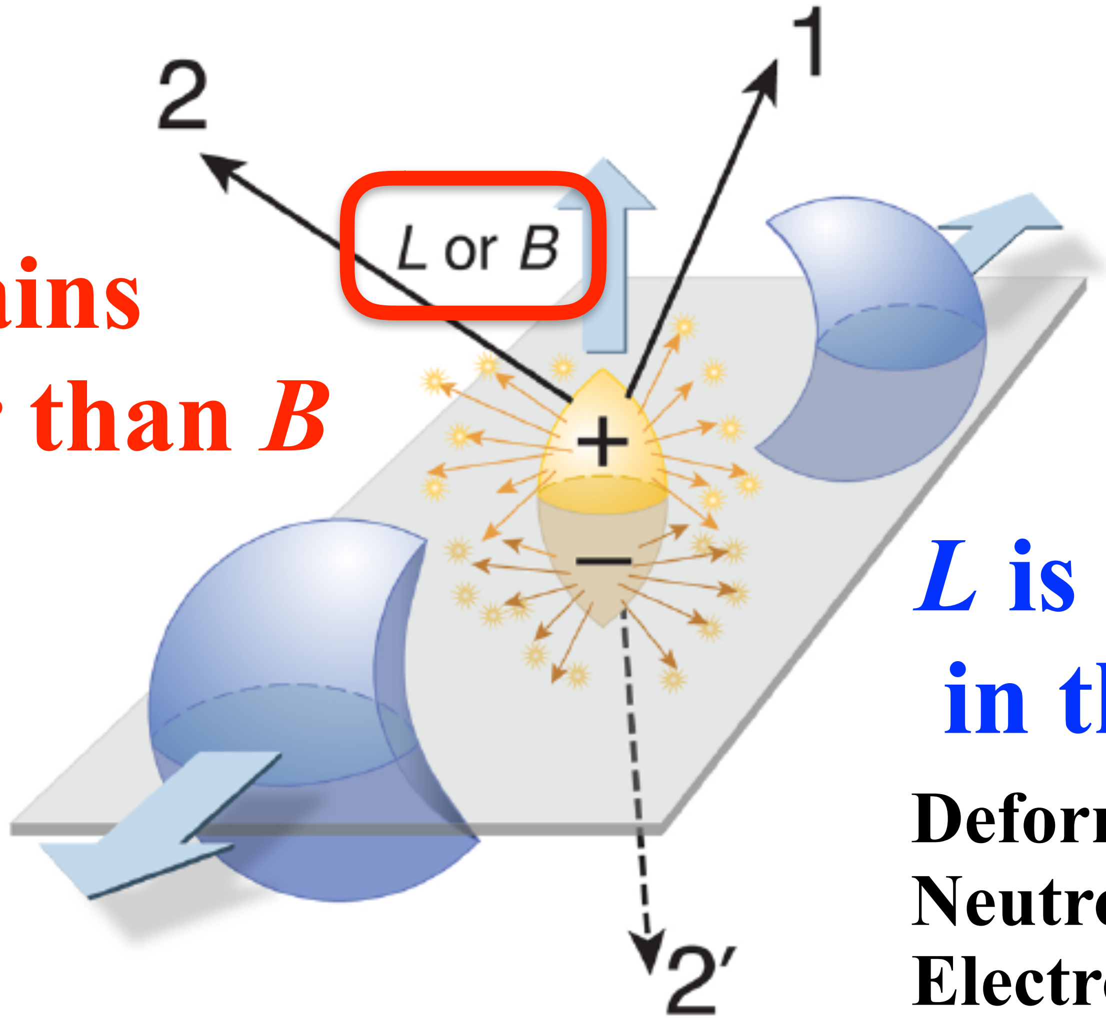
Quark Matter under Rotation

Kenji Fukushima

The University of Tokyo

— Theory of Hadronic Matter under Extreme Conditions —

Rotation in Collisions



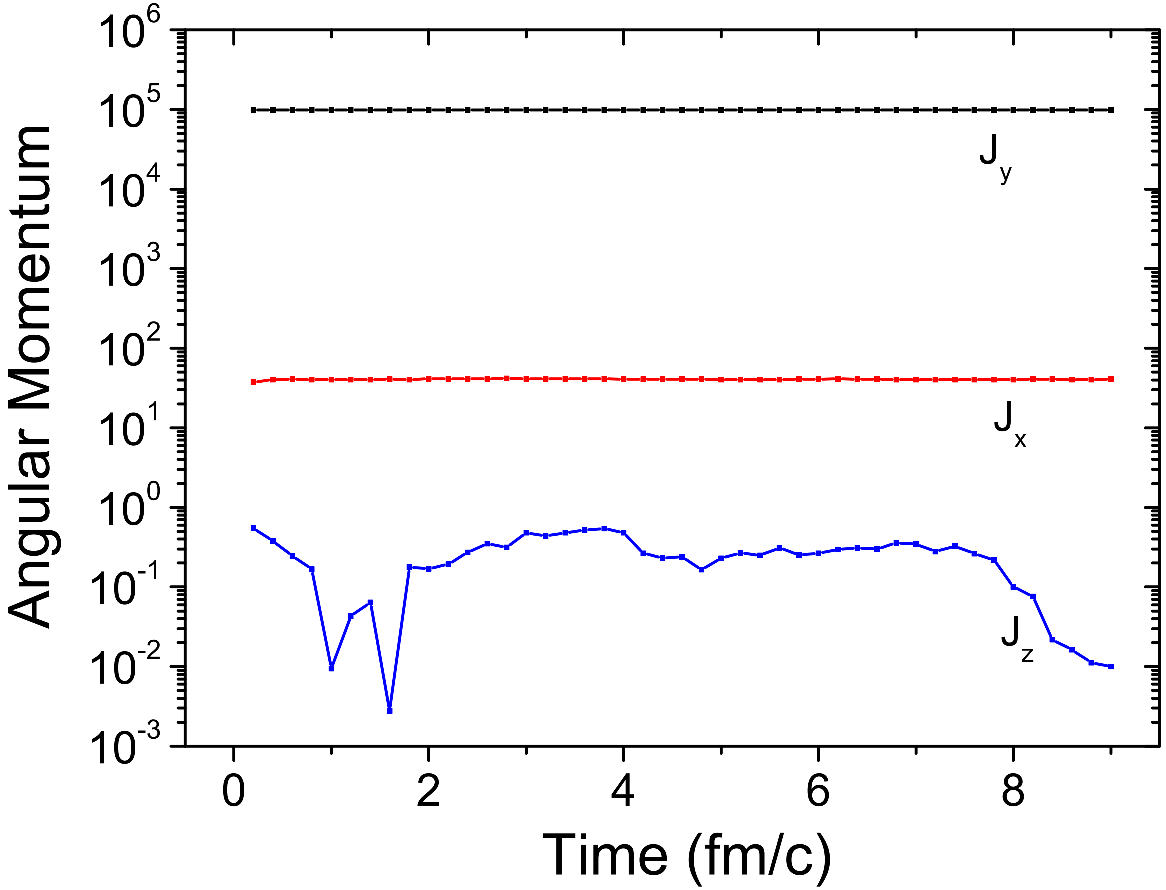
L **remains longer than *B***

L **is ubiquitous in the nature**

Deformed Nuclei
Neutron Stars
Electron Vortices …

Rotation in Collisions

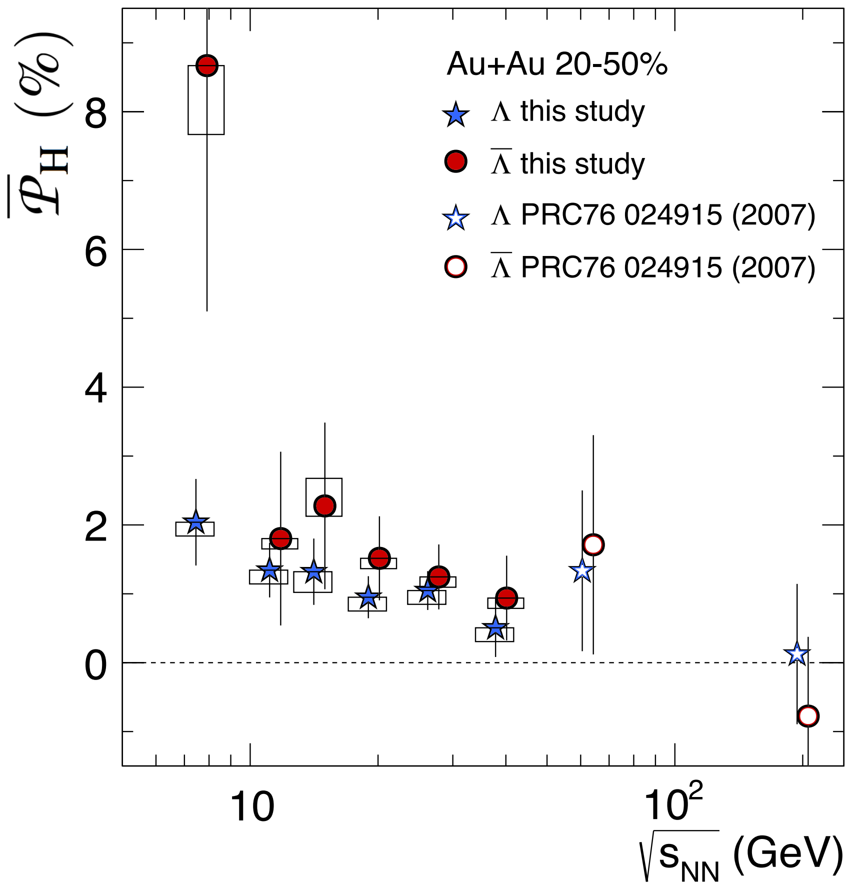
L **is an *intrinsic* property of matter**



Jiang-Lin-Liao, PRC (2016)

Rotation in Collisions

Global Polarization of *Λ*



$$P_{\rm Vortical} = \frac{1}{2}(P_\Lambda + P_{\bar{\Lambda}})$$

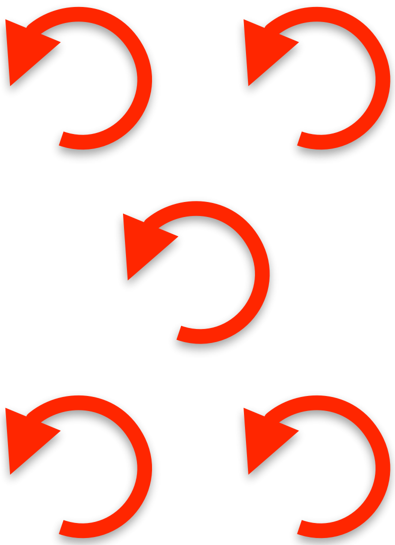
$$P_{\rm Magnetic} = \frac{1}{2}(P_\Lambda - P_{\bar{\Lambda}})$$

Becattini, Csernai, Wang, …

STAR: 2016

Theoretical Formulation

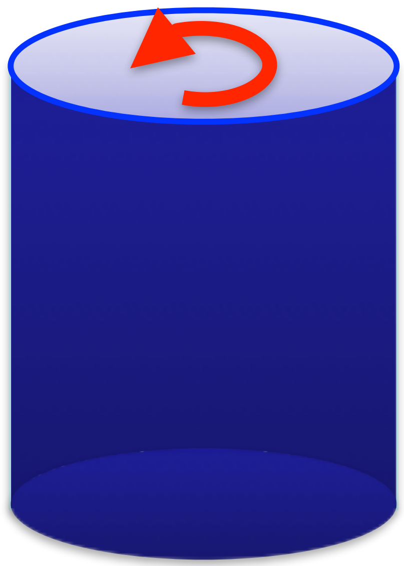
Fluid



$$\nabla \times \boldsymbol{u}$$

v.s.

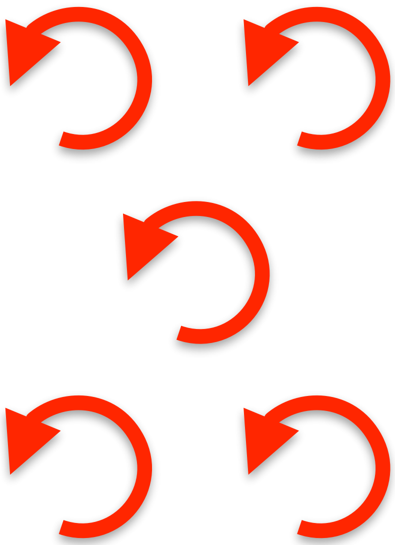
Rotating QFT



Coordinate Transformation
Finite Size (causality)

Theoretical Formulation

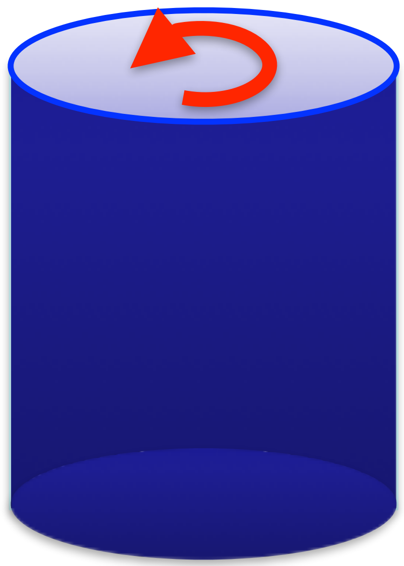
Fluid



$\nabla \times \boldsymbol{u}$

v.s.

Rotating QFT



Coordinate Transformation
Finite Size (causality)

Theoretical Formulation

$$[i\gamma^{\mu}(\partial_{\mu} + \Gamma_{\mu}) - m]\psi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Solve this in a finite cylinder (radius R)

Not only the affine connection but gamma's changed

Vierbeins are needed !

Theoretical Formulation

Theoretical treatment for deformed nuclei

Cranking Hamiltonian $H_{\mathrm{rot}} = H \underline{- \omega J_z}$

Chemical Potential Like

J **Looks like** ***B***

μ

Angular Momentum

Theoretical Formulation

Theoretical treatment for deformed nuclei

Cranking Hamiltonian $H_{\mathrm{rot}} = H \underline{- \omega J_z}$

Chemical Potential Like

J **Looks like** ***B***

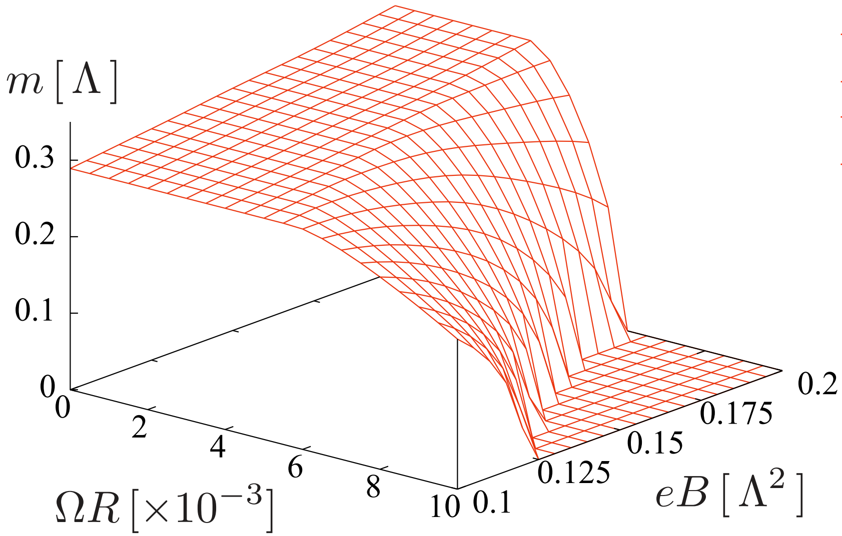
μ

Angular Momentum

Rotation ~ Density

Chen-KF-Huang-Mameda, PRD (2015)

Inverse Magnetic Catalysis driven by rotation



More interestingly,
Rotation+*B* = (Genuine) Density

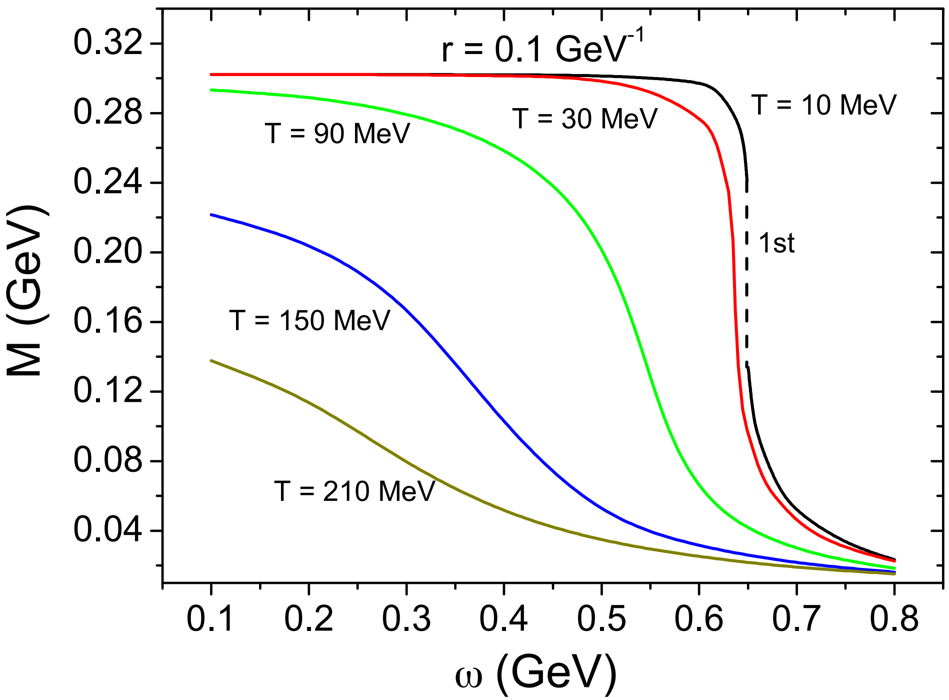
$$n = -\frac{\partial\Omega}{\partial\mu}\bigg|_{\mu=0} = \frac{eB\omega}{4\pi^2}$$

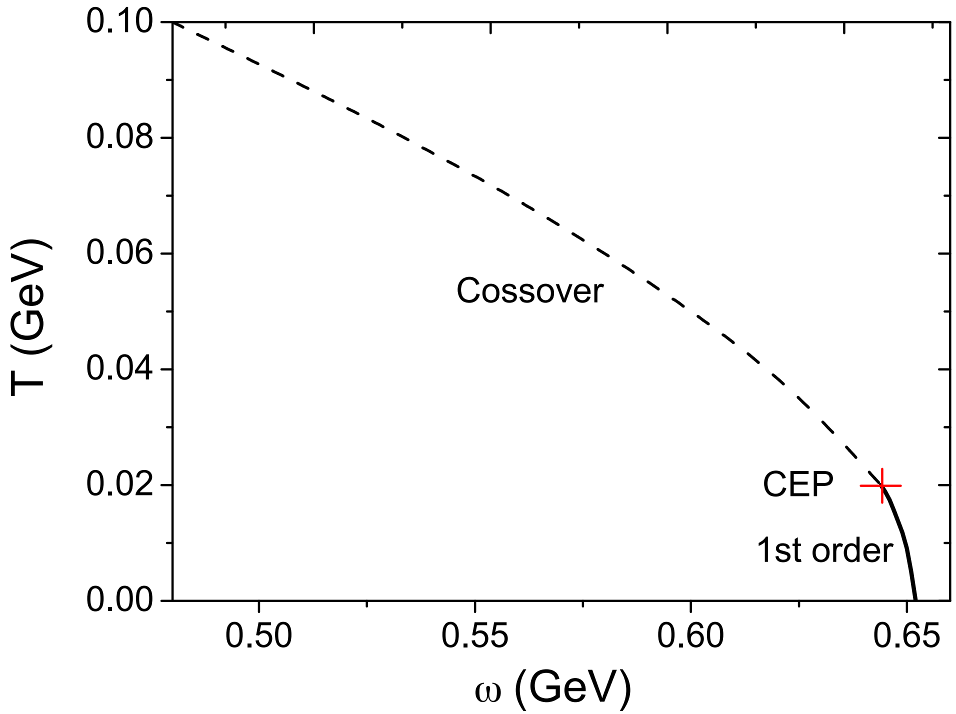
interpreted as **anomaly**
Hattori-Yin, PRL (2016)

Can be given another interpretation from the Floquet theory

Rotation ~ Density

Jiang-Liao, PRL (2017)





Completely analogous to chemical potential… BUT!

Rotation ~ Density

Is it really possible to change the QCD vacuum just by rotation ???

The answer is negative:

Ebihara-Fukushima-Mameda, PLB (2017)

Causality $\omega R < 1$

System size should be finite ~ *R*

Energy dispersion should be gapped ~ *J*/*R*

Induced chemical potential ~ *ωJ*

Gap is always bigger than the chemical potential

Rotation ~ Density

Is it really possible to change the QCD vacuum just by rotation ???

The answer is negative:

Ebihara-Fukushima-Mameda, PLB (2017)

If one wants to see nontrivial effects of rotation, it should be coupled with…

μ	B	T
Gauge CVE	**Chiral Pumping Effect**	**Gravity CVE**

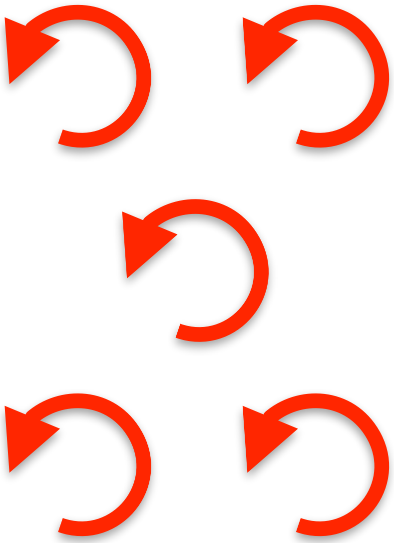
There are still lots of interesting challenges in physics and theoretical computations!

But, these are mostly technical issues, and there seems to be no conceptual problem.

Let’s move on to a more subtle thing now…

Switch the gear into...

Fluid



$\nabla \times \boldsymbol{u}$

$\boldsymbol{J}$

leading to

Looks like

$\boldsymbol{B}$

Magnetization

Two Choices

Hydrodynamics with Local Vorticity Vectors

Derivative expansion ? (vorticities are second order)

Discrimination of *L* and *S* ?

Kinetic Equations with Local Vorticity Vectors

$$\varepsilon_{\rm rot} = p - \boldsymbol{\omega} \cdot \left(\boldsymbol{x} \times \boldsymbol{p} + \hbar\lambda\hat{\boldsymbol{p}} \right)$$

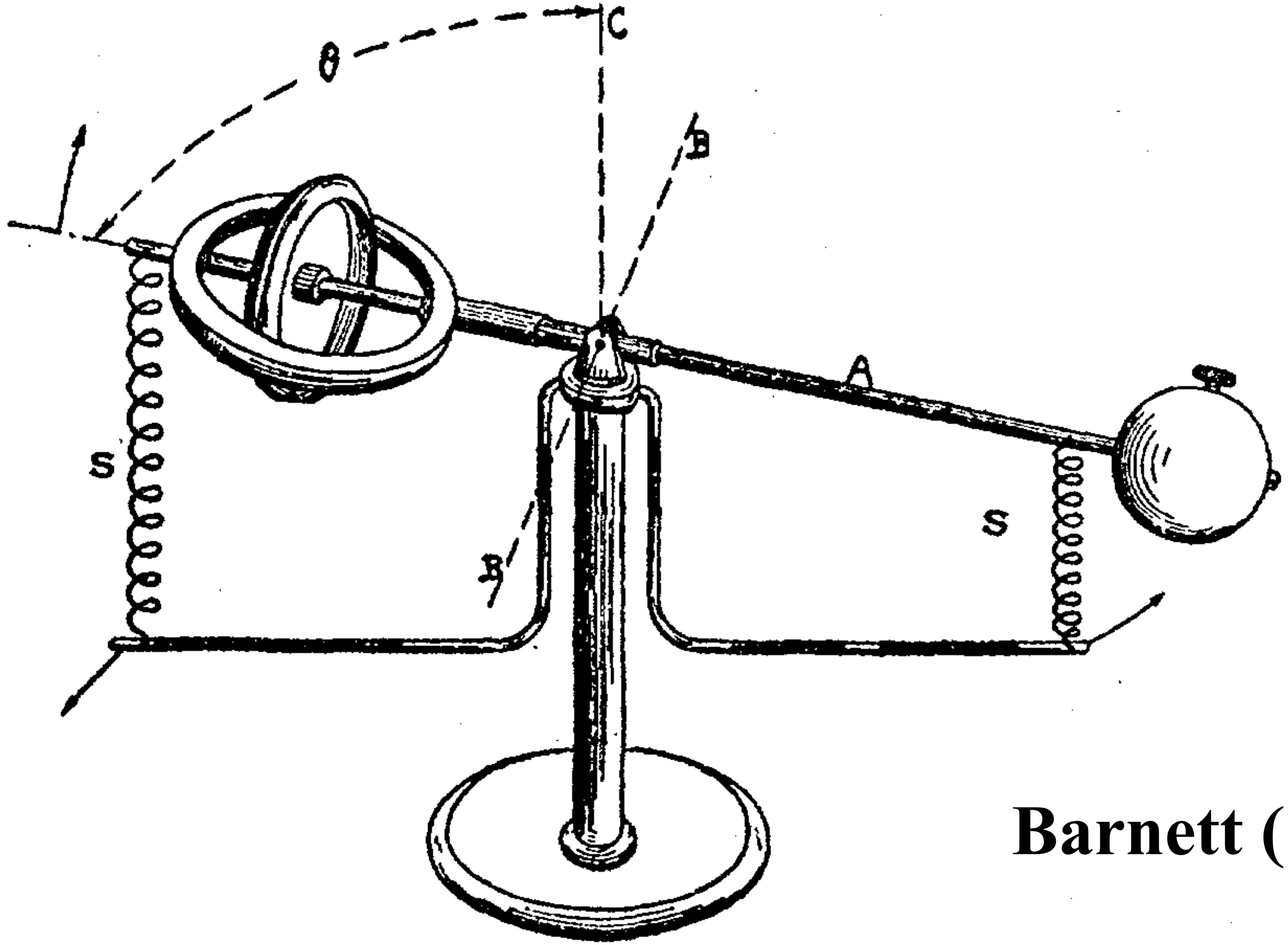
$$= \boldsymbol{\omega} \cdot \boldsymbol{J}$$

$$f(\varepsilon) \;\to\; f(\varepsilon_{\rm rot})$$

Corrections in the Kinetic Eqs.?

Barnett Effect

Gyromagnetic Effect



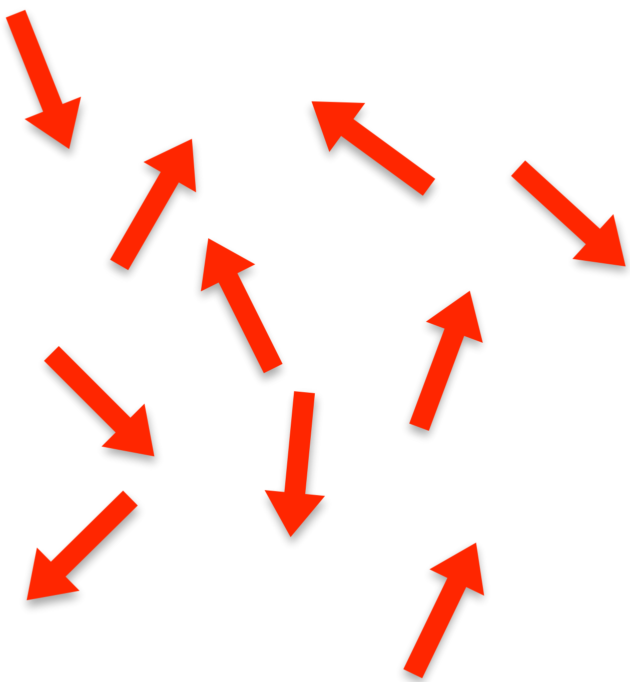
Barnett (1935)

Barnett Effect

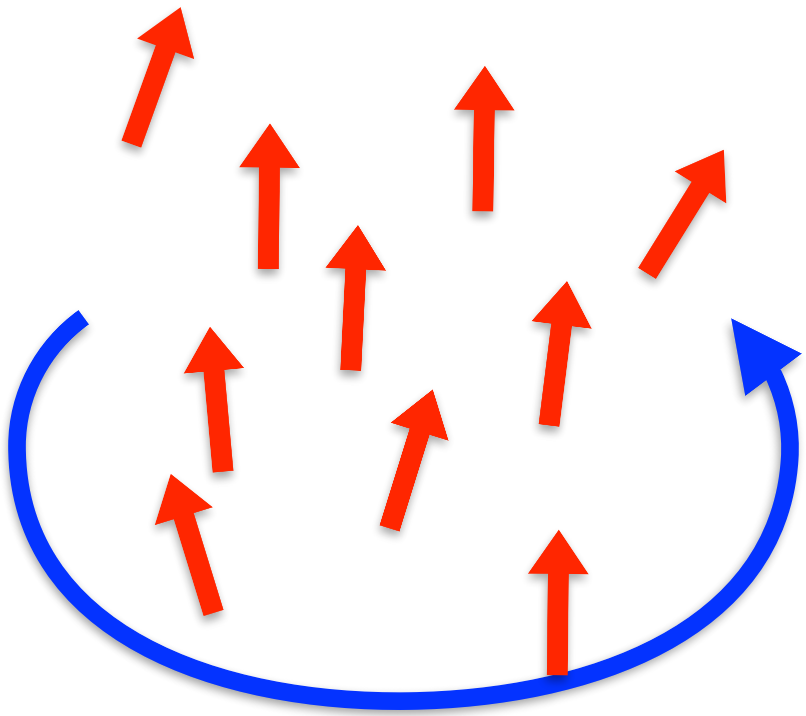
Gyromagnetic Effect

Barnett Effect

Spin Alignment in response to Rotation



"Gyroscopic" Motion



Barnett Effect

Quickest Derivation

$$\boldsymbol{\omega} \cdot \boldsymbol{J} \;=\; \boldsymbol{\mu} \cdot \boldsymbol{B}$$

Magnetization $\boldsymbol{M} = \chi_B \boldsymbol{B}$

magnetic susceptibility

Magnetic moment $\boldsymbol{\mu} = \gamma \boldsymbol{J}$

gyromagnetic ratio

→ $$M = \frac{\chi_B}{\gamma}\omega$$ **Standard formula for the Barnett effect**

Roughly speaking, the Barnett effect is a transport from the orbital to the spin angular momentum.

To make this phenomenon well-defined, the orbital and the spin components must be well separated.

HOW?

Decomposition of L and S

Angular Momentum
= Noether Current from Rotational Symmetry

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu}$$

$L^{\lambda\mu\nu}$: $\bar{\psi}\, i\hbar(\gamma^\lambda x^\mu \partial^\nu - \gamma^\lambda x^\nu \partial^\mu)\psi$

$S^{\lambda\mu\nu}$: $\frac{1}{2}\bar{\psi}\, i\hbar \gamma^\lambda \gamma^\mu \gamma^\nu \psi$

Neither *L* nor *S* conserved separately

$$\partial_\lambda L^{\lambda\mu\nu} = -\partial_\lambda S^{\lambda\mu\nu} = \bar{\psi}\, i\hbar(\gamma^\mu \partial^\nu - \gamma^\nu \partial^\mu)\psi$$

Decomposition of L and S

Different decomposition

$$\tilde{L}^{\lambda\mu\nu} = \frac{1}{2} L^{\lambda\mu\nu} + \frac{1}{2}\bar{\psi}\, i\hbar \big[(x^\mu \gamma^\nu - x^\nu \gamma^\mu) \partial^\lambda \big] \psi$$

$$\tilde{S}^{\lambda\mu\nu} = J^{\lambda\mu\nu} - \tilde{L}^{\lambda\mu\nu}$$

$$\Longrightarrow \quad \partial_\lambda \tilde{L}^{\lambda\mu\nu} = \partial_\lambda \tilde{S}^{\lambda\mu\nu} = 0$$

Separately conserved?

Belinfante angular momentum
(Only the orbital part remains, and the spin part turns out to be trivial…)

Decomposition of L and S

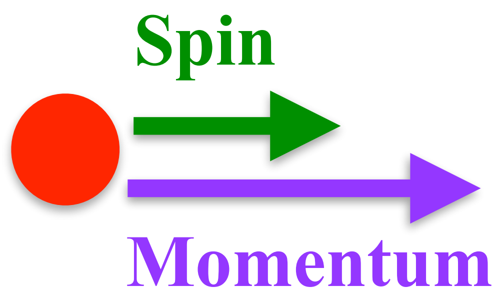
We believe the former decomposition makes sense:

1) Reduced to ordinary *L* and *S* in non-rela limit

2) *S* is related to the axial current

$$S^{0ij} = \epsilon^{ijk}\,\frac{\hbar}{2}\,\bar{\psi}\gamma^k\gamma_5\psi = \epsilon^{ijk}\,\frac{j_5^k}{2}$$

Corresponding Spin Operator:



$$\boldsymbol{S} \;\rightarrow\; \hbar\lambda\left(\hat{\boldsymbol{p}} - \underline{\hbar\lambda\frac{\hat{\boldsymbol{p}}}{2p}\times\boldsymbol{\nabla}}\right)$$

Torque from gyromagnetic effect

Chen-Son-Stephanov, PRL (2015)

Relativistic Barnett Effect

Spin Expectation Value

Energy in a rotating fluid $\varepsilon_{\text{rot}} = p - \boldsymbol{\omega} \cdot \left(\boldsymbol{x} \times \boldsymbol{p} + \hbar\lambda\hat{\boldsymbol{p}}\right)$

$$\langle \boldsymbol{S} \rangle = \int_{\boldsymbol{p}} \lambda\hbar \left(\hat{\boldsymbol{p}} - \lambda\hbar \frac{\hat{\boldsymbol{p}}}{2p} \times \boldsymbol{\nabla} \right) f(\varepsilon_{\text{rot}})$$

Vilenkin (1978)

$$= -\hbar\lambda(\boldsymbol{\omega} \times \boldsymbol{x}) \int_{\boldsymbol{p}} \frac{p}{3} f'(p) - \hbar^2 \boldsymbol{\omega} \int_{\boldsymbol{p}} f'(p)$$

$\langle \boldsymbol{S} \rangle_{\perp}$ → **"Transverse" Barnett Effect**

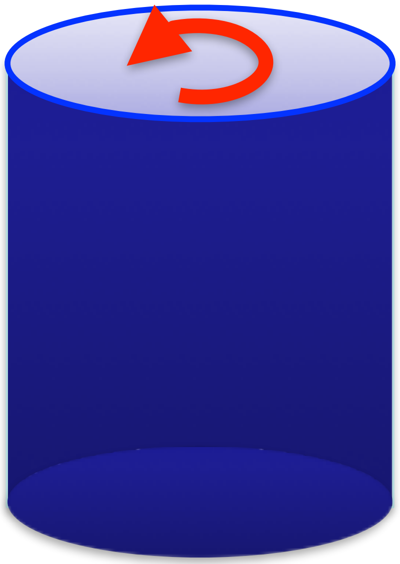
Chiral Vortical Effect
~ Barnett Effect

Relativistic Barnett Effect

Spin Expectation Value

$$\langle \boldsymbol{S} \rangle_\perp = -\hbar \sum_{R,L} \lambda(\boldsymbol{\omega} \times \boldsymbol{x}) \int_{\boldsymbol{p}} \frac{p}{3} f'_\lambda(p)$$

$$= \frac{\hbar}{2}(\boldsymbol{\omega} \times \boldsymbol{x}) \int_{\boldsymbol{p}} [f_R(p) - f_L(p)] = \frac{\hbar}{2}(\boldsymbol{\omega} \times \boldsymbol{x})\, n_5$$



$$\boldsymbol{j}_5 = n_5 \, \boldsymbol{v}$$

Transverse Barnett appears for massless and chirally imbalanced fermions

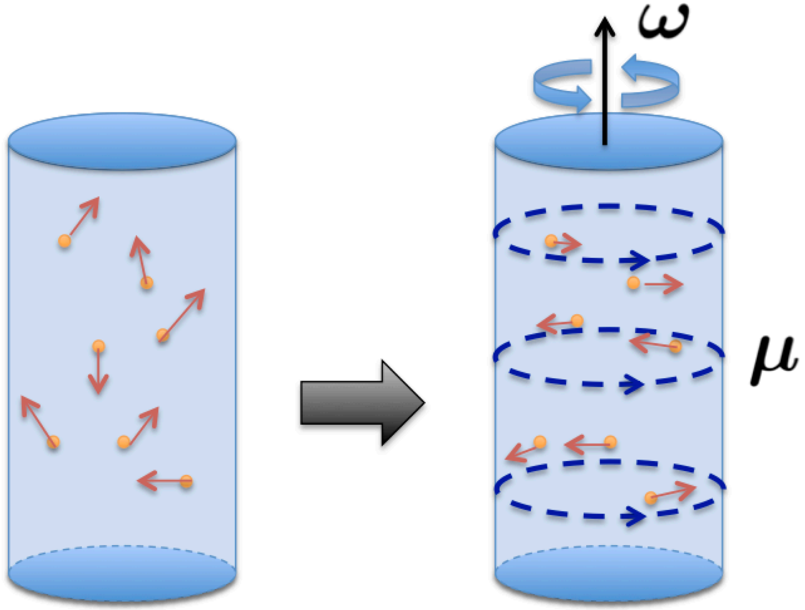
Relativistic Barnett Effect

Magnetic moment

$$\langle \boldsymbol{\mu} \rangle = \langle \boldsymbol{\mu}_L \rangle + \langle \boldsymbol{\mu}_S \rangle$$

$$= \langle \boldsymbol{\mu}_L \rangle_{\text{mech}} - \hbar \lambda \frac{q_e}{6} (\boldsymbol{\omega} \times \boldsymbol{x}) \int_{\boldsymbol{p}} f'(p)$$

up to $O(\hbar^1)$



Eddy magnetic moment

Fukushima-Pu-Qiu, PRA (2018)

Eddy Magnetization

Possible Evidence for Free Precession of a Strongly Magnetized Neutron Star in the Magnetar 4U 0142+61

K. Makishima,[1,2,3] T. Enoto,[4,5] J. S. Hiraga,[2] T. Nakano,[1] K. Nakazawa,[1] S. Sakurai,[1] M. Sasano,[1] and H. Murakami[1]

[1]*Department of Physics, Graduate School of Science, the University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

[2]*Research Center for the Early Universe, Graduate School of Science, the University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

[3]*MAXI team, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*

[4]*High Energy Astrophysics Laboratory, RIKEN Nishina Center, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*

[5]*NASA Goddard Space Flight Center, Astrophysics Science Division, Code 662, Greenbelt, MD 20771, USA*

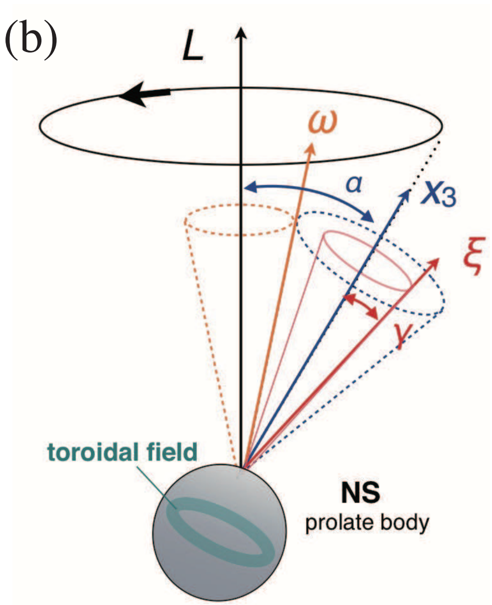
(Dated: February 19, 2018)

Magnetars are a special type of neutron stars, considered to have extreme *dipole* magnetic fields reaching $\sim 10^{11}$ T. The magnetar 4U 0142+61, one of prototypes of this class, was studied in broadband X-rays (0.5–70 keV) with the *Suzaku* observatory. In hard X-rays (15–40 keV), its 8.69 sec pulsations suffered slow phase modulations by ± 0.7 sec, with a period of $\sim$ 15 hours. When this effect is interpreted as free precession of the neutron star, the object is inferred to deviate from spherical symmetry by $\sim 1.6 \times 10^{-4}$ in its moments of inertia. This deformation, when ascribed to magnetic pressure, suggests a strong *toroidal* magnetic field, $\sim 10^{12}$ T, residing inside the object. This provides one of the first observational approaches towards toroidal magnetic fields of magnetars.

PRL112, 171102 (2014)

Eddy Magnetization

Precession has been measured



Spherically non-symmetric moment of inertia inferred

Deformation is assumed to be sustained by (toroidal) field energy

$$B \sim 10^{16}\ \mathrm{gauss}$$

Stronger than the surface *B* of magnetar !
How can it be created? ← Chiral Barnett Effect?

Subtlety in Hydro

Can we compute the same quantities in hydro?

YES, BUT NO !

$$T^{\mu\nu}_{\rm hydro} = (E+P)u^\mu u^\nu - P\, g^{\mu\nu} + \hbar\, n_5 (u^\mu \omega^\nu + u^\nu \omega^\mu)$$

$$L^{ij}_{\rm hydro} = x^i T^{0j}_{\rm hydro} - x^j T^{0i}_{\rm hydro}$$

$$\boldsymbol{L}_{\rm hydro} = (E+P)(\boldsymbol{x}\times\boldsymbol{u}) - \hbar\, n_5 (\boldsymbol{\omega}\times\boldsymbol{x})$$

Is this consistent with kinetic theory results?

Subtlety in Hydro

$$\boldsymbol{L}_{\text{hydro}} = (E+P)(\boldsymbol{x}\times\boldsymbol{u}) - \hbar\, n_5(\boldsymbol{\omega}\times\boldsymbol{x})$$

$$P = E/3$$

$$\boldsymbol{x}\times(\boldsymbol{\omega}\times\boldsymbol{x})\frac{4}{3}\int_{\boldsymbol{p}} p(f_R + f_L)$$

$$= \langle \boldsymbol{L}\rangle_{\text{mech}}$$

OK

$$-\,2\langle \boldsymbol{S}\rangle_{\perp}$$

Twice bigger !?

The difference comes from the energy momentum tensor.
In hydro the energy momentum tensor is a symmetrized one.
Belinfante form? Should be pseudo-gauge invariant…

Becattini-Florkowski-Speranza, PLB (2019)

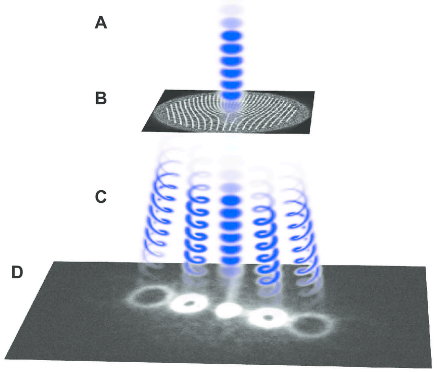
Summary of the Talk

Rotation ~ Density

- □ Phase Diagram
- □ Finite-size System / Inhomogeneous Condensates

Rotation ~ Magnetic Field

- □ Barnett Effect expected
- □ Chiral Vortical Effect is nothing but the Barnett Effect.
- □ Decomposition of *L* and *S* still assumed…
- □ **Ideas testable in optics and electron vortex systems**



Works along these lines ongoing