The II Int. workshop on Theory of Hadronic Matter under Extreme Conditions, Dubna, Sept. 16-19, 2019

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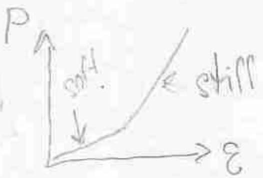
THE ONSET OF CRITICAL PHENOMENA NEAR THE PHASE TRANSITION AT FINITE DENSITY

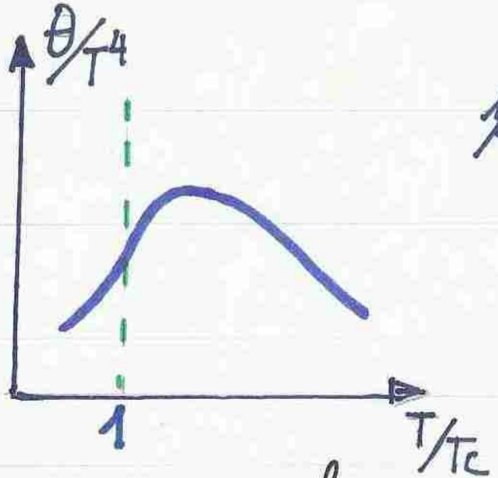
1. PRE-CRITICAL FLUCTUATIONS AT FINITE DENSITY
2. CURRENT-CURRENT AND PRESSURE-PRESSURE CORRELATION FUNCTIONS
3. FLUCTUATING QUARK PAIR FIELD
4. ELECTRICAL CONDUCTIVITY AND SOFT PHOTON EMISSION
5. CRITICAL BEHAVIOR OF BULK VISCOSITY AND SOUND ATTENUATION
6. RAYLEIGH CAVITATION IN MAGNETIC FIELD

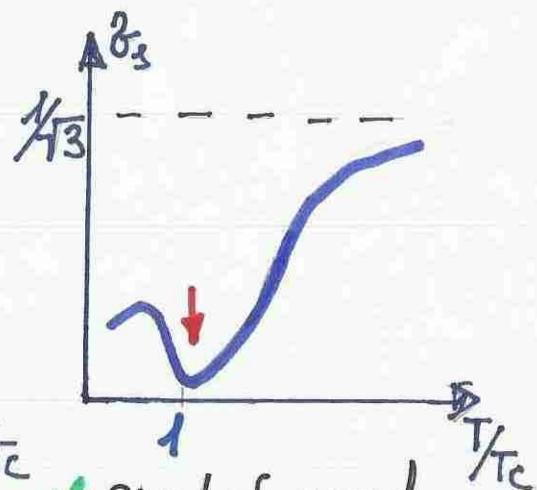
Anomalies near the QCD Phase Transition

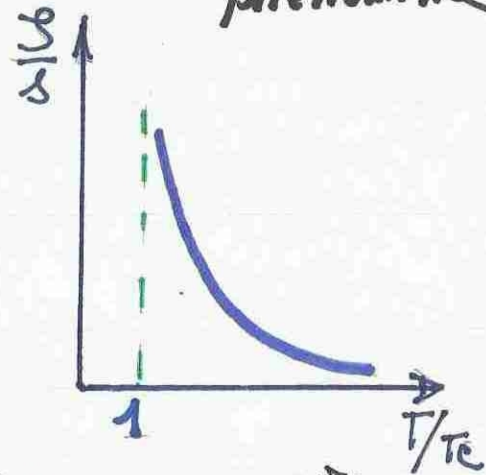


$C_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{\partial p/\partial T}{\partial \varepsilon/\partial T} = \frac{s}{c_V}$

near T_c quark matter is expected to exhibit several spectacular phenomena







(a) Trace anomaly

$$\theta/T^4 = \frac{\varepsilon - 3P}{T^4}$$

(b) Speed of sound
$T = T_c \to$ the softest EoS point

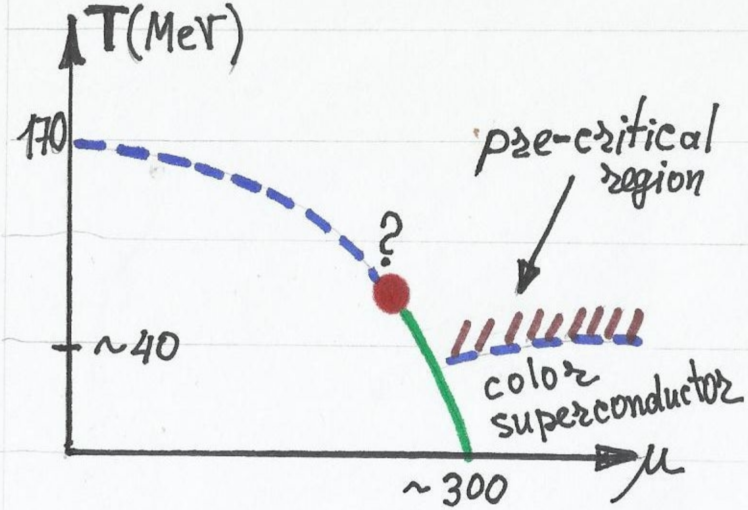
(c) $P = P_{equil} - \zeta \vec{\nabla}\vec{v}$
Bulk viscosity ζ divergence

(d) Sound attenuation anomaly (e) Electrical conductivity growth

(f) Excess of soft photon emission

Physics depends on the (T, μ) values – QCD phase diagram

("Theorist's science fiction" – Owe Philipsen)



Focus of this talk – finite density $\mu_q \sim 300$–400 MeV, T approaching T_c from above, $T \to T_c$ (~ 40 MeV)

We show that the following phenomena

- sound attenuation anomaly
- bulk viscosity divergence
- electrical conductivity growth
- enhanced soft photon emission

have the same dynamical origin – collective soft mode of the diquark field

- **Pre-critical fluctuations – inherent in the II-nd or weak I-st order phase transitions**

Wide fluctuation region $Gi = \frac{\delta T}{T_c} \sim 10^8 \left(\frac{\delta T}{T_c}\right)_{BCS}$

What are we calculating and how?

$\sigma_{el}, \omega \frac{dR_\gamma}{d^3p}$ $\qquad\qquad$ γ, ζ

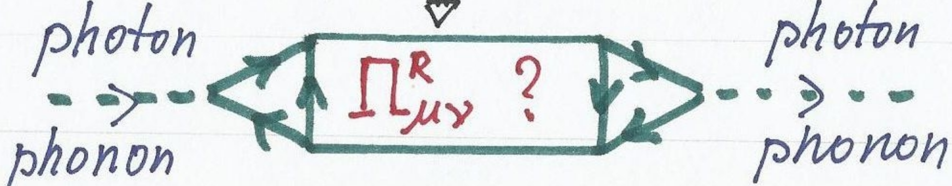
γ – sound absorption

ζ – bulk viscosity

$\underbrace{\sigma_{el}, \omega \frac{dR_\gamma}{d^3p}}$ $\longrightarrow$ $\langle j^{em}_\mu(t,\vec{x}), j^{em}_\nu(0,\vec{0})\rangle$

$\underbrace{\gamma, \zeta}$ $\longrightarrow$ $\langle T^i_i(t,\vec{x}), T^i_i(0,\vec{0})\rangle$

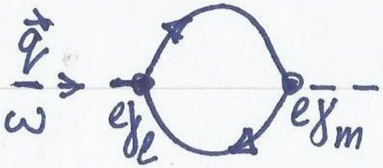
$\Downarrow$



$\Pi^R_{\mu\nu}$ – retarded responce operator, Wightman correlator, polarization tensor. Not calculable beyond perturbation theory

$\Pi^R_{\mu\nu}$ – one loop contribution, Matsubara formalism

- Photon – Drude-Lorentz conductivity



$$\Pi_{lm}(\vec q,\omega) = T\sum_{\tilde\varepsilon_n}\int\frac{d\vec p}{(2\pi)^3}\,\mathrm{tr}\left[G(\vec p,\tilde\varepsilon_n)\gamma_l \times G(\vec p+\vec q,\tilde\varepsilon_n+\omega_k)\gamma_m\right] =$$

$$= \frac{1}{3}\vec v^2\tau\nu\omega_k\int\frac{d\Omega}{4\pi}\frac{1}{1+|\omega_k|\tau+i\vec q\vec v\tau}$$ → Hard density loop, $q \ll p$, $T \ll \mu$

$\tilde\varepsilon_n = \pi T(2n+1)+\frac{1}{2\tau}\,\mathrm{sgn}\,\varepsilon_n$, τ – momentum relaxation time, $\tau \simeq 0.3$ fm

$\nu = \frac{\mu p_F}{\pi^2}$ replaces g^2T^2 in HTL τ regulates collinear singularities

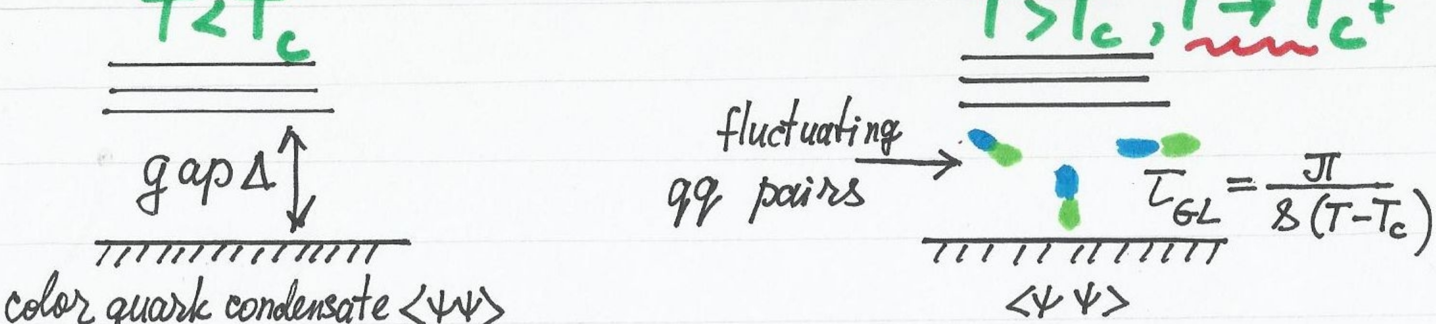
vs. HTL $\Pi_{\mu\nu}(\vec q,\omega) = m_D^2\,\omega\int\frac{d\Omega}{4\pi}\frac{v_\mu v_\nu}{\omega-\vec v\vec q+i\eta}$

$m_D^2 \sim g^2T^2$

σ_{el} (one-loop) is temperature independent, single-particle effect, no dependence on temperature Fermi surface blurring

- Near T_c slow relaxation collective mode dominates

Fluctuating quark pair field

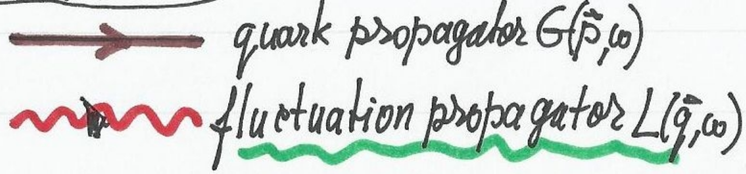


$$\tau_{GL}=\frac{\pi}{8(T-T_c)}$$

Wide fluctuation region, $Gi=\frac{\delta T}{T_c}\sim 10^{8-10}\left(\frac{\delta T}{T_c}\right)_{BCS}$

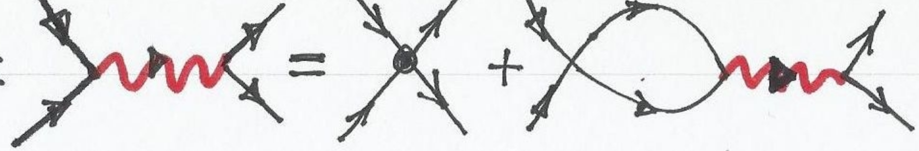
photon/phonon

The famous Aslamazov-Larkin diagram



Derivation of $L(\vec{q},\omega)$

- Dyson Equation

$L(\vec{q},\omega) =$ 

- Time dependent Ginzburg-Landau with Langevin force

$$F[\psi] = \left(t|\psi|^2 + \frac{\pi}{8T_c}\mathcal{D}|\vec{\nabla}\psi|^2\right) \to \nu\left(t|\psi|^2 + \frac{\pi}{8T_c}\mathcal{D}\vec{q}^{\,2}\right)$$

$t = \frac{T-T_c}{T_c}$, $\mathcal{D}$ - diffusion coefficient

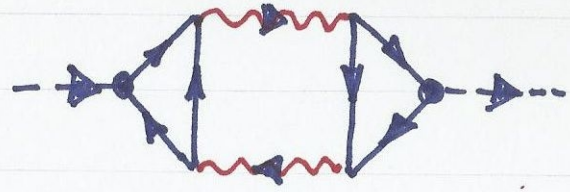
$$-\gamma\frac{\partial\psi(\vec{r},t)}{\partial t} = \frac{\delta F}{\delta\psi^*} + \xi(\vec{r},t),$$ ξ - Langevin forces, thermodynamic fluctuations

$$\hat{L} = -\left[\gamma\frac{\partial}{\partial t} + \nu\left(t + \frac{\pi}{8T_c}\mathcal{D}\vec{q}^{\,2}\right)\right]^{-1} \to \frac{-1}{\nu}\,\frac{1}{t + \frac{\pi}{8T_c}\left(-i\omega + \mathcal{D}\vec{q}^{\,2}\right)}$$

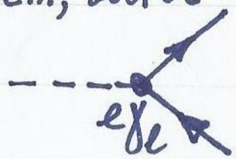
Ornstein-Zernike

At $T \to T_c$, ω, q - small, $\hat{L}$ is arbitrary large and rapidly varying $\Rightarrow$ AL diagram dominates

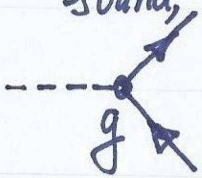
Aslamazov-Larkin Diagram (AL)

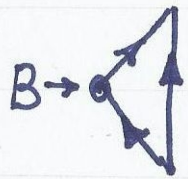
$\Pi(\vec{q},\omega) \sim$ 

em, vector



sound, scalar





$\sim \sum_{\varepsilon_n} \int \frac{d\vec{p}}{(2\pi)^3} GGG$

AL diagram is twice singular at T_c due to two $\hat{L}$

Electrical Conductivity and Soft Photon Emission

$$\sigma_{el} = -\frac{1}{\omega_k} \mathrm{Im}\,\Pi^R_{\mu\mu}, \quad \omega \frac{dR_\gamma}{d^3p} = -\frac{2}{(2\pi)^3} \frac{1}{e^{\omega/T}-1} \mathrm{Im}\,\Pi^R_{\mu\mu} \simeq$$

$$\simeq\Big|_{\omega \ll T} \simeq \frac{2}{(2\pi)^3} T \sigma_{el}$$

Very close to T_c the rise is suppressed by the interaction between fluctuations

Results for $Gi = t = \frac{T-T_c}{T_c} = 10^{-2}$

$$\sigma_{el}(AL) = 0.08\,fm^{-1} \propto \left(\frac{T-T_c}{T_c}\right)^{-1/2} \quad \omega \frac{dR_\gamma}{d^3p} = 1.3 \cdot 10^{-2}\,fm^{-4}\,GeV^{-2}$$

$$\sigma_{el}(\text{Karsch et al}) = 0.02 - 0.06\,fm^{-1} \quad \omega \frac{dR_\gamma}{d^3p}(\text{Karsch}) = (0.3 - 1.0)\,fm^{-4}\,GeV^{-2}$$

An excess of low invariant mass dielectrons has been observed by CERES, PHENIX, ALICE. Significant enhancement of soft photons is expected in future experiments at NICA.

Sound Absorption Anomaly and Bulk Viscosity Divergence

AL diagram with scalar vertices g (instead of $e\gamma_\ell$) and two fluctuation propagators $\hat{L}$ and energy-dependent density of states near the Fermi surface

$$A(x) = A_0 e^{-\gamma x}, \quad \gamma \text{ – attenuation constant}$$

γ from Dyson equation for phonon:

$$D^{-1}(\omega, k) = D_0^{-1}(\omega, k) - \Pi^R(\omega, k), \quad D_0(\omega, k) = \frac{\omega_0^2}{\omega^2 - \omega_0^2 + i0}$$

$$\operatorname{Im} \Pi^R = \omega^2 g^2 R\, t^{-3/2} \rightarrow \gamma = \omega^2 g^2 \frac{R}{2} t^{-3/2}$$

$$\gamma \sim \omega^2 \zeta \rightarrow \zeta \sim t^{-3/2}$$

Near T_c sound absorption is anomalously large and bulk viscosity diverges as $t^{-3/2}$ (vs. $t^{-1/2}$ for σ_{el} and $\omega\, dR_\gamma/d^3p$).

Bulk viscosity near T_c in other approaches

A difficult problem. Not yet completely solved.

Ising-like universality

Mode coupling theory

Renormalization group

$d=4-\varepsilon$ regularization

$$\gamma, \zeta \sim \xi^{z-\alpha/\nu}, \quad \xi = |t|^{-\nu} \simeq |t|^{-0.61}$$

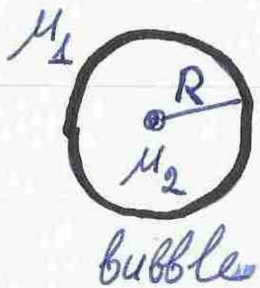
correlation length

$$\gamma, \zeta \sim |t|^{-1.69} \text{ vs. } t^{-3/2} \text{ here}$$

RAYLEIGH INSTABILITY

Near T_c $\zeta/s \sim 1 \gg \eta/s \to \zeta$ driven pressure P becomes $P < 0$
$\to$ fluid falls into droplets, "holes" are formed near the edges of the system $\to$ **cavitation, Rayleigh collapse**

(Torrieri; Tomasik and Mishustin; Rajagopal and Tripuraneni; Denicol, Gale and Jeon; Shuryak and Staig; ...)



Lord Rayleigh: cavitation damage of ship propellers

With only liquid inertia mattered $R\ddot{R} + \frac{3}{2}\dot{R}^2 = 0$

$\dot{R}(t) \propto (t_* - t)^{-3/5}$ – collapse, sound shoks from collapsing bubbles, sonoluminiscence

Effect of viscosity $R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{4\eta\dot{R}}{\rho R}$ (Shuryak and Staig) Brenner review 2002

However, near T_c, $\eta/s \ll 1$.

Magnetic field $R\ddot{R} + \frac{3}{2}\dot{R}^2 = -4\eta\dot{R}/\rho R - \frac{\sigma B^2}{\rho} R\dot{R}$ (assuming $\mu_1 \gg \mu_2$)

μ – medium magnetic permeability

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{4\eta\dot{R}}{\rho R} - \frac{\sigma B^2}{\rho}R\dot{R} + \text{surface tension} + \ldots$$

Near T_c $\eta/s \ll 1$, $\sigma \propto \left(\frac{T-T_c}{T_c}\right)^{-1/2}$, $B > 10\, m_\pi^2$

Soft landing instead of collapse?

Shuryak and Staig: η may stop the velocity from divergence

Is B capable of stopping the collapse?

In fact both η and B terms diverge at slower rate than the inertial term $\dot{R}^2$

$$\dot{R}^2 \propto (t_* - t)^{-6/5}$$

$$\frac{4\eta\dot{R}}{\rho R} \propto (t_* - t)^{-1}$$

$$\frac{\sigma B^2}{\rho}R\dot{R} \propto (t_* - t)^{-1/5}$$

The kinetic energy in the collapse ends up as sound waves and light. Possible bubble rebound calls for more accurate solution near the singularity – multiple publications

SUMMARY

- The properties of quark matter in finite-density pre-critical region are determined by the soft mode
- The soft mode is described by the fluctuation propagator singular at the critical temperature
- Fluctuation propagator brings a dominant contribution into transport coefficients and soft photon emission around the transition line
- Sound attenuation constant and bulk viscosity diverge as $t^{-3/2}$ at $T \to T_c$
- Electical conductivity and soft photon emission are enhanced as $t^{-1/2}$
- Magnetic field plays an important role in cavitation

Thank you for attention!