

# The II International Workshop on Theory of Hadronic Matter Under Extreme Conditions

Dubna, JINR  
16-19 September 2019

## Symmetries, topology and the fate of the $\eta'$ in the QGP

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INFN Firenze

Florian Burger, Ernst-Michael Ilgenfritz, MpL and Anton Trunin Phys. Rev. D 98, 094501 (2018)

Andrey Kotov, MpL, Anton Trunin, Phys.Lett. B794 (2019)



Istituto Nazionale di Fisica Nucleare  
SEZIONE DI FIRENZE



# Motivation I:

QCD Lagrangian symmetries:

Always exact

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_A \times U(1)_B$$

Breaking/restoration  
at  $T_c$

studied a lot  
on the lattice

Always broken if topological charge  
fluctuates!

DOES IT?

BUT:

the 'amount' of breaking,  
may  
depend on temperature!

IMPLICATIONS?

HOW ARE THESE RELATED??

# Motivation II

PHYSICAL REVIEW D

VOLUME 53, NUMBER 9

1 MAY 1995

## Return of the prodigal Goldstone boson

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(Received 14 July 1995)

We propose that the mass of the  $\eta'$  meson is a particularly sensitive probe of the properties of finite energy density hadronic matter and quark-gluon plasma. We argue that the mass of the  $\eta'$  excitation in hot and dense matter should be small, and, therefore, that the  $\eta'$  production cross section should be much increased relative to that for  $pp$  collisions. This may have observable consequences in dilepton and diphoton experiments.

# Plan

Introduction

Results:

Thermodynamics with twisted Wilson fermions

Topology (with a detour on the QCD axion)

$\eta'$

Conclusions&Outlook

# QCD topology and phenomenology

Hadron cosmology:

Origin of mass

**Almost all hadrons can be described taking into account chiral symmetry breaking and confining potential**

Quarks

Hadrons

Nuclei

*time*

QCD transition

Nucleosynthesis

Chiral symm. breaking  
Confinement:

Chiral perturbation theory +  
Potential models  
=  
Hadron spectrum

# Hadron cosmology: Origin of mass

Almost all hadrons can be described  
taking into account  
chiral symmetry breaking  
and confining potential

**With an important  
exception**



QCD transition

Nucleosynthesis

Chiral symm. breaking  
Confinement:  
  
Chiral perturbation theory +  
Potential models  
=  
Hadron spectrum

Pseudoscalar light spectrum:  
eight pseudoGoldstones

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$$

$\chi PT$  predicts

$$m_\pi^2 \propto (m_u + m_d) \Lambda_{QCD}$$

$$m_K^2 \propto (m_s + m_{u,d}) \Lambda_{QCD}$$

$$m_\eta^2 \propto \frac{1}{3} (m_u + m_d + 4m_s) \Lambda_{QCD} ,$$

Particle name	Particle symbol $\blacklozenge$	Antiparticle symbol $\blacklozenge$	Quark content	Rest mass (MeV/c <sup>2</sup> ) $\blacklozenge$
Pion <sup>[6]</sup>	$\pi^+$	$\pi^-$	$u\bar{d}$	139.570 18 $\pm$ 0.000 35
Pion <sup>[7]</sup>	$\pi^0$	Self	$\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$ [a]	134.9766 $\pm$ 0.0006
Eta meson <sup>[8]</sup>	$\eta$	Self	$\frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$ [a]	547.862 $\pm$ 0.018
Eta prime meson <sup>[9]</sup>	$\eta'(958)$	Self	$\frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}}$ [a]	<u>957.78 <math>\pm</math> 0.06</u>
Kaon <sup>[12]</sup>	$K^+$	$K^-$	$u\bar{s}$	493.677 $\pm$ 0.016
Kaon <sup>[13]</sup>	$K^0$	$\bar{K}^0$	$d\bar{s}$	497.614 $\pm$ 0.024

$U(1)_A$

should be broken  
as well  
producing a 9th  
Goldstone BUT:

← **Exception!**

$\eta'$  is too heavy

Topology,  $\eta'$  and the  $U_A(1)$  problem:

The  $U_A(1)$  symmetry  $q \rightarrow e^{i\alpha\gamma_5} q$

would be broken by the (spontaneously generated)  $\bar{q}q$  :

the candidate Goldstone is the  $\eta'$

too heavy!! (900 MeV)

BUT:

the divergence of the current  $j_5^\mu = \bar{q}\gamma_5\gamma_\mu q$ ,

contains a mass independent term

$$\partial_\mu j_5^\mu = m\bar{q}\gamma_5 q + \frac{1}{32\pi^2} F\tilde{F}.$$

IF 
$$\frac{1}{32\pi^2} \int d^4x F\tilde{F} \neq 0$$

The  $U_A(1)$  symmetry is **explicitly** broken

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c <sup>2</sup> )
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Topology,  $\eta'$  and the  $U_A(1)$  problem:

It can be proven that

$$\frac{1}{32\pi^2} \int d^4x F \tilde{F} = Q$$

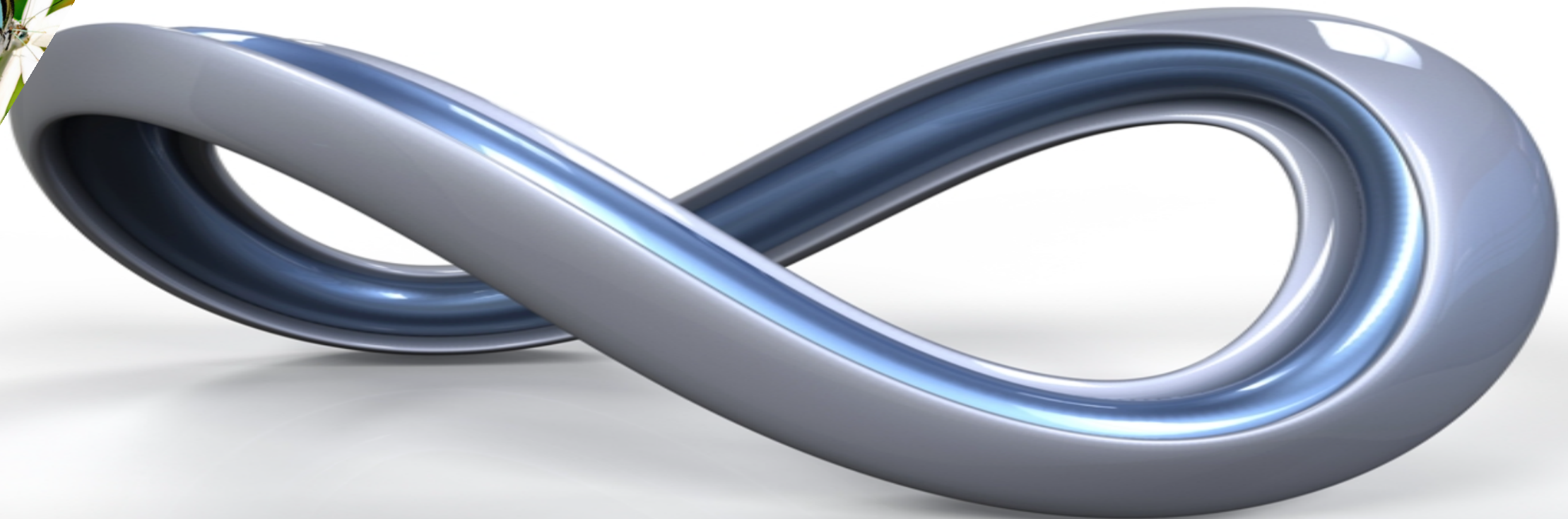
**Gluonic definition**

and

$$Q = n_+ - n_-$$

**Fermionic definition**

$F \tilde{F}$



Topology,  $\eta'$  and the  $U_A(1)$  problem:

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**Gluonic definition**

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$$Q = n_+ - n_-$$

**Fermionic definition**

The  $\eta'$  mass may now be computed from the decay of the correlation

$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x) \tilde{F}(x) F(y) \tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2} \chi_t^{\text{qu}}$$

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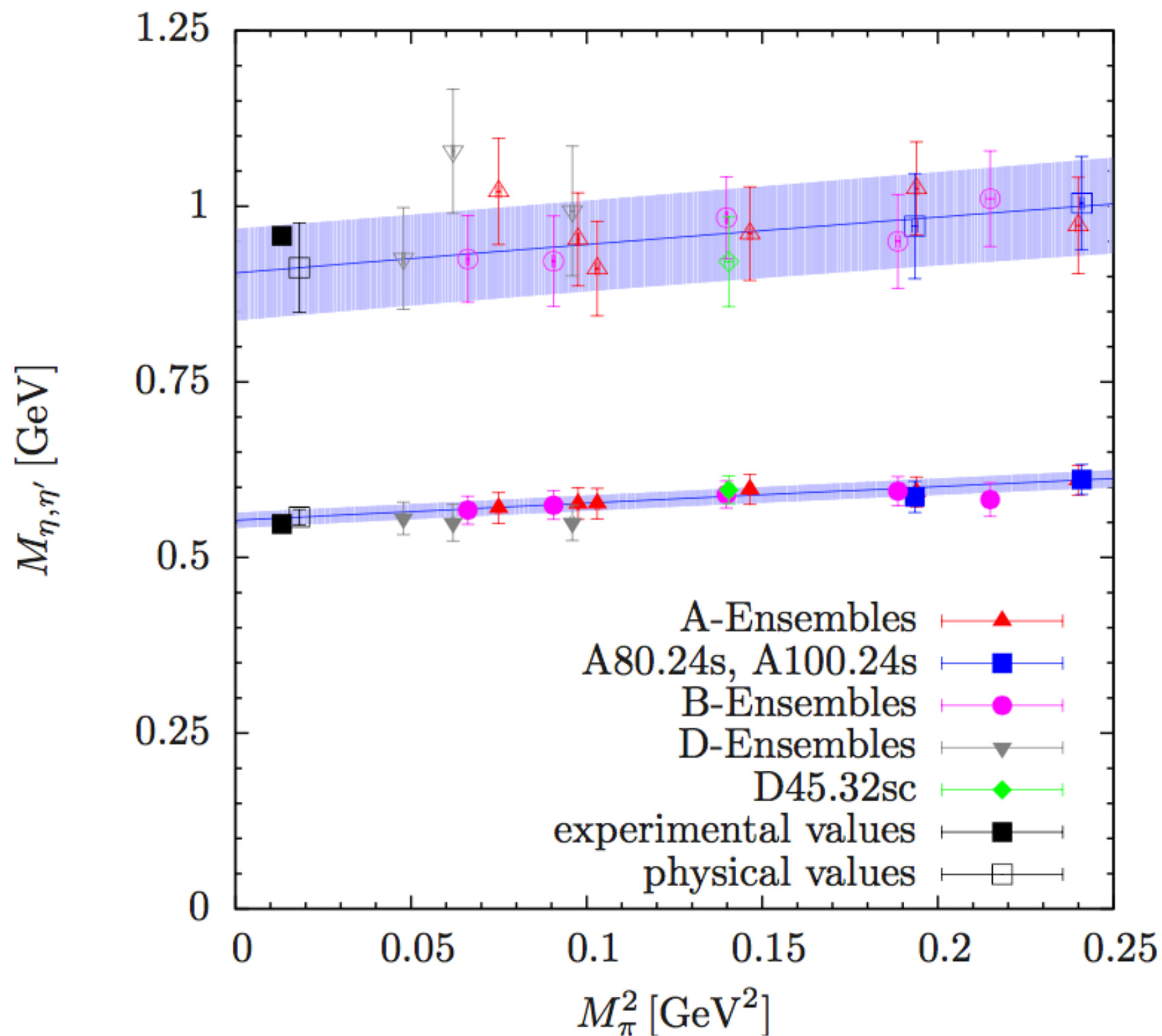
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**Successful  
at T=0**

# Topology, $\eta'$ and the $U_A(1)$ problem... ~~problem~~ solution



ETMC 2017

# Overview of Thermodynamics

Twisted mass Wilson Fermions,  $N_f=2+1+1$

# Wilson fermions with a twisted mass term

Frezzotti Rossi 2003

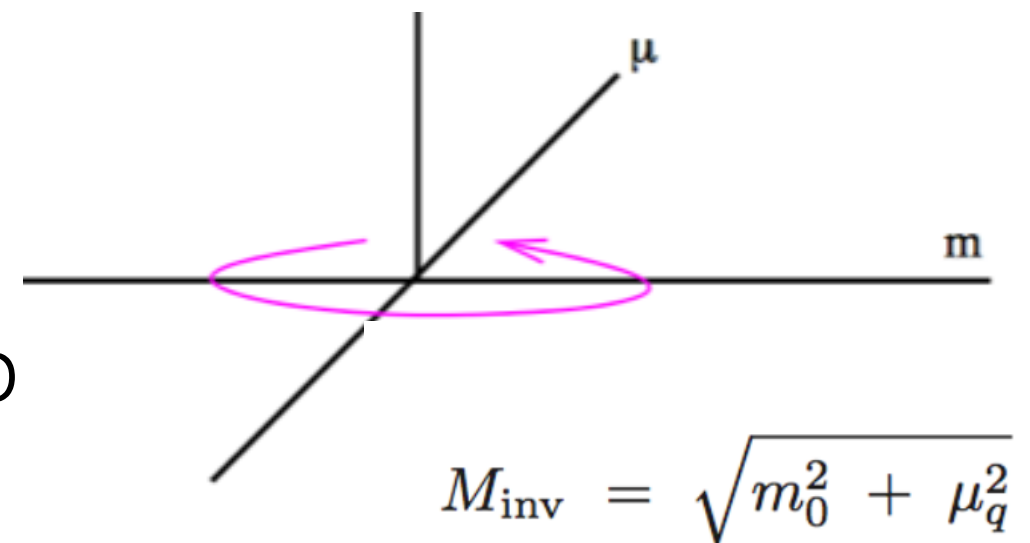
A twisted mass term in flavor space:

$i\mu\tau_3\gamma_5$  for two degenerate light flavors

is added to the standard mass term in the Wilson Lagrangian

Consequences:

- simplified renormalization prop
- automatic  $O(a)$  improvement
- control on unphysical zero modes


$$M_{\text{inv}} = \sqrt{m_0^2 + \mu_q^2}$$

Successful phenomenology at  $T=0$

ETMC collaboration 2003—

# Why $N_f = 2 + 1 + 1$ ?

$T_c$

340 – 380 MeV  
RHIC AuAu  
200 GeV

420-480 MeV  
LHC  
2.76 TeV

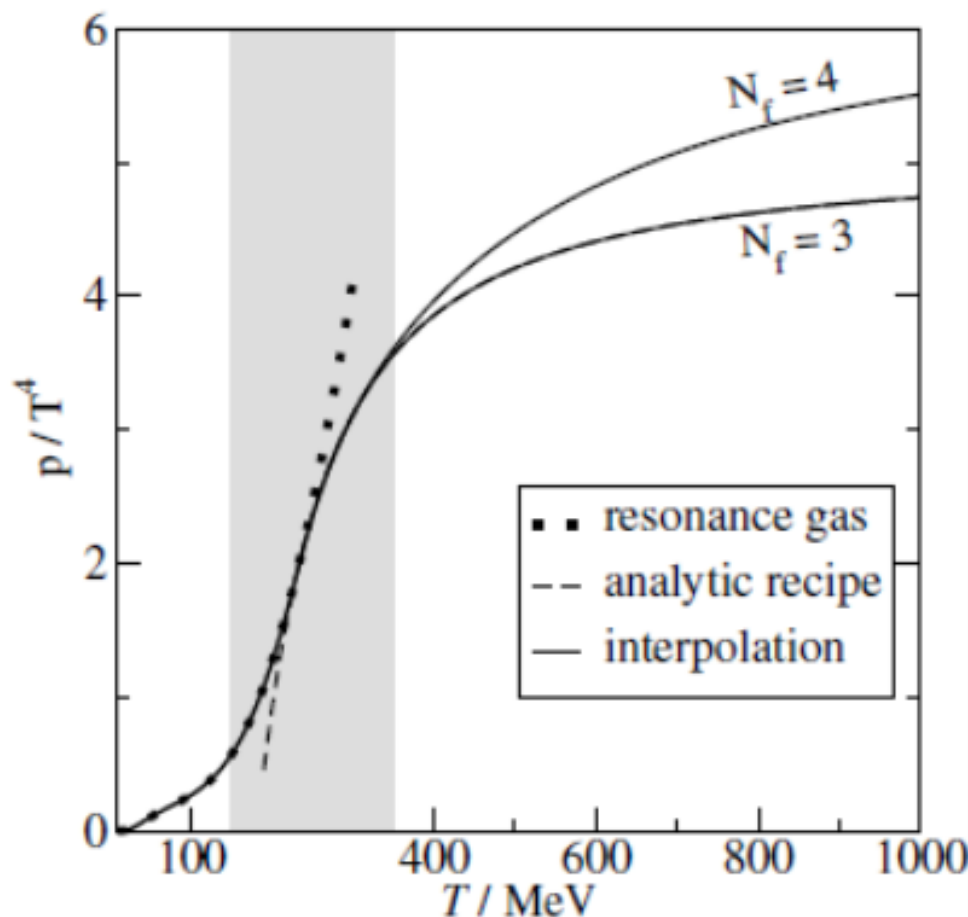
500- 600 MeV  
LHC hot spots  
2.76 TeV

1 GeV  
LHC  
7 TeV



$\approx 200 \text{ MeV}$

## Quark Gluon Plasma @ Colliders



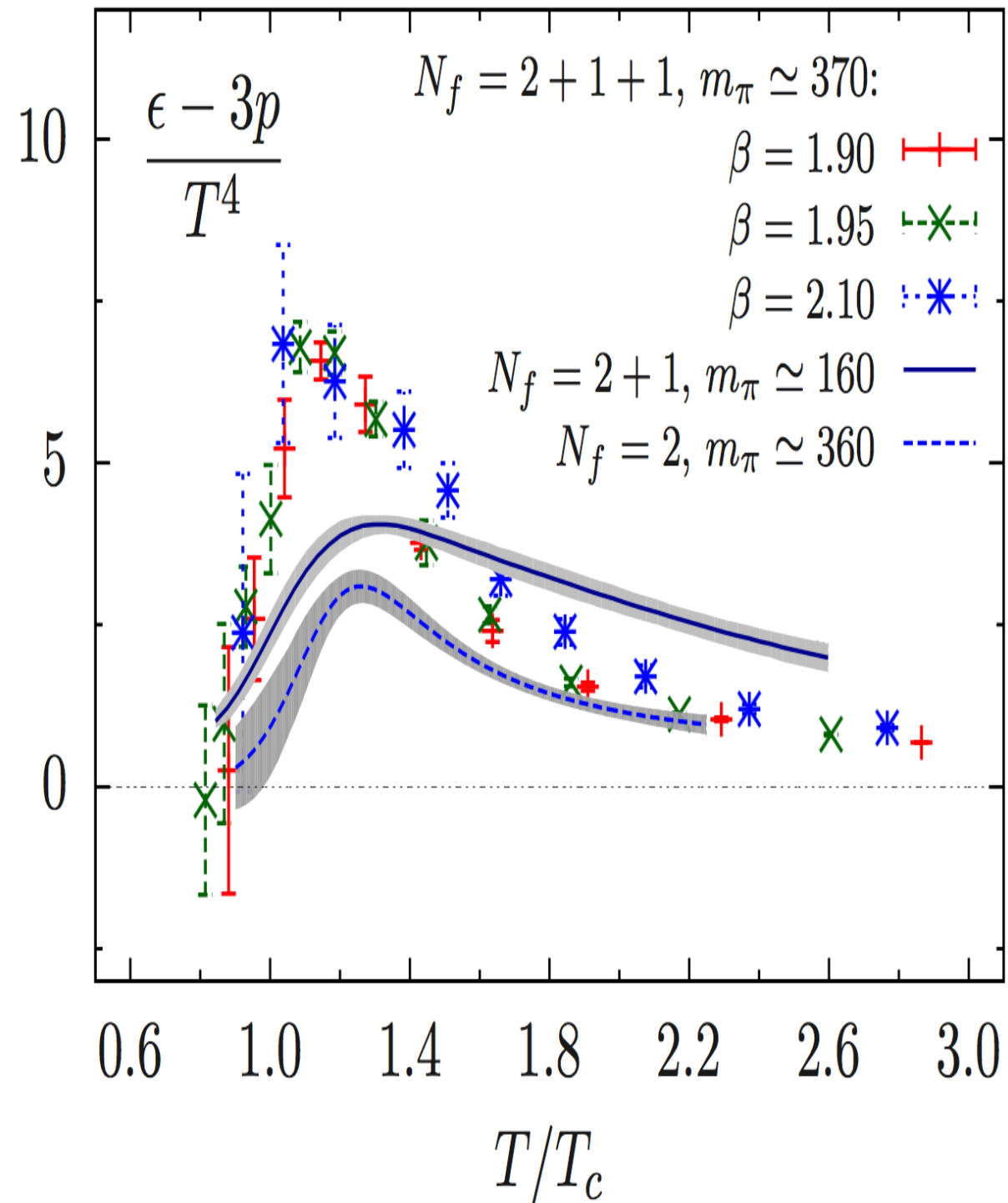
Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

Laine Schroeder 2006

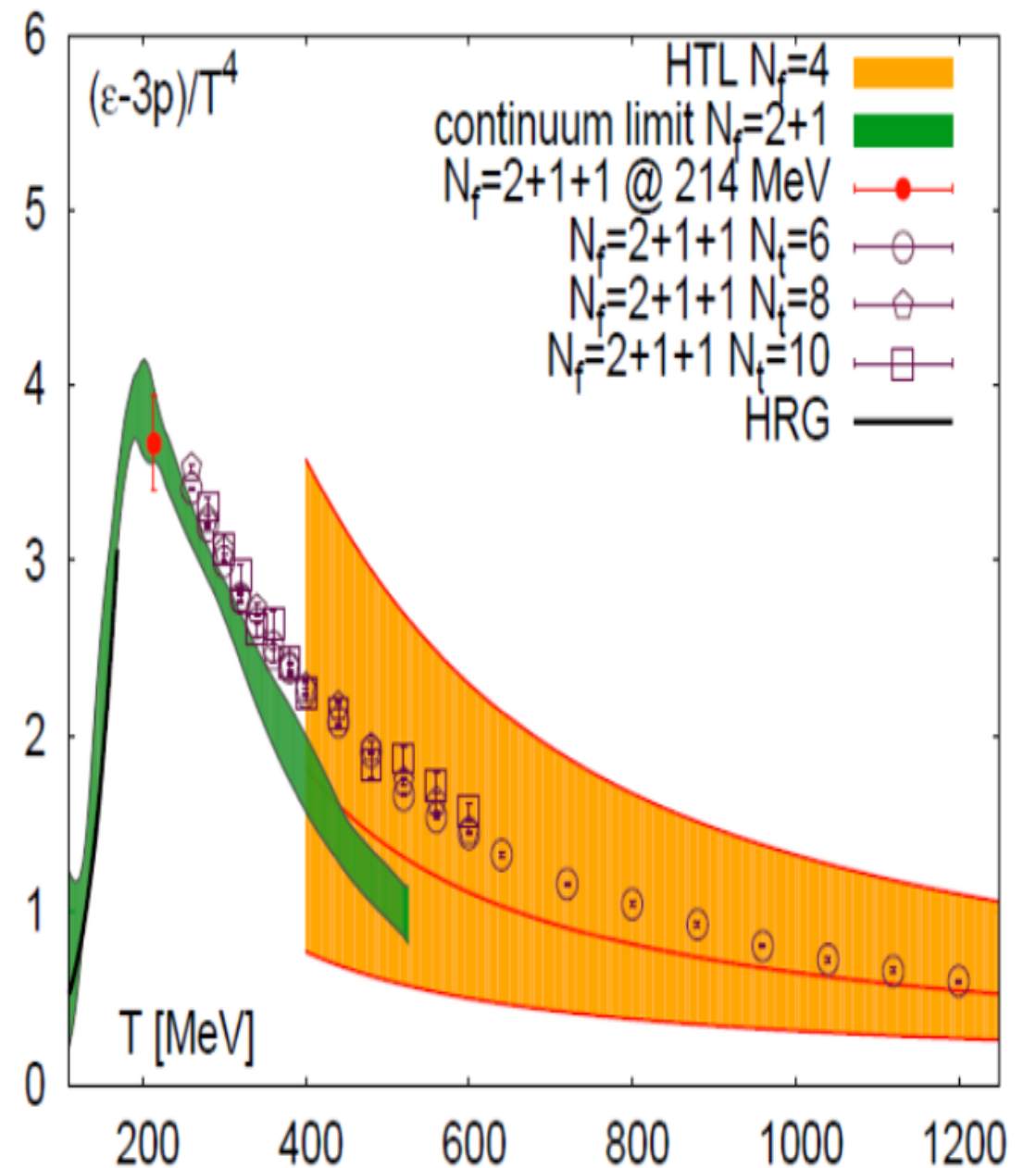


# Trace anomaly: effects of a dynamical charm

Tmft



Wuppertal-Budapest



Staggered



Fixed  
varying  
scale

For each lattice  
spacing we explore  
a range of  
temperatures  
150MeV — 500  
MeV by varying  $N_t$

We repeat this for  
three different lattice  
spacings following  
ETMC T=0  
simulations.

Four pion  
masses

Advantages: we  
rely on the setup of  
ETMC T=0  
simulations. Scale is  
set once for all.

Disadvantages:  
mismatch of  
temperatures - need  
interpolation before  
taking the  
continuum limit

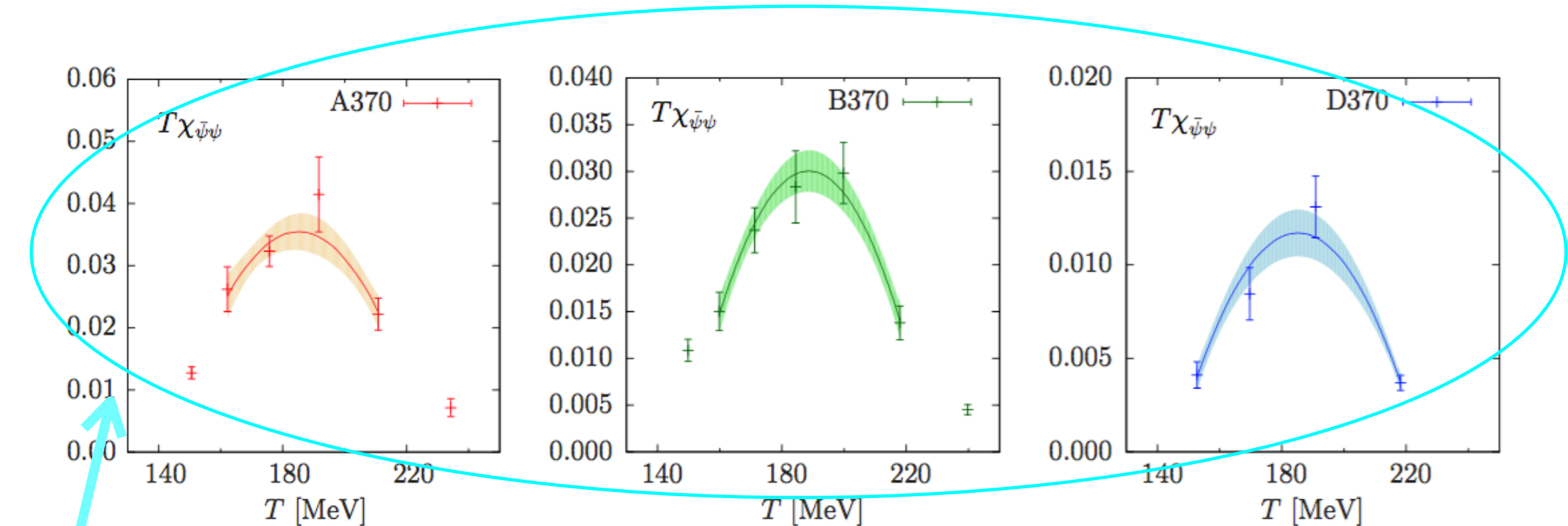
Number of flavours	$m_{\pi^\pm}$
	210
$N_f = 2 + 1 + 1$	260
	370
	470
$N_f = 2$	360
	430

# Nf = 2 + 1 + 1 Setup

$T = 0$ (ETMC) nomenclature	$\beta$	$a$ [fm] [6]	$N_\sigma^3$	$N_\tau$	$T$ [MeV]	# confs.
A60.24	1.90	0.0936(38)	$24^3$	5	422(17)	585
				6	351(14)	1370
				7	301(12)	341
				8	263(11)	970
				9	234(10)	577
				10	211(9)	525
				11	192(8)	227
			$32^3$	12	176(7)	1052
				13	162(7)	294
				14	151(6)	1988
B55.32	1.95	0.0823(37)	$32^3$	5	479(22)	595
				6	400(18)	345
				7	342(15)	327
				8	300(13)	233
				9	266(12)	453
				10	240(11)	295
				11	218(10)	667
				12	200(9)	1102
				13	184(8)	308
				14	171(8)	1304
				15	160(7)	456
				16	150(7)	823
D45.32	2.10	0.0646(26)	$32^3$	6	509(20)	403
				7	436(18)	412
				8	382(15)	416
				10	305(12)	420
				12	255(10)	380
				14	218(9)	793
			$40^3$ $48^3$	16	191(8)	626
				18	170(7)	599
				20	153(6)	582

# Overview of Chiral observables

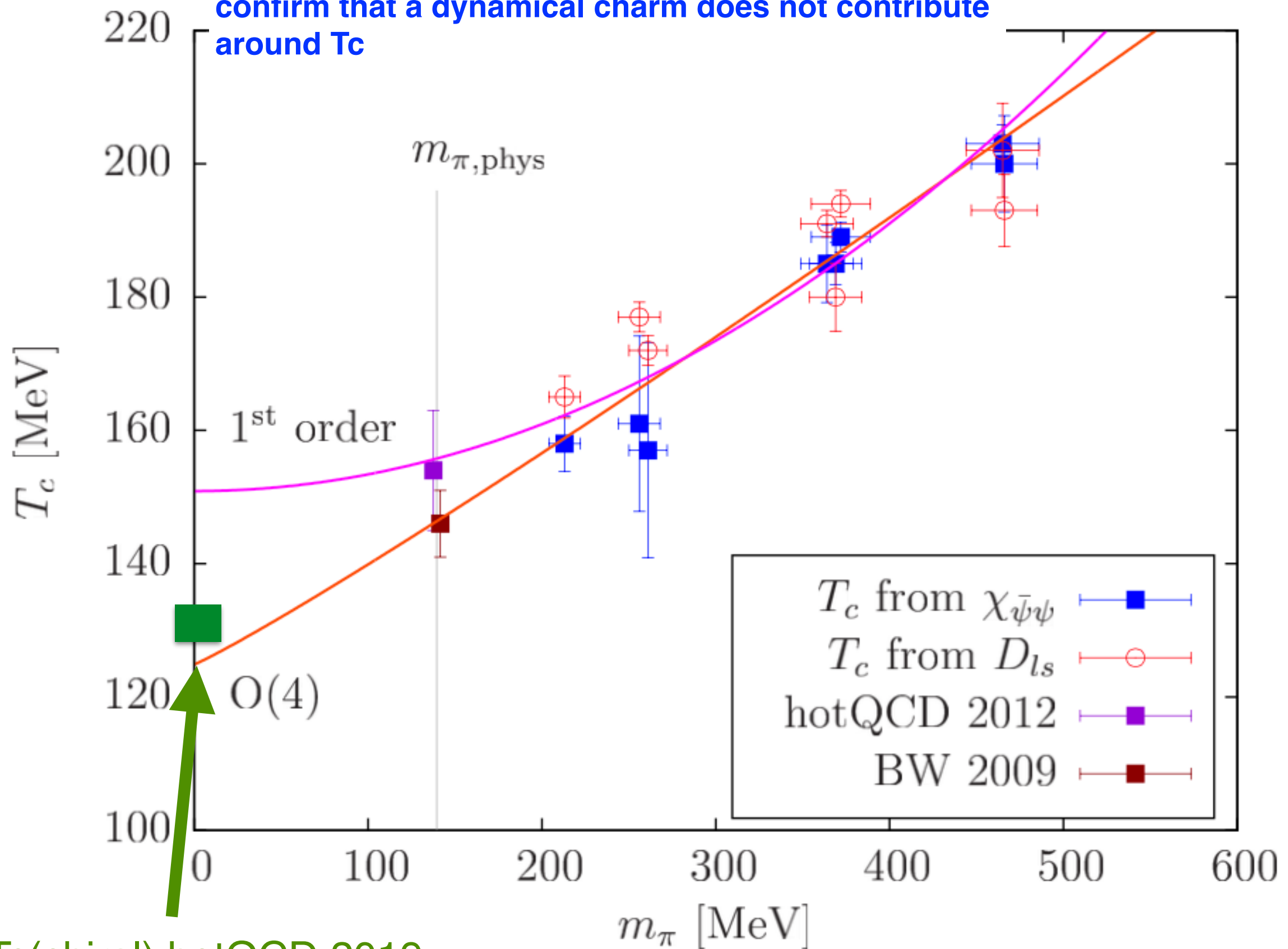
## Nf 2 + 1 + 1



$a$ [fm]	$m_\pi$ [MeV]	$T_\chi$ [MeV]	$T_\Delta$ [MeV]	$T_{\text{deconf}}$ [MeV]
0.065	213	158(1)(4)	165(3)(1)	176(8)(8)
0.094	261	157(8)(14)	172(2)(1)	188(6)(1)
0.082	256	161(13)(2)	177(2)(1)	192(9)(2)
0.094	364	185(5)(3)	191(2)(0)	202(3)(0)
0.082	372	189(2)(1)	194(2)(0)	201(6)(0)
0.065	369	185(1)(3)	180(5)(1)	193(13)(2)
0.094	466	200(4)(6)	193(5)(2)	205(4)(2)
0.082	465	203(2)(2)	202(7)(1)	212(6)(1)

spacing effects below statistical errors

Outcome: twisted mass ok; and the results confirm that a dynamical charm does not contribute around  $T_c$



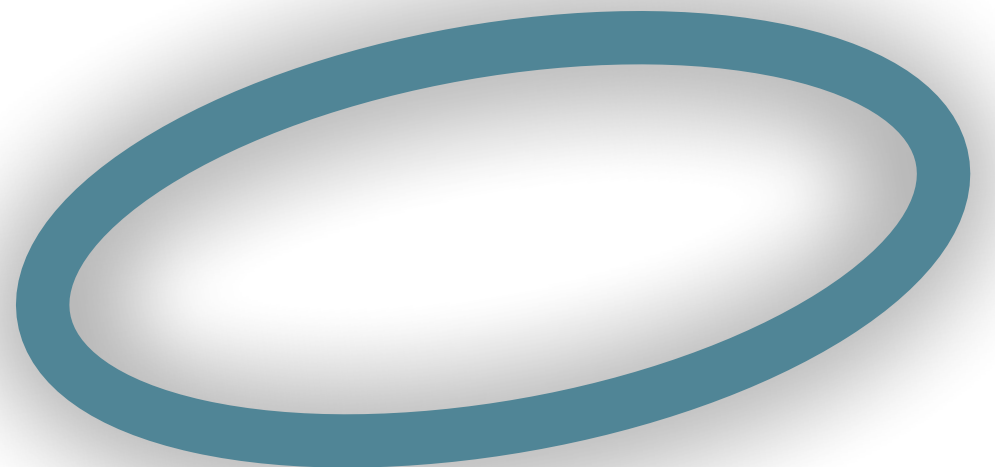
$T_c(\text{chiral})$  hotQCD 2019

Topology

# Topology from low to high Temperature

In the hadronic phase topology solves the puzzle by explicit breaking  $U(1)_A$   $\eta'$

What happens to topology in the Quark Gluon Plasma?



# Topological and chiral susceptibility

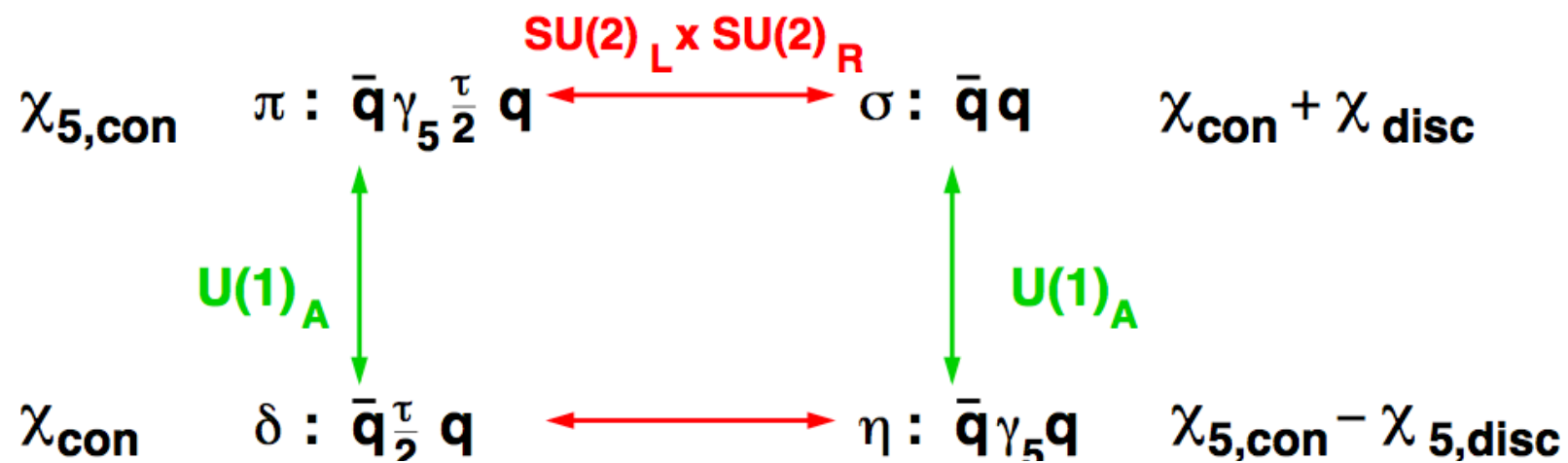
Kogut, Lagaë, Sinclair 1999

HotQCD, 2012

$$\chi_{top} = \langle Q_{top}^2 \rangle / V = m_l^2 \chi_{5,disc}$$

From:

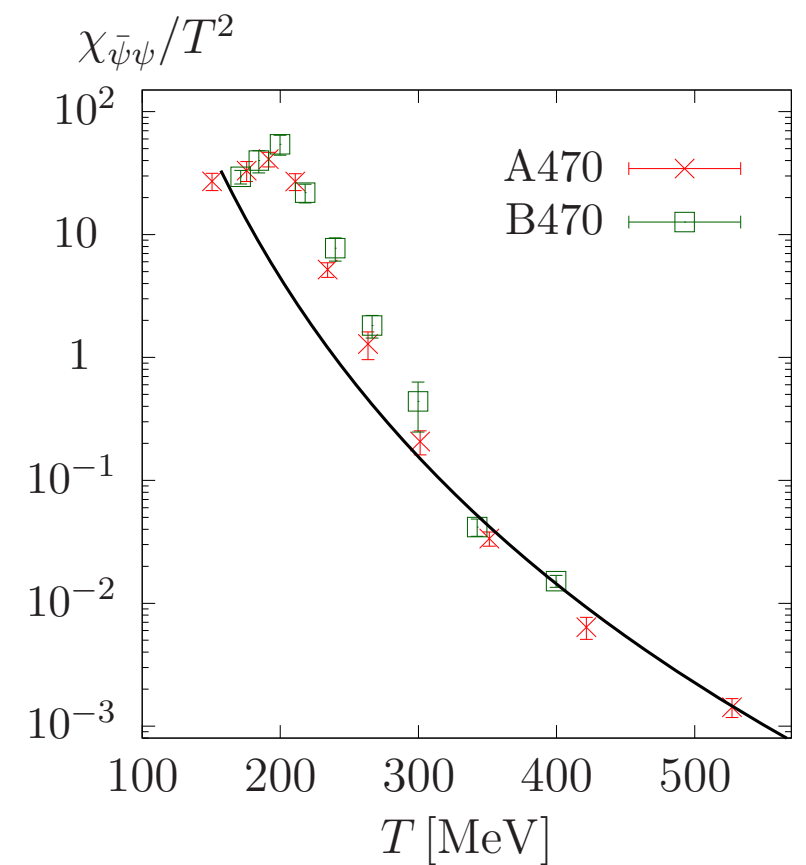
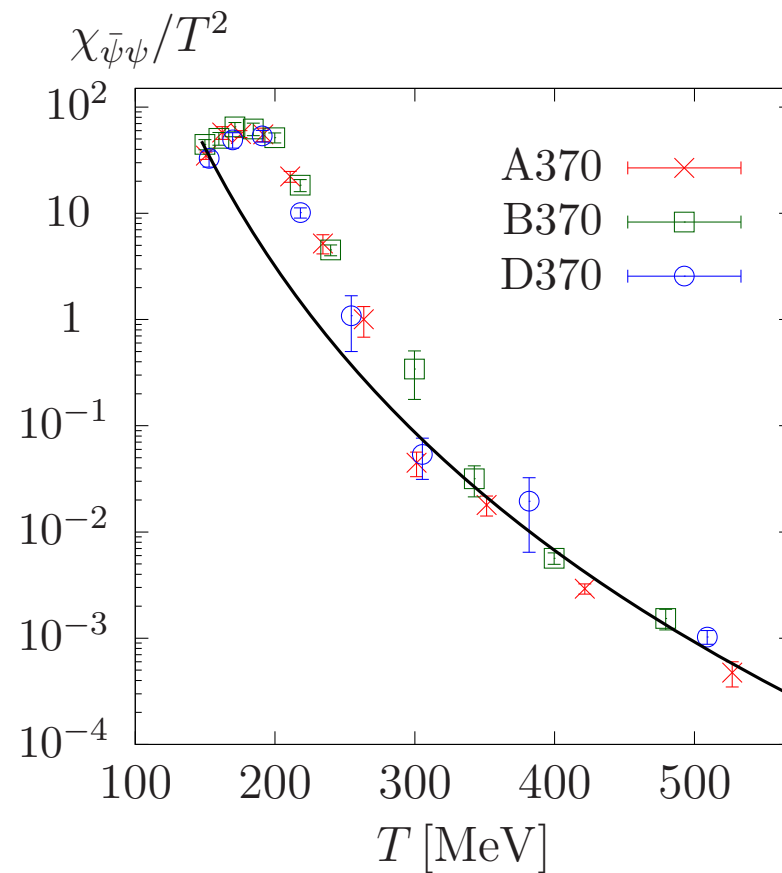
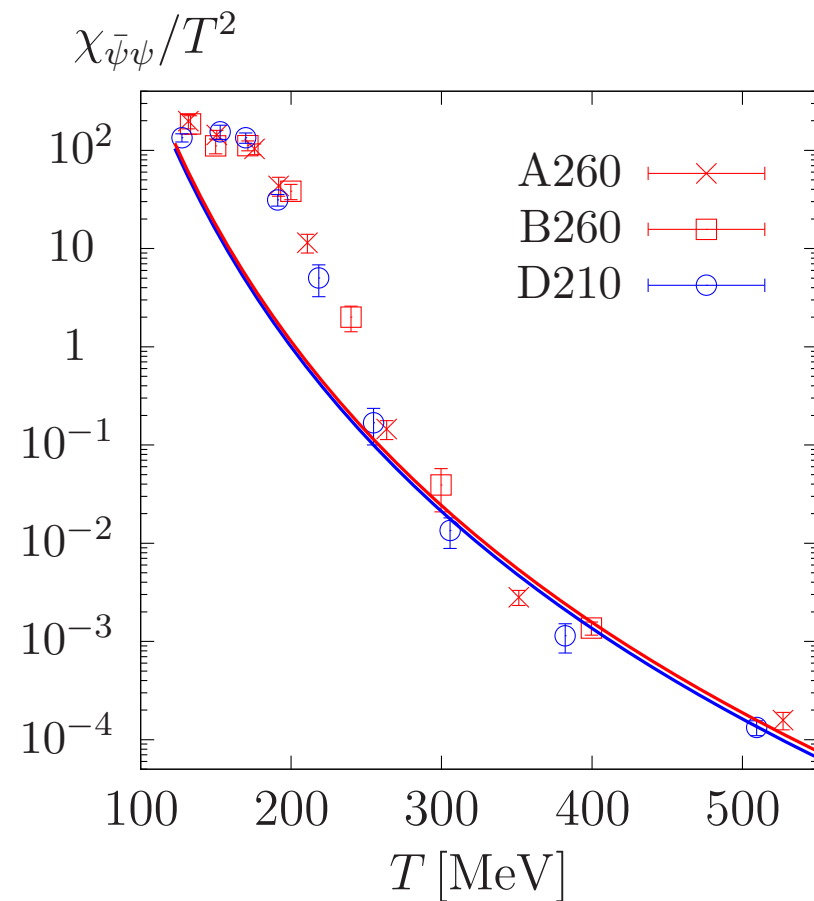
$$m \int d^4x \bar{\psi} \gamma_5 \psi = Q_{top}$$



$$\chi_{\pi} - \chi_{\delta} = \chi_{disc} = \chi_{5,disc} , \quad \text{for } T \geq T_c , m_l \rightarrow 0$$

$$\chi_{top} = \langle Q_{top}^2 \rangle / V = m_l^2 \chi_{disc}$$

# Chiral susceptibility



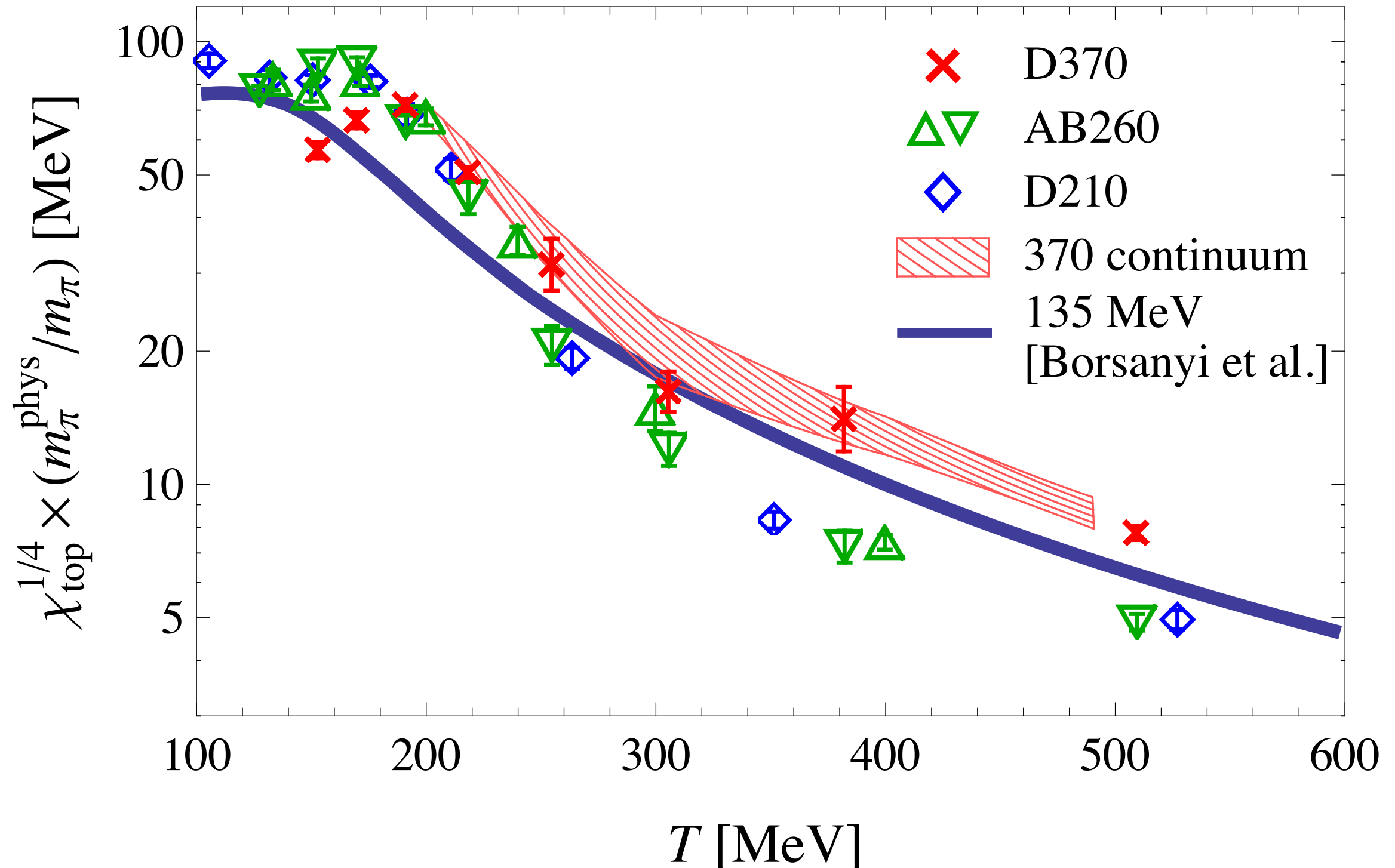
Within errors, no discernable spacing dependence

# Topological Susceptibility:

## Results for physical pion mass

Rescaled according to

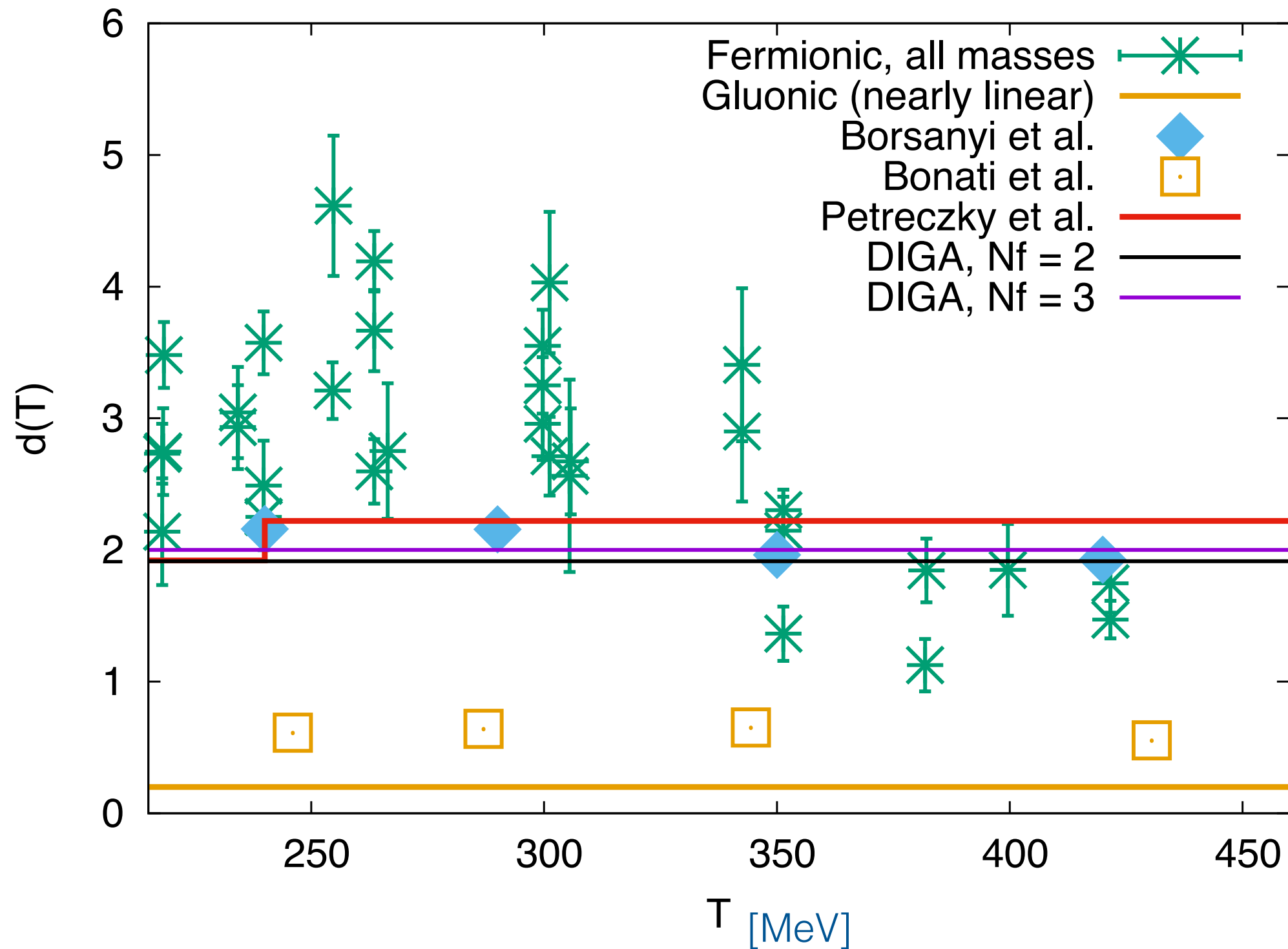
$$\chi_{\text{top}} = m_l^2 \chi_{\bar{\psi}\psi}^{\text{disc}} = \sum_{n=0} a_n m_{\pi}^{4(n+1)}.$$





Effective exponent  $d(T)$ :

$$\chi_{top}^{1/4} = aT^{-d(T)}$$



<- Revised?

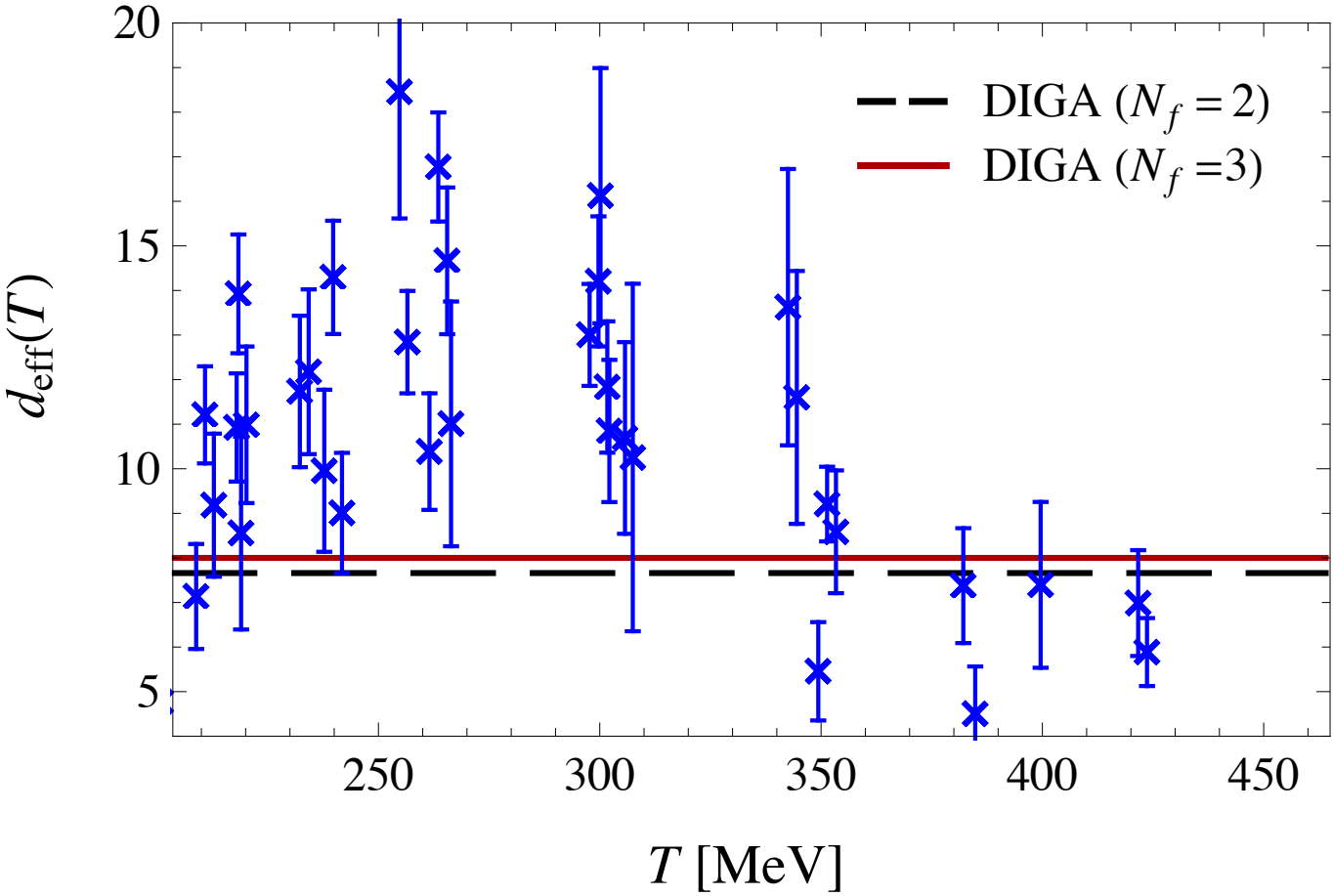
# Power-law decay?

For instanton gas

$$\chi^{0.25}(T) = aT^{-d(T)}$$

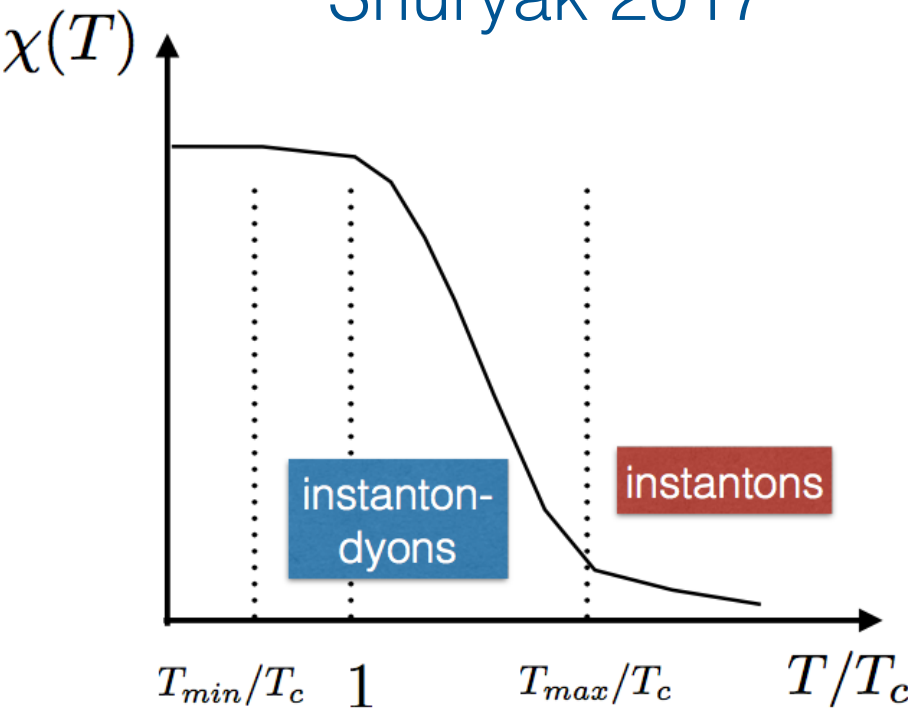
$$d(T) \equiv const \simeq (7 + \frac{N_f}{3})$$

$$d(T) = -T \frac{d}{dT} \ln \chi^{0.25}(T)$$

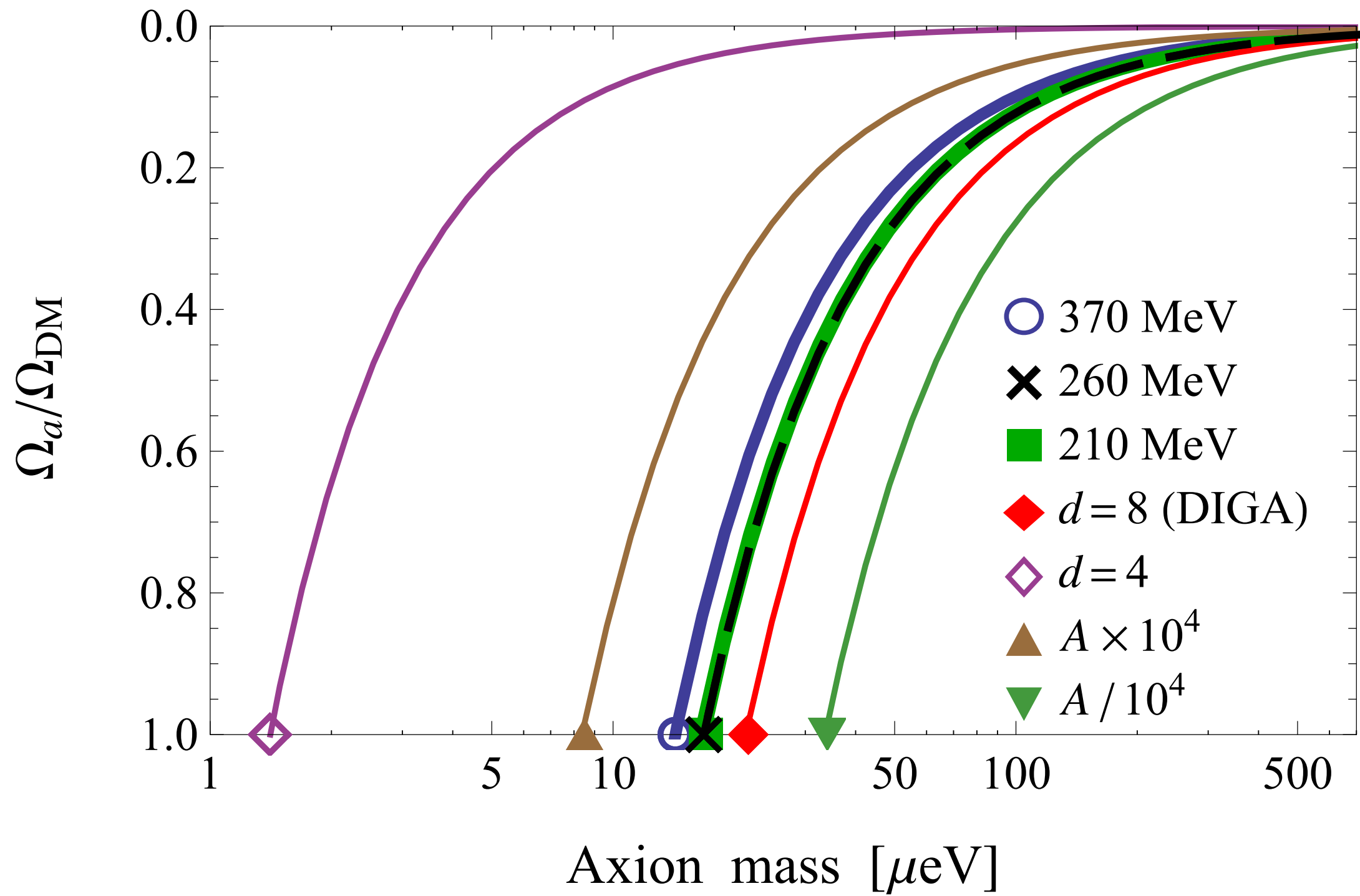


Faster decrease before DIGA sets in

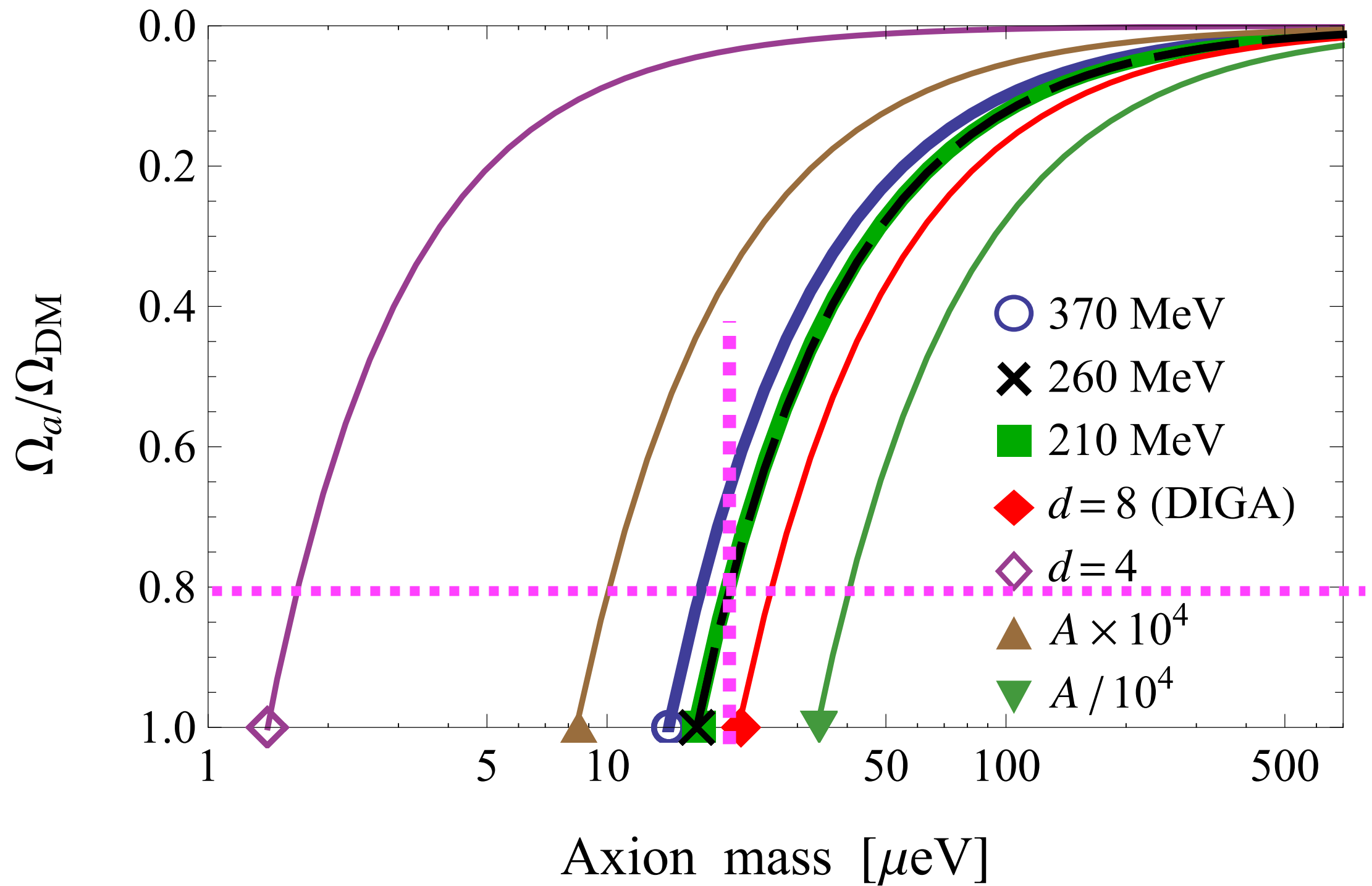
Possibly consistent with instant-dyon?  
Shuryak 2017



[ Axion

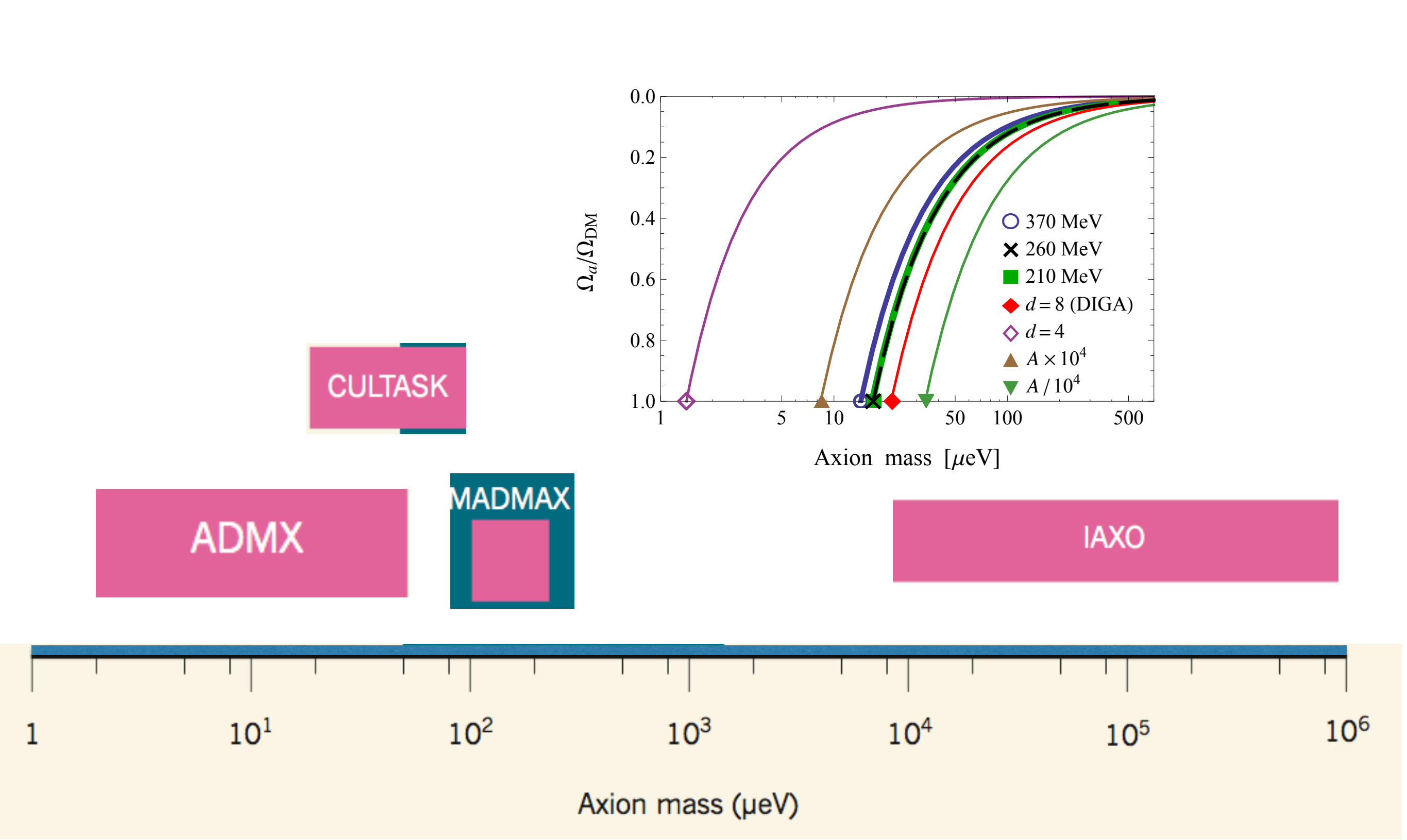


$$\Omega_a = \frac{\rho_{a,0}}{\rho_c},$$



$$\Omega_a = \frac{\rho_{a,0}}{\rho_c};$$

Example: if axions constitute 80% DM,  
our results give a lower bound for the  
axion mass of  $\simeq 30\mu\text{eV}$



Adapted from MpL, Nature N&V 2016

]

$\eta'$



## Return of the prodigal Goldstone boson

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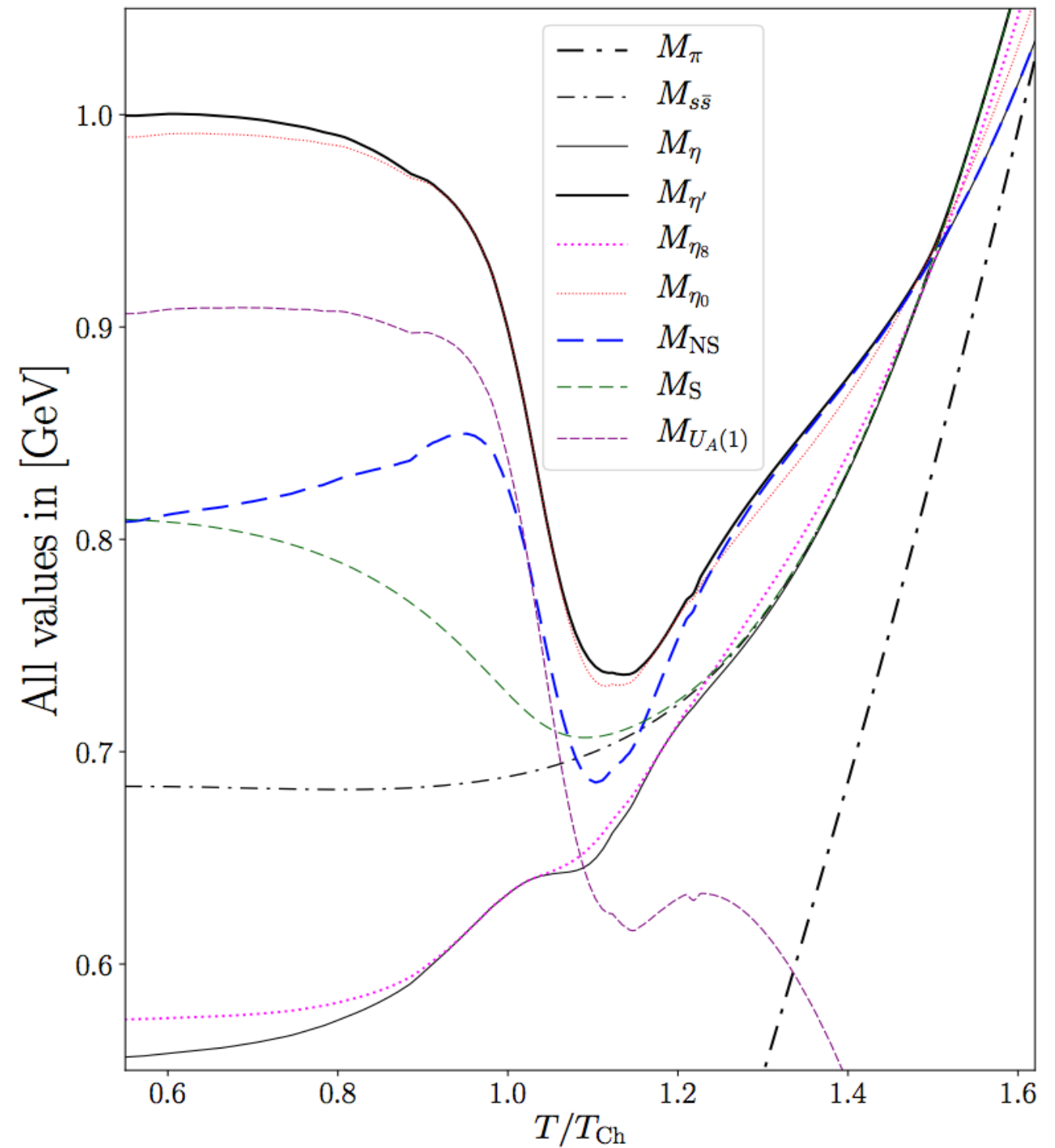
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# $\eta'$ in the QGP

So far, only results from model's studies



Horvatic et al. 2018

# Different mechanisms leading to $\eta'$ (900 MeV) mass reduction

Adopting the basis

$$I \equiv \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$S \equiv s\bar{s}$$

The mass matrix of the  $\eta$  complex is:

$$\begin{pmatrix} m_\pi^2 + m_A^2 & m_A^2 / \sqrt{2} \\ m_A^2 / \sqrt{2} & 2m_K^2 - m_\pi^2 + m_A^2 / 2 \end{pmatrix}$$

Veneziano, 1981

$$m_A^2 = 2 \frac{N_f}{f_0^2} \chi^2$$

Non anomalous:

$$\eta' \simeq 700 \text{ MeV} \\ (\text{strange only})$$

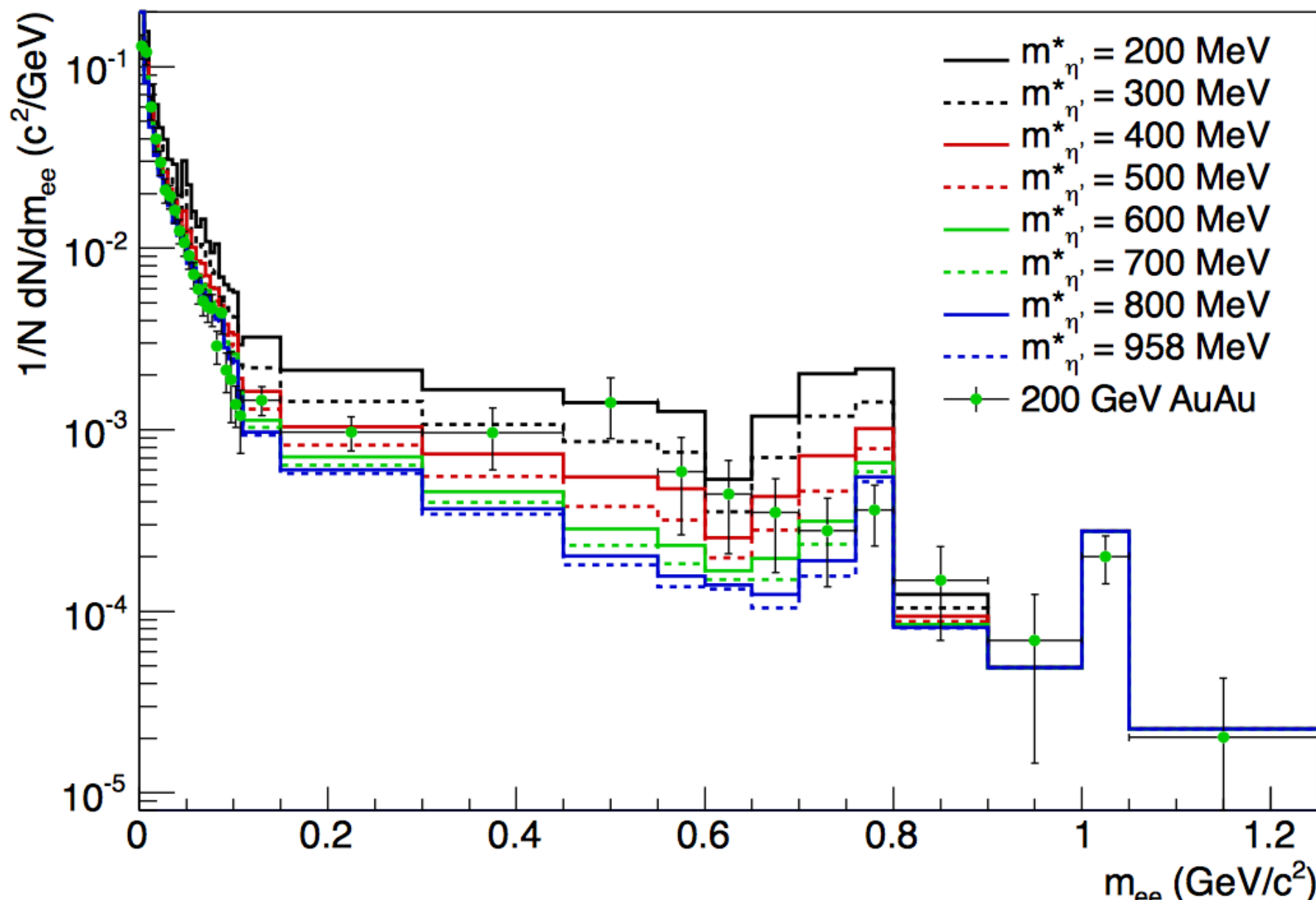
However: also sensitive to  $SU(2) \times SU(2)$

# Indication of topology suppression in PHENIX

**NICA?**

Effects of chain decays, radial flow and  $U_A(1)$  restoration on the low-mass dilepton enhancement in  $\sqrt{s_{NN}}=200$  GeV Au+Au reactions

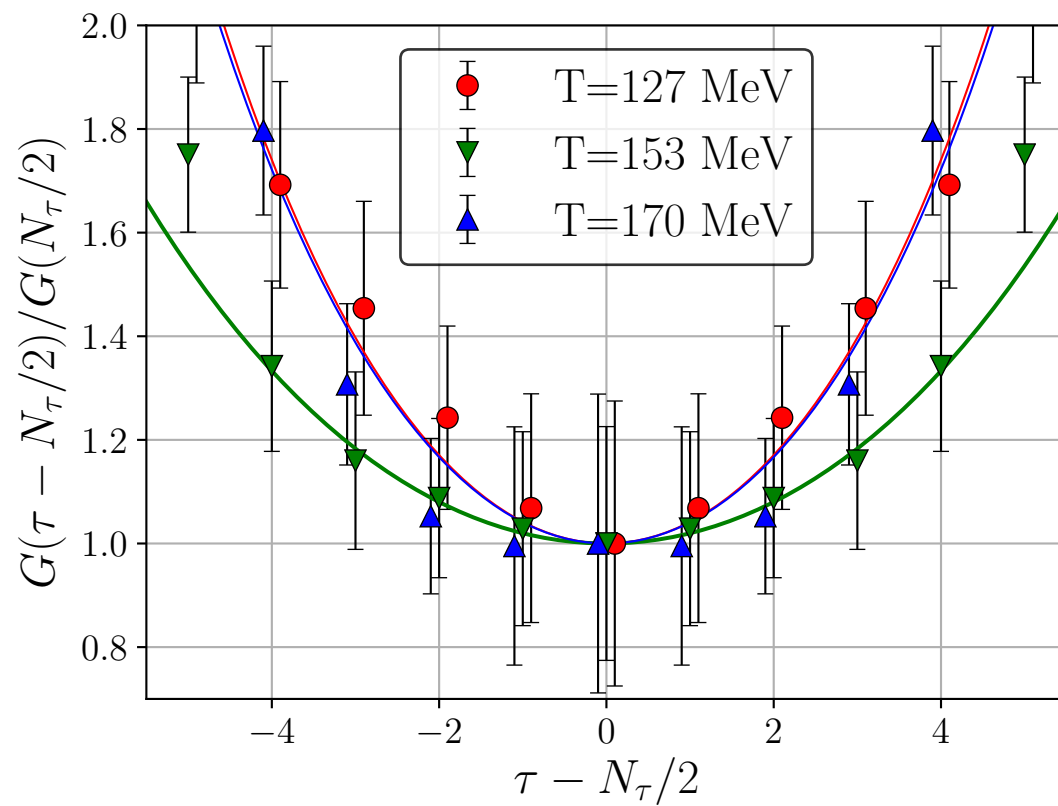
Márton Vargyas<sup>a,b,1</sup>, Tamás Csörgő<sup>b,2</sup>, Róbert Vértesi<sup>b,c,3</sup>



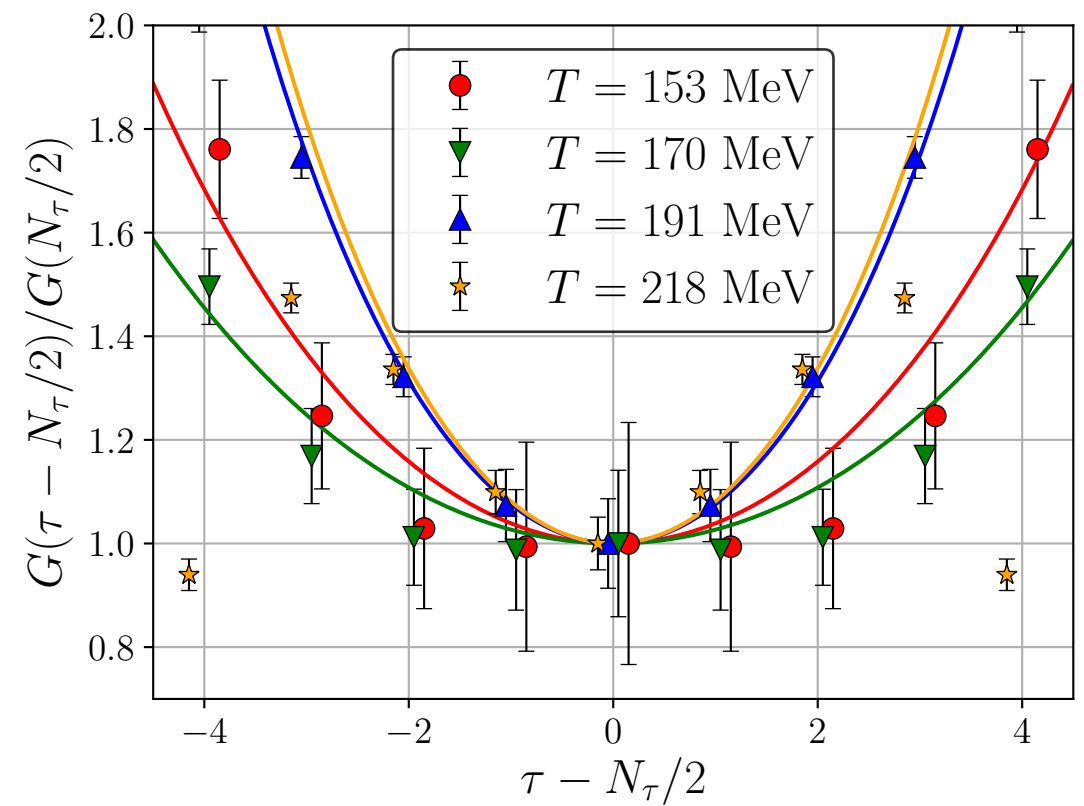
This is  
at small  
but nonzero  
density!

# $\eta'$ mass from topological charge correlators

$$G(\tau) = \int d^3 \bar{x} q(0) q(\tau, \bar{x}) = \int d^3 \bar{x} \frac{1}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}(0) \times \\ \times \frac{1}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}(\tau, \bar{x}) \simeq e^{-m_{\eta'} \tau}$$

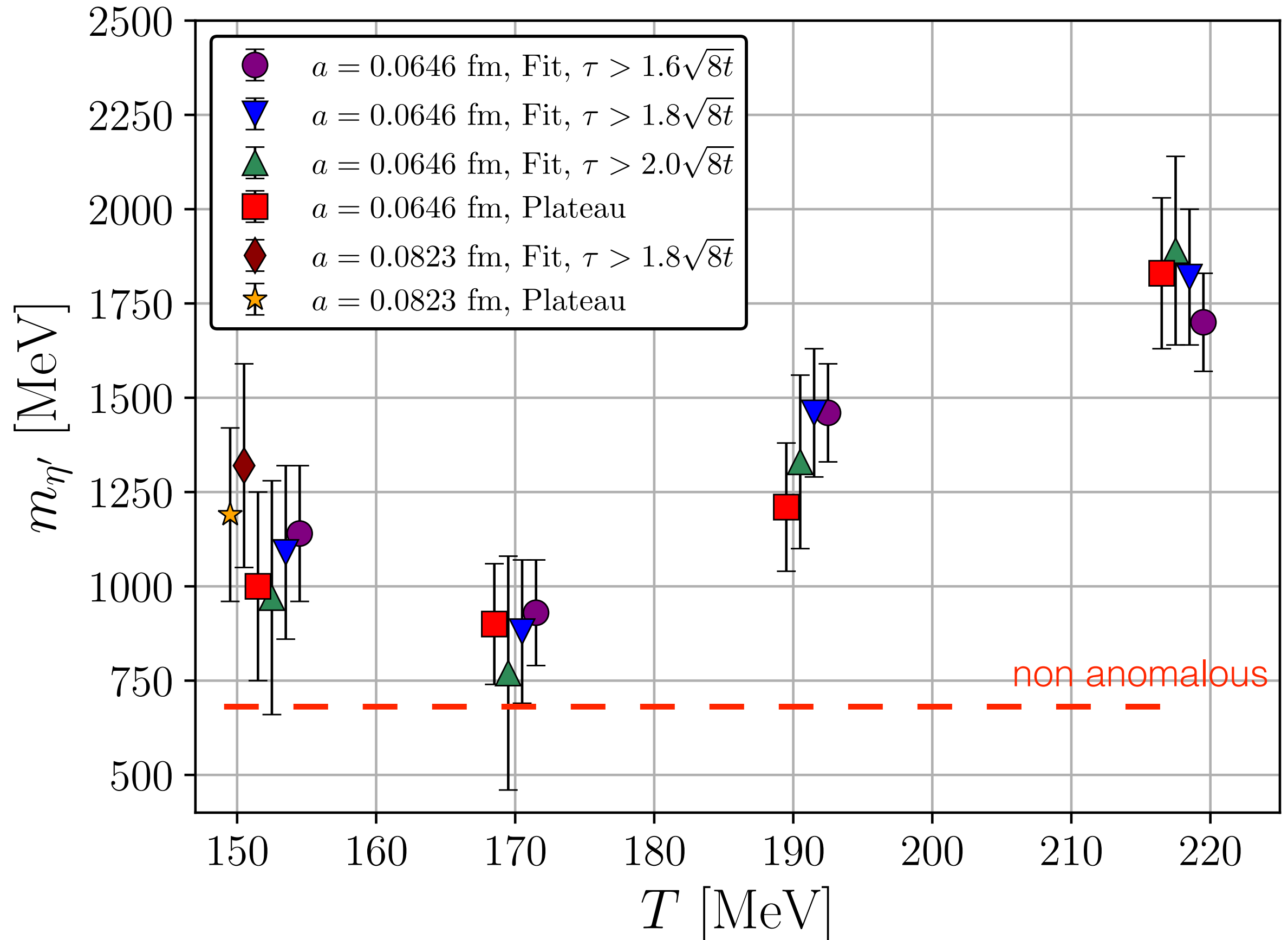


Pion mass 210 MeV

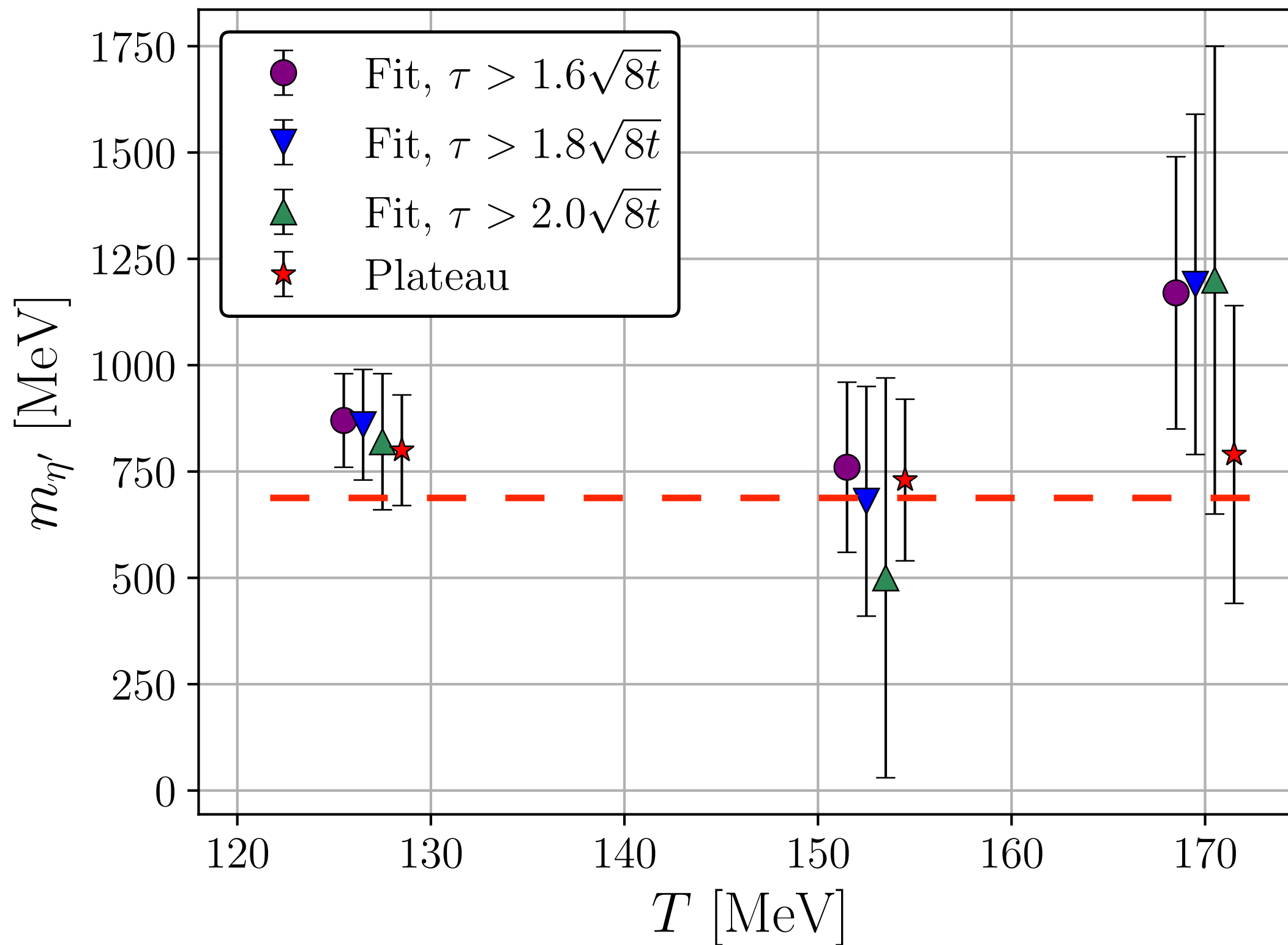


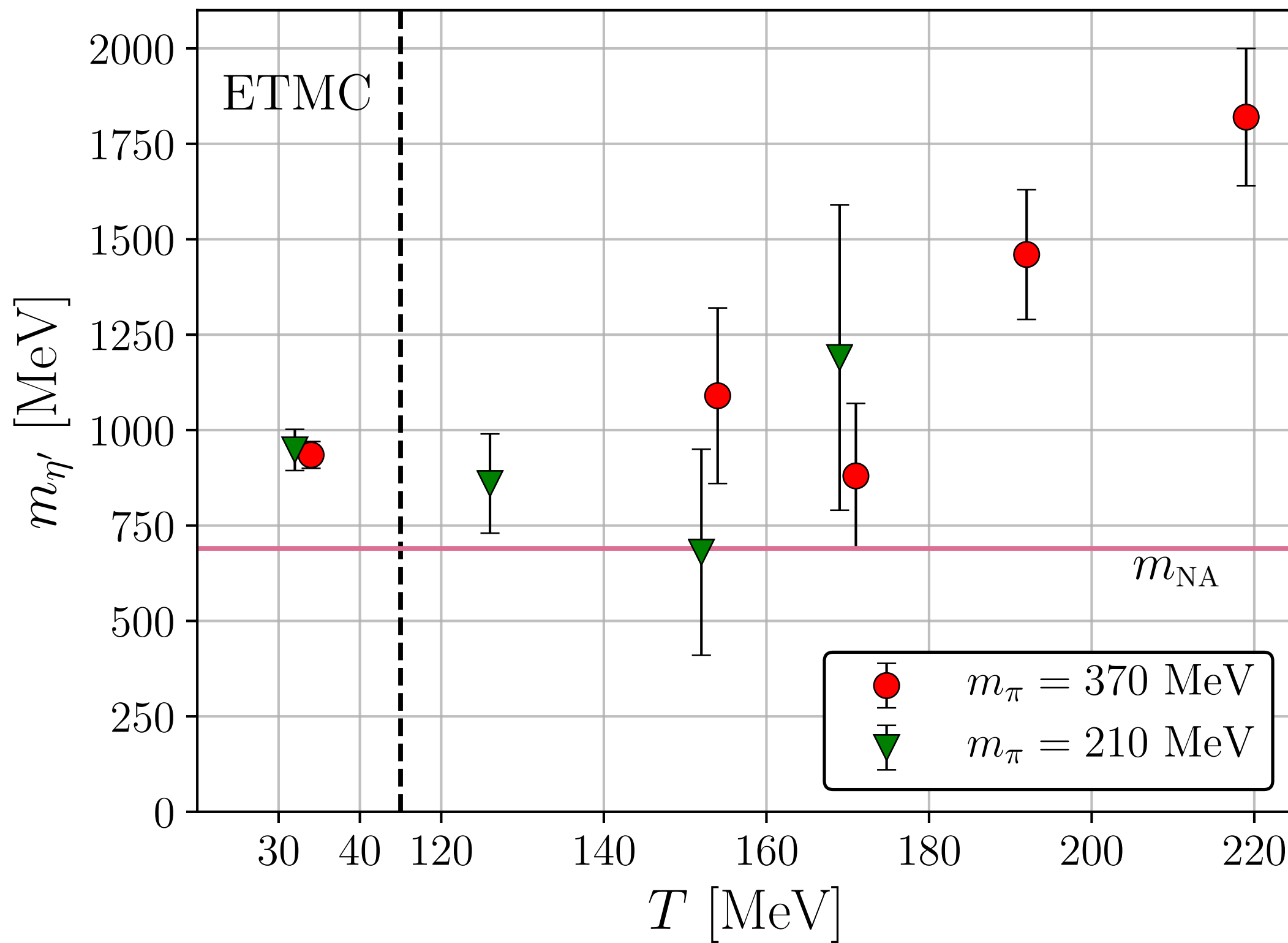
Pion mass 370 MeV

Pion mass = 370 MeV



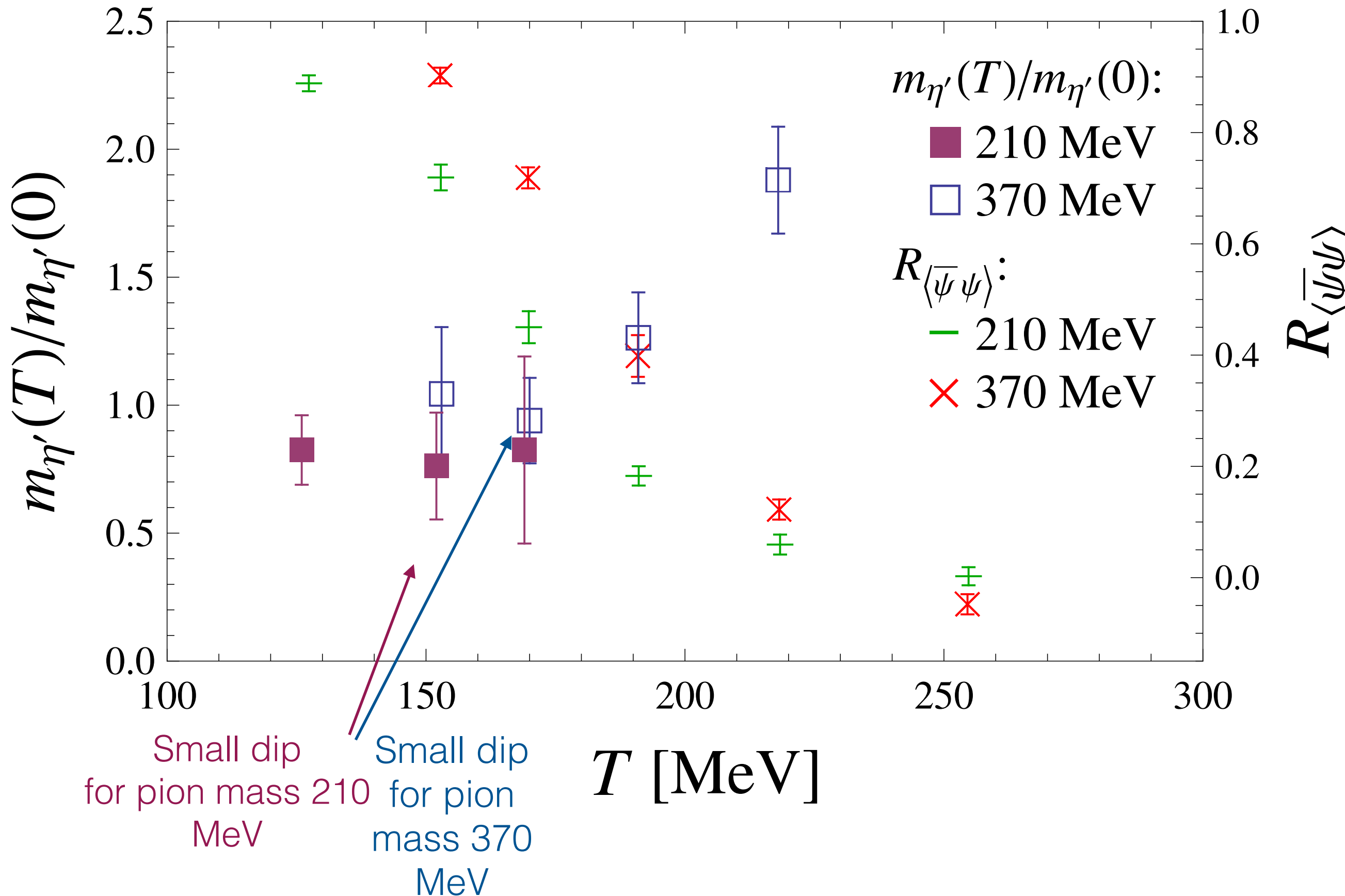
Pion mass 210 MeV







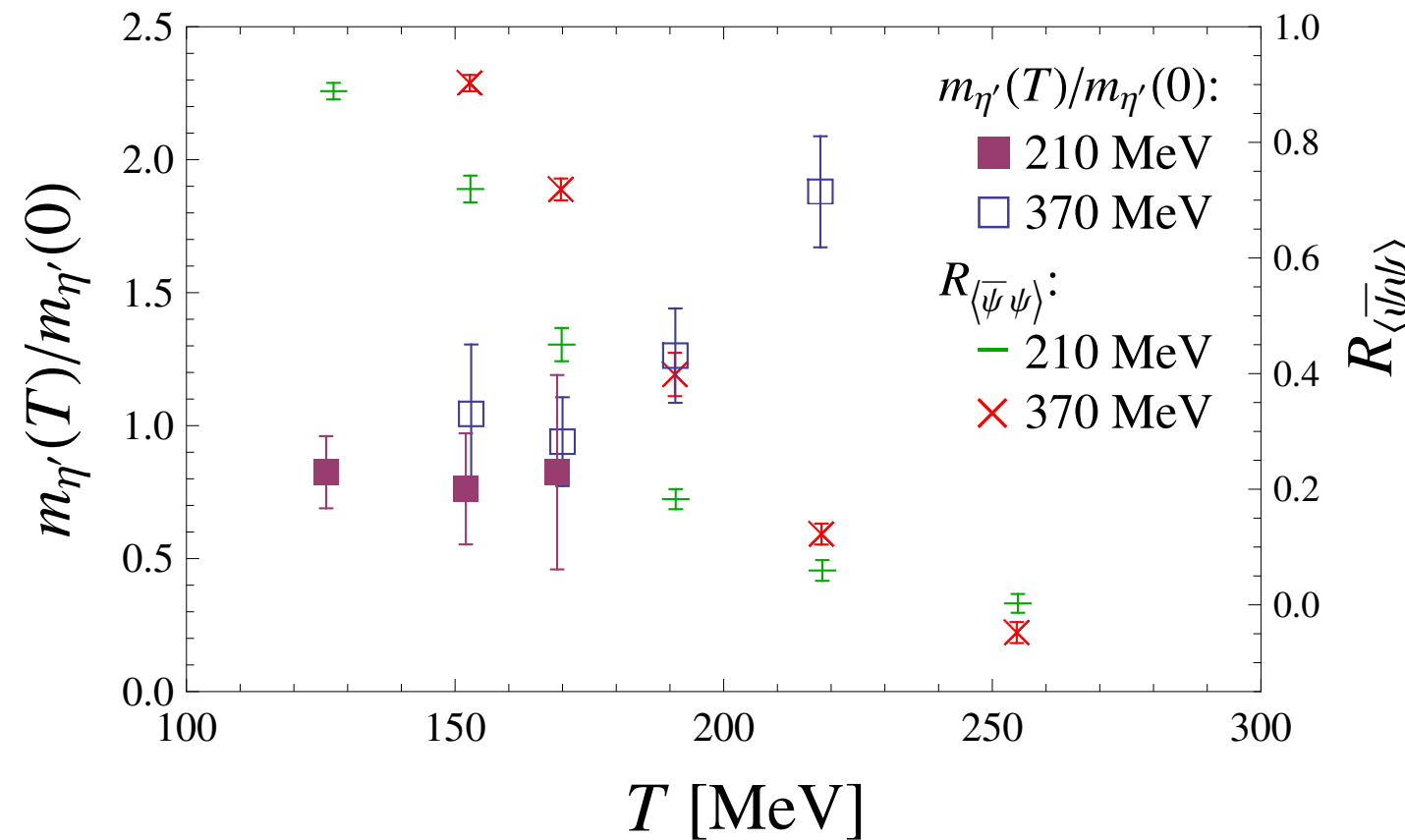
# Correlations?



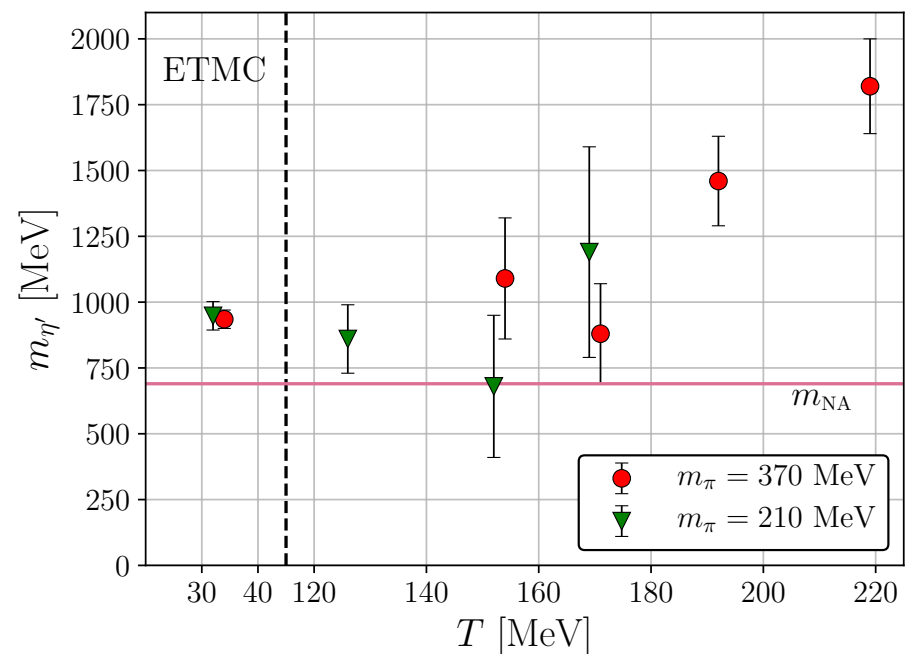
Minimum of the  $\eta'$

Approx. correlated with  $T_\chi$

Ensemble	$a$ [fm]	$m_\pi$ [MeV]	$T_\chi$ [MeV]	$T_{\eta'} \text{ [MeV]}$
D210	0.065	213	158(1)(4)	$\simeq 150$
A260	0.094	261	157(8)(14)	
B260	0.082	256	161(13)(2)	
A370	0.094	364	185(5)(3)	$\simeq 170$
B370	0.082	372	189(2)(1)	
D370	0.065	369	185(1)(3)	
A470	0.094	466	200(4)(6)	
B470	0.082	465	203(2)(2)	



Consistent with suppression of the anomalous contribution



## Summary & **Outlook**

The QCD topological susceptibility at high temperature gives a strict lower bound on the QCD axion mass.

The  $\eta'$  meson is an important probe of axial symmetry and of its interplay, or lack thereof, with chiral symmetry.

Connection with confinement [S.Nedelko et al. ]

—> **to be discussed**

The correlators of the QCD topological charge afford an estimate of the  $\eta'$  mass, which appears to be correlated with signals of chiral symmetry restoration.

The results so far have been obtained by studying the correlators of the topological charge, which does not give access to  $\eta$  and to the mixing angle -> **in progress.**

Future measurements (FAIR, NICA, BNL, CERN)?