Z_3 meta-stable states in PNJL model.

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Outline

- Meta-stable states in the PNJL model.
- Bounce solution for the decay of meta-stable states.
- Meta-stable states in heavy-ion collisions(?).

Z(N) symmetry in SU(N) gauge theories

Euclidean $SU(N)$ action for the gauge fields

$$S = \int_V d^3x \int_0^\beta d\tau \left\{ \frac{1}{2} Tr\left(F^{\mu\nu} F_{\mu\nu}\right) \right\}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu], \qquad A_\mu = A_\mu^a T^a.$$

In Euclidean theory,

$$A_\mu^a(\vec{x}, 0) = A_\mu^a(\vec{x}, \beta)$$

The transformation of the gauge fields under $SU(N)$ is

$$A_\mu \longrightarrow U A_\mu U^{-1} + \frac{1}{g}\left(\partial_\mu U\right) U^{-1}.$$

The invariance of the pure gauge action and the periodicity of the gauge fields can be satisfied by,

$$U(\vec{x}, \tau = 0) = z U(\vec{x}, \tau = \beta).$$

$z \in Z_N$ and Z_N is the center of the gauge group $SU(N)$

Polyakov Loop

- The Polyakov loop is defined as

$$L(\vec{x}) = \frac{1}{3}\mathrm{Tr}\left(\mathcal{P}\exp\left[ig\int_0^\beta d\tau A_0(\vec{x},\tau)\right]\right)$$

- Under a Z_3 gauge transformation, the Polyakov loop, transforms as $L(\vec{x}) \to zL(\vec{x})$. The thermal as well as volume average of the Polyakov loop $L(\vec{x})$,

$$L(T) = \left\langle \frac{1}{V}\int L(\vec{x})d^3x \right\rangle,$$

  is related to the free energy $F_{\bar{Q}Q}(r)$ of a static (infinitely heavy) quark-anti-quark pair at infinite separation.

$$|L(T)|^2 = \exp\left[-\beta F_{\bar{Q}Q}(r=\infty)\right].$$

- In pure $SU(N)$ gauge theories, this is the ideal candidate for an order parameter for confinement - deconfinement transition since it is zero in the confined phase and acquires non-zero value in the deconfined phase.
- Thus in deconfined phase the Z_N symmetry is spontaneously broken and we have N degenerate vacua.

At high temperatures, SU(N) gauge theories with fermions have Z(N) metastable states.

- "Metastability in SU (N) gauge theories at high temperatures"
  V. Dixit and M. C. Ogilvie, Phys. Lett. B 269, 353 (1991).
- "Cosmological QCD Z(3) Phase Transition in the 10 Tev Temperature Range"
  J. Ignatius, K. Kajantie and K. Rummukainen, Phys. Rev. Lett. 68, 737 (1992).
- "ZN domains in gauge theories with fermions at high temperatures"
  V. M. Belyaev, I. I. Kogan, G. W. Semenoff and N. Weiss, Phys. Lett. B 277, 331 (1992).
- "Meta-stable States in Quark-Gluon Plasma"
  M. Deka, S. Digal and A. P. Mishra, Phys. Rev. D 85, 114505 (2012)
  2 flavor case, metastable states appear above **750 MeV**
- "Z(3) metastable states in Polyakov Quark Meson model"
  H. Mishra and R. K. Mohapatra, Phys. Rev. D 95, 094014 (2017) Metastable states appear above **310 MeV**

Polyakov Loop potential.

The effective potential for the Polyakov loop (Dumitru, Pisarki):

$$U(\bar{\Phi}, \Phi, T) = T^4 \left[-\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6}(\bar{\Phi}^3 + \Phi^3) + \frac{1}{4}(\bar{\Phi}\Phi)^2 \right] b_4$$

$$\mathrm{b}_2(T) = (1 - 1.11(T_0/T))\,(1 + 0.265(T_0/T))^2\,(1 + 0.3(T_0/T))^3 - 0.487$$

$$\Phi = (\Phi_1, \Phi_2)$$

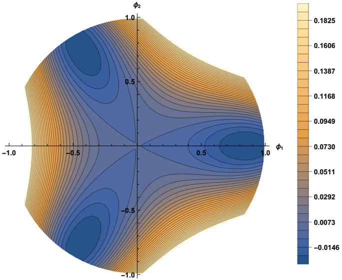
The parameters rescaled such that the expectation value of order parameter goes to unity as T goes to infinity

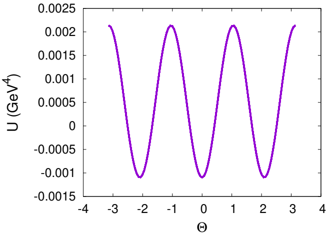
$b_3 = 2.0$ and $b_4 = 0.6016$
$T_0 = 270$ MeV.

$$U(\bar{\Phi}, \Phi, T) = T^4 \left[-\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\bar{\Phi}^3 + \Phi^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2 \right]$$

$$b_2(T) = a_0 + a_1 \left(\frac{T}{T_0}\right) + a_2 \left(\frac{T}{T_0}\right)^2 + a_3 \left(\frac{T}{T_0}\right)^3$$

Polyakov Loop potential





Thermodynamic potential at $1.3\,T_0$

Polyakov Loop Nambu-Jona-Lasinio (PNJL) model.

- NJL model - effective theory of nucleons and mesons constructed from interacting Dirac fermions.
- NJL model incorporates chiral symmetry breaking but not confinement
- It has been extended to include the Polyakov loop to study the deconfinement phase transition.

$$\mathcal{L}_{PNJL} = \sum_f \bar{\psi}_f (i\gamma^\mu D_\mu - m_f)\psi_f + G_s[(\bar{\psi}_f \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \tau \psi_f)^2] - U(\bar{\Phi}, \Phi, T)$$

$D_\nu = \partial_\nu - i(gA_\nu + \delta_0^\nu \mu_f)$, $A_\nu = A_\nu^a \tau^a 2$.

- Using MFA, $(\bar{\psi}\psi)^2 = (\langle\bar{\psi}\psi\rangle + \delta(\bar{\psi}\psi))^2$, thermodynamic potential $\Omega = -\frac{T}{V}\ln Z$ is written.

Thermodynamic potential

$$\Omega = U(\bar{\Phi}, \Phi, T) + 6 \sum_{f=u,d} \int \frac{d^3p}{(2\pi)^3} E_f \, \Theta(\Lambda - |\vec{p}|) + 2G_s\sigma^2 - \sum_{f=u,d} \int_0^\infty \frac{d^3p}{(2\pi)^3} 2T(\ln[x1] + \ln[x2])$$

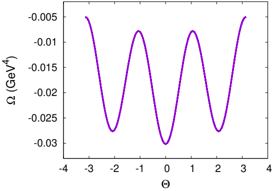
$$x1 = 1 + 3\Phi e^{-\beta(E_f - \mu_f)} + 3\bar{\Phi} e^{-2\beta(E_f - \mu_f)} + e^{-3\beta(E_f - \mu_f)}$$
$$x2 = 1 + 3\bar{\Phi} e^{-\beta(E_f + \mu_f)} + 3\Phi e^{-2\beta(E_f + \mu_f)} + e^{-3\beta(E_f + \mu_f)}$$

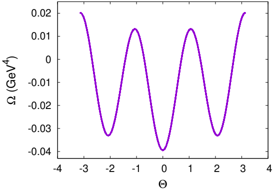
$E_f = \sqrt{p^2 + \Sigma_f^2}$; $\Sigma = m_0 - 2G_s\sigma$; $\sigma = \langle \bar{\Psi}\Psi \rangle$; $\mu_u = \mu_d = 0$; $G_s = 10.08 \text{GeV}^{-2}$;
$m_0 = 5$ MeV; $T_0 = 190$ MeV

Minimise:

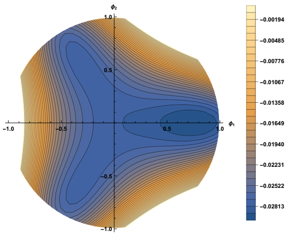
$$\frac{\partial \Omega}{\partial \Phi_1} = 0, \quad \frac{\partial \Omega}{\partial \Phi_2} = 0, \quad \frac{\partial \Omega}{\partial \sigma} = 0.$$

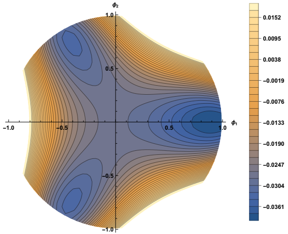
We see meta-stable states at and above the temperature $T_m \sim 193$ MeV.

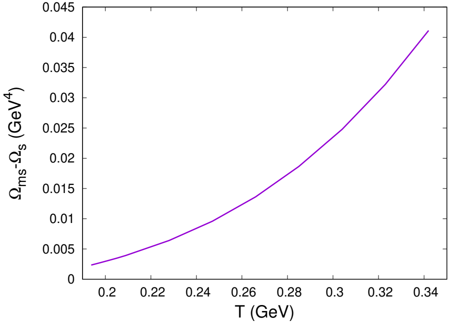




Thermodynamic potential at temperature 199.5 MeV 247 MeV.







0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.2
0.22
0.24
0.26
0.28
0.3
0.32
0.34
T (GeV)
$\Omega_{ms}-\Omega_s$ (GeV^4)

Bounce solution for the decay of meta-stable states.

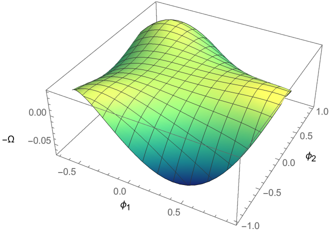
- If a system is in the meta-stable state, it can decay to the stable state by nucleation of bubbles.
- In this case, the bubble nucleation picture is not related to any phase transition but to the fact that the theory has meta-stable states above a certain temperature and they can tunnel into the stable state.
- The decay rate of the false vacuum (meta-stable state) can be calculated in the semi classical approximation where the dominant contribution comes from the configurations with the least action.

The bounce:

$$\frac{d^2\Phi_1}{dr^2} + \frac{2}{r}\frac{d\Phi_1}{dr} = \frac{d\Omega}{d\Phi_1}$$

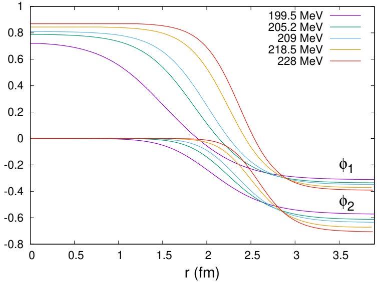
$$\frac{d^2\Phi_2}{dr^2} + \frac{2}{r}\frac{d\Phi_2}{dr} = \frac{d\Omega}{d\Phi_2}$$

$\Phi_i \to \Phi_i^m$ as $r \to \infty$ where $i = 1, 2$ and Φ_i^s and Φ_i^m, the stable and metastable values of the field.

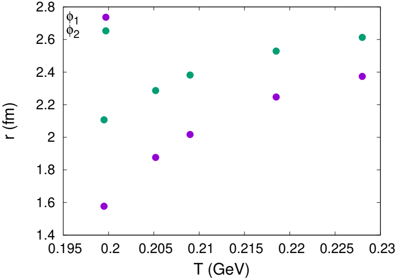


Inverted potential in the Polyakov loop plane at 247 MeV.

Bubble profiles



Bubble radii vs temperature



Bubble action

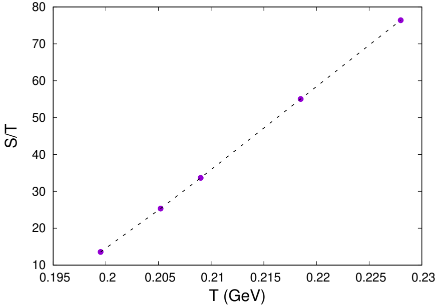
$$S = \int 4\pi r^2 dr \left[\frac{1}{2} \alpha T^2 \left\{ \left(\frac{d\Phi_1}{dr} \right)^2 + \left(\frac{d\Phi_2}{dr} \right)^2 \right\} + \frac{1}{2} G_s^2 \left(\frac{d\sigma}{dr} \right)^2 + \Omega(\Phi_1, \Phi_2, \sigma) \right]$$

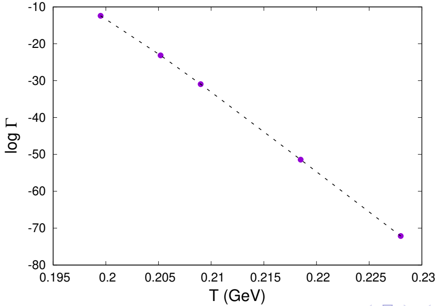
Bubble nucleation probability

$$\Gamma = T^4 \left(\frac{S}{2\pi T} \right)^{3/2} \exp(-S/T)$$

The number of bubbles nucleated in volume V during the time t when the temperature drops to T is given by

$$N(t) = V \int_{t_i}^{t} \Gamma(t) dt$$

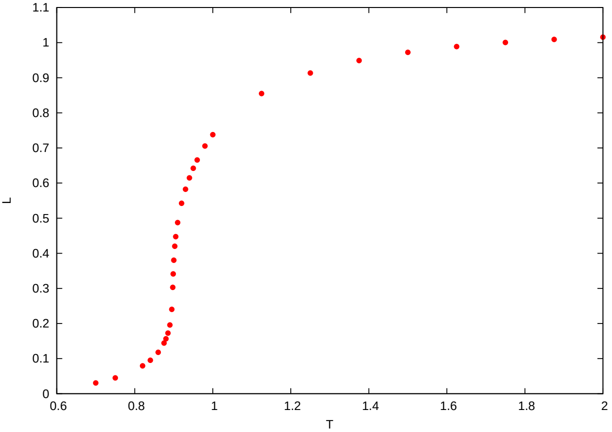

S/T
T (GeV)
80
70
60
50
40
30
20
10
0.195
0.2
0.205
0.21
0.215
0.22
0.225
0.23


log Γ
T (GeV)
-10
-20
-30
-40
-50
-60
-70
-80
0.195
0.2
0.205
0.21
0.215
0.22
0.225
0.23

Summary

- The Z_3 meta-stable states exist at and above the temperature $T_m \sim 193$ MeV in PNJL model.
- The probability of these states decaying by tunnelling into stable states is zero in the case of heavy-ion collisions.
- So these states will decay only when the system cools down below T_m and metastable states become unstable. It is like a spinodal decomposition scenario where the field rolls down to the stable state.
- This can lead to large angular oscillations of the Polyakov loop field, which can have interesting consequences in the dynamics of flow and coherent emission of particles.

Thank you



1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
L
0.6
0.8
1
1.2
1.4
1.6
1.8
2
T