Phase structure of QC_2D at T=0 and large quark density

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OUTLINE

- Introduction
- Lattice setup
- Confinement-deconfinement transition at zero temperature. Flux tube
- Spatial string tension
- Polyakov loop and its correlators. Quark number density
- Screening mass
- Conclusions and Outlook

Motivation

- Detailed understanding of QC_2D at $\mu > 0$ should give insight into phenomena expected in QCD at $\mu > 0$

- Results are useful for other approaches, e.g. DSE or approaches using uncontrolled approximations (effective actions like PNJL model, massive YM)

Related Talks

- Nikita Astrahantsev

Study of confinement/deconfinement transition in cold dense quark matter in QCD-like theories

(evening session)

- Roman Rogalyov

Gluon propagators in QC_2D at high baryon density

(evening session)

Other lattice studies of $\mathbf{QC_2D}$

$N_f = 4$, staggered

- Kogut, Toublan and Sinclair, The phase diagram of four flavor SU(2) lattice gauge theory at nonzero chemical potential and temperature, Nucl. Phys. B 642 (2002) 181

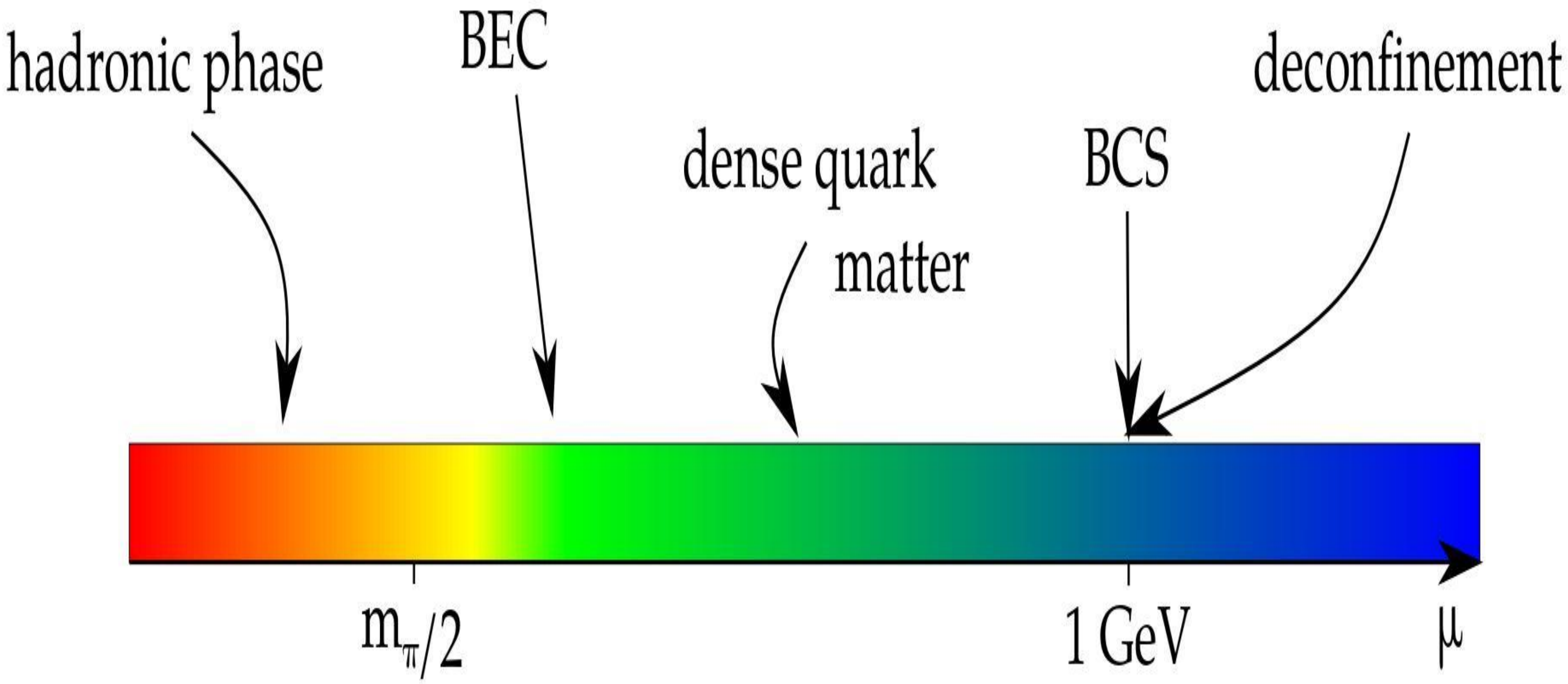
$N_f = 2$, staggered

- Braguta, Ilgenfritz, Kotov, Molochkov, Nikolaev, Study of the phase diagram of dense two-color QCD within lattice simulation, Phys. Rev. D 94 (2016)114510
- Holicki, Wilhelm, Smith, Wellegehausen and von Smekal, Two-colour QCD at finite density with two flavours of staggered quarks, PoS(LATTICE2016)052

$N_f = 2$, Wilson

- Cotter, Giudice, Hands and Skullerud, Towards the phase diagram of dense two-color matter, Phys. Rev. D 87 (2013) 034507

Phase Diagram of QC_2D at T=0



Simulation settings

- SU(2) lattice QCD with $N_f = 2$ staggered Dirac operator
- Lattice size 32^4
- Lattice spacing $a = 0.044$ fm
- Pion mass $m_\pi = 740(40)$ MeV
- Range of μ values: $0 \le a\mu \le 0.5$

or

$0 \le \mu \lesssim 2000$ MeV

Simulation settings

Lattice fermion action:

$$S_F = \sum_{x,y} \bar{\psi}_x M(\mu, m)_{x,y} \psi_y + \frac{\lambda}{2} \sum_x \left(\psi_x^T \tau_2 \psi_x + \bar{\psi}_x \tau_2 \bar{\psi}_x^T \right)$$

M is the staggered lattice Dirac operator,

λ- term is needed to make the di-quark condensate nonzero

Partition function:

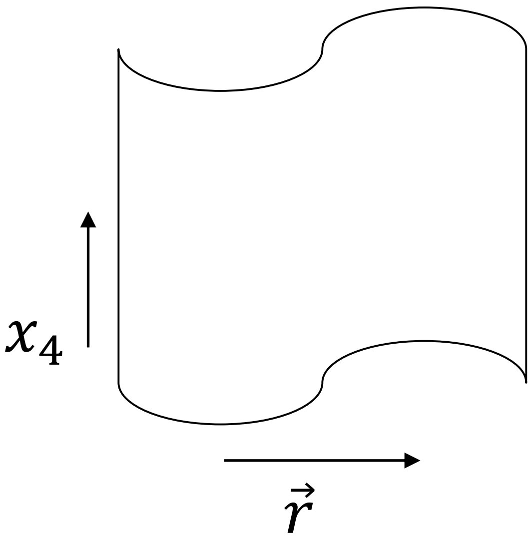
$$Z = \int DU e^{-S_G} \cdot \left(\det(M^\dagger M + \lambda^2) \right)^{\frac{1}{4}}$$

Definitions

Wilson loop

$$W(C) = \frac{1}{N_c} Tr\, \{ \, P \exp(i \oint_C \; dx_\mu \, A_\mu(x)) \, \}$$

To compute $V_{\bar{q}q}(r)$ the contour *C* is



$$< W(r,t) >= \; C_0 e^{-E_0(r)t} + \; C_1 e^{-E_1(r)t} + \; ...$$

$$E_0(r) = V_{\bar{q}q}(r)$$

$$V_{\bar{q}q}(r) = \; -\lim_{t \to \infty} \frac{1}{t} \log < W(r,t) >$$

Spectral representation of WL.

Confinement phase:

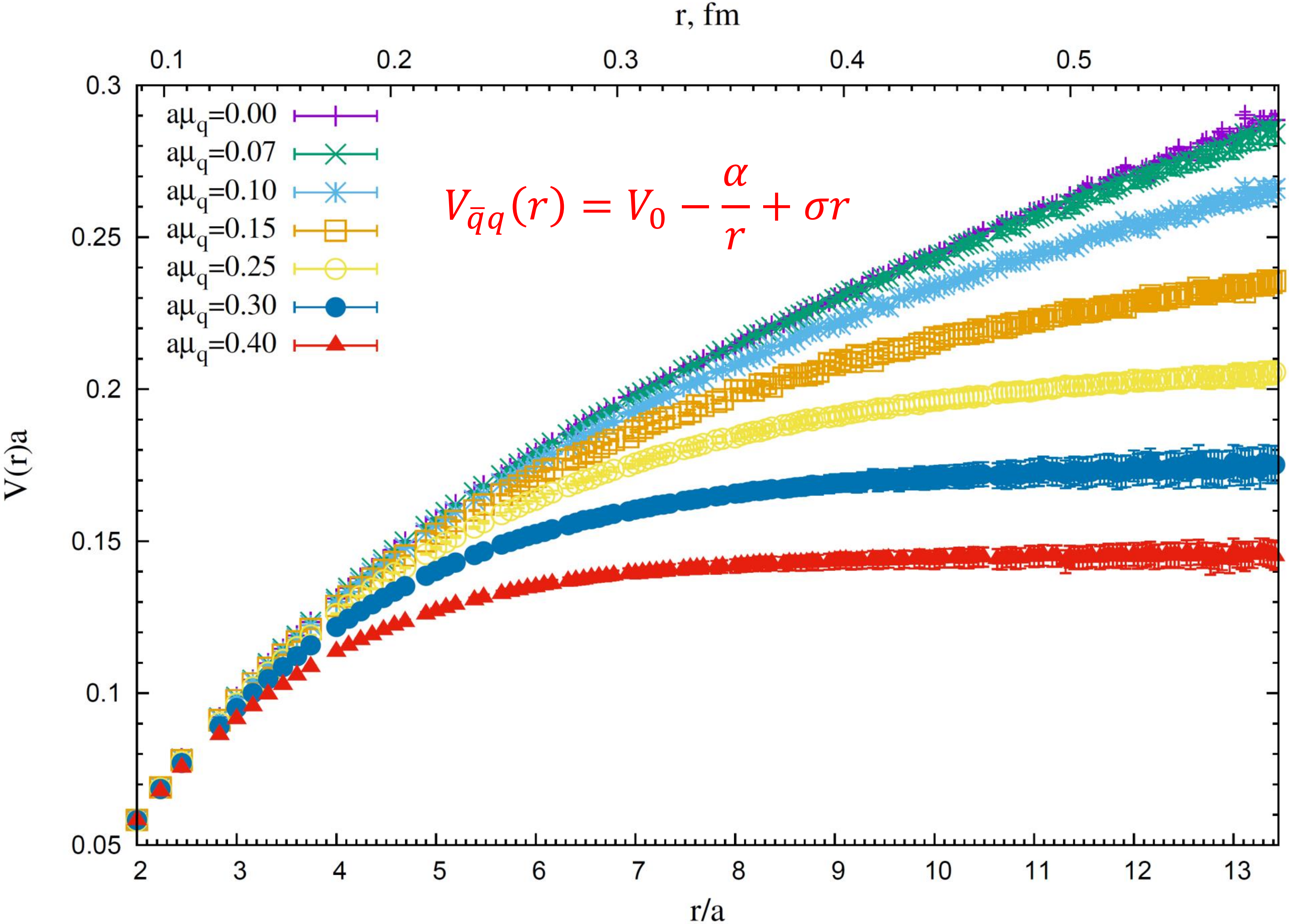
Ground state – hadron string up to distance r_sb, then – 2 h-l mesons

But WL has very small overlap with h-l meson state, C_hl <<1

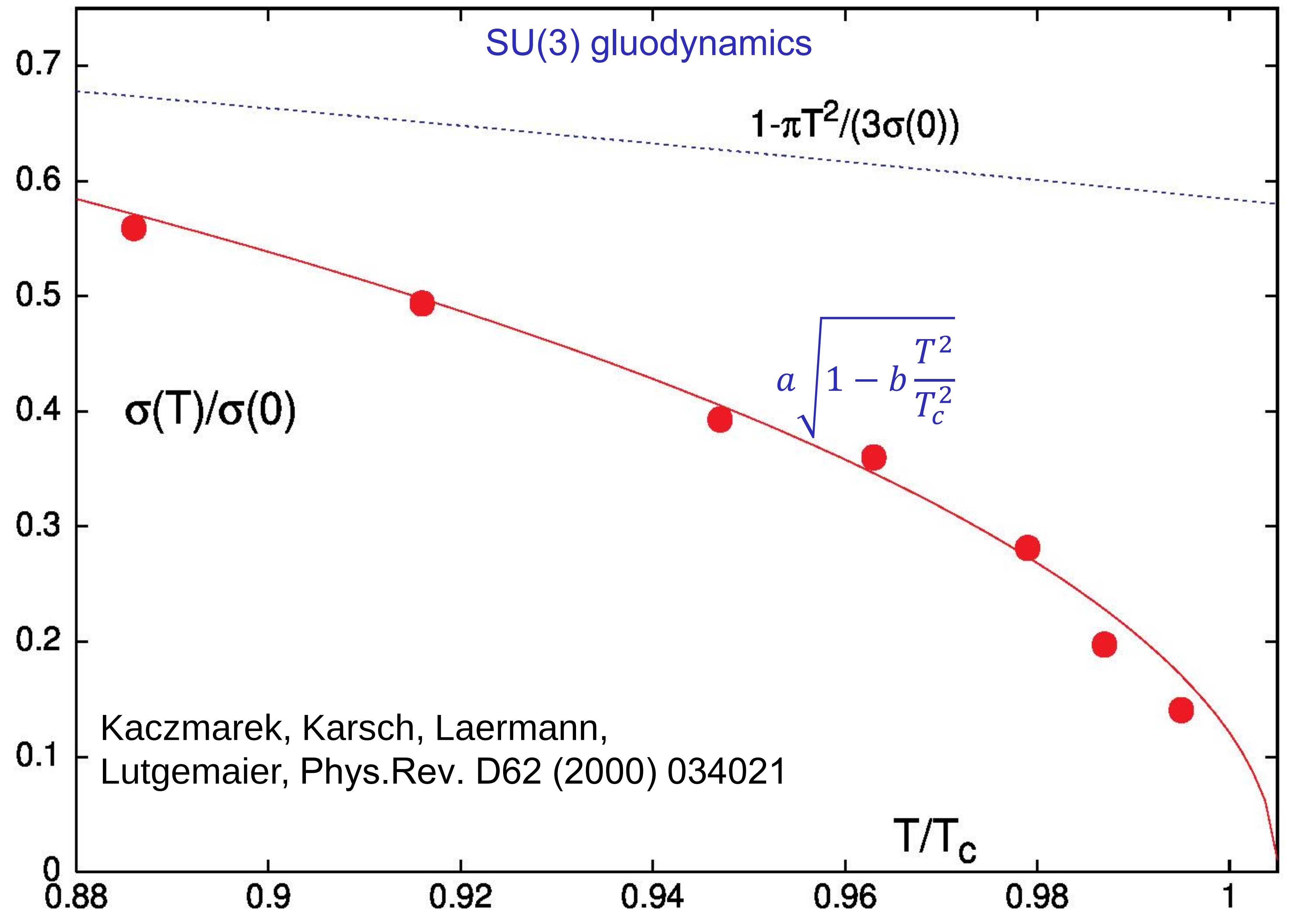
For this reason we do not see string breaking, but clearly see hadron string state

Deconfinement phase:

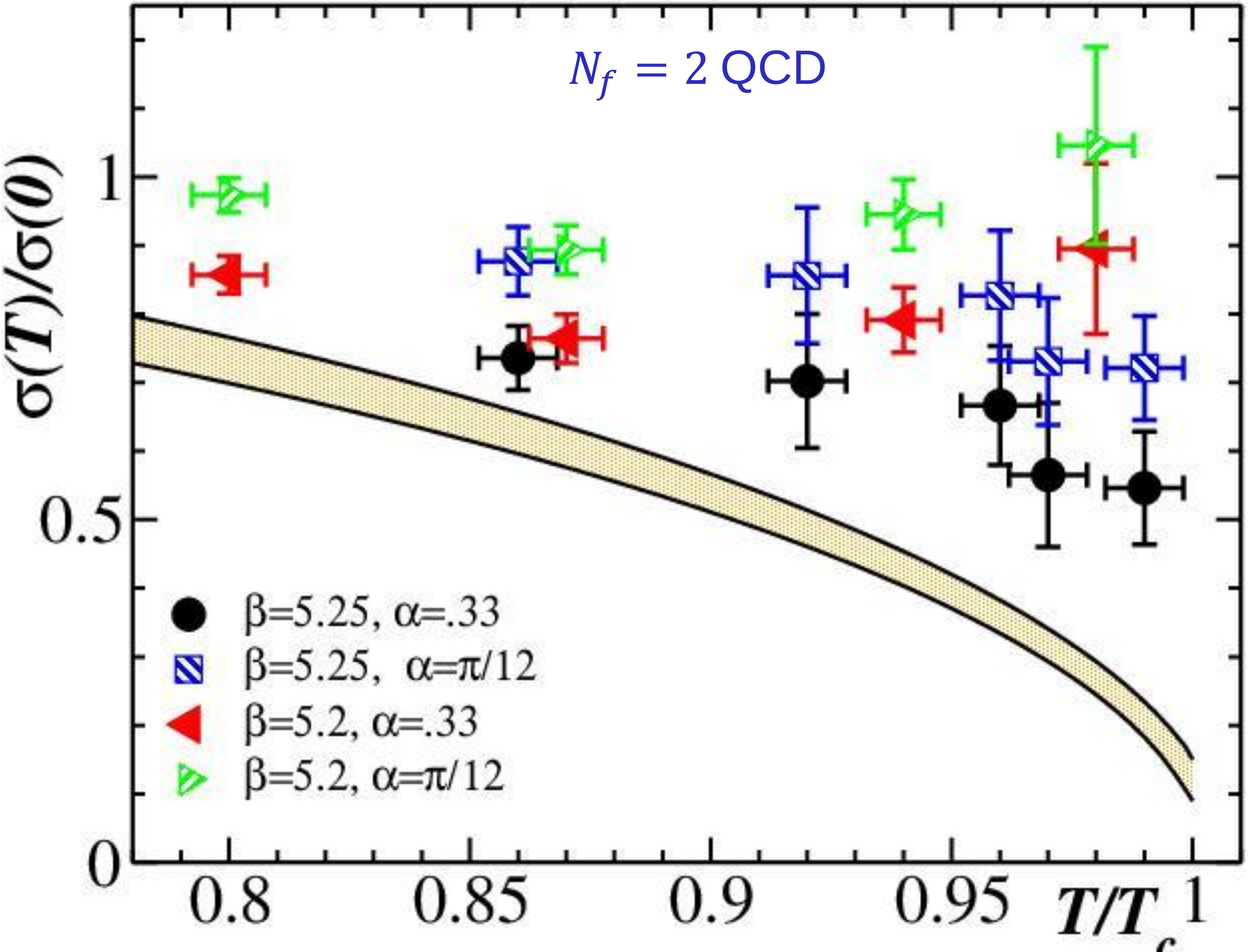
Ground state – color interaction is screened, Debye screening



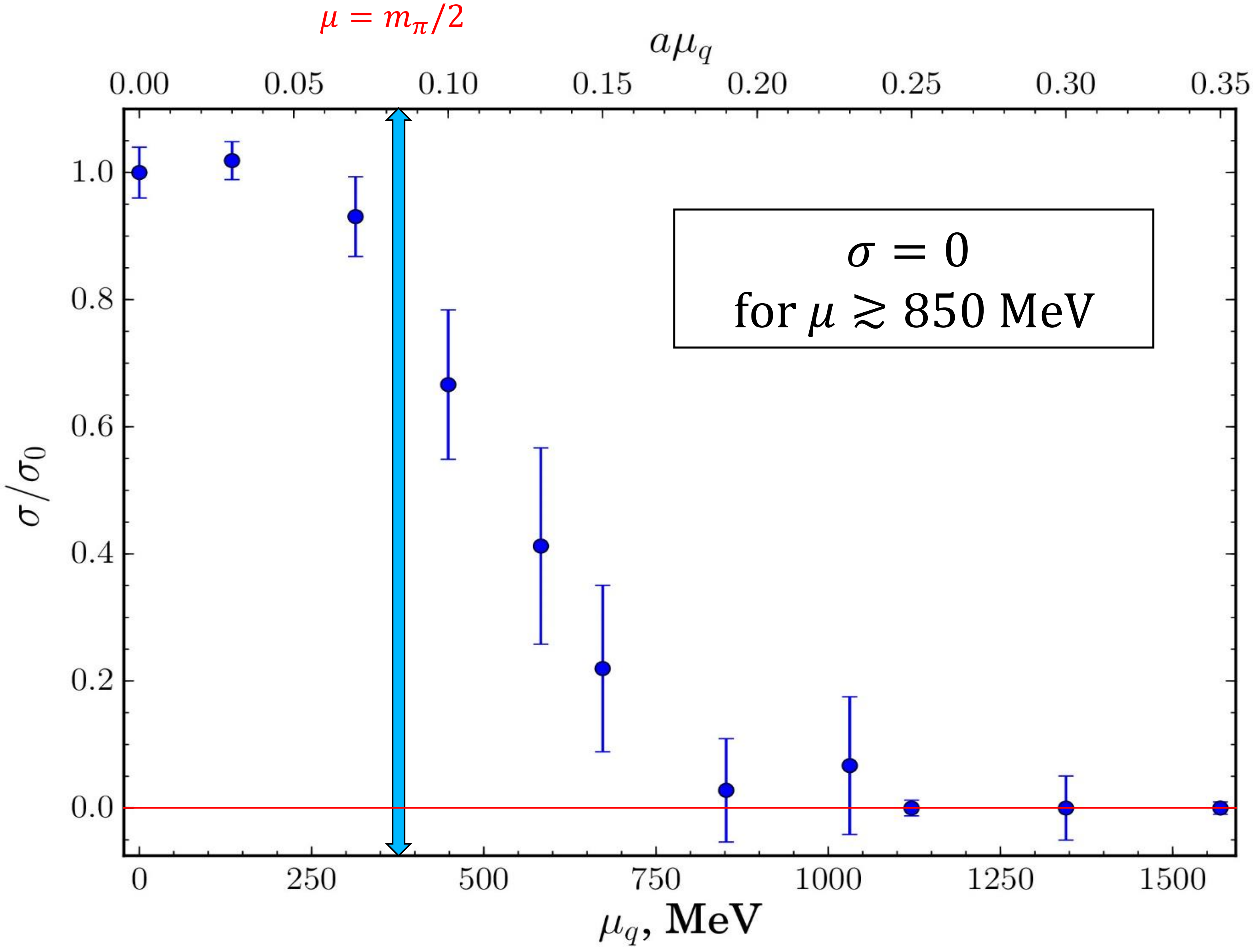
r, fm
0.1
0.2
0.3
0.4
0.5
0.3
0.25
0.2
0.15
0.1
0.05
V(r)a
aμq=0.00
aμq=0.07
aμq=0.10
aμq=0.15
aμq=0.25
aμq=0.30
aμq=0.40
$V_{\bar{q}q}(r) = V_0 - \frac{\alpha}{r} + \sigma r$
2
3
4
5
6
7
8
9
10
11
12
13
r/a



SU(3) gluodynamics
1-πT²/(3σ(0))
σ(T)/σ(0)
a √(1 − b T²/T_c²)
Kaczmarek, Karsch, Laermann,
Lutgemaier, Phys.Rev. D62 (2000) 034021
T/T_c
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.88
0.9
0.92
0.94
0.96
0.98
1



DIK collaboration, 2004



String tension vs. μ

The confinement-deconfinement transition thus happens in the range

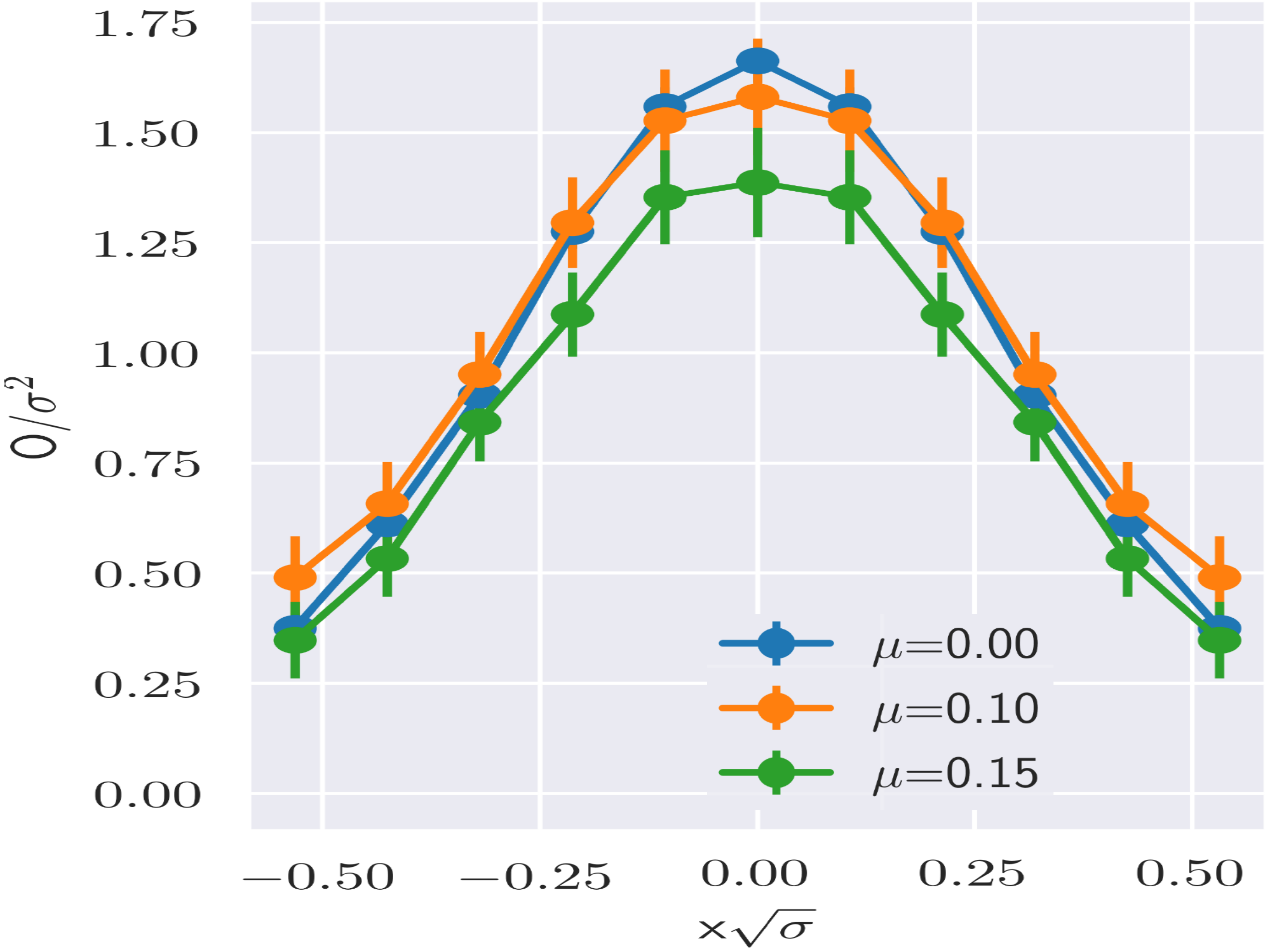
$$850 \text{ MeV} < \mu < 1100 \text{ MeV}$$

(we find screening at $\mu > 1100$ MeV)

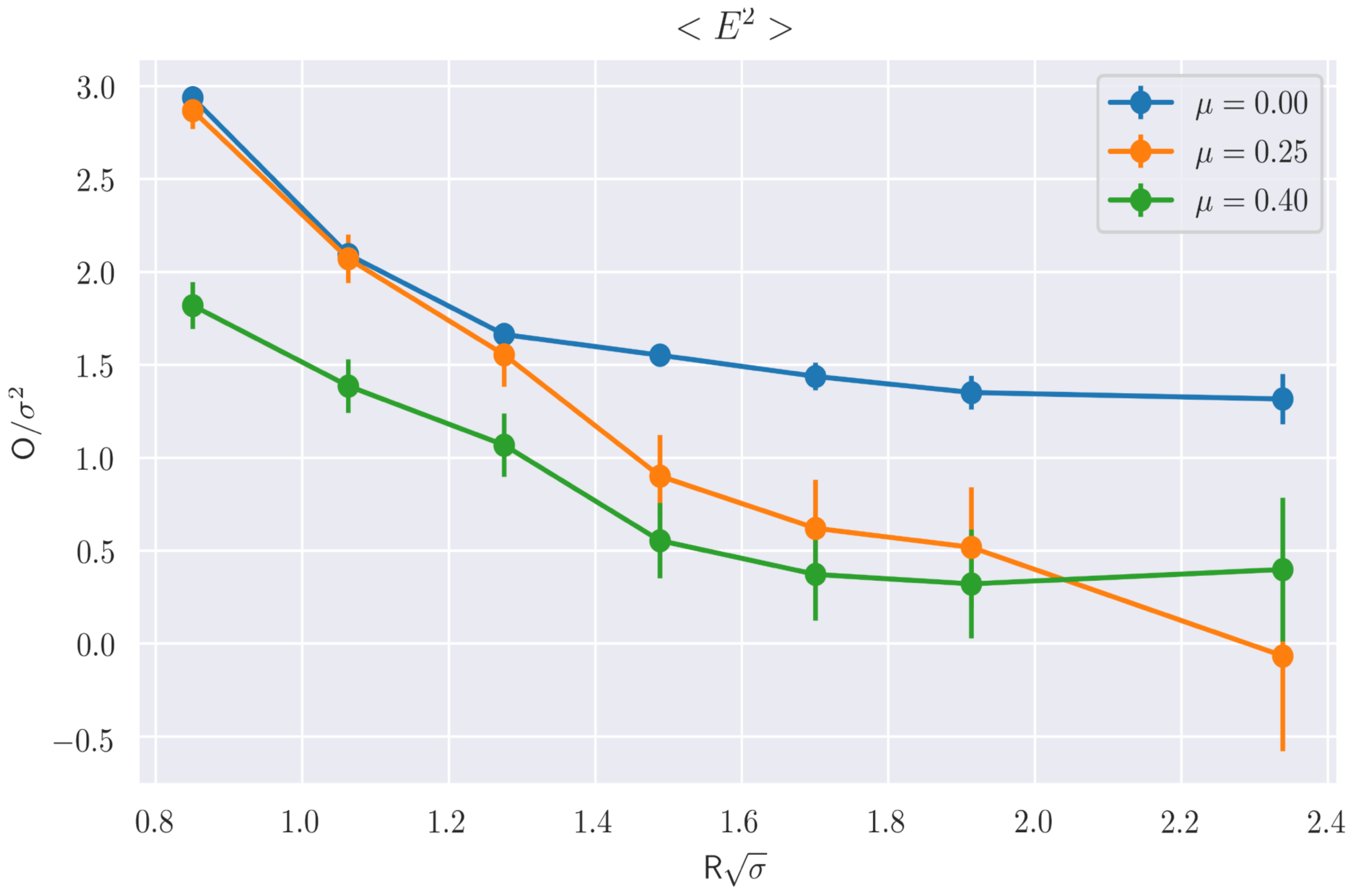
Flux tube

$$< E^2 >_{q\bar{q}} \;, \quad < B^2 >_{q\bar{q}}$$

$$< P >_W = \frac{< W(R,T)\; P(T/2) >}{< W(R,T) >} - < P >$$



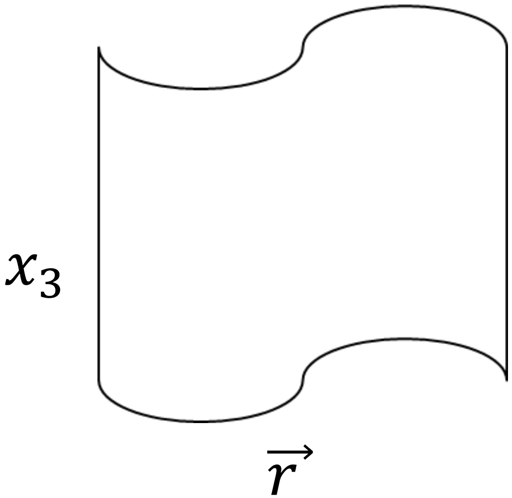
Transverse profile of $< E^2 >_{q\,\bar{q}}$ at distance $\mathrm{R}/a = 12$



$< E^2_{max} >$ $vs. q\bar{q}$ distance *R.* Flux tube is absent at large μ

Spatial string tension

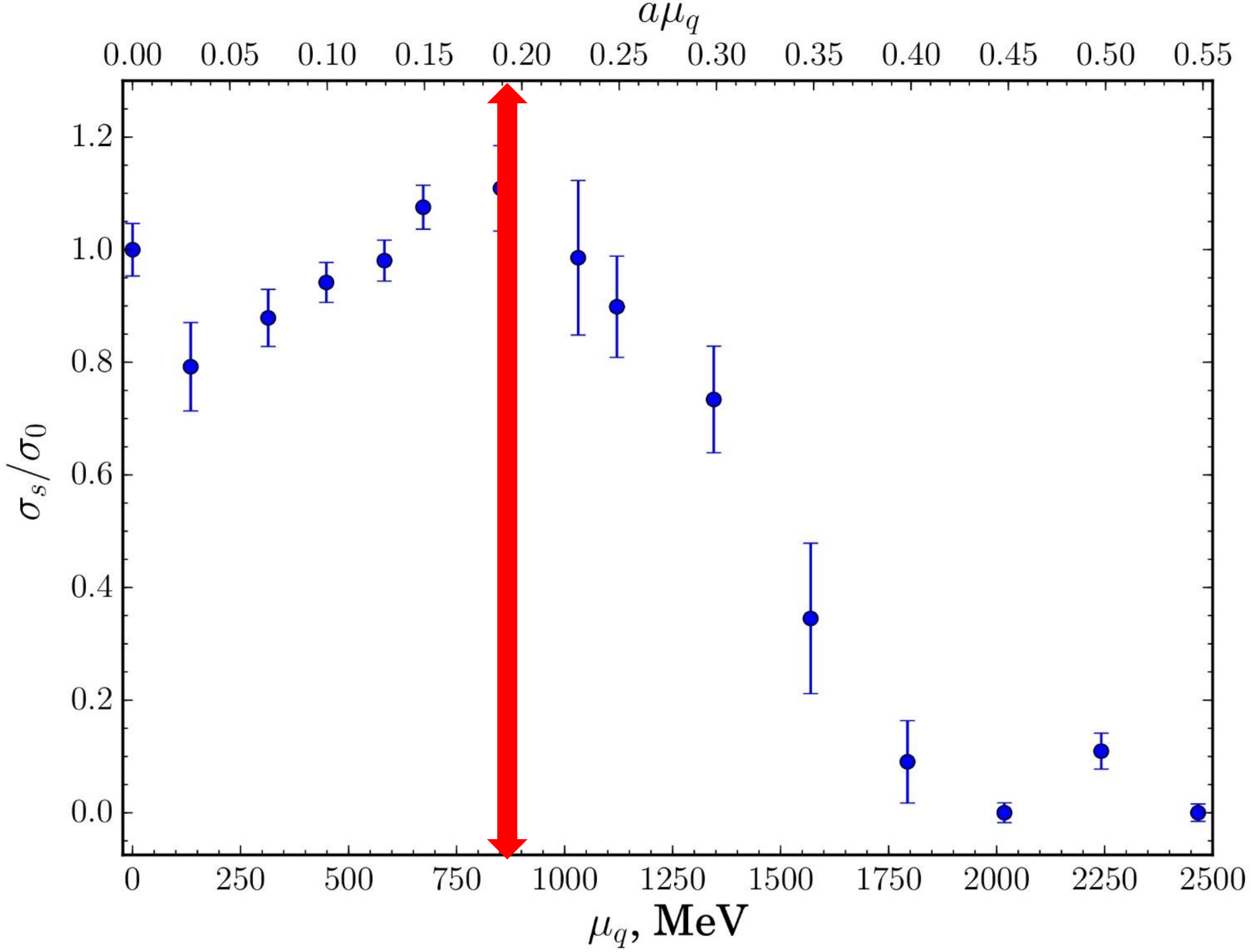
Spatial Wilson loop



$$V_s(r) = V_{0s} - \frac{c}{r} + \sigma_s r$$

At $T > T_c$ σ_s is increasing $\sim g^2 T$ both in SU(2) and SU(3) theories

This is different in QC_2D, see next slide



Spatial string tension

Polyakov loop correlators

(order parameter for heavy quarks)

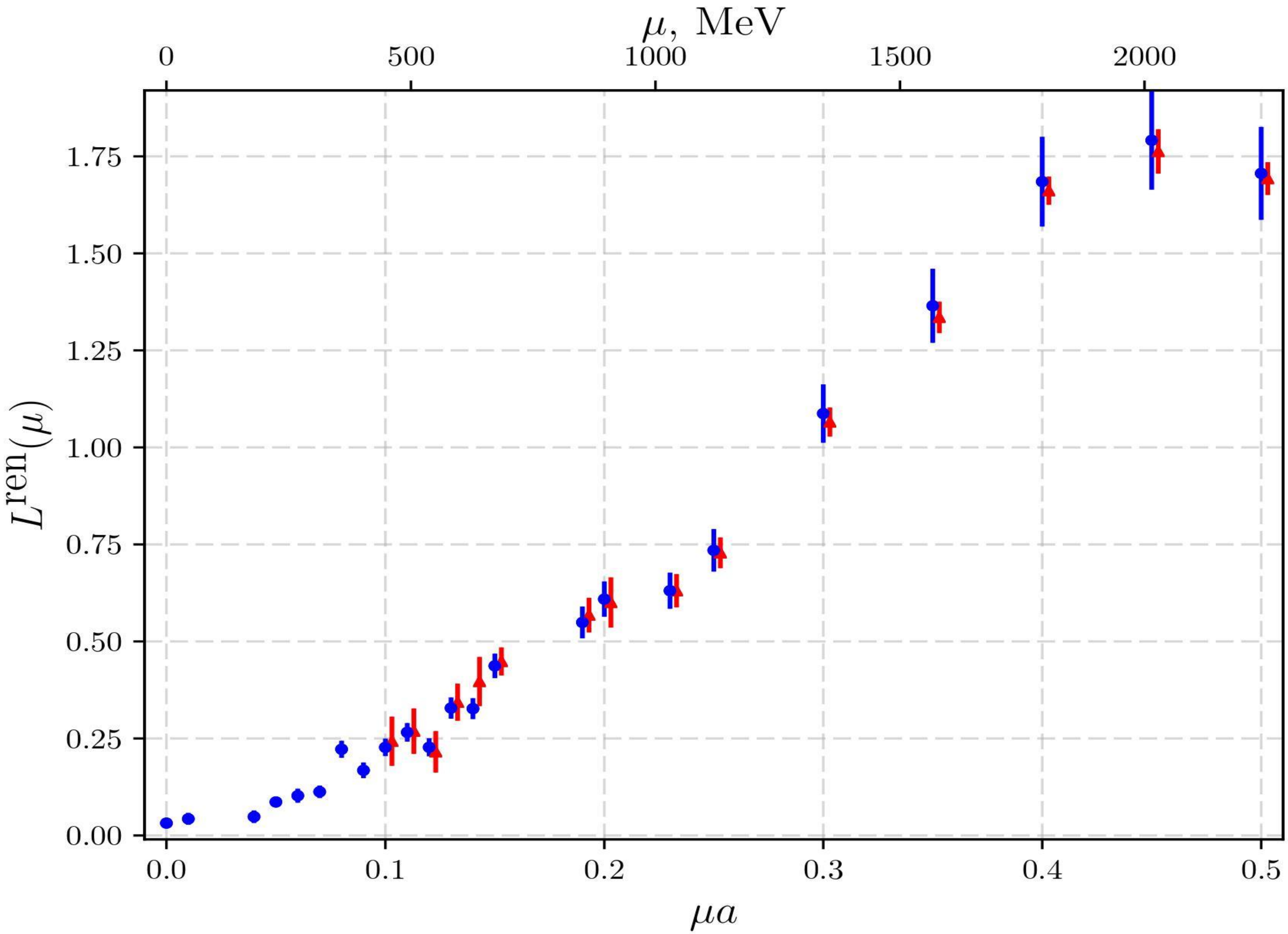
$$L(\vec{r}) = \mathrm{P}\exp\left\{ i \int dx_4\, A_4(\vec{r}, x_4) \right\}$$

$$< L > = < \frac{1}{2}\, TrL(\vec{r}) >$$

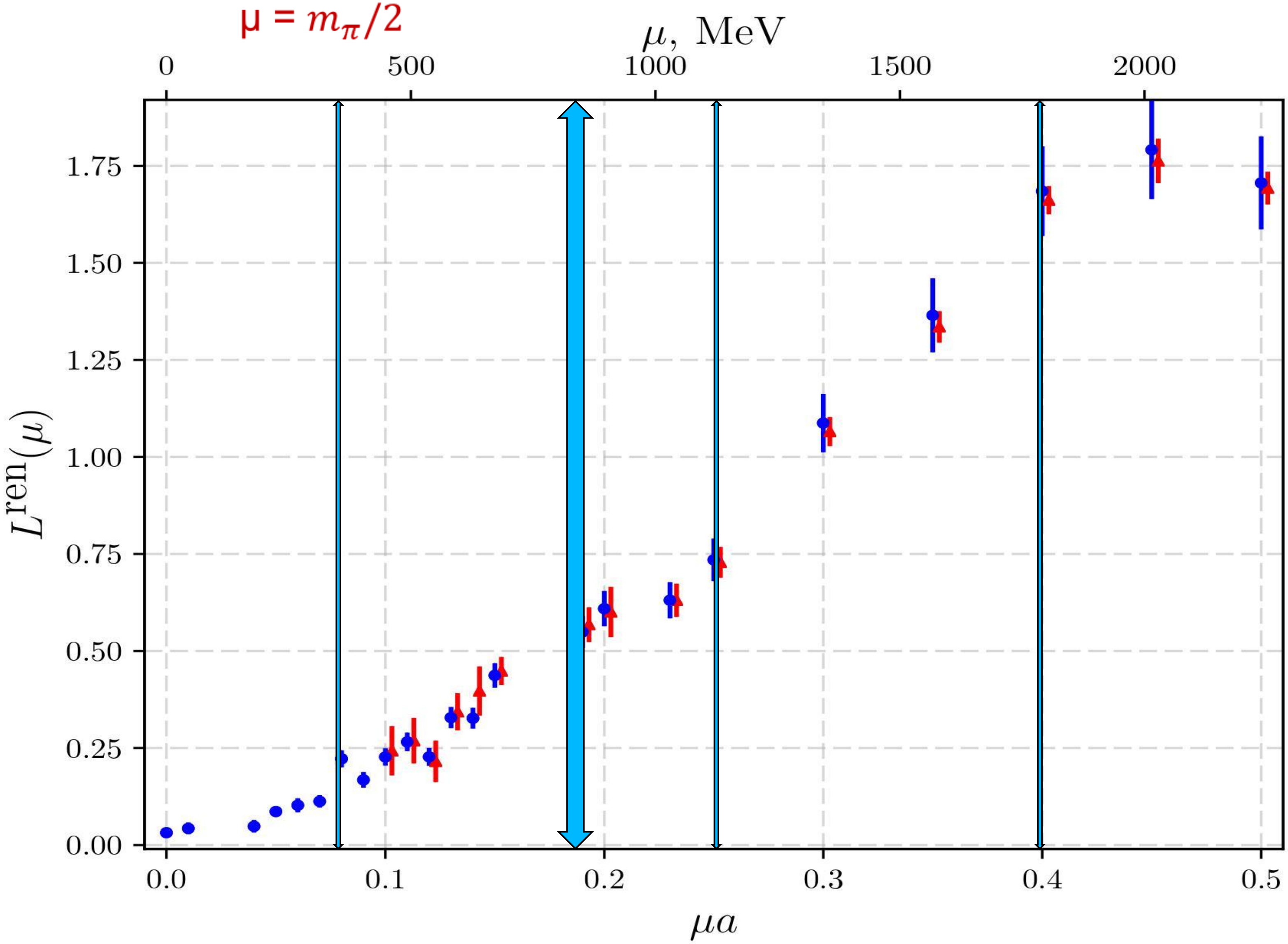
At T>0 it determines free energy of a static source $F_q(T)$.

At $\mu_q > 0$ it determines grand potential $\Omega(\mu_q$,T).

Polyakov loop vs. μ



Polyakov loop vs. μ



Polyakov loop correlators allow to study static quarks interaction in vacuum or in medium as in our case

$$\exp\left[-\frac{\Omega_{\bar{q}q}(r,\mu)}{T}\right] = \frac{1}{4}\left\langle \mathrm{Tr}L(\vec{r})\mathrm{Tr}L^\dagger(0)\right\rangle ,$$

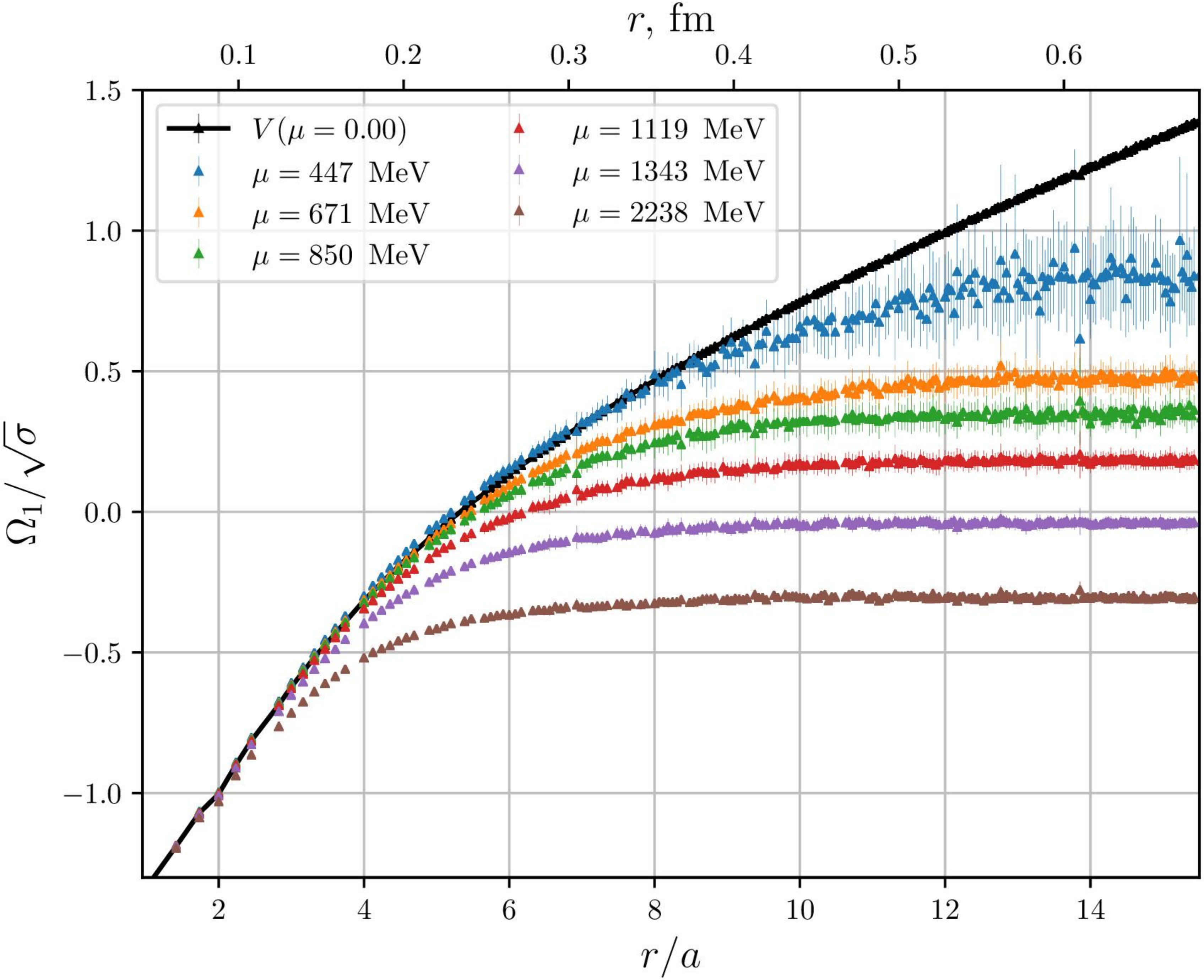
$$\exp\left[-\frac{\Omega_1(r,\mu)}{T}\right] = \frac{1}{2}\left\langle \mathrm{Tr}L(\vec{r})L^\dagger(0)\right\rangle ,$$

$$\exp\left[-\frac{\Omega_3(r,\mu)}{T}\right] = \frac{1}{3}\left\langle \mathrm{Tr}L(\vec{r})\mathrm{Tr}L^\dagger(0)\right\rangle - \frac{1}{6}\left\langle \mathrm{Tr}L(\vec{r})L^\dagger(0)\right\rangle$$

S. Nadkarni, Phys. Rev. D 34 (1986) 3904, O. Philipsen, Phys. Lett. B 535 (2002) 138

In perturbation theory

$$\Omega_1(r,\mu) = -3\ \Omega_3(r,\mu) = -\frac{g^2(r)}{8\pi r}$$



r, fm
0.1
0.2
0.3
0.4
0.5
0.6
1.5
1.0
0.5
0.0
−0.5
−1.0
$\Omega_1/\sqrt{\sigma}$
$V(\mu = 0.00)$
$\mu = 447$ MeV
$\mu = 671$ MeV
$\mu = 850$ MeV
$\mu = 1119$ MeV
$\mu = 1343$ MeV
$\mu = 2238$ MeV
2
4
6
8
10
12
14
r/a

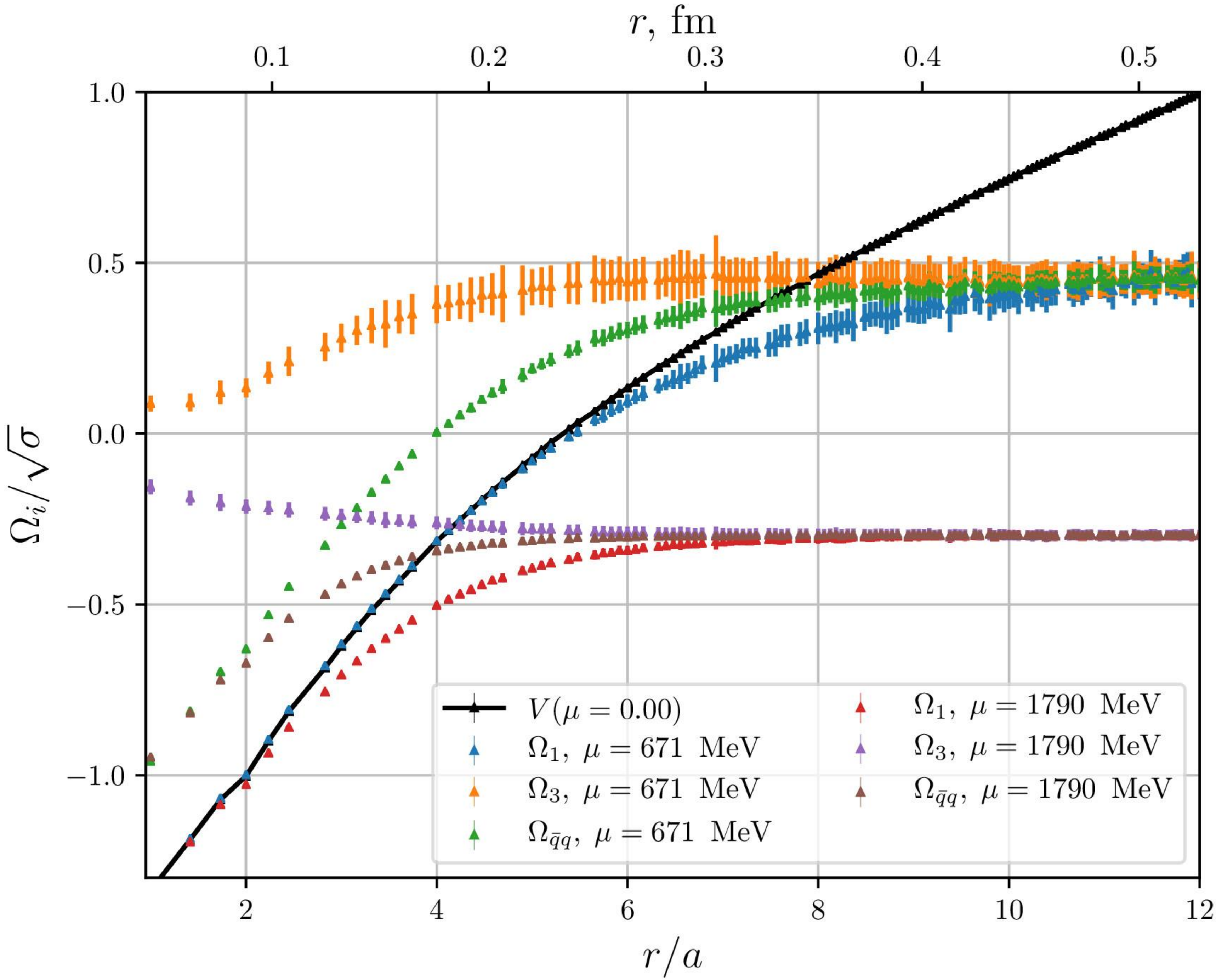
Behavior of $\Omega_1(\mu, r)$ is similar to that of free energy $F_1(T, r)$:

- At small distances they agree with $V(r)$, this agreement stops at smaller distances for larger μ or T

- At large distances they flatten. This flattening signals string breaking in the confinement phase and screening in the deconfinement phase

Thus at μ=0.447 MeV and 671 MeV we observe **string breaking** at **T=0** in a **theory with fundamental fermions** using **Polyakov loop correlator**.

So far to observe string breaking at T=0 operators mixing hadronic string and static-light mesons were used.



r, fm
0.1
0.2
0.3
0.4
0.5
1.0
0.5
0.0
−0.5
−1.0
$\Omega_i/\sqrt{\sigma}$
2
4
6
8
10
12
r/a
$V(\mu = 0.00)$
$\Omega_1,\ \mu = 671$ MeV
$\Omega_3,\ \mu = 671$ MeV
$\Omega_{\bar{q}q},\ \mu = 671$ MeV
$\Omega_1,\ \mu = 1790$ MeV
$\Omega_3,\ \mu = 1790$ MeV
$\Omega_{\bar{q}q},\ \mu = 1790$ MeV

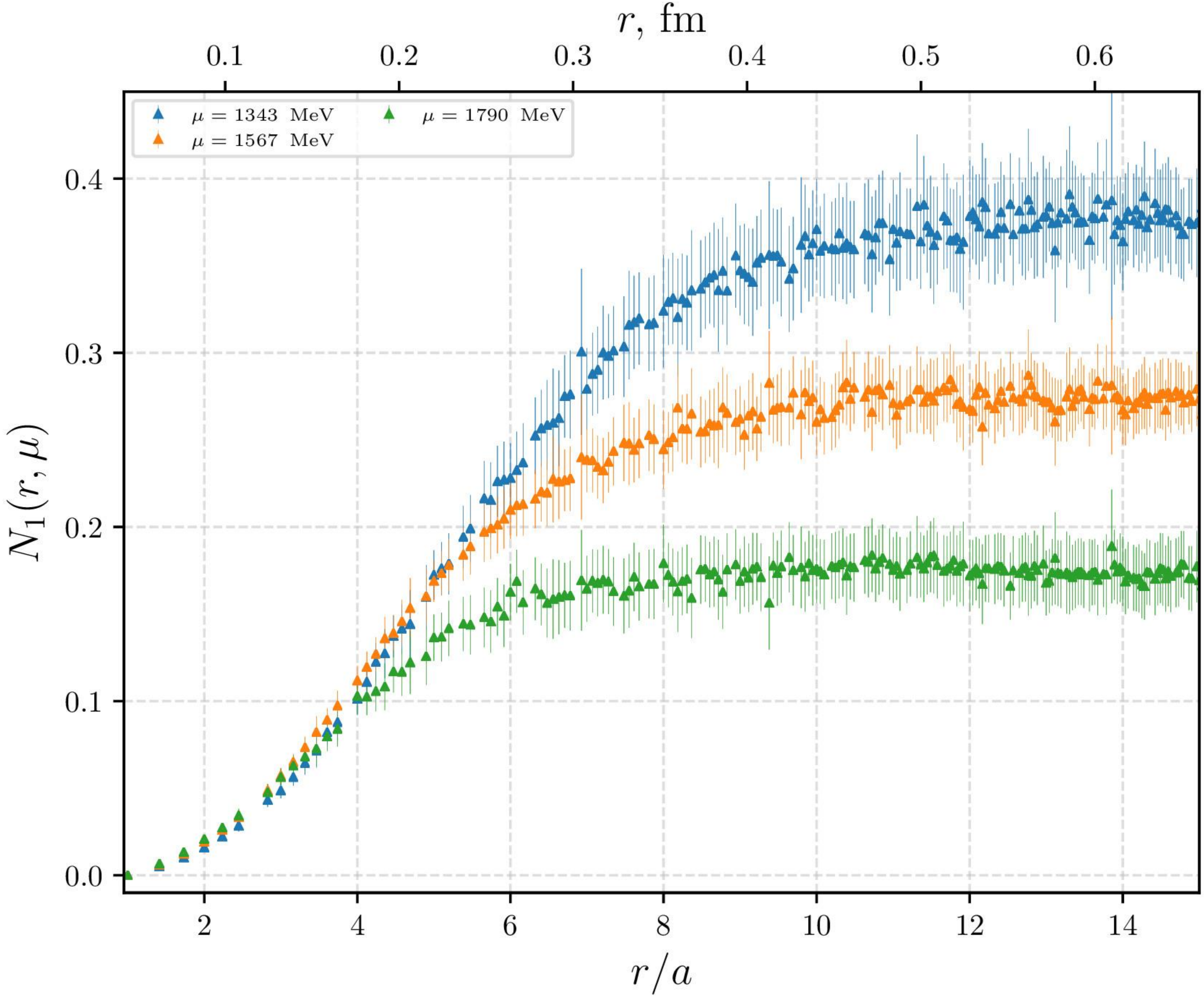
Number density

$$\Omega_1(r,\mu,T) = U_1(r,\mu,T) - TS_1(r,\mu,T) - \mu N_1(r,\mu,T)$$

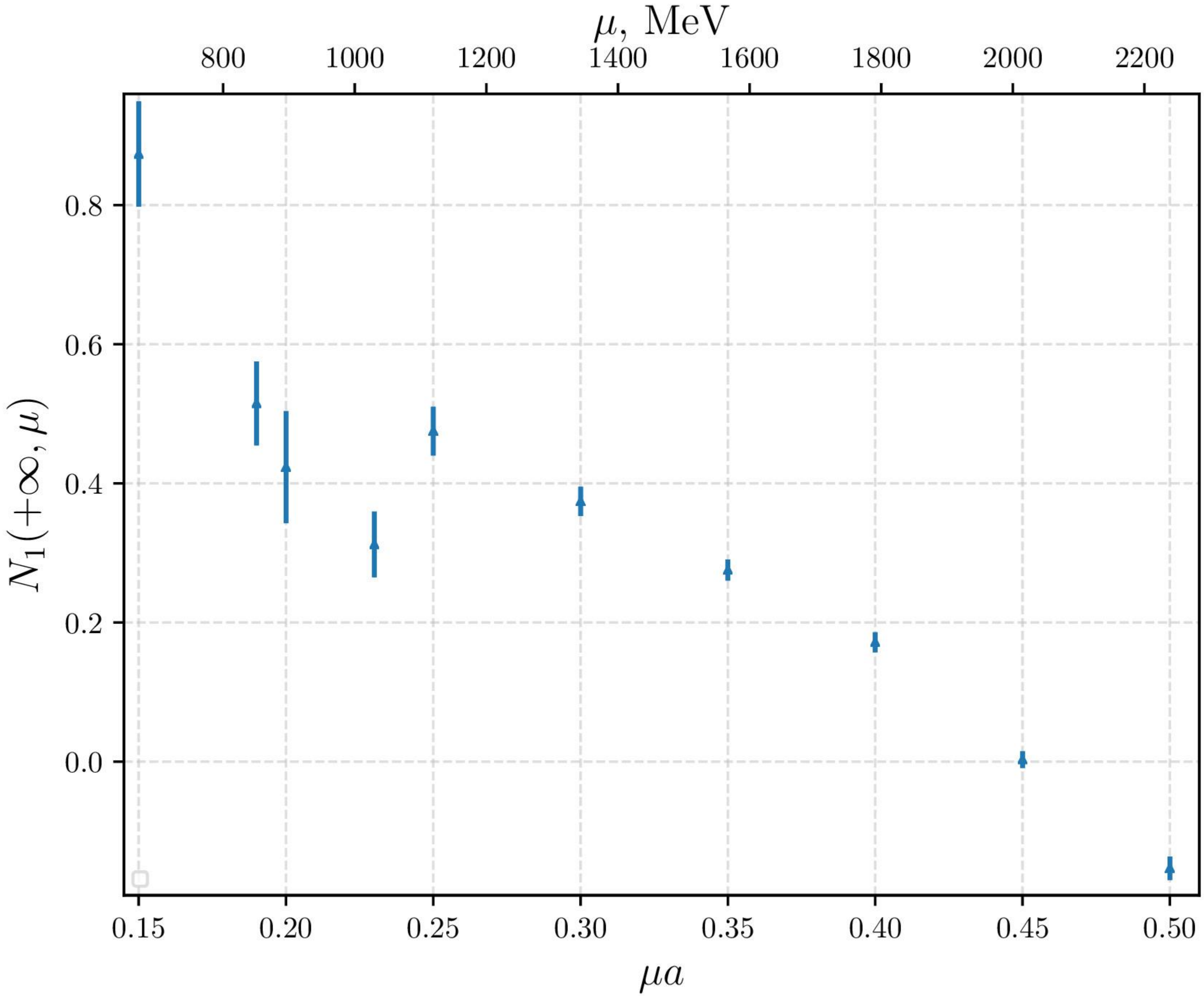
$$N_1(r,\mu) = -\frac{\partial \Omega_1(r,\mu)}{\partial \mu}$$

(numerical differentiation)

Important quantity $N_1(\infty,\mu)$ - determines variation of the Polyakov loop. Expected to have maximum at transition.



r, fm
0.1
0.2
0.3
0.4
0.5
0.6
$\mu = 1343$ MeV
$\mu = 1567$ MeV
$\mu = 1790$ MeV
$N_1(r, \mu)$
0.0
0.1
0.2
0.3
0.4
2
4
6
8
10
12
14
r/a



μ, MeV
800
1000
1200
1400
1600
1800
2000
2200
0.8
0.6
0.4
0.2
0.0
$N_1(+\infty, \mu)$
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
μa

Screening length

We introduce screening length R_{sc} defined for all μ (in analogy with T>0 case) as

$$V_{\mu=0}(R_{sc}) = \Omega_1(\infty, \mu)$$

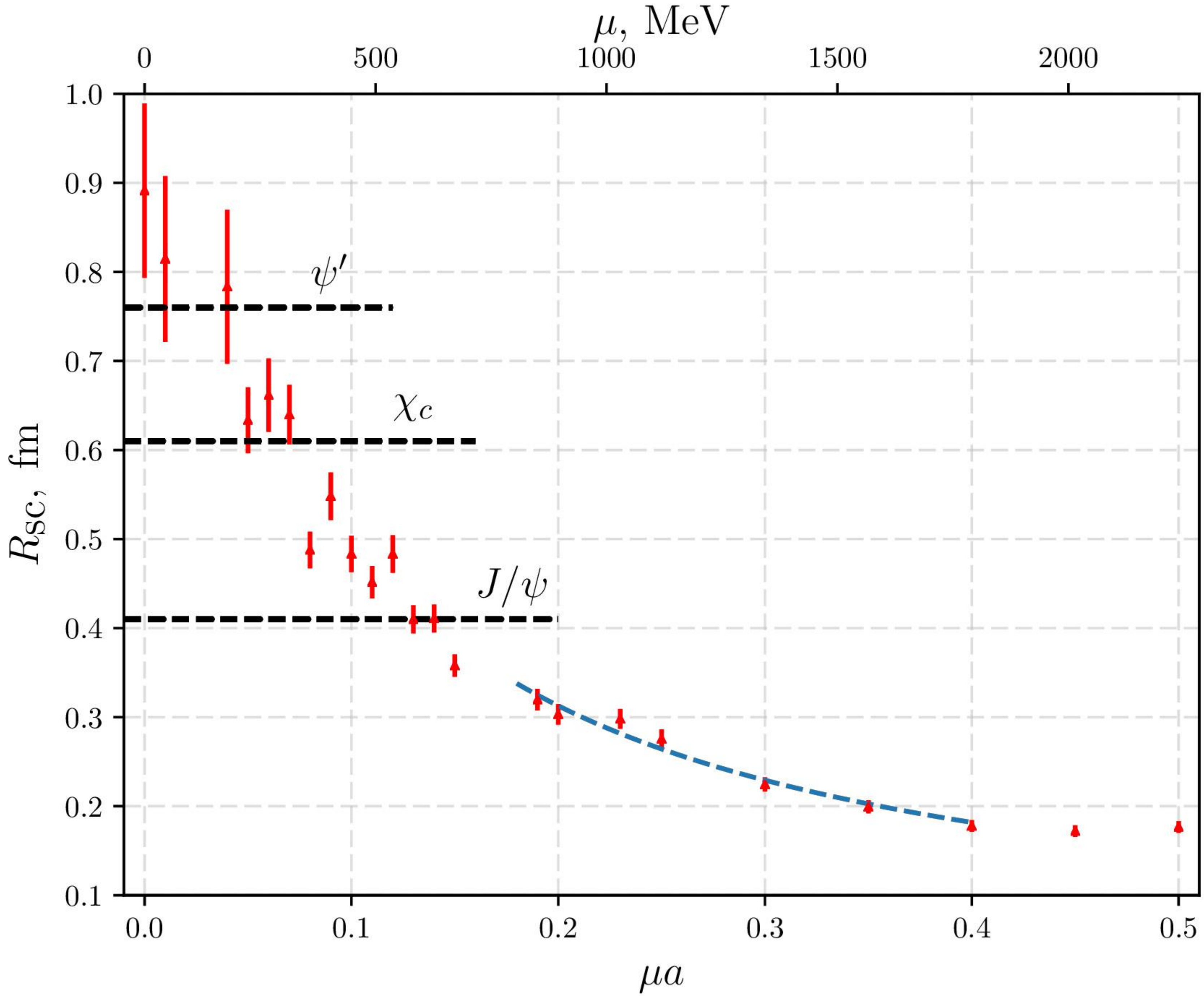
Kaczmarek, Karsch, Petreczky, Zantow Phys.Lett. B543 (2002) 41

Perturbation theory gives for screening mass

$$m_D^2(\mu) = \frac{4}{\pi}\alpha_s(\mu)\mu^2$$

for large μ we fit R_{sc} by

$$R_{sc} = \frac{1}{A m_D(\mu)}$$

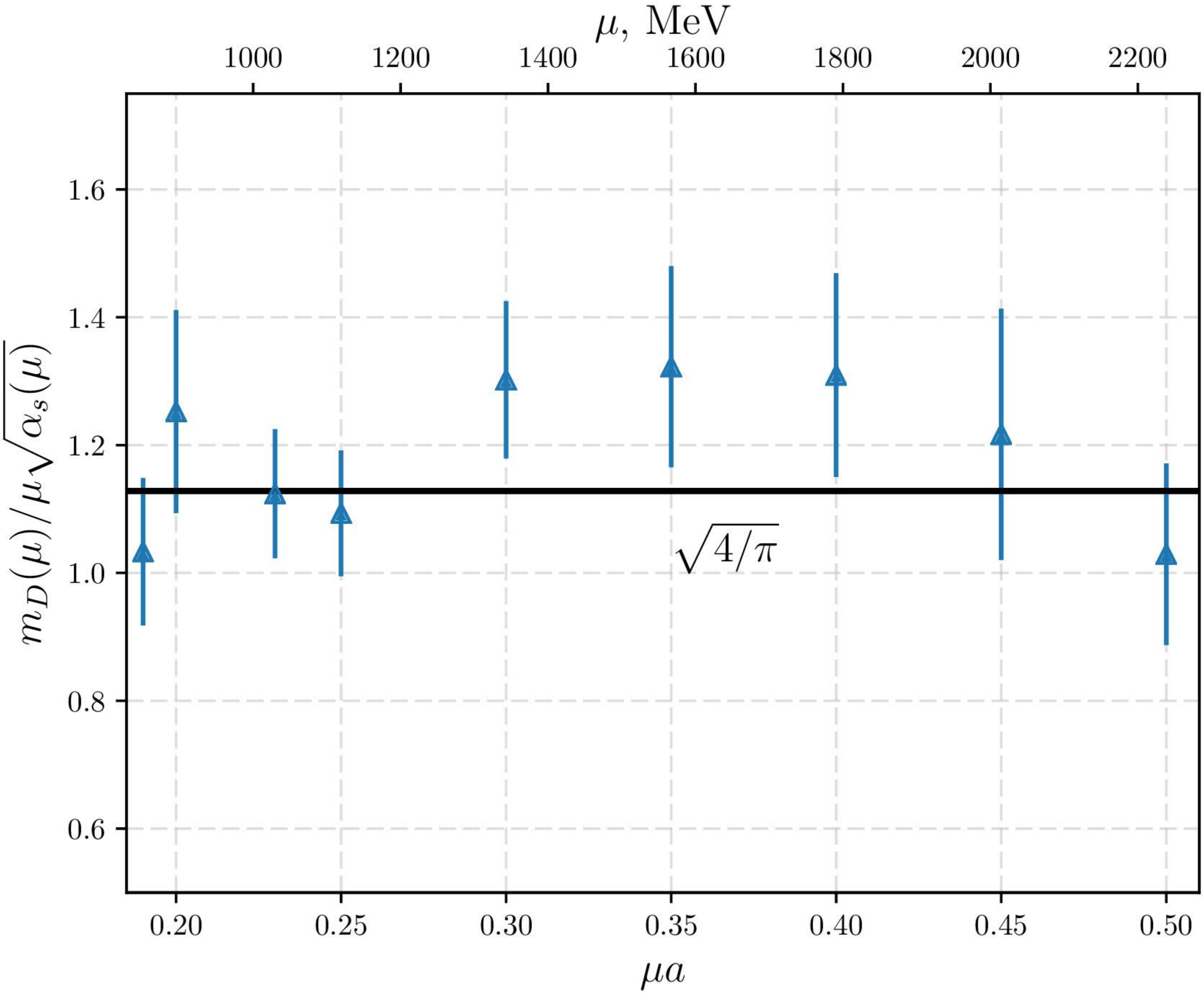


μ, MeV
0
500
1000
1500
2000
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$R_{\rm SC}$, fm
ψ'
χ_c
J/ψ
0.0
0.1
0.2
0.3
0.4
0.5
μa

Screening mass

$$\Omega_1(r,\mu) = \Omega_1(\infty,\mu) - \frac{3}{4}\frac{\alpha_s(\mu)}{r}e^{-m_D r}$$

$$m_D^2(\mu) = \frac{4}{\pi}\alpha_s(\mu)\mu^2$$



μ, MeV
1000
1200
1400
1600
1800
2000
2200
1.6
1.4
1.2
1.0
0.8
0.6
$m_D(\mu)/\mu\sqrt{\alpha_s(\mu)}$
$\sqrt{4/\pi}$
0.20
0.25
0.30
0.35
0.40
0.45
0.50
μa

Conclusions

- Confinement-deconfinement transition range of μ_q values was determined by string tension computation:

850 MeV < μ_q < 1100 MeV

- It was discovered that the spatial string tension σ_s goes to zero in the deconfinement phase at $\mu_q > 2000$ MeV

- Number density and internal energy were computed for static pair of quark and anti-quark

- String breaking distance and Debye screening length were computed. Some agreement with perturbation theory was found

Outlook

Simulations in $N_f = 2$ *SU(2)* QCD are planned

- with improved staggered Dirac operator (stout smearing)

- small lattice spacing (~0.05 fm)
- bigger volume (~2 fm)
- smaller pion mass

To confirm our findings, to clarify possible volume effects, to uncover physics at $\mu > 2000$ MeV

Study of mixed effects of temperature and chemical potential is also planned

Study of the analytic continuation from imaginary μ to real μ is going on