Holographic non-conformal models for QCD and effective quark-antiquark potentials.

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based on works with

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Outline

Conventional picture of QGP dynamics

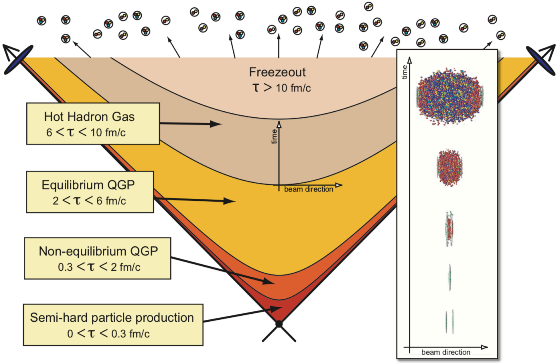


Figure: from Strickland 1410.5786

Holographic picture

- d-dim Gauge theory at strong coupling $\Leftrightarrow$ $(d+1)$-dim Gravity
  Particularly, $4d$ $\mathcal{N}=4$ SYM $\Leftrightarrow$ SUGRA in AdS_5
- The scenario of a heavy-ion collision can be represented as a shock wave collision (Lorentz contracted pancakes) in which trapped surface (the black hole) is formed.
- After the collision the shocks slowly decay, leaving the plasma described by hydrodynamics .
- The temperature of the the Yang-Mills theory is identified with the Hawking temperature of the black hole.
- NOTE: At high T QCD is nearly conformal theory: the AdS/CFT correspondence.
- The viscosity-to-entropy ratio for QGP

$$\eta/s = \frac{1}{4\pi}$$

Policastro,Son,Starinets' 01

Lattice calculations

Lattice shows that QCD exhibits a quasi-conformal behavior at $T > 300$MeV and the equation of state $\sim E = 3P$ (a traceless conformal energy-momentum tensor).

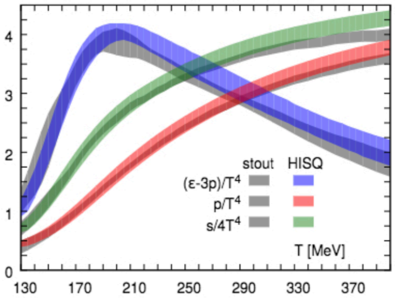
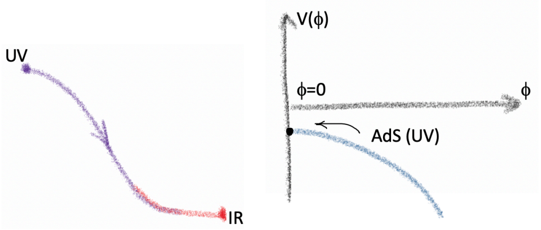


Figure: from Bazavov et.al.'1407.6387. The comparison for the trace anomaly, the pressure, and the entropy density stout.

Holographic picture for deviations from confomality



- d-dim CFT has a description in terms of $d+1$-dim gravity in AdS
- An operator $\mathcal{O}(x)$ corresponds to a dynamical bulk field $\phi(x,u)$
- $\phi(x,0)$ – a source for the $\mathcal{O}$ in the CFT

$$S = \int dx^d du \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] + S_{GYH}.$$

- $\phi(x,u) = \alpha u^{d-\Delta} + \ldots \Leftrightarrow S_{CFT} = S_0 + \int d^4x \alpha \mathcal{O}(x)$
- $\alpha = 0$ – undeformed CFT, bulk scalar – const., spacetime is AdS
- $\alpha \neq 0$ corresponds to relevant coupling for the CFT; deform. AdS

The holographic bottom-up models

- Buchel, Heller, Myers '1503.07114 $N = 2^*$ SYM
- Janik, Plewa, Soltanpanahi, Spalinski'1503.07149
  $V = cosh(\phi) + \phi^2 + \phi^4 + \phi^6.$
- Improved holographic QCD Gursoy,Kiritsis' 07, Gubser'08

For asymptotically AdS UV $\lambda \to 0 \quad V(\lambda) \; = \; V_0 + v_1\lambda + v_2\lambda^2 + \ldots$

For confinement in the IR $\lambda \to \infty \quad V(\lambda) \; \sim \; \lambda^Q (\log\lambda)^P$

- Perturbative analysis near extrema of the potential Gursoy et al.'17,Kiritsis et al' 16'17'18'19
- Single exponent potential $V = V_0(1 - X^2)e^{-\frac{8}{3}X\phi}$, $X < 0$ Gursoy, Jarvinen, Policastro'16
- Two exponent potential $V = C_1 e^{2k\phi} + C_2 e^{\frac{32}{9k}\phi}$,
  $C_1 < 0, C_2 > 0, k > 0$ Aref'eva, AG, Policastro'19

Outline

The gravity duals

$$S = \frac{1}{2\kappa^2}\int d^4x \int du\sqrt{-g}\left(R - \frac{4}{3}(\partial\phi)^2 + V(\phi)\right) - \frac{1}{\kappa^2}\int_{\partial} d^4x\sqrt{-\gamma},$$

$$V_{GJP} = V_0(1-X^2)e^{-\frac{8}{3}X\phi}, \quad X < 0.$$

$$V_{AGP} = C_1 e^{2k\phi} + C_2 e^{\frac{32}{9k}\phi}; C_1 < 0, C_2 > 0, k > 0$$

- V_{GJP} : $\quad ds^2 = e^{2A(u)}\left(-f(u)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{du^2}{f(u)}$, Chamblin, Reall'99

  $$e^A = e^{A_0}\lambda^{\frac{1}{3X}}, \quad f = 1 - C_2\lambda^{-\frac{4(1-X^2)}{3X}}, \quad \lambda = e^{\phi} = (C_1 - 4X^2\frac{u}{\ell})^{\frac{3}{4X}}.$$

- V_{AGP} : new holographic backgrounds Aref'eva, AG, Policastro'19

$$
\begin{aligned}
ds^2 &= F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-dt^2 + d\vec{y}^{\,2}\right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2 \\
\phi &= -\frac{9k}{9k^2-16}\log F_1 + \frac{9k}{9k^2-16}\log F_2
\end{aligned}
$$

- $$
\begin{aligned}
F_1 &= \sqrt{\left|\frac{C_1}{2E_1}\right|}\sinh(\mu_1|u-u_{01}|), \mu_1 = \sqrt{\left|\frac{3E_1}{2}(k^2-\frac{16}{9})\right|}, \\
F_2 &= \sqrt{\left|\frac{C_2}{2E_2}\right|}\sinh(\mu_2|u-u_{02}|), \mu_2 = \sqrt{\left|\frac{3E_2}{2}((\frac{16}{9})^2\frac{1}{k^2}-\frac{16}{9})\right|}, \\
E_1 &+ E_2 = 0, \quad E_1 < 0, \quad E_2 > 0
\end{aligned}
$$

- $$
\begin{aligned}
F_1 &= \sqrt{\frac{3}{4}(k^2-\frac{16}{9})C_1}(u-u_{01}), F_2 = \sqrt{\frac{4}{3}(\frac{16}{9k^2}-1)C_2}|u-u_{02}|, \\
E_1 &= E_2 = 0
\end{aligned}
$$

- two types of backgrounds: **1)** on $u \in (u_{02}, u_{01})$, **2)** $u \in (u_{01}, +\infty)$.
- boundaries of the solutions correspond to fixed points.
- $u_{01} = u_{02} \Rightarrow$ special flow : $u \in (u_0, +\infty)$ with AdS UV fixed point

The phase trajectories

$$X(\phi) = \frac{\beta(\lambda)}{3\lambda} = \frac{\phi'}{A'}$$

$\lambda = e^{\phi}$ – the running coupling.

The X-function on exact solutions AGP'19

$$X = \frac{1}{3}\left(\frac{F_2}{F_1}\right)^{\frac{9k}{16-9k^2}} \frac{\lambda'}{\mathcal{A}'}.$$

Holographic RG flow equation Kiritsis et al.'0812.0792

$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2\right)\left(1 + \frac{3}{8X}\frac{d\log V}{d\phi}\right)$$

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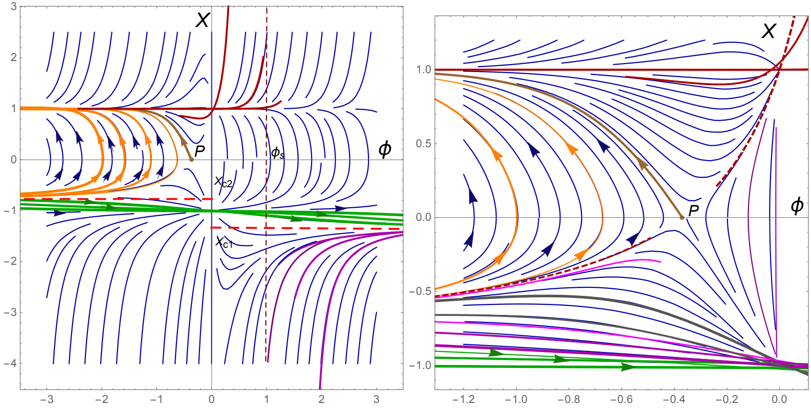

Holographic running coupling

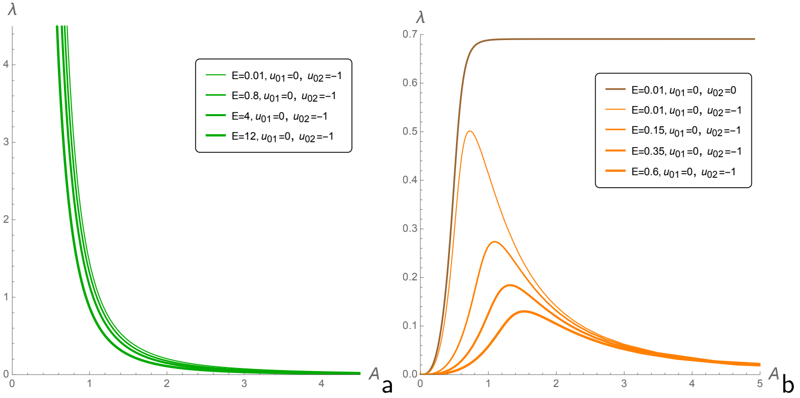

Figure: The coupling constant on the energy A on the dilaton plotted using the solutions for $\mathcal{A}$ and ϕ: a) $0 < u < u_{01}$; b) $u > u_{01}$.

The holographic Wilson loops

The expectation value of the holographic WL can be defined through the Nambu-Goto action $\mathcal{S}_{NG}$

$$\langle W(\mathcal{C})\rangle \sim e^{-\mathcal{S}_{NG}}$$

Maldacena'98

The expectation value of the WL of size $T \times \ell$ is related with $q\bar{q}$-potential

$$\langle W\rangle \sim e^{-V_{q\bar{q}}(\ell)T}$$

The potential of the quark antiquark interaction as

$$V_{q\bar{q}} = \frac{1}{T}\mathcal{S}_{NG}$$

The Nambu-Goto action is defined as

$$\mathcal{S}_{NG} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-\det h}, \quad h_{\alpha\beta} = e^{\frac{4}{3}\phi}G_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu,$$

$G_{\mu\nu}$ is the background metric, the world-sheet coordinates σ^α, $\alpha = 0, 1$, and the embedding functions $X^\mu = X^\mu(\sigma^\alpha)$

Holographic Wilson loops AG&Vu Nguen'19 TMPh

We choose the following gauge

$$\sigma^0 = t, \quad \sigma^1 = x_1, \quad u = u(x_1)$$

The Nambu-Goto action in the string frame

$$\frac{\ell}{2} = \int du \frac{ce^{3\mathcal{A}}}{\sqrt{e^{4\mathcal{A}+\frac{8}{3}\phi} - c^2}}$$

and for the Nambu-Goto action we have the following relation

$$\frac{\mathcal{S}_{NG}}{2} = \frac{T}{2\pi\alpha'} \int du \frac{e^{7\mathcal{A}+\frac{8\phi}{3}}}{\sqrt{e^{4\mathcal{A}+\frac{8\phi}{3}} - c^2}}.$$

Let us define the so-called effective potential with $u' = 0$ as

$$V_{eff} = e^{2\mathcal{A}+\frac{4}{3}\phi} = F_1^{\frac{4(2-3k)}{9k^2-16}} F_2^{\frac{3k(3k-8)}{2(16-9k^2)}}.$$

In terms of V_{eff} the quark-antiquark distance can be presented as

$$\frac{\ell}{2} = \int du\, e^{-2\phi} \frac{V_{eff}(u)\sqrt{V_{eff}(u)}}{\sqrt{\frac{V^2_{eff}(u)}{V^2_{eff}(u_*)} - 1}}$$

and the string action is given by

$$\frac{\mathcal{S}_{NG}}{2} = \frac{T}{2\pi\alpha'} \int du \frac{e^{-2\phi} V^3_{eff}(u)\sqrt{V_{eff}(u)}}{\sqrt{V^2_{eff}(u) - V^2_{eff}(u_*)}}.$$

• Require the function V_{eff} to be decreasing on the region (u_{01}, u_*).
• To observe a confinement in the IR region, that implies $\ell \to +\infty$ with $\mathcal{S}_{NG} \to +\infty$, V_{eff} needs have a local minimum. Expanding in the Taylor series at the point u_{min}, with $u_{min} = u_*$ one has

$$\frac{V^2_{eff}(u)}{V^2_{eff}(u_{min})} = 1 + \frac{V''(u_{min})}{V_{eff}(u_{min})}(u - u_{min})^2 + \mathcal{O}(u - u_{min})^2.$$

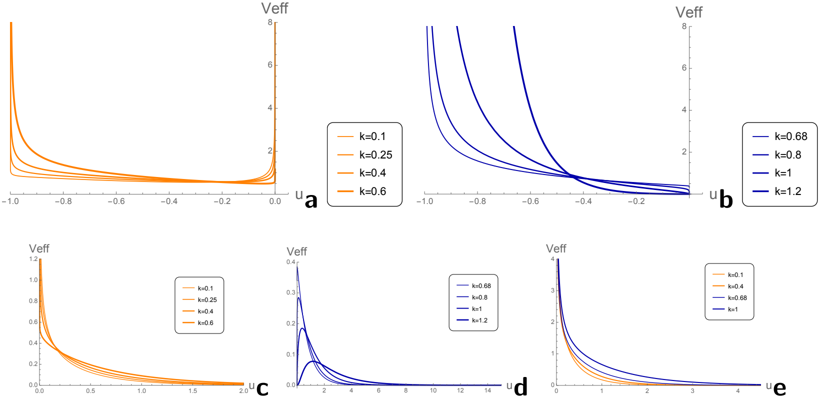

Figure: V_{eff} as a function of u for the holographic RG flows: **a),b)** defined on $(u_{02}; u_{01})$ with $u_{02} = -1$, $u_{01} = 0$ for small and big k, correspondingly; **c),d)** defined on $(u_{01}; +\infty)$ with $u_{01} = 0$ for small and big k, correspondingly; **e)** with coinciding points $u_{01} = u_{02} = 0$.

The effective $q\bar{q}$-potential

- The holographic background on (u_{02}, u_{01}) is confining with $k < 2/3$

$$\frac{\ell}{2} = \int_{u_{min}}^{u_{01}} \frac{e^{-2\phi} V_{eff}^2(u_{min}) du}{\sqrt{V''_{eff}(u_{min})(u_{min} - u)}} \sim \frac{e^{-2\phi(u_{min})} V_{eff}^2(u_{min})}{\sqrt{V''_{eff}(u_{min})}} \log(u_{min} - u),$$

  and the Nambu-Goto action is given by

$$\mathcal{S}_{NG} = \int_{u_{min}}^{u_{(\Lambda)}} \frac{e^{-2\phi} V_{eff}^3(u_{min}) du}{\sqrt{V''_{eff}(u_{min})(u_{min} - u)}} \sim \frac{e^{-2\phi(u_{min})} V_{eff}^3(u_{min})}{\sqrt{V''_{eff}(u_{min})}} \log(u_{min} - u),$$

  so $\ell \to +\infty$ with $\mathcal{S}_{NG} \to +\infty$ as $u \to u_{min}$.

$$V_{q\bar{q}\,\ell \to +\infty} \sim \sigma \ell, \quad \sigma = V_{eff}(u_{min})$$

  At large distance one has a linear growth of the quark potential.

- The holographic background on $(u_{01}, +\infty)$ is non-confining for all k.

Black hole solution, $u = +\infty$ is the horizon, $u_{01} = 0$

- $\mu_2 = \mu_1 = \mu$
- Hawking temperature: $T = \frac{1}{2\pi} \frac{\mu}{\mathcal{C}^{3/2}}$.

$$
\begin{aligned}
ds^2 &= \mathcal{C}\,\mathcal{X}\Big(- e^{-2\mu u} dt^2 + d\vec{y}^2 \Big) + \mathcal{C}^4\,\mathcal{X}^4 e^{-2\mu u} du^2, \\
\mathcal{X} &= \left(1 - e^{-2\mu u}\right)^{-\frac{8}{16-9k^2}} \left(1 - e^{-2\mu(u-u_{02})}\right)^{\frac{9k^2}{2(16-9k^2)}}, \\
\mathcal{C} &\equiv 2^{\frac{16}{(16-9k^2)}} (3\mu)^{\frac{1}{2}} \left|C_1\right|^{\frac{8}{2(9k^2-16)}} \left(\frac{C_2}{k} e^{-2\mu u_{02}}\right)^{\frac{9k^2}{4(16-9k^2)}} (16-9k^2)^{-\frac{1}{4}}.
\end{aligned}
$$

$$
\phi = \frac{9k}{9k^2-16} \log\left[\sqrt{\left|\frac{E_1 C_2}{E_2 C_1}\right|} \frac{\sinh(\mu(u-u_{02}))}{\sinh(\mu u)}\right].
$$

Free energy through the holographic on-shell action

$$\frac{I_{reg}^{on-shell}}{\beta V_3} = -\left(6\mathcal{A}'(u) + \frac{f'(u)}{f(u)}\right)|_{u=\epsilon}\,.$$

$$\mathcal{F} \sim -\frac{1}{2}\left(\mu - \frac{27k^2}{16-9k^2}(\sqrt{\Lambda^2+\mu^2}-\Lambda)\right), \frac{\mu}{\Lambda} = \sinh(-\mu u_{02}).$$

Free energy through black hole thermodynamics

$$d\mathcal{F} = -sdT.$$

$$\mathcal{F} = -\frac{V_3}{8\pi}\left(\mu - \frac{27k^2}{16-9k^2}(\sqrt{\Lambda^2+\mu^2}-\Lambda)\right)\,,$$

$u_{02} \to 0$, $\Lambda \to 0$ the free energy $\mathcal{F} = -\frac{V_3}{8\pi}\mu$.

Free energy

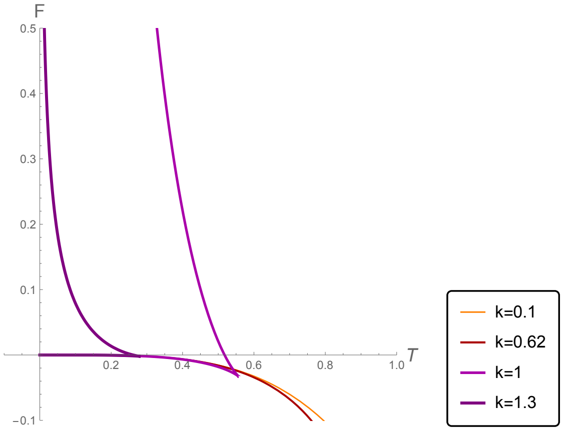

Figure: The dependence of the free energy F on the temperature T for the different shapes of the potential (different k, $C_1 = -2$, $C_2 = 2$).

Holographic WL for $T \neq 0$

The effective potential

$$V_{eff} = \mathcal{C} e^{-\mu u} \left(\frac{4e^{-\mu u_{02}}}{3k} \sqrt{\frac{C_2}{|C_1|}} \right)^{\frac{12k}{9k^2-16}} \left(1 - e^{-2\mu(u-u_{02})}\right)^{\frac{3k(8-3k)}{2(9k^2-16)}} \left(1 - e^{-2\mu u}\right)^{\frac{4(2-}{9k^2-}}$$

The distance between quarks and the Nambu-Goto action can represented in terms of V_{eff} as

$$\frac{\ell}{2} = \int_0^{u_*} du \, \frac{e^{-2\phi} e^{\frac{\mu}{2}u} V_{eff} \sqrt{V_{eff}}}{\sqrt{\frac{V_{eff}^2(u)}{V_{eff}^2(u_*)} - 1}}$$

and

$$\mathcal{S}_{NG} = \frac{T}{\pi\alpha'} \int_0^{u_*} du \, \frac{e^{-2\phi} e^{\frac{\mu}{2}u} V_{eff}^3 \sqrt{V_{eff}}}{\sqrt{V_{eff}^2(u) - V_{eff}^2(u_*)}},$$

correspondingly.

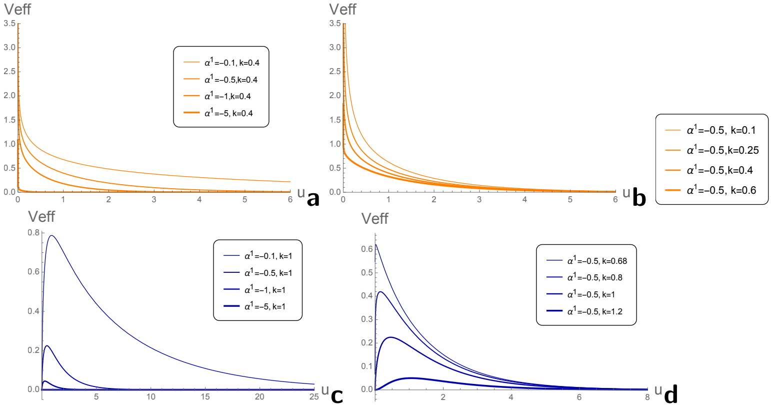

Figure: V_{eff} as a function of u for the holographic RG flows at finite temperature: **a),c)** we fix k varying $\alpha^1 = -\frac{3}{4}\mu$, **b),d)** we fix $\alpha^1 = -\frac{3}{4}\mu$ varying k.

Outline

New results

- New viable holographic backgrounds with confinement
- Holographic running coupling mimic QCD
- $q\bar{q}$-potential has an area law
- Holographic backgrounds with AdS fixed point
- Black hole backgounds work like IR cut-off

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In progress

- Is it possible to reproduce the whole Cornell potential?
- The hadronic spectrum
- Phase transitions (Aref'eva&Rannu'18, Aref'eva,Rannu&Slepov'19)
- More precise studies of thermal case
- Generalization on non-zero baryonic density

Thank you for attention!