Lattice study of electromagnetic conductivity of quark-gluon plasma in external magnetic field

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Outline

- Introduction
- Details of the calculation
- Few words about CME in Dirac semimetals
- CME in QCD
- Conclusion

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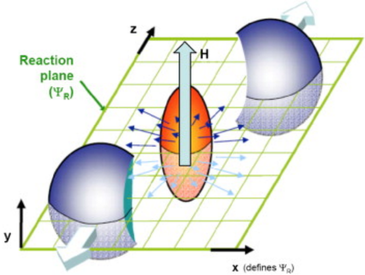
CME in Dirac semimetals: Annals Phys. 396 (2018) 78-86, CME in QCD: to be published

Chiral Magnetic Effect

"A system with a nonzero chirality responds to a magnetic field by inducing a current along the magnetic field. This is the Chiral Magnetic Effect."
[K. Fukushima, D. Kharzeev, H.J. Warringa, 2008]



$$\vec{J}_{CME} = \frac{e^2}{2\pi^2}\mu_5 \vec{B}$$

Dynamical CME is manifested through electromagnetic conductivity

Conductivity in external magnetic field

- $\vec{E}, \vec{B}$
- $\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2}(\vec{E}, \vec{B}) - \frac{\rho_5}{\tau}$, τ - chirality-changing scattering time

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- $\vec{J} = \sigma\vec{E} + \frac{e^2}{2\pi^2}\vec{B} \times \mu_5$
- $\sigma_{\parallel}^{CME} \sim eB\tau$
- **Manifestation of CME: rise of the conductivity with B**
- Anomaly related quantum phenomenon (classically $\sigma_{\parallel}^{CME} = 0$)

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  Q. Li et al., Nature Phys. 12 (2016) 550-554

  H. Li et al., Nat. Comm. 7, 10301 (2016)

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  H. Li et al., Nat. Comm. 7, 10301 (2016)
- Observed in heavy-ion collision experiments(?)

  B. I. Abelev et al. (STAR), Phys.Rev.Lett. 103, 251601 (2009)

  B. Abelev et al. (ALICE), Phys.Rev.Lett. 110, 012301 (2013)

Lattice simulations of strongly correlated system

- Dirac semimetals (strongly correlated system)
  - Two Fermi-points (Na_3Bi, Cd_3As_2)
  - Dispersion relation: $E^2 = v_{\parallel}^2(k_x^2 + k_y^2) + v_{\perp}^2 k_z^2, v_{\parallel}, v_{\perp} \sim 10^{-3}$
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Lattice simulation can be used to study both systems

Details of the simulations

- Staggered fermions (four Fermi points)

$$S_f = \bar{\Psi}_x D_{x,y} \Psi_y =$$

$$\sum_x \left(m\bar{\Psi}_x \Psi_x + \frac{1}{2}\eta_{\mathbf{4}}(x)[\bar{\Psi}_x \Psi_{x+\hat{\mathbf{4}}} - \bar{\Psi}_{x+\mathbf{4}}\Psi_x] + \frac{1}{2}\sum_{i=\mathbf{1}}^{\mathbf{3}} \eta_i(x)[\bar{\Psi}_x \Psi_{x+\hat{\imath}} - \bar{\Psi}_{x+\imath}\Psi_x] \right)$$

- For Dirac Semimetals: two Fermi-points

$$Z = \int DU\, exp(-S_G(U))\, \left(\mathbf{det}(\hat{D}(U) + m)\right)^{\mathbf{1/2}}$$

- For QCD: u,d,s–quarks at physical quark masses

$$Z = \int DU\, exp(-S_G(U))\, \left(\mathbf{det}(\hat{D}_u(U) + m_u)\right)^{\mathbf{1/4}} \left(\mathbf{det}(\hat{D}_d(U) + m_d)\right)^{\mathbf{1/4}} \left(\mathbf{det}(\hat{D}_s(U) + m_s)\right)^{\mathbf{1/4}}$$

Conductivity in lattice simulations

- $J_i = \sigma_{ij} E_j$

- Electromagnetic conductivity
  $\sigma_{ij} = \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x \; e^{i\omega t} \langle [J_i(x), J_j(0)] \rangle$
  $\rho_{ij} = -\frac{1}{\pi} Im G_R^{ij}(\omega, \vec{k} = 0)$
  $\sigma_{ij} = \pi \lim_{\omega \to 0} \frac{1}{\omega} \rho_{ij}(\omega)$

- Analytic continuation
  $G_E(\omega, \vec{p}) = -G_R(i\omega, \vec{p}), \quad \omega > 0$

- On lattice we measure
  $C_E(\tau) = \int d^3x \langle J_i(\tau, \vec{x}) J_j(0, \vec{0}) \rangle$
  $C_E(\tau) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega\tau\right)}{sh\left(\frac{\omega}{2T}\right)}, \quad \tau \in \left(0, \frac{1}{T}\right)$

Conductivity with staggered fermions

- Correlation function for staggered fermions

$$C_{ij}(\tau) = \frac{1}{L_s^3} \langle J_i(\tau) J_j(0) \rangle,$$

$$J_i(\tau) = \frac{1}{4} e \sum_f q_f \sum_{\vec{x}} \eta_i(x) (\bar{\Psi}_x^f U_{x,i} \Psi_{x+i}^f + \bar{\Psi}_{x+i}^f U_{x,i}^{+} \Psi_x^f)$$

- Two branches of staggered correlator

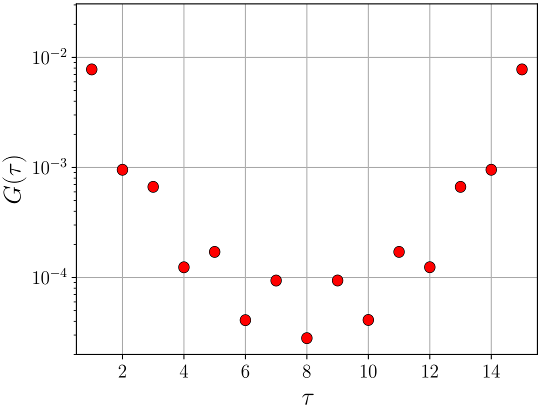
$$C_{ij}^e(\tau = 2n \times a) = \int d^3 y \big(\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle - \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle \big)$$

$$C_{ij}^o(\tau = (2n+1) \times a) = \int d^3 y \big(\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle + \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle \big)$$

$$A_i = e \sum_f q_f \bar{\psi}^f \gamma_i \psi^f, \quad B_i = e \sum_f q_f \bar{\psi}^f \gamma_5 \gamma_4 \gamma_i \psi^f$$

Conductivity with staggered fermions

- Typical plot for the staggered correlation function



Conductivity with staggered fermions

The strategy of the calculation

- Measure $C_E^{even,odd}(\tau)$ on two branches
- Reconstruct the $\rho^{even,odd}(\omega)$ (Backus-Gilbert method)

$$C_E^{even,odd}(t) = \int_0^\infty d\omega \rho^{even,odd}(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Calculate $\rho(\omega) = \frac{1}{2}(\rho^{even}(\omega) + \rho^{odd}(\omega))$

  (what corresponds to the $\langle J_{el}(\tau) J_{el}(0) \rangle$)
- Calculate the conductivity $\sigma = \pi \frac{\rho(\omega)}{\omega}\Big|_{\omega \sim 0}$

Backus-Gilbert method for the spectral function

- Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i,\omega), \quad K(x_i,\omega) = \frac{ch\left(\frac{\omega}{2T} - \omega x_i\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega},\omega)$ - resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega},\omega)\rho(\omega)$$

- Let us expand $\delta(\bar{\omega},\omega)$ as

$$\delta(\bar{\omega},\omega) = \sum_i b_i(\bar{\omega}) K(x_i,\omega) \quad \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

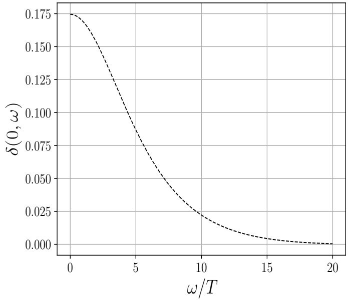
- Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i,\omega)(\omega - \bar{\omega})^2 K(x_j,\omega), R_i = \int d\omega K(x_i,\omega)$$

- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1-\lambda) S_{ij}, \quad 0 < \lambda < 1$$

Backus-Gilbert method for the spectral function



- We calculate the estimator of the spectral function

$$\bar{\rho}(\bar{\omega}) = \int d\omega \delta(\omega, \bar{\omega})\rho(\omega)$$

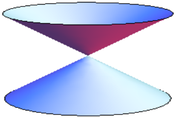
- BG method average the spectral function over the width $\sim 3.5 \times T$
- For very narrow spectral density BG method underestimates conductivity,
- But lattice studies give the width $\sim 4T$ or larger
  - G. Aarts et al, JHEP02, 186 (2015)
  - B. B. Brandt et al,Phys. Rev.D93, 054510 (2016)
  - H.-T. Ding, et al, Phys. Rev.D94, 034504 (2016)

Chiral symmetry breaking in Dirac semimetals

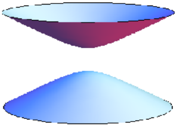
Phase transition in Dirac semimetal at $\alpha_{eff}^c \simeq 1.14$

Phys. Rev. B94, 205147 (2016), Annals Phys. 391 (2018) 278-292

- Semimetal phase $\alpha_{eff} < 1.14$: $\langle\bar{\psi}\psi\rangle = 0$, $M = 0$, $E^2 = v_F^2\vec{p}^2$

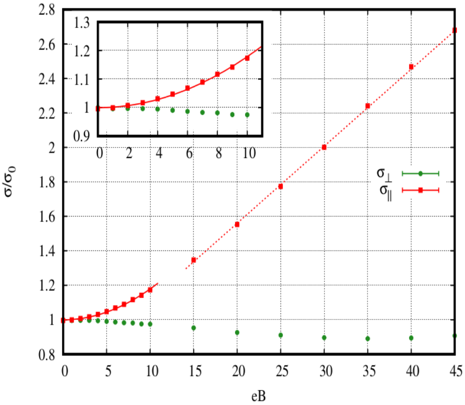


- Insulator phase $\alpha_{eff} > 1.14$: $\langle\bar{\psi}\psi\rangle \neq 0$, $M \neq 0$, $E^2 = v_F^2\vec{p}^2 + M^2$



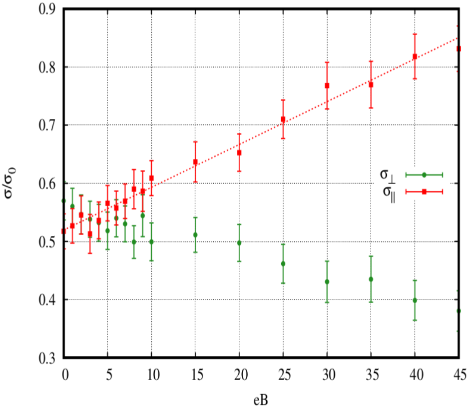
We can study CME in both phases

Conductivity in the semimetal phase



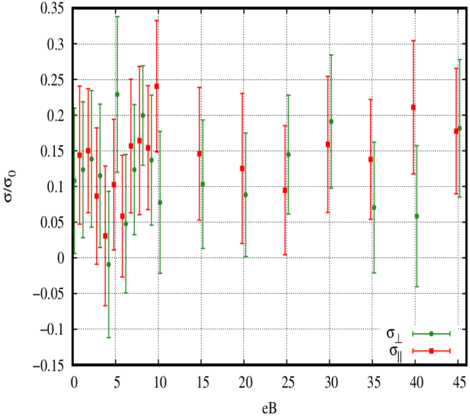
- Rise of $\sigma_{||}$ with $B \Rightarrow$ **we observe CME**
- Decrease of $\sigma_\perp$ with $B \Rightarrow$ **we observe magnetoresistance**

Conductivity in the transition region



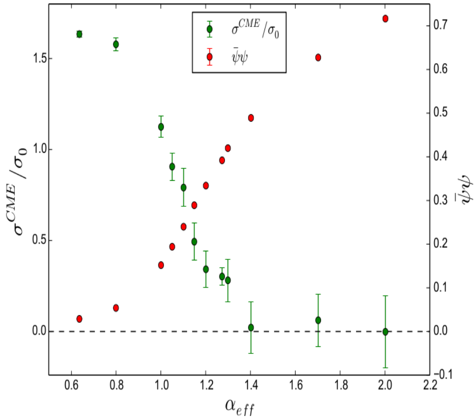
- CME in the the transition region has smaller magnitude

Conductivity in the insulator phase



- We don't observe CME in the insulator phase

CME and chiral symmetry breaking



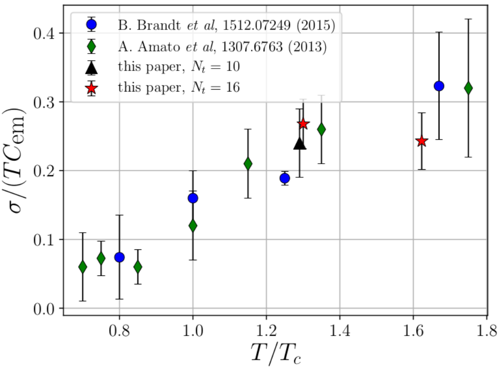
- Chiral symmetry is important for CME

Lattice simulation of QCD: details

- Stout smeared staggered $2+1$ fermions
- Physical pion m_π and strange m_s quark masses
- $T \approx 200$, 250 MeV
- Lattice sizes and steps:

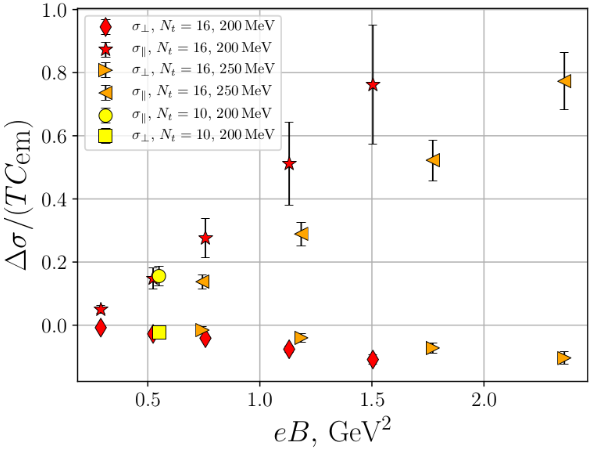
a, fm	L_s	N_t
0.988	48	10
0.0618	64	16

Conductivity at zero magnetic field $eB = 0$



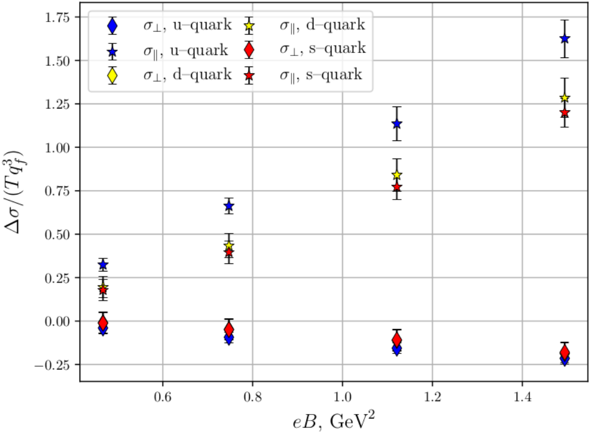
- **First calculation of the conductivity at physical pion mass**
- Agreement with previous papers

Conductivity at nonzero magnetic field $eB \neq 0$



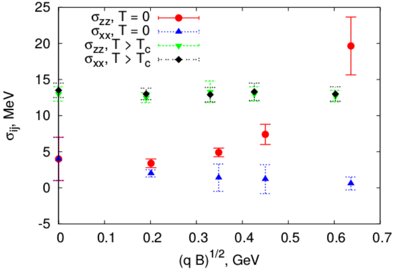
- We observe CME and magnetoresistance in QGP

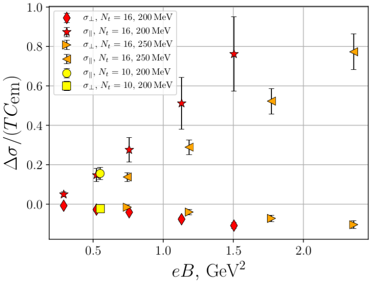
The contribution of different quarks



- The conductivity scale as q_f^3
- $\sigma_d/q_d^3 \simeq \sigma_s/q_s^3$, $\sigma_u/q_u^3 > \sigma_{d,s}/q_{d,s}^3$ $(|q_u| = \frac{2}{3}, |q_d| = |q_s| = \frac{1}{3})$
- Large mass of s-quark does not influence the conductivity

Comparison with other studies





P.V. Buividovich et al., Phys.Rev.Lett. 105 (2010) 132001

- No CME in QGP and there is CME in the confinement phase
- Disagreement with our results (small magnetic fields? $eB < 0.36\,\mathrm{GeV}^2$; complicated spectral function in confinement)

Conclusion:

- Observe CME in Dirac semimetals (semimetal phase)
- No CME in the phase with broken chiral symmetry
- The first calculation of the conductivity in QCD at physical pion mass
- Observe CME and magnetoresistance in QGP