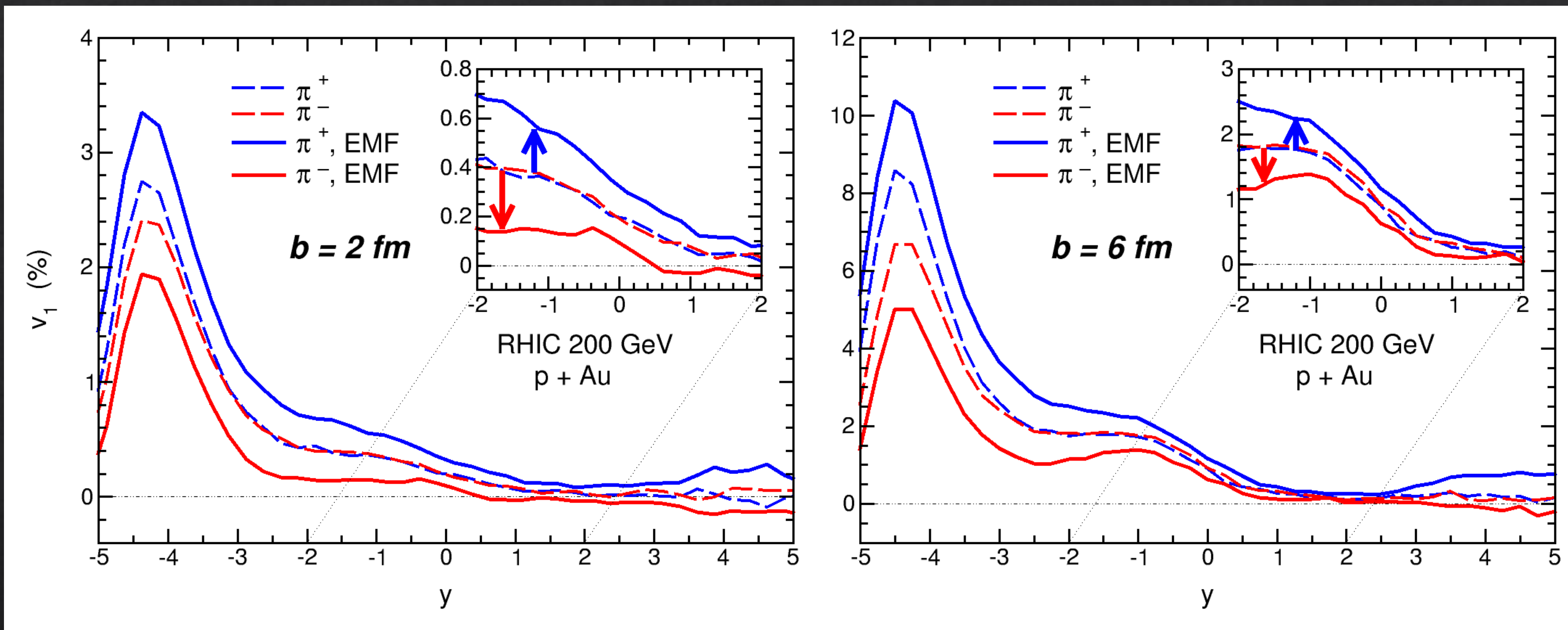
LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770

Splitting of π^+ and π^- induced by the electromagnetic field

p+Au: directed flow

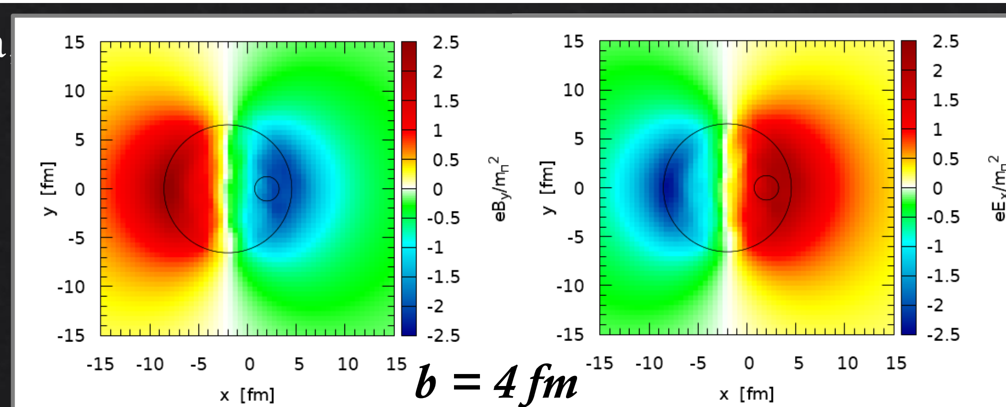
rapidity dependence of the
DIRECTED FLOW OF PIONS

$$v_1(y) = \langle \cos[\varphi(y)] \rangle = \langle p_x / p_T \rangle$$



LO, Moreau, Voronyuk and Bratkovskaya

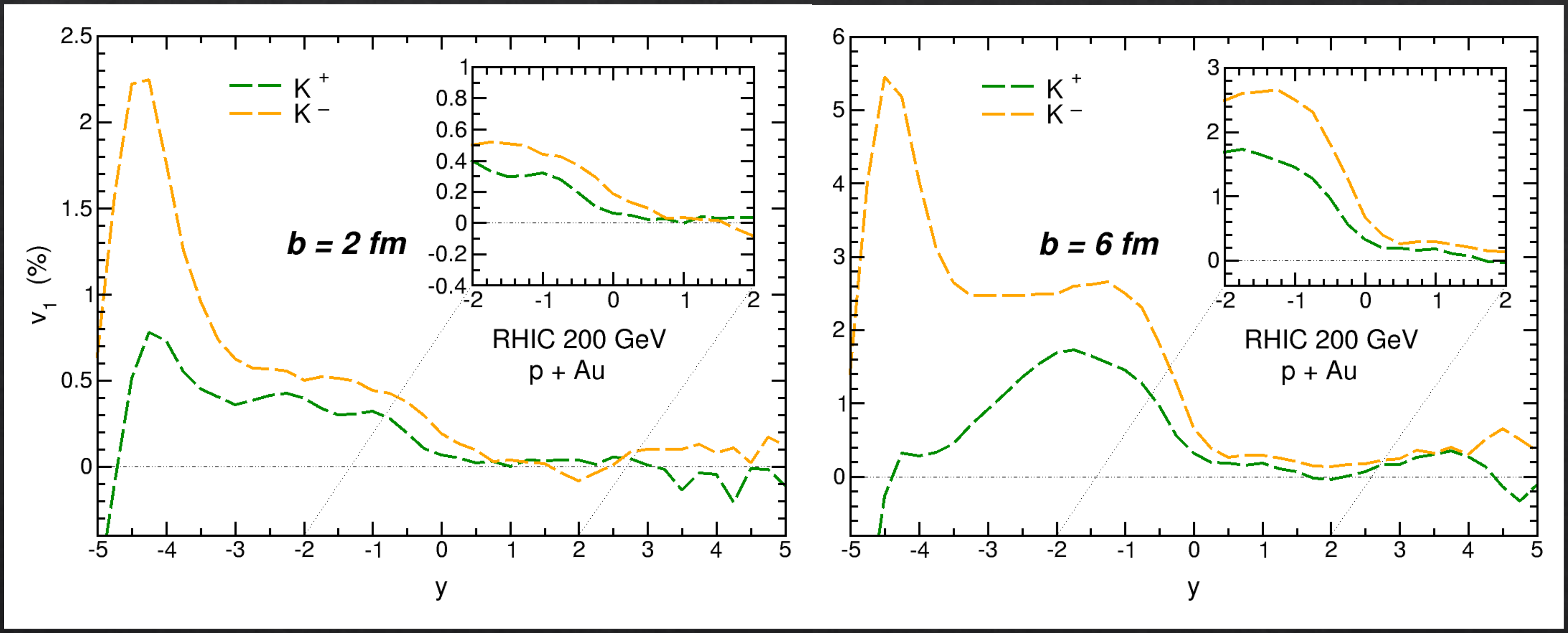
Splitting of π^+ and π^- induced by electric and magnetic field



p+Au: directed flow

rapidity dependence of the
DIRECTED FLOW OF KAONS

$$v_1(y) = \langle \cos[\varphi(y)] \rangle = \langle p_x / p_T \rangle$$

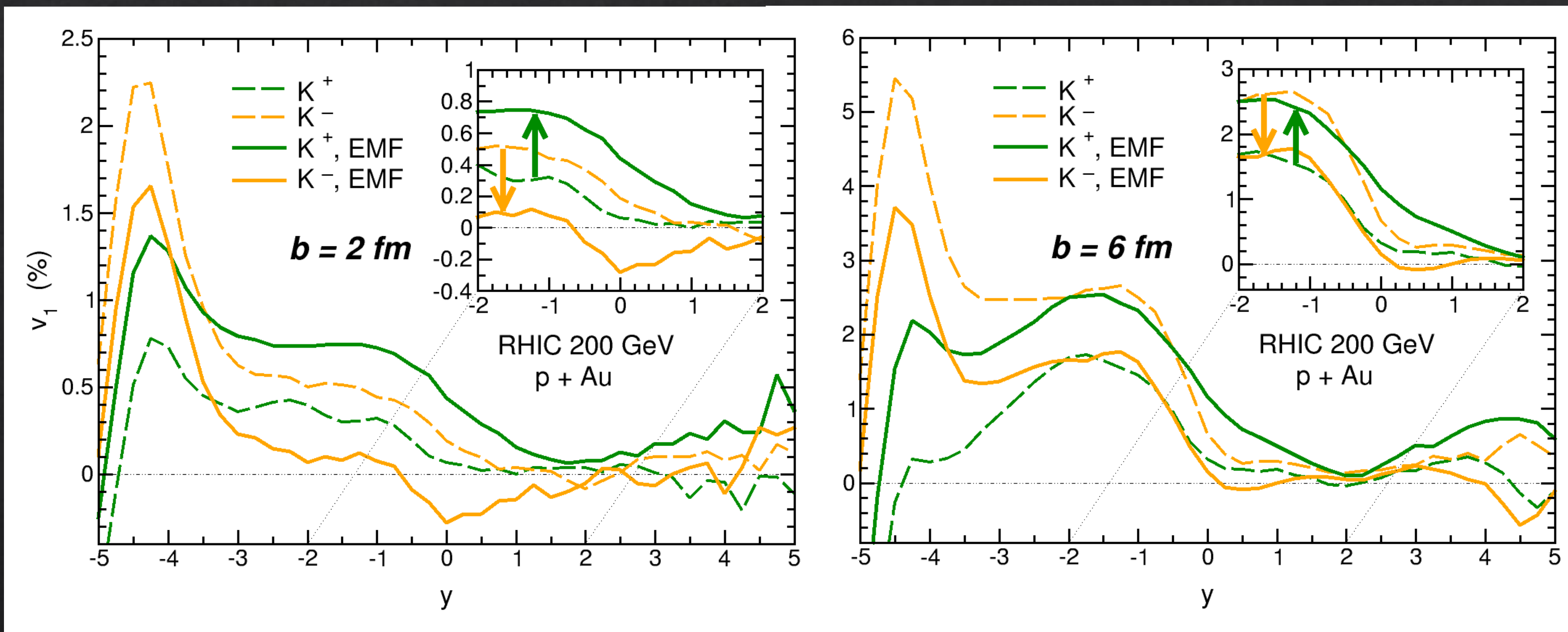


LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770

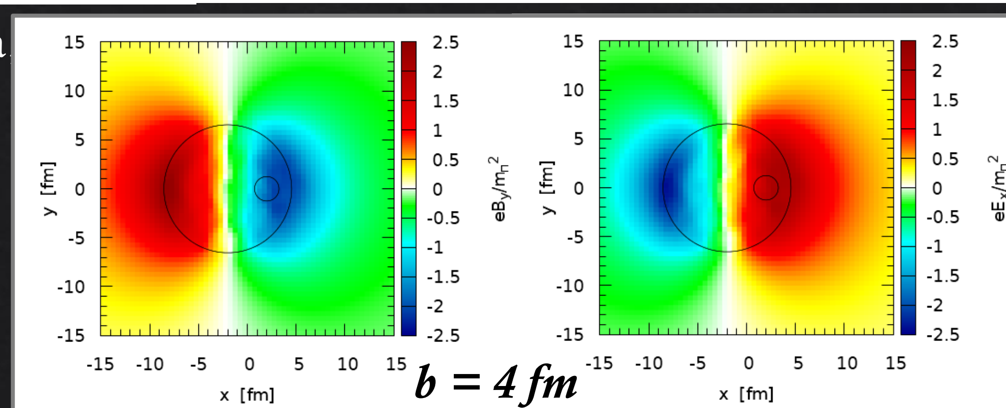
p+Au: directed flow

rapidity dependence of the
DIRECTED FLOW OF KAONS

$$v_1(y) = \langle \cos[\varphi(y)] \rangle = \langle p_x / p_T \rangle$$



LO, Moreau, Voronyuk and Bratkovskaya



Splitting of K^+ and K^- induced by electric and magnetic field

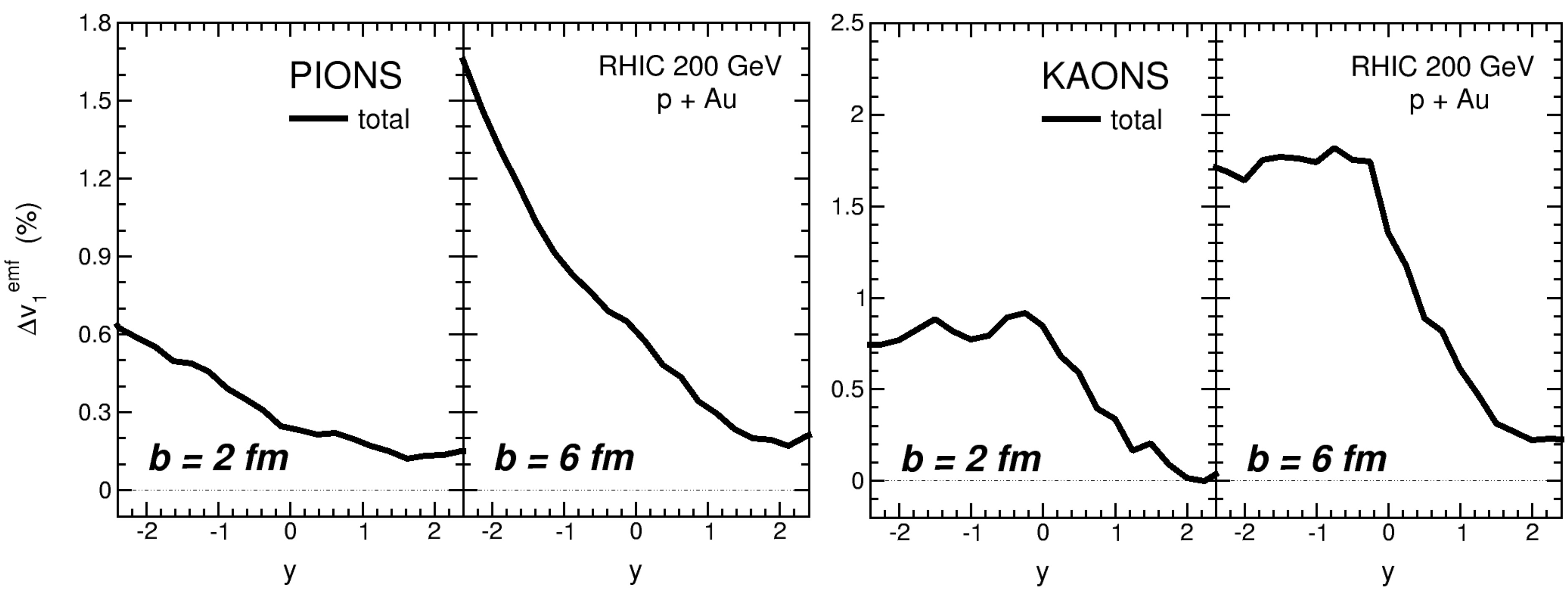
p+Au: directed flow

ELECTROMAGNETICALLY-INDUCED SPLITTING in the directed flow of hadrons with same mass and opposite charge

$$\Delta v_1{}^{emf} \equiv \Delta v_1{}^{(PHSD+EMF)} - \Delta v_1{}^{(PHSD)}$$

$$\Delta v_1 \equiv v_1^+ - v_1^-$$

LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770



- **magnitude increasing with impact parameter**
- **larger splitting for kaons than for pions**

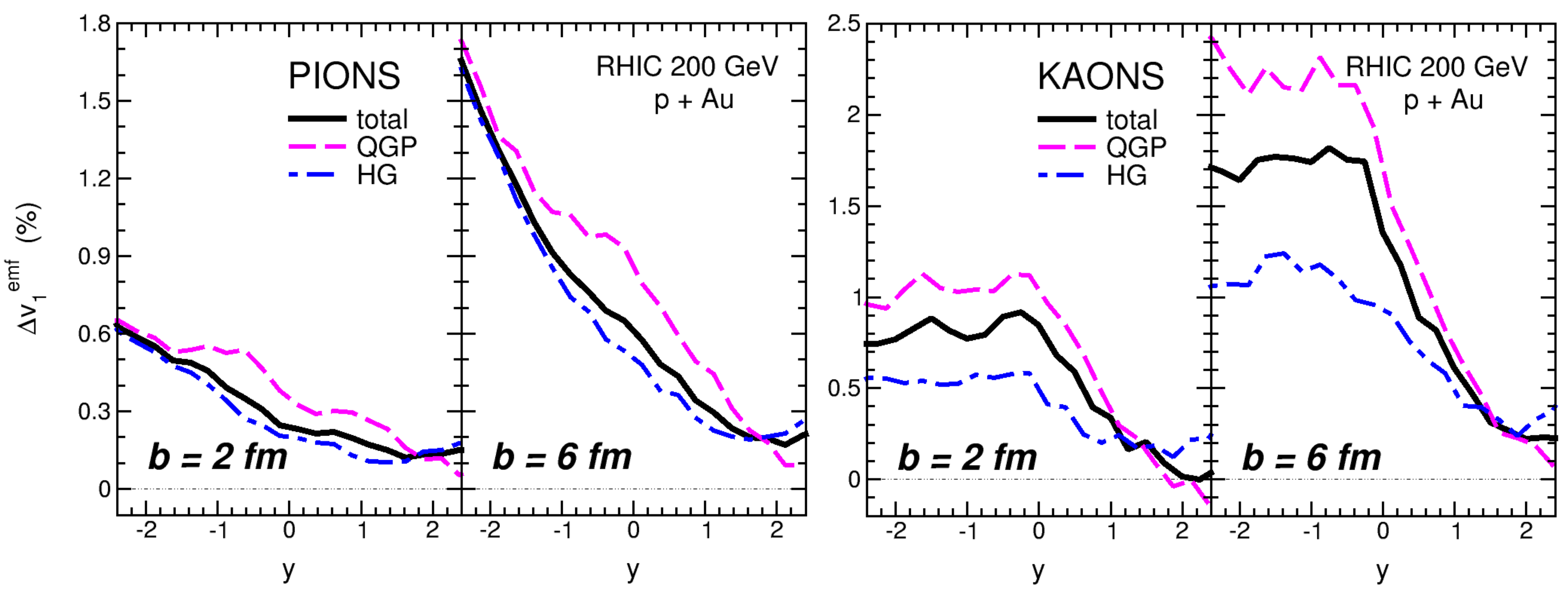
p+Au: directed flow

ELECTROMAGNETICALLY-INDUCED SPLITTING in the directed flow of hadrons with same mass and opposite charge

$$\Delta v_1{}^{emf} \equiv \Delta v_1{}^{(PHSD+EMF)} - \Delta v_1{}^{(PHSD)}$$

$$\Delta v_1 \equiv v_1^+ - v_1^-$$

LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770



- **splitting generated at partonic level higher than that induced in the hadronic phase, especially for kaons**

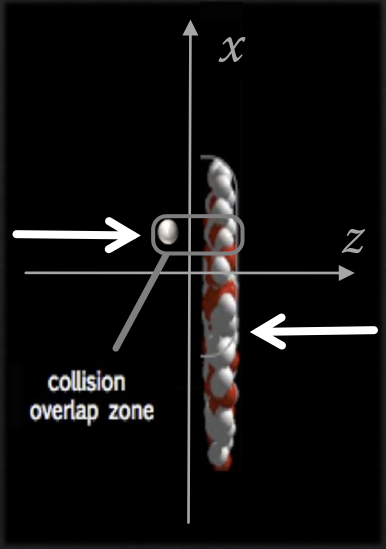
CONCLUDING….

The Parton-Hadron-String-Dynamics (PHSD) approach describes the entire dynamical evolution of large and small colliding systems within one single theoretical framework

PHSD includes in a consistent way the intense electromagnetic fields produced in the very early stage of the collision

Study of p+Au collisions at top RHIC energy:

- ✓ the electric field is strongly asymmetric inside the overlap region
- ✓ asymmetry of charged-particle rapidity distributions increasing with centrality
- ✓ collectivity as signal of quark-gluon plasma formation
- ✓ effect of electromagnetic fields in directed flow of mesons: splitting between positively and negatively charged particle
- ✓ Electromagnetically-induced splitting generated in the deconfined phase larger than that produced in the hadronic phase



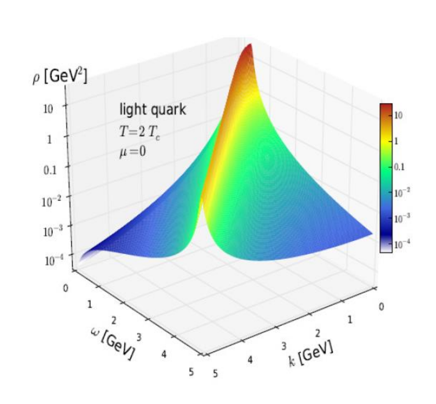
Thank you

for your attention!

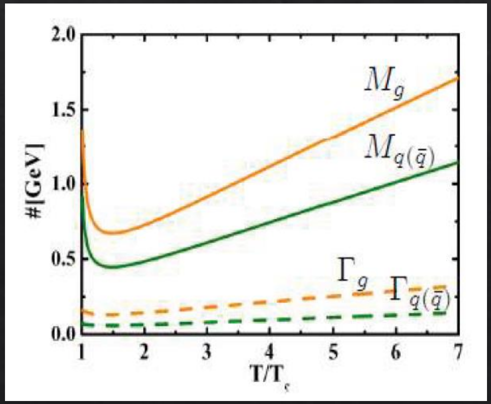
DQPM: Dynamical QuasiParticle Model

The QGP phase is described in terms of interacting quasiparticle: massive quarks and gluons ($g, q, \bar{q}$) with Lorentzian spectral functions

$$\rho_j(\omega, \mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2}\right) \equiv \frac{4\omega\gamma_j}{\left(\omega^2 - \mathbf{p}^2 - M_j^2\right)^2 + 4\gamma_j^2\omega^2}$$



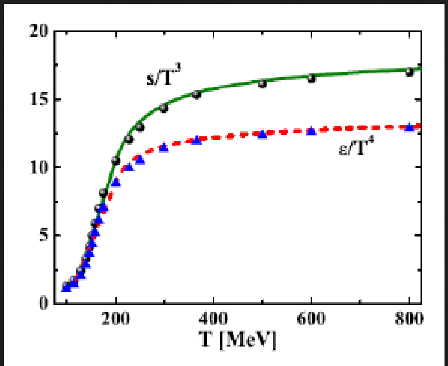
	QUARKS	GLUONS
MASSES	$m^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$	$M^2(T) = \frac{g^2}{6}\left((N_c + \frac{1}{2}N_f)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$
WIDTHS	$\gamma_q(T) = \frac{N_c^2 - 1}{2N_c}\frac{g^2 T}{4\pi} \ln\frac{c}{g^2}$	$\gamma_g(T) = N_c \frac{g^2 T}{4\pi} \ln\frac{c}{g^2}$



$$g^2(T/T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln(\lambda^2(T/T_c - T_s/T_c)^2}$$

RUNNING COUPLING

parameters from fit of lattice QCD thermodynamics



Peshier, PRD 70 (2004) 034016
Peshier and Cassing, PRL 94 (2005) 172301
Cassing, NPA 791 (2007) 365; NPA 793 (2007)

retarded electromagnetic fields

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}$$

General solution of the wave equation for the electromagnetic potentials

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{4\pi}\int \frac{\mathbf{j}(\mathbf{r}',t')\ \delta(t-t'-|\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}\ d^3r'dt'$$

$$\Phi(\mathbf{r},t) = \frac{1}{4\pi}\int \frac{\rho(\mathbf{r}',t')\ \delta(t-t'-|\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}\ d^3r'dt'$$

$$\mathbf{r}' \equiv \mathbf{r}(t')$$

$$t' = t - \frac{\mathbf{r}-\mathbf{r}'}{c}$$

Liénard-Wiechert potentials for a moving point-like charge

$$\Phi(\mathbf{r},t) = \frac{e}{4\pi}\left[\frac{1}{R(1-\mathbf{n}\cdot\boldsymbol{\beta})}\right]_{\mathrm{ret}} \qquad \mathbf{A}(\mathbf{r},t) = \frac{e}{4\pi}\left[\frac{\boldsymbol{\beta}}{R(1-\mathbf{n}\cdot\boldsymbol{\beta})}\right]_{\mathrm{ret}}$$

ret: evaluated at the times t'

$$\mathbf{R} = \mathbf{r} - \mathbf{r}'$$

$$\mathbf{n} = \frac{\mathbf{R}}{R}$$

$$\boldsymbol{\beta} = \frac{\mathbf{v}}{c}$$

Voronyuk *et al.*, PRC 83 (2011) 054911

retarded electromagnetic fields

Retarded electric and magnetic fields for a moving point-like charge

$$\mathbf{E}(\mathbf{r},t)=\frac{e}{4\pi}\left[\frac{\mathbf{n}-\boldsymbol{\beta}}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3\gamma^2R^2}+\frac{\mathbf{n}\times\left((\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right)}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3cR}\right]_{\mathrm{ret}}\qquad \mathbf{B}(\mathbf{r},t)=[\mathbf{n}\times\mathbf{E}(\mathbf{r},t)]_{\mathrm{ret}}$$

elastic Coulomb scatterings

inelastic bremsstrahlung processes

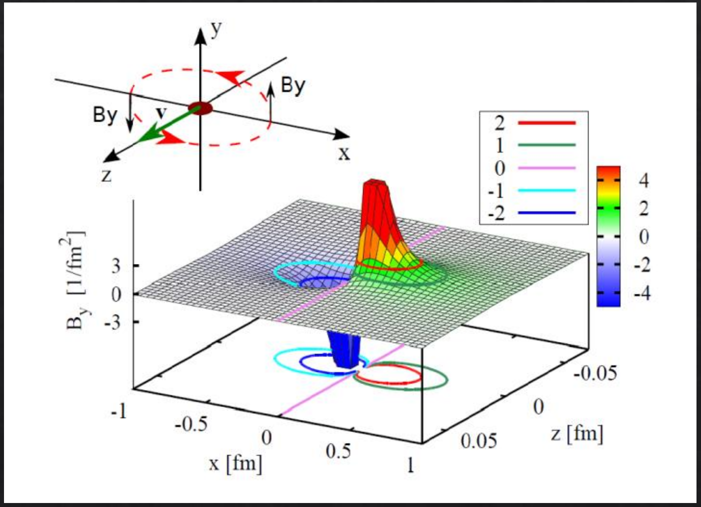
$$\mathbf{R}=\mathbf{r}-\mathbf{r}'\qquad \mathbf{n}=\frac{\mathbf{R}}{R}\qquad \boldsymbol{\beta}=\frac{\mathbf{v}}{c}$$

Neglecting the acceleration

$$e\mathbf{E}(t,\mathbf{r})=\alpha_{em}\frac{1-\beta^2}{\left[(\mathbf{R}\cdot\boldsymbol{\beta})^2+R^2\left(1-\beta^2\right)\right]^{3/2}}\mathbf{R}$$

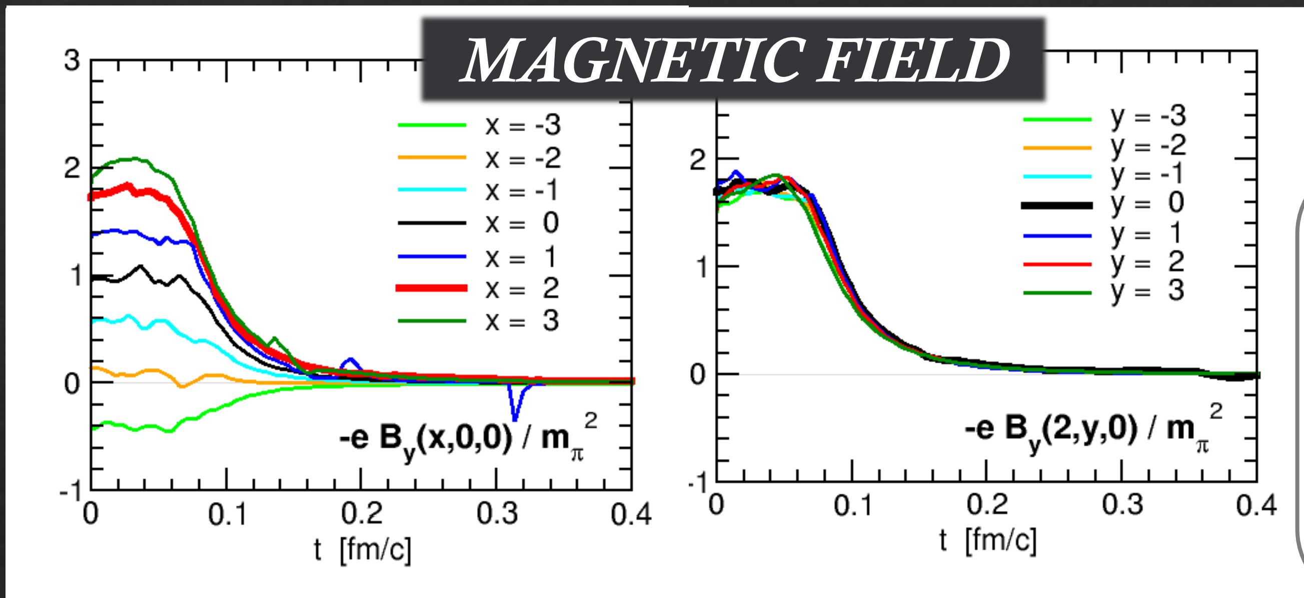
$$e\mathbf{B}(t,\mathbf{r})=\alpha_{em}\frac{1-\beta^2}{\left[(\mathbf{R}\cdot\boldsymbol{\beta})^2+R^2\left(1-\beta^2\right)\right]^{3/2}}\boldsymbol{\beta}\times\mathbf{R}$$

magnetic field created by a single freely moving charge

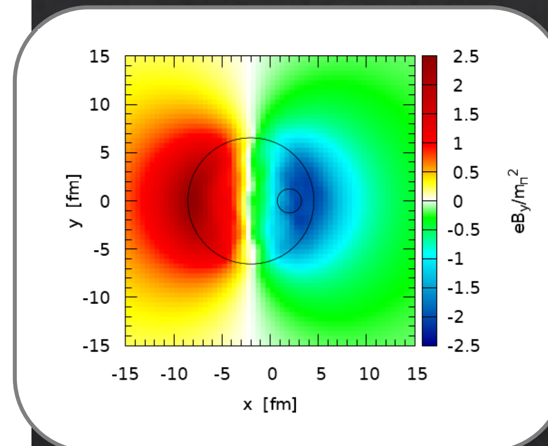


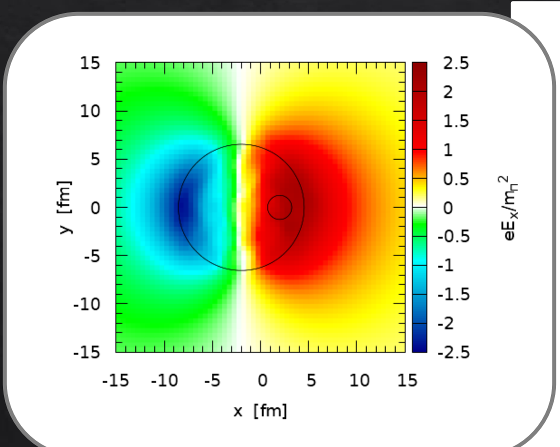
Voronyuk *et al.*, PRC 83 (2011) 054911

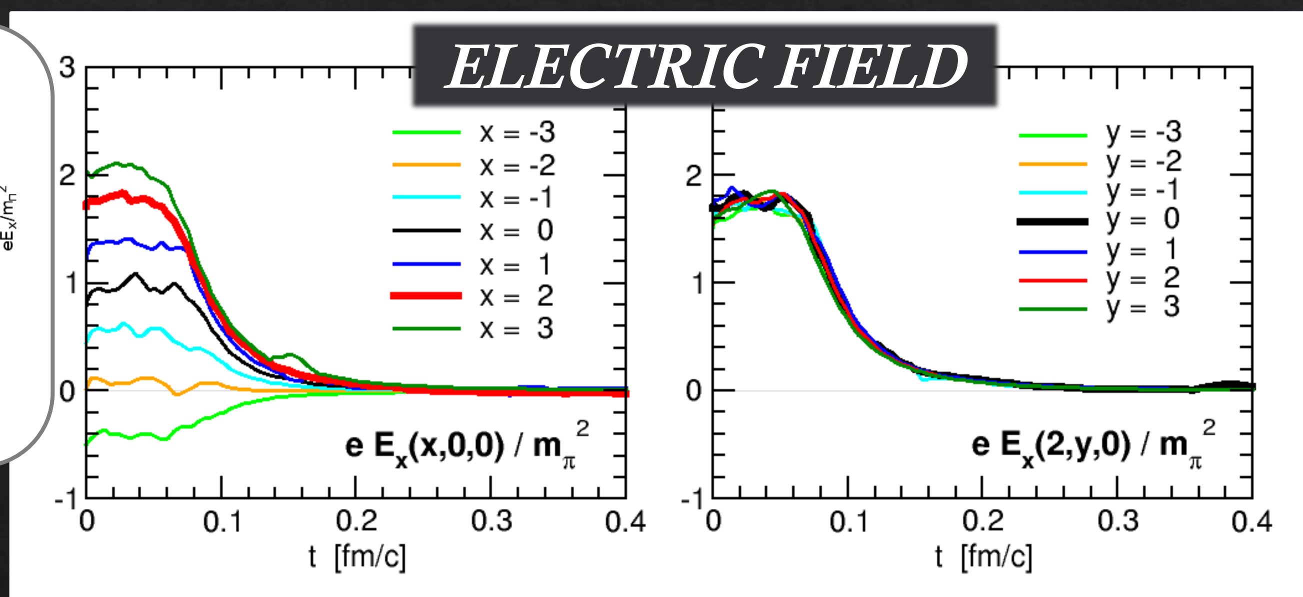
p+Au collisions @RHIC 200GeV b=4 fm



B_y



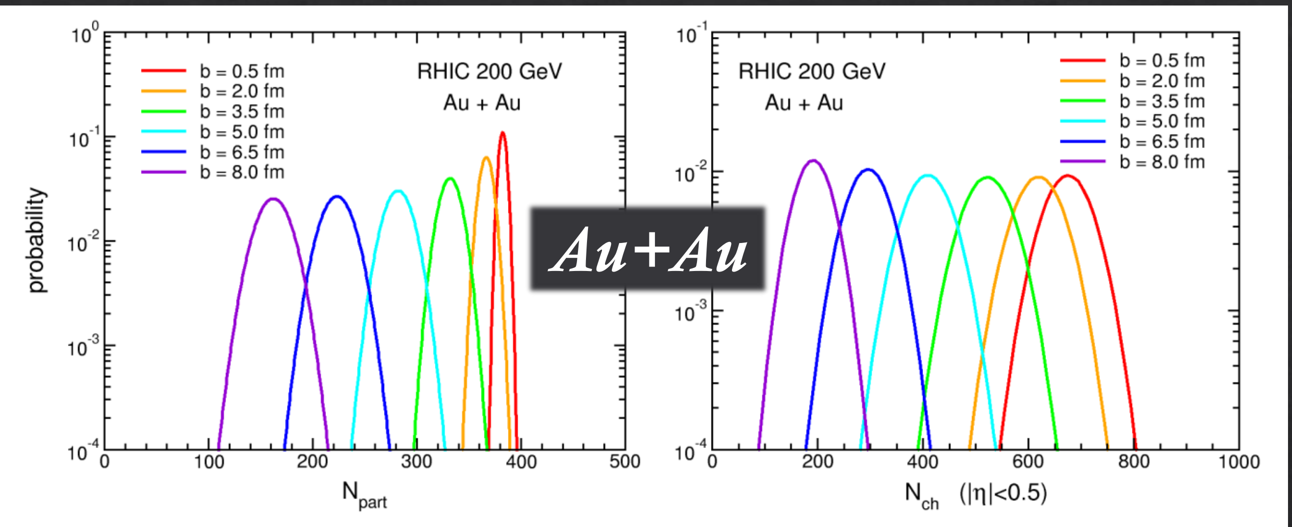




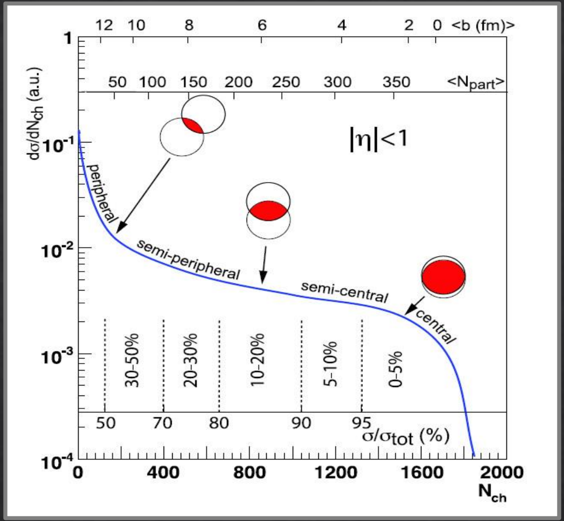
E_x

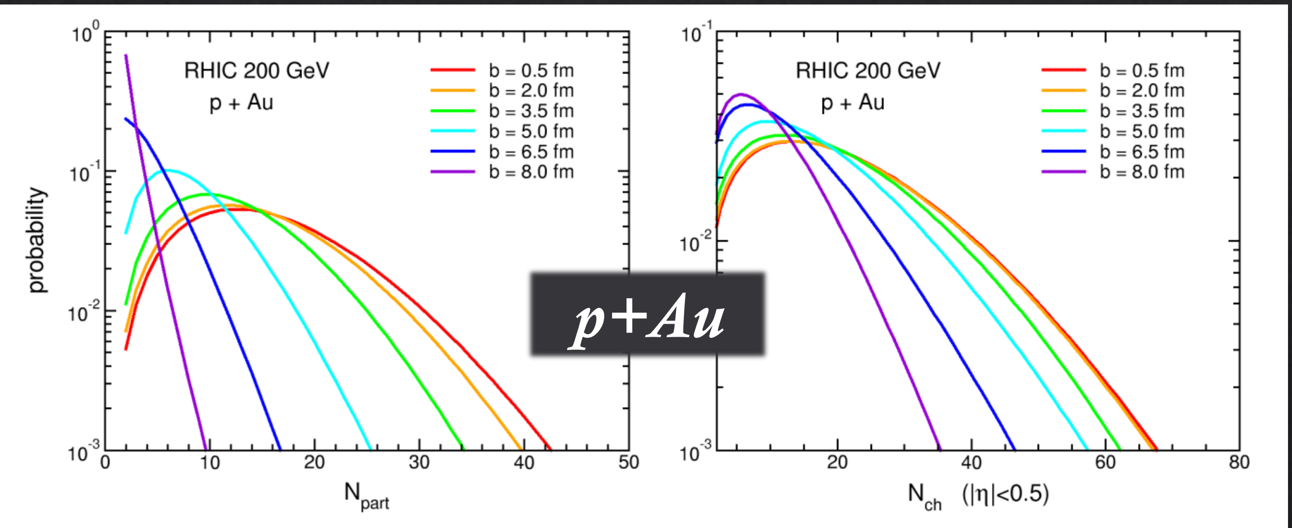
Centrality in small systems

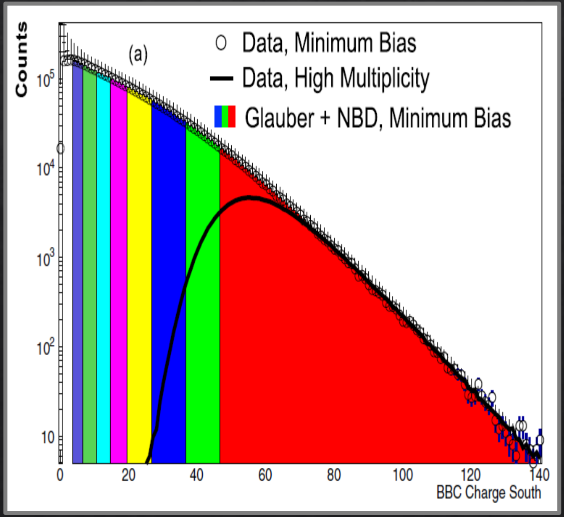
LO, Moreau, Voronyuk and Bratkovskaya, 1909.06770



Miller *et al.*, ARNPS 57 (2007) 205





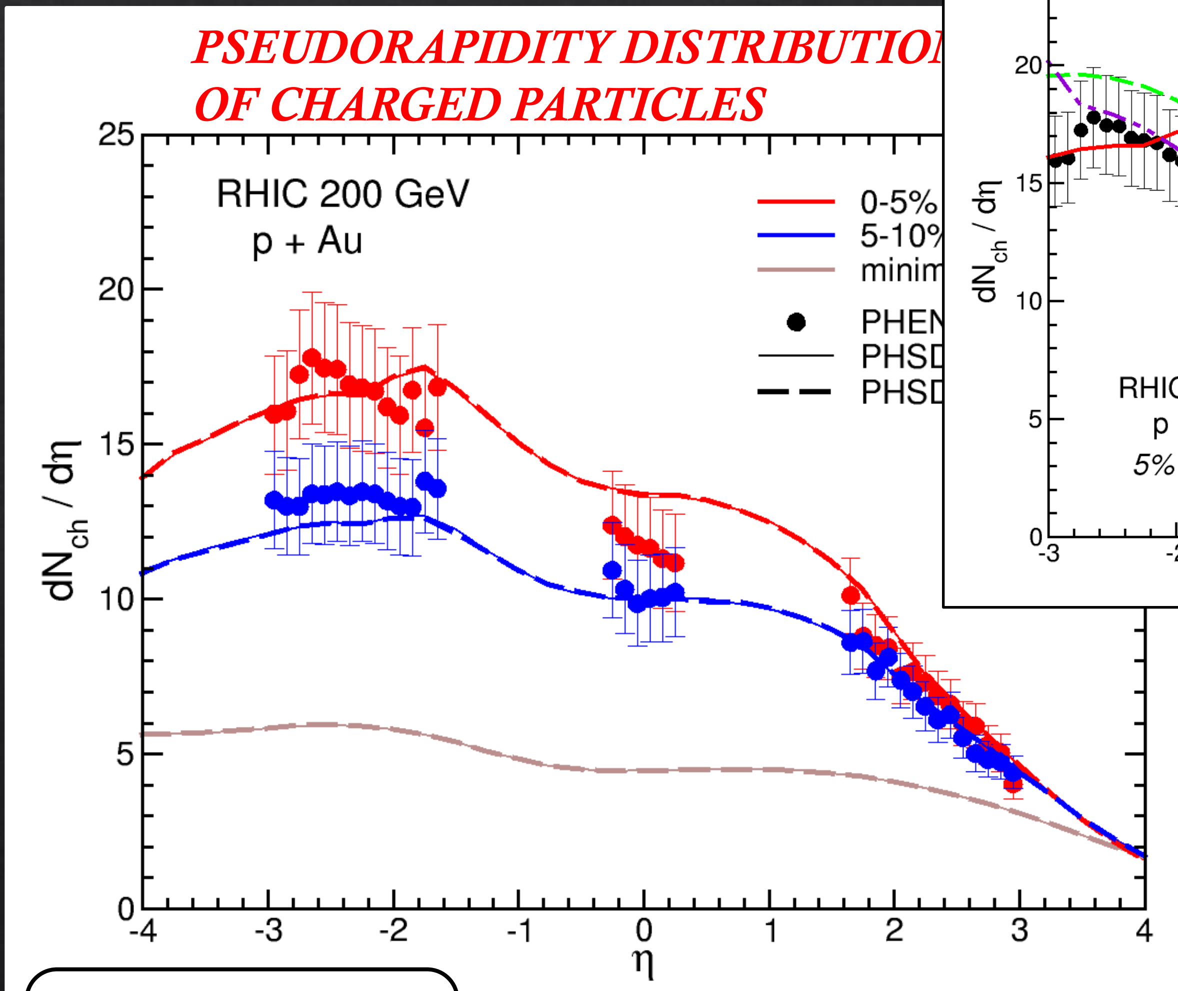


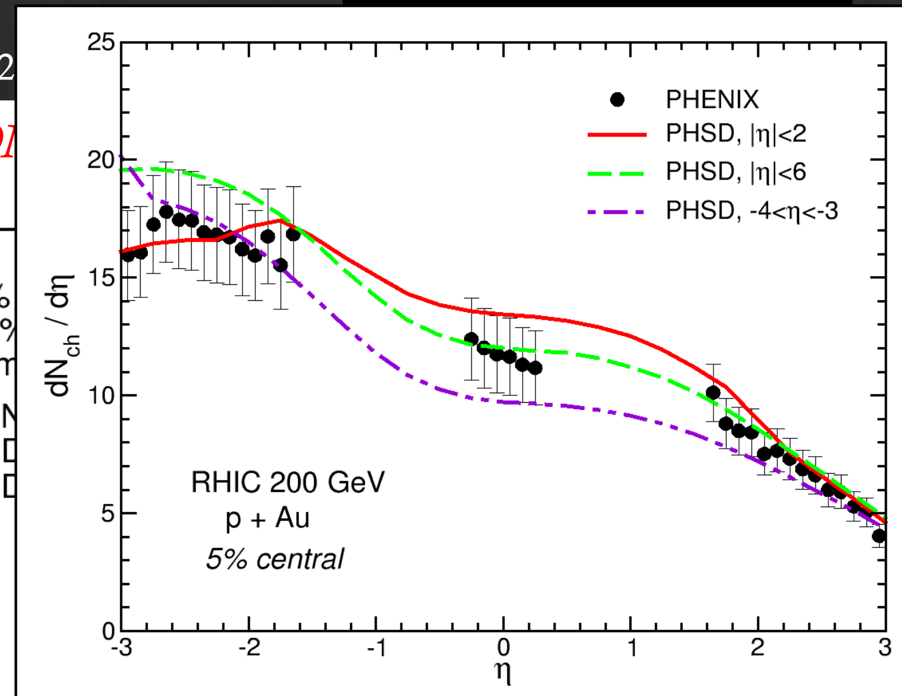
PHENIX, PRC 95 (2017) 034910

Multiplicity fluctuation in the final state mixes events from different impact parameters!

p+Au collisions @ RHIC 200 GeV

Exp. Data: PHENIX Collaboration, PRL 121 (2018) 222





- enhanced particle production in the Au-going directions
- asymmetry increases with centrality of the collision

$$\eta = -\ln\left(\tan\frac{\theta}{2}\right)$$

Anisotropic radial flow

A DEEPER INSIGHT…FINITE EVENT MULTIPLICITY

azimuthal particle distributions
w.r.t. the reaction plane

$$\frac{dN}{d\varphi} \propto 1 + \sum_n 2 v_n(p_T) \cos[n(\varphi - \Psi_n)]$$

n-th order
flow harmonics

$$v_n = \frac{\langle \cos[n(\varphi - \Psi_n)] \rangle}{Res(\Psi_n)}$$

event-plane angle resolution
(three-subevent method)

n-th order
event-plane angle

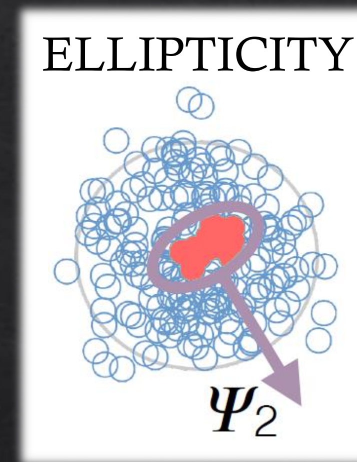
$$\Psi_n = \frac{1}{n} \operatorname{atan2}\left(Q_n^y, Q_n^x\right)$$

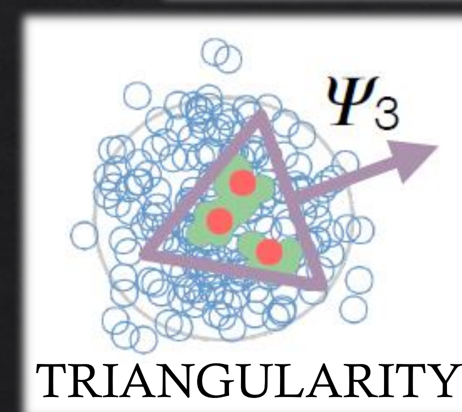
$$Q_n^x = \sum_i \cos[n\varphi_i]$$

$$Q_n^y = \sum_i \sin[n\varphi_i]$$

Since the finite number of particles produces limited resolution in the determination of Ψ_n, the v_n must be corrected up to what they would be relative to the real reaction plane

Poskanzer and Voloshin,
PRC 58 (1998) 1671

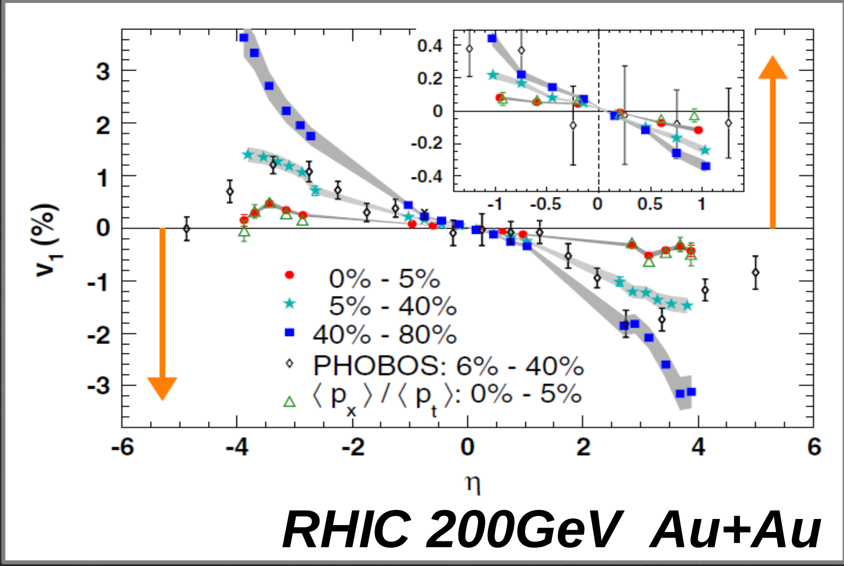




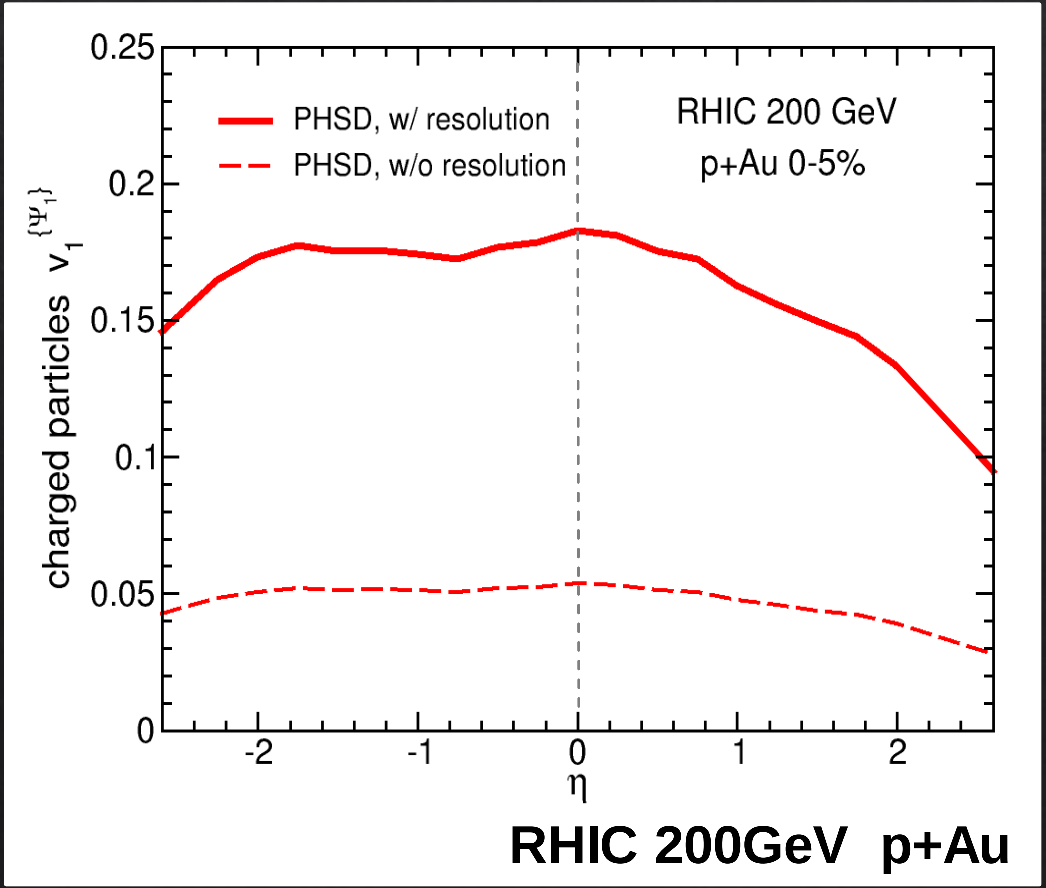
Important especially for small colliding system, e.g. p+A

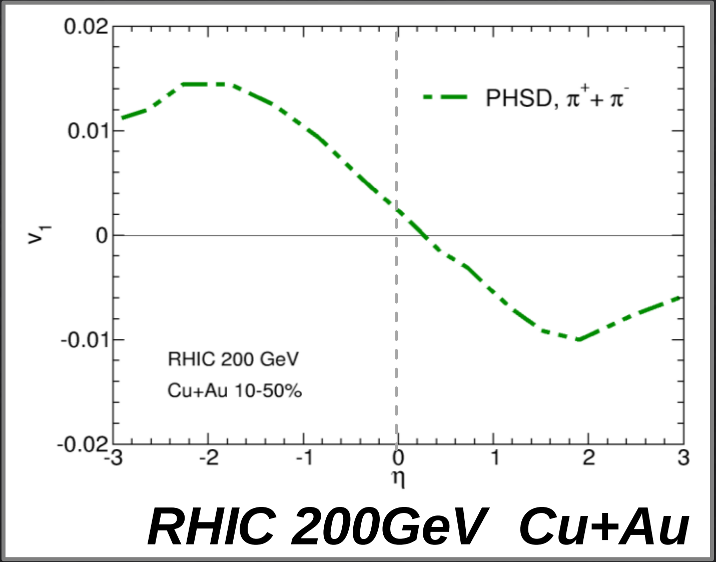
p+Au collisions @ RHIC 200 GeV

STAR Collaboration, PRL 101 (2008) 252301



pseudorapidity dependence of the
DIRECTED FLOW OF CHARGED PARTICLES





Voronyuk *et al.*, PRC 90 (2014) 064903
Toneev *et al.*, PRC 95 (2017) 034911