Unruh Instability

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Original Papers

Original statements are based on common work with George Prokhorov and Oleg Teryaev (JINR, Dubna)

“Instability at Unruh temperature as manifestation of singularity in complex momentum plane”arXiv:1906.03529

and “ Unruh effect for fermions from the Zubarev density operator” arXiv:1903.09697 [hep-th]

and as well as on some other papers by the same authors: arXiv:1807.03584, 1805.12029 ...

see also theTalk by G. Prokhorov at this Conference

Explanation of words

One considers a sequence of states (media), accelerated ($\vec{a} \neq 0$) and thermal ($T \neq 0$).

beginnng with $T > T_{Unruh}$ where

$$T_{Unruh} \equiv \frac{a}{2\pi}$$

and ending at $T < T_{Unruh}$

Below $T = T_{Unruh}$ quantum corrections give negative contribution to the energy density

Conjectured "Unruh instability" of these states is their decay into Minkowskian vacuum plus particles

Reservations

Motivation

We consider thermodynamics of accelerated ($\vec{a} \neq 0$) media and develop a technique to obtain one-loop exact results in Quantum Statistical Physics,

evaluation of the Unruh temperature being an example

Consideration of the"Unruh instability" is the first attempt on nonperturbative application of the technique

Predecessors: the topic "Fast thermalization and Unruh temperature" was pioneered by D. Kharzeev (2005) and Castorina+D.Kharzeev+H. Satz (2007)
Also, discussed in holography (see, e,g, I. Aref'eva)

At a closer look, our pictures differ considerably

Outline of the Talk

- Anomalies and quantum statistical physics (QSP)
- Finite-size perturbative expansions in QSP
- Instability at $T < T_{Unruh}$

Quantum effects and hydrodynamics ("recent")

Pioneering paper: D.T. Son, P. Surowka arXiv:0906.5044

Hydro: expansion in derivatives plus conservation laws:

$$\partial_\mu s^\mu \geq 0, \partial_\mu J_V^\mu \ = \ 0, \ \partial_\mu J_A^\mu \ = \ e^2 C_{anom}(\vec{E}\cdot\vec{H}), \partial_\mu \Theta^{\mu\nu} = \ldots$$

(s_μ is entropy current, $\vec{E}, \vec{H}$ electric, magnetic fields)

Result: extra pieces in currents uniquely fixed by C_{anom}

$$(J^\mu_{Axial})_{hydro} \ = \ n_A \cdot u^\mu + \mu^2 C_{anom} \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma \ + \ldots$$

where μ is the chemical potential.

No superfluidity assumed, ideal fluids

Trading equilibrium for new interaction

In thermodynamics,

$$\hat{H} \;\rightarrow\; \hat{H} - \mu \hat{Q}$$

Instead, could think in terms of new 4d interaction:

$$\mu \hat{Q} \;\rightarrow\; \mu u_\alpha \hat{J}^\alpha, \quad \text{or} \quad eA_\alpha \;\rightarrow\; eA_\alpha + \mu u_\alpha$$

where μ is chem.potential, u_α is 4-velocity of the fluid

(A.V. Sadofyev, V.I. Shevchenko, V.I.Z., arXiv:1012.1958)

Starting from the triangle anomaly
chiral effects of Son&Surowka are reproduced

Some details

In U(1) case chiral anomaly can be reformulated as conservation of “extended current”:

$$\partial_\alpha \left(J_5^\alpha - e^2 C_{anom} \epsilon^{\alpha\beta\gamma\delta} A_\beta \partial_\gamma A_\delta \right) = 0$$

Making the substitution $eA_\alpha \to eA_\alpha + \mu u_\alpha$ reproduces all the chiral effects in the current

The example cannot be systematically generalized since ideal fluid corresponds to (unknown) strong interactions between constituents.

If one starts from free particles one can hope to develop systematic pert. theory in, say, μ, Ω. A first example was elaborated in fact long time ago.

“Statistical perturbation theory”, systematic

Alexander Vilenkin (1980), rotation case

$$< J^\mu(\vec{x}) > = Tr\Big(\hat{\rho} J^\mu(\vec{x}, t)\Big)$$

where $J^\mu = \frac{1}{2}[\bar{\Psi}, \gamma^\mu \Psi]$ is the current density operator and

$$\hat{\rho} = C \exp\Big(- \beta(\hat{H} - \hat{\vec{M}} \cdot \vec{\Omega} - \Sigma_i \mu_i \hat{N}_i)\Big)$$

$\hat{\rho}$ is the statistical operator, built on conserved operators

$\beta \equiv T^{-1}$,$\hat{\vec{M}}$ is angular momentum, $\vec{\Omega}$ is angular velocity

μ_i is chemical potential, $\hat{N}_i$ is number of charged particles

Chiral vortical effect ($\vec{J}^A \sim \vec{\Omega}$) is the same as for fluid

Hint on unversality of the results of “statistical pert.th.”

(Polynomial) Sommerfeld Integrals

Statistical average $< j^5 >$ reduces to a Sommerfeld integral
:

$$< J^5 > = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \epsilon^2 d\epsilon \cdot \left(\frac{1}{1 + e^{\beta(\epsilon - (\mu + \Omega/2))}} - \frac{1}{1 + e^{\beta(\epsilon - (\mu - \Omega/2)}} \right)$$

Linear in Ω term is polynomial in μ, reproduces anomaly:

$$< \vec{J}_5 > = \frac{\mu^2}{4\pi^2} \vec{\Omega}$$

Polynomial Sommerfeld integrals in QSPh play role of anomalies in QFTh, producing exact one-loop results

We build up our next steps on this observation.

Zubarev density operator

In the spirit of the discussion above one introduces the Lorentz-invariant density operator

$$\hat{\rho} \;=\; \frac{1}{Z}\exp\Big(-\beta_\alpha \hat{P}^\alpha + \frac{1}{2}\bar{\omega}_{\alpha\beta}\hat{J}^{\alpha\beta} + \zeta\hat{Q}\Big)$$

where $\beta_\alpha = u_\alpha/T$, $\bar{\omega}_{\alpha\beta} = -1/2\big(\partial_\alpha\beta_\beta - \partial_\beta\beta_\alpha\big)$, $\zeta = \mu/T$
D. N. Zubarev, TMF (1979)....reviewed F.Becattini et al.
e-Print: arXiv:1704.02808
Consider first no rotation, no charge case.

$$\hat{\rho} \;=\; \frac{1}{Z}\exp\Big(-\beta_\mu \hat{P}^\mu - \alpha_z \hat{K}^z\Big)$$

where $\alpha_\mu \;=\; u^\alpha\partial_\alpha u_\mu/T$, $\hat{K}^z$ is the boost operator

Inclusion of acceleration $a \neq 0$ as a challenge

The density operator looks rather paradoxical:

- constant a implies horizon, but we work in Minkowski
- first-order interaction is exponentiated, but can be put to zero by choice of coordinates
- Not included into L&L textbook

Perturbative technique is developed by F. Becattini et al
see, F. Becattini, e-Print: arXiv:1712.08031 + references

We find new polynomial Sommerfeld integrals and new exact one-loop results

Quasi-dispersive representation for ϵ

Within the approach developed, the energy density for massless fermions is given by:

$$\epsilon = \frac{a^4}{120\pi^2} + + \frac{T^4}{\pi^2}\Big(\int_{-\infty}^{+\infty} \frac{x^3 dx}{e^{x+iy}+1} + (y \to -y)\Big) \quad (1)$$

$$+2iy\frac{T^4}{\pi^2}\Big(\int_{-\infty}^{\infty} \frac{x^2 dx}{e^{x+iy}+1} - (y \to -y)\Big) \quad (2)$$

where "dispersive integrals" are in fact noval polynomial Sommerfeld integrals
the "subtraction term" is calculated perturbatively, via use of the Zubarev density operator No free parameters left

"Emergent horizon"

$$\epsilon = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2}$$

In field theory:

$$< T_0^0 > = \frac{7\pi^2}{60\beta_R^4\rho^4} + \frac{1}{24\beta_R^2\rho^4} - \frac{17}{960\pi^2\rho^4}$$

where $\rho = 1/a$ is distance to the horizon, $\beta = a/T$, a, T are independent at price of conical singularity.
Casimir effect due to the horizon is reproduced by statistical pert.th. in Minkowskian space

$$\epsilon = \frac{1}{120}\left(T^2 - (\frac{a}{2\pi})^2\right)(17a^2 + 28\pi^2 T^2)$$

ϵ vanishes at the Unruh temperature, as it should do

Acceleration as imaginary chemical potential

Our "central equation" above incorporates generalization of Fermi-distribution to the cases of $\vec{\Omega}, \vec{a} \neq 0$.
For simplicity consider $\vec{\Omega}||\vec{a}$

Rotation (as mentioned also by some other authors):

$$\mu \ \rightarrow \ \mu \pm \frac{\Omega}{2}$$

Acceleration:

$$\mu \ \rightarrow \ \mu \pm \frac{\Omega}{2} \pm i\frac{|a|}{2}$$

as is suggested by field theory
When expanded in ia, odd powers vanish.
However, non-perturbatively there is a pole at $a/T = 2\pi$

“Unruh Instability”

We used the same non-pert. expression for ϵ to analytically continue to negative ϵ The observations/speculations are:

- Energy of quantum levels becomes negative. Suggesting instability of the state. By analogy with, say, super-radiance
- If applied to heavy-ion-collision physics, would imply decay of the state with large acceleration (mechanical energy) into a thermal state
- Energy becomes negative smoothly, discontinuity in the second derivative of the energy with respect to temperature. Somewhat similar to second-order phase transition

Statistical pert. th. versus field-theory expansion

After submission of our paper we found out that (somewhat) similar ideas are contained in
"Vacua on the Brink of Decay"
G. L. Pimentel, A. M. Polyakov, G. M. Tarnopolsky
e-Print: arXiv:1803.09168

who considered 2d problems with external e-m or grav. field when lowest quantum level becomes negative. Rather striking similarity of conclusions.

In particular, the main conclusion is that the decay of the "old" vacuum is soft, or controllable, through analytic contnuation

“Aanalyticity in integer numbers”

Smoothness of the transitions in the two cases is similar However, unlike the standard analyticity the “thermodynamic analyticity” in terms of finite-size pert. expansion does not have imaginary part or singularities, like log. Instead, for example, the polynomial changes from containing even powers of acceleration into containing odd powers.

Mystery of the quantum-classical similarity

In both cases, the classical solution seems to be unstable as well.

In our case, in negative region we need a conical ingularity with angle larger than 2π.

In case of Plimentel et al., they get $g_{00} < 0$

However, Plimentel et al. make a point that collapse in their case is controllable, i.e. smooth. And enjoy similarity of the critical exponents, classical and quantum. By classical instability they understand the Choptuik observations on collapse of matter into a back hole.

Conclusions

Concentrating on the instability:

- Analytical continuation to negative energies of quantum corrections has been worked out within "finite-size perturbative expansions in statistical physics"
- There are rather reasonable argument in favor of an instability at the Unruh temperature if temperature is smaller that the acceleration
- The transition seems to be smooth
- There are interesting parallels with stanard FT approach to instabilities