Lattice study of QCD–lite theories

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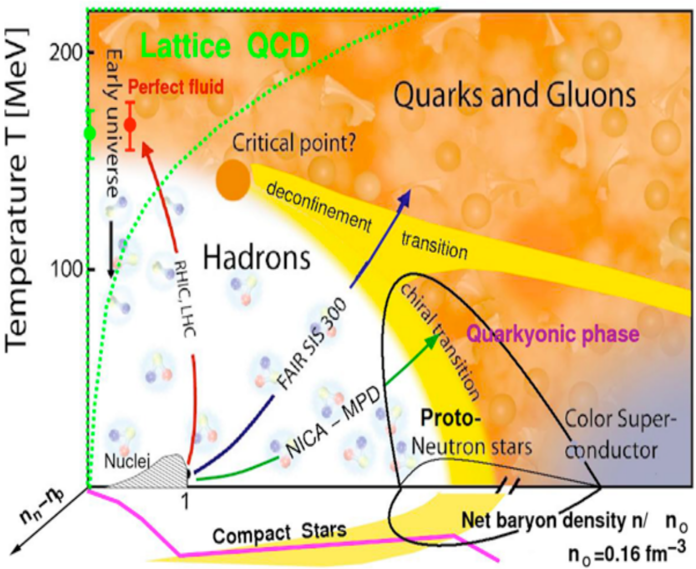
JHEP 1803 (2018) 161
JHEP 1905 (2019) 171

Joint Institute for Nuclear Research

Theory of Hadronic Matter Under Extreme Conditions, 2019

QCD phase diagram



$SU(3)$ QCD

- $Z = \int DU D\bar{\psi} D\psi \ \exp\left(-S_G - \int d^4x \bar{\psi}(\hat{D}+m)\psi\right) =$
  $= \int DU \exp\left(-S_G\right) \times \det\left(\hat{D}+m\right)$
- Eigenvalues go in pairs $\hat{D}: \ \pm i\lambda \Rightarrow$
  $\det\left(\hat{D}+m\right) = \prod_\lambda (\lambda^2 + m^2) > 0$
  i.e. one can use lattice simulation
- Introduce chemical potential:
  $\det\left(\hat{D}+m\right) \rightarrow \det\left(\hat{D} - \mu\gamma_4 + m\right) \Rightarrow$
  the determinant becomes complex (**sign problem**)

$SU(2)$ QCD

- $(\gamma_5 C \tau_2) \cdot D^* = D \cdot (\gamma_5 C \tau_2)$
- Eigenvalues go in pairs $\hat{D} - \mu\gamma_4$: λ, λ^*
- For even N_f $\det\left(\hat{D} - \mu\gamma_4 + m\right) > 0 \ \Rightarrow$ **free of sign problem**

Differences between $SU(3)$ and $SU(2)$ QCD

- $SU(2)$ QCD symmetry: $SU(2N_f)$
  $SU(3)$ QCD symmetry: $SU_R(N_f) \times SU_L(N_f)$
- $SU(3)$: baryons are fermions of 3 quarks, $SU(2)$: baryons are bosons of two quarks

However, at large μ_q this becomes unimportant (see later).

Similarities

- There are transitions: confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of observables are very close:

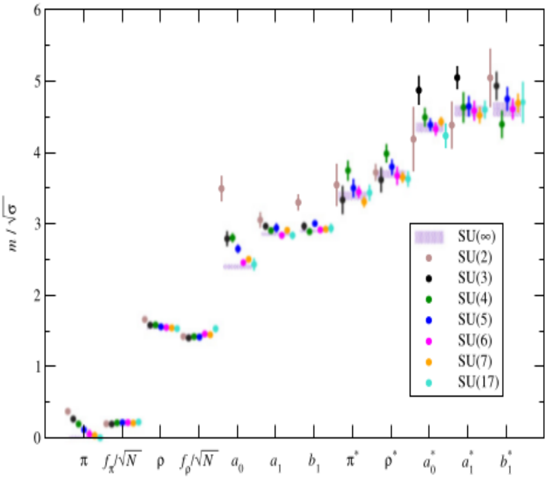
Topological susceptibility (Nucl.Phys.B715(2005)461):
$\chi^{1/4}/\sqrt{\sigma} = 0.3928(40)\ (SU(2)), \quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35)\ (SU(3))$

Critical temperature (Phys.Lett.B712(2012)279):
$T_c/\sqrt{\sigma} = 0.7092(36)\ (SU(2)), \quad T_c/\sqrt{\sigma} = 0.6462(30)\ (SU(3))$

Shear viscosity
$\eta/s = 0.134(57)\ (SU(2)), \quad \eta/s = 0.102(56)\ (SU(3))$

JHEP 1509(2015)082 Phys.Rev. D76(2007)101701

Spectroscopy (Phys.Rep.529(2013)93)



Phase diagram of dense $SU(N_c)$ for $N_c \to \infty$

L. McLerran, R. D. Pisarski, Nucl.Phys. A796 (2007) 83-100

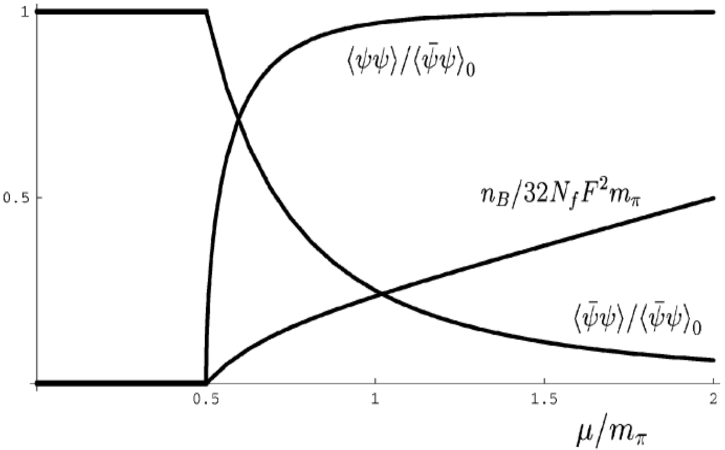
- Hadron phase $\mu < M_N/N_c$ $(p \sim O(1))$
- Dilute baryon gas $\mu > M_N/N_c$ (width $\delta\mu \sim \frac{\Lambda_{QCD}}{N_c^2}$)
- Quarkyonic phase $\mu > \Lambda_{QCD}$ $(p \sim N_c)$
    - Degrees of freedom:
        - Baryons (on the surface)
        - Quarks (inside the Fermi sphere $|p| < \mu$)
    - No chiral symmetry breaking
    - The system is in confinement phase
- Deconfinement $(p \sim N_c^2)$

Small and moderate chemical potentials

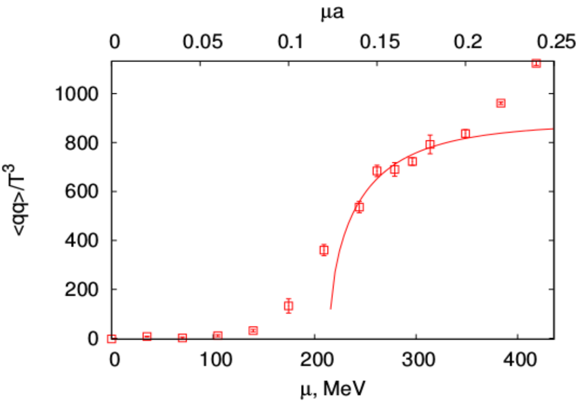
Results of Phys.Rev.D94(2016)no.11:

- Staggered fermions
- 2-rooting corresponds to $N_f = 2$
- Diquark source in the action $\delta S \sim \lambda \psi^T (C\gamma_5) \times \sigma_2 \times \tau_2 \psi$, $\lambda \to 0$ explicitly breaks $U_V(1)$ symmetry to observe $\langle qq \rangle$ on finite lattice
- Wilson gauge action
- $a = 0.11\,\mathrm{fm}$
- $m_\pi = 362(40)$ MeV
- Lattice: $16^3 \times 32$

Predictions of ChPT

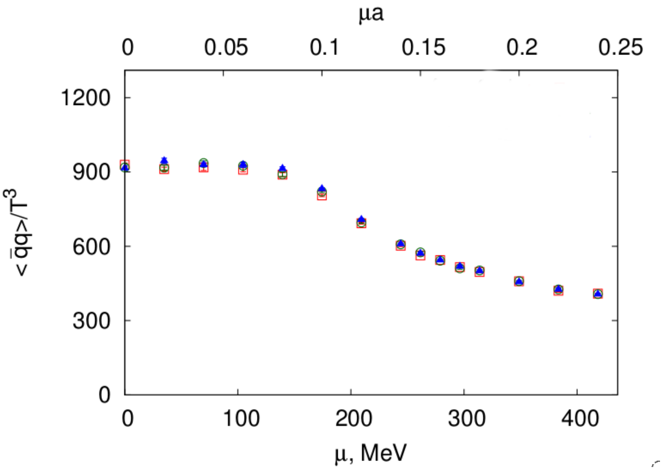


Diquark condensate



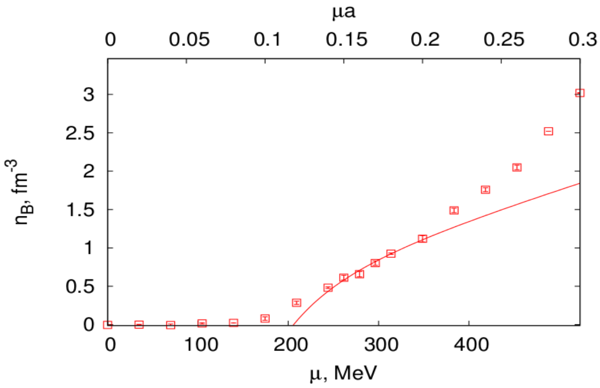
- Good agreement with ChPT: $\langle\psi\psi\rangle/\langle\bar{\psi}\psi\rangle_0 = \sqrt{1 - \frac{m_\pi^4}{\mu^4}}$
- Phase transition at $\mu \sim m_\pi/2$
- Bose Einstein condensate (BEC) phase $\mu \in (200, 350)$ MeV

Chiral condensate



Good agreement with ChPT

Baryon density



- Good agreement with ChPT $n \sim \mu - \frac{m_\pi^4}{\mu^3}$
- Transition from *dilute baryon gas* to *dense quark matter* is visible

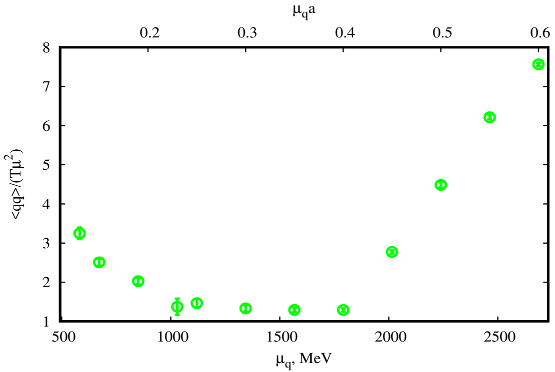
Large chemical potentials

Study of QCD at ultrahigh densities

The results are presented in JHEP 1803 (2018) 161:

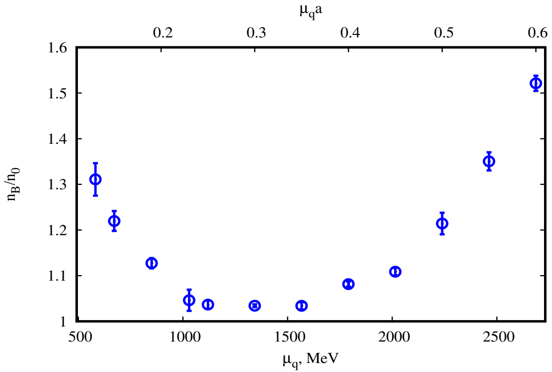
- Tree-level improved gauge action
- $a = 0.044\,\text{fm}$: closer to continuum limit, allows for reaching large μ without lattice artifacts
- $m_\pi = 740(40)\,\text{MeV}$
- Lattice: $32^3 \times 32$

Diquark condensate



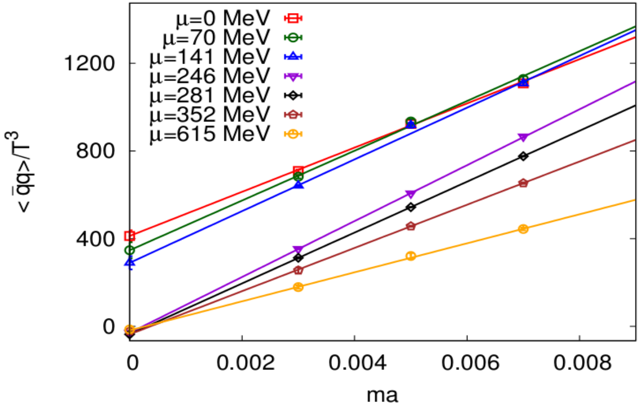
- Bardeen–Cooper–Schrieffer (BCS) phase $\mu > 800$ MeV, $\langle\psi\psi\rangle \sim \mu^2$
- **Baryons (on the surface)**

Baryon density



- Free quarks $n_0 = N_f \times N_c \times (2s+1) \times \int \frac{d^3p}{(2\pi)^3} \theta(|p| - \mu) = \frac{4}{3\pi^2}\mu^3$
- **Quarks inside Fermi sphere**
- Quarks inside Fermi sphere dominate over the surface: $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$

Chiral condensate (chiral limit $m \to 0$)



Chiral symmetry breaking restoration at $\mu > 256\,\mathrm{MeV}$

Phase structure

- $\mu < \mu_c = m_{\pi/2} \sim 200\,\text{MeV}$: **hadronic phase**, confinement, chiral symmetry broken, no diquark condensate and baryon density, dof — Goldstone bosons
- $\mu_c < \mu < \mu_d \approx 350\,\text{MeV}$: **BEC phase**, Bose-Einstein condensate of scalar diquarks, confinement, dof — Goldstone bosons, *chiral symmetry restoration*
- $\mu > \mu_d$: ChPT deviations: **transition from dilute baryon gas to dense matter**, transition μ is close for $SU(2)$ and $SU(3)$.
- $\mu \in (500, 1000)\,\text{MeV}$: **BCS phase**, quarks within Fermi-sphere, duqiarks on the surface, chiral symmetry restored: less differences between $SU(2)$ and $SU(3)$.

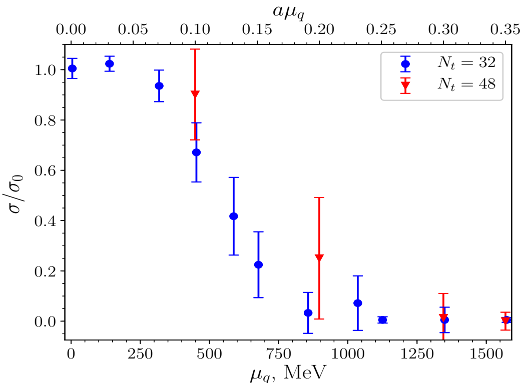
Quarkyonic phase:

- Baryons (on the surface) ✓
- Quarks (inside the Fermi sphere $|p| < \mu$) ✓
- No chiral symmetry breaking ✓
- The system is in confinement phase?

Study of deconfinement in dense two-color QCD

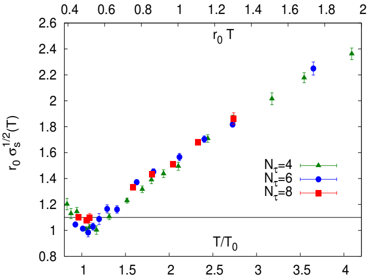
arXiv:1808.06466

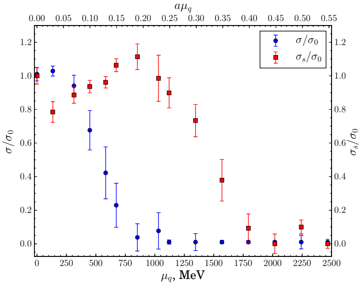
String tension



- Good fit by the Cornell potential: $V(r) = A + \frac{B}{r} + \sigma r \quad \mu \leq 1100$ MeV
- Good fit by the Debye potential: $V(r) = A + \frac{B}{r} e^{-m_D r} \quad \mu \geq 850$ MeV
- Confinement/deconfiniment transition in $\mu \in (850, 1100)$ MeV

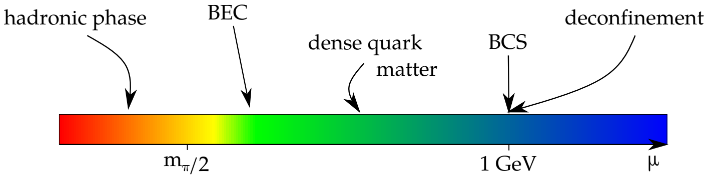
Spatial string tension



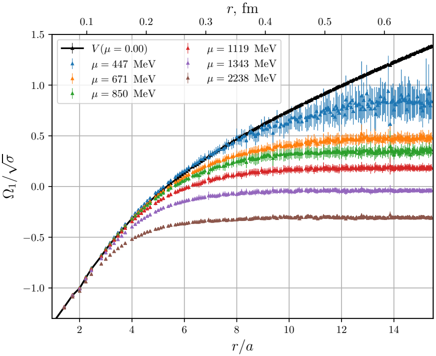


- Deconfinement at $\mu > 900 - 1100\,\text{MeV}$?
- Spatial string tension disappears at $\mu \geq 1800$ MeV ($a\mu > 0.4$)
- Different behaviors of spatial string tension at finite temperature and finite density

Tentative phase diagram

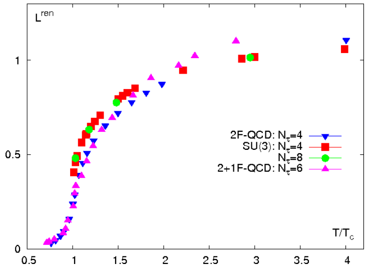


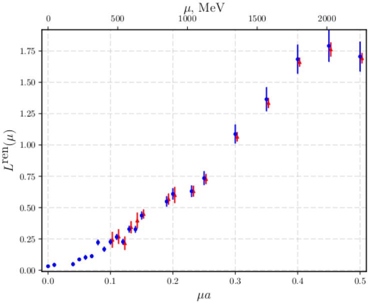
The grand potential



- The $L = \exp\Big(-\Omega(r=\infty, T, \mu)/2T\Big)$ is the Polyakov line of single quark/antiquark
- The Polyakov line L is an approximate order parameter:
  - In confinement phase $\Omega(r, T, \mu) = \infty$ (due to string breaking it is finite but large)
  - In deconfinement $\Omega(r, T, \mu) = finite$

The Polyakov lines at finite temperature and density

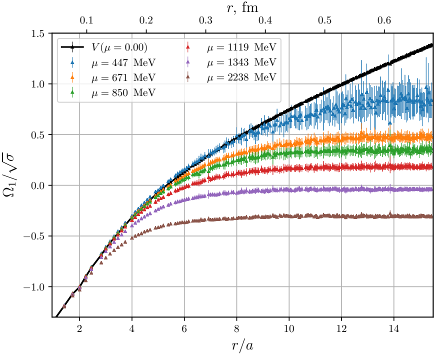




Polyakov lines and confinement/deconfinement transition

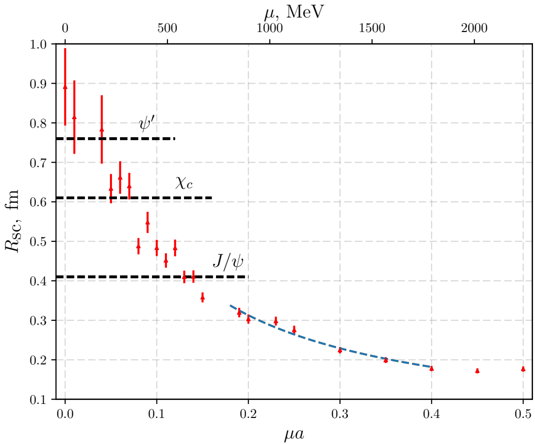
- Rapid transition at finite T
- Smooth transition at finite μ
- Nontrivial physics at $\mu > 2000$ MeV

String breaking in cold dense quark matter



- The plateau in the grand potential is the manifestation of the string breaking
- The screening length $\Omega(\infty, \mu) = V_{\mu=0}(R_{sc})$

Screening length and quarkonia dissociation

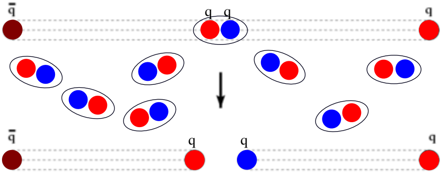


Quarkonia dissociation even in confinement! The fit is $R(\mu) = 1/Am_D(\mu)$ with one-loop expression for $m_D(\mu)$.

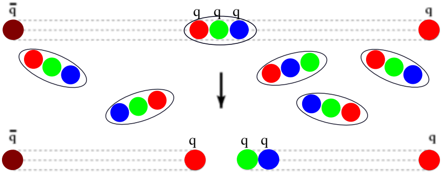
At $\mu \in (900, 1300)$ MeV Coulomb starts to dominate over the maximum string energy: deconfinement.

String breaking in dense medium

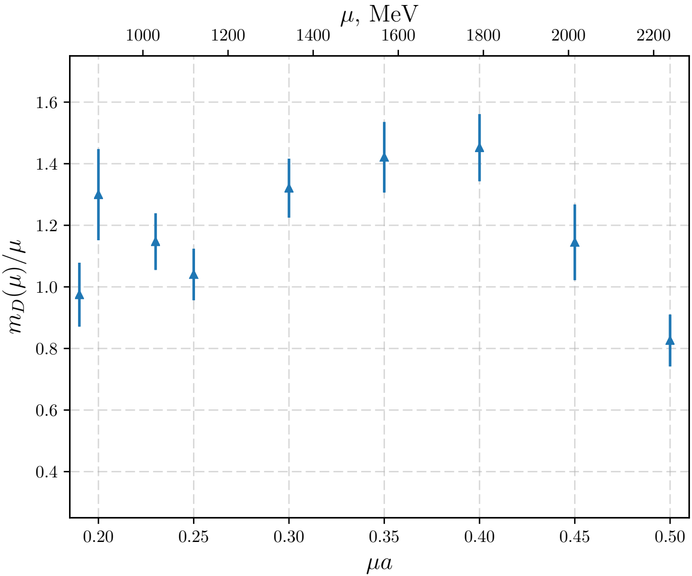
In SU(2) QCD:



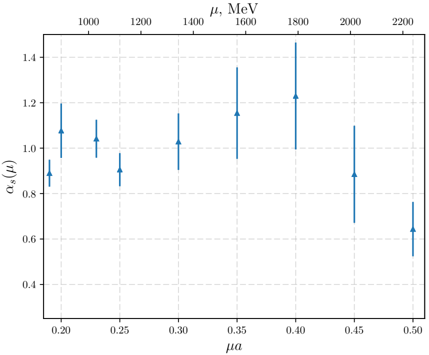
Analogous mechanism may be proposed in SU(3) QCD:



Debye mass in cold dense matter

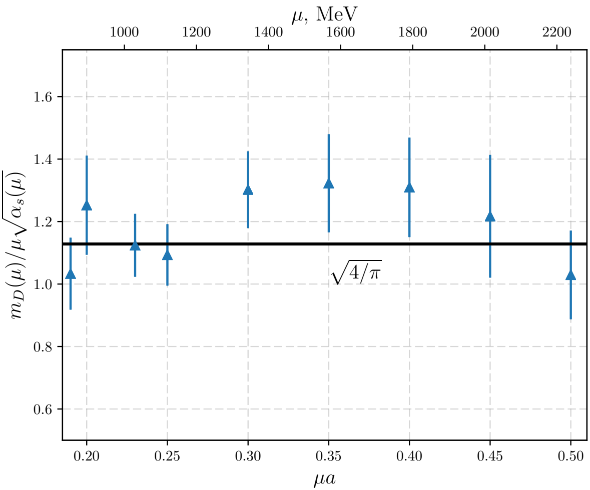


Effective coupling constant in cold dense matter



$\alpha_s \sim 1$ i.e. even at high density QCD is strongly correlated

One-loop formula for the Debye mass



- The one-loop formula: $m_D^2(\mu) = \frac{4}{\pi}\alpha_s(\mu)\mu^2 \Rightarrow \frac{m_D(\mu)}{\mu\sqrt{\alpha_s(\mu)}} = \sqrt{\frac{4}{\pi}}$
- The one-loop formula works well even for the $\alpha_s \sim 1$

Conclusions

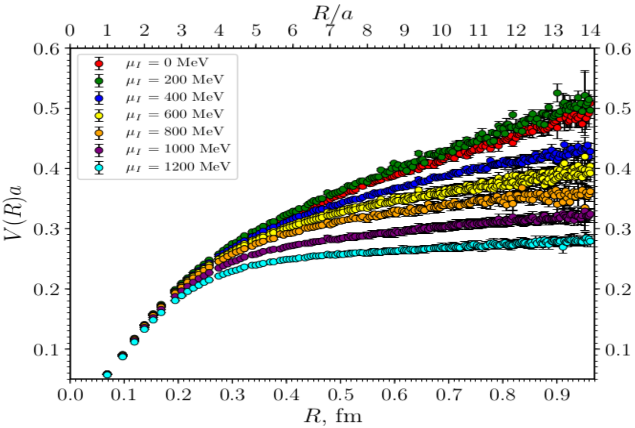
- **Observation of deconfinement in dense medium**
- Difficult to determine critical chemical potential $\mu_c \in (850, 1100)\,\mathrm{MeV}$
- Spatial string tension disappears $\mu \geq 1800\,\mathrm{MeV}$
- Deconfinement at finite density is different to deconfimenent at finite temperature
- String breaking distance decreases with density
- Heavy quarkonia dissociate at moderate densities due to string breaking
- We observe Debye screening phenomenon in deconfimenent phase

Study of $SU(3)$ QCD at nonzero isospin density

- stout staggered fermions;
- dynamical $\mathrm{u, d, s}$–quarks;
- tree-level improved gauge action;
- $a = 0.069\,\mathrm{fm}$; one can reach larger isospin density without lattice artifacts $\mu > 1000\,\mathrm{MeV}$;
- $m_\pi \simeq 380\,\mathrm{MeV}$, m_s at its physical value
- Lattice: $28^3 \times 28$

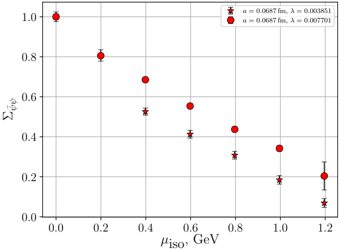
Preliminary results!

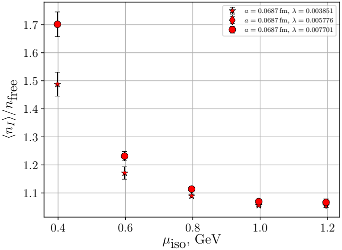
Potential between static quark-antiquark pair



We observe deconfinement at large isospin density!

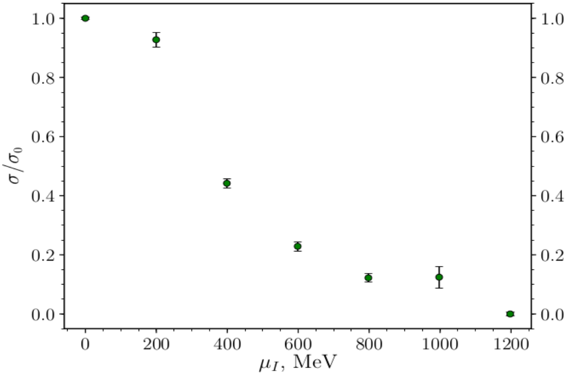
Fermionic observables





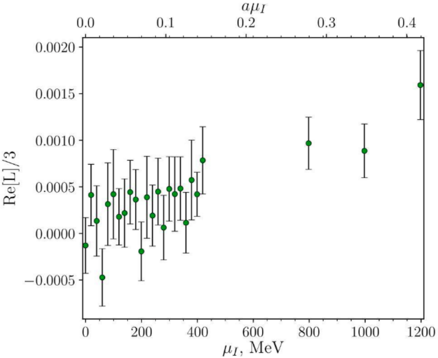
The situation is similar to the $SU(2)$ with μ case.

String tension



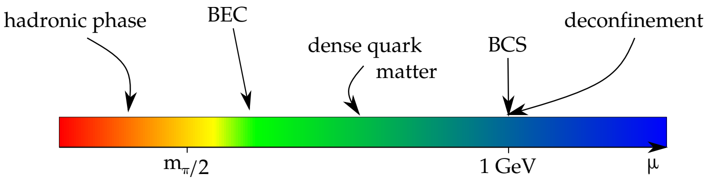
Deconfinement takes place at $\mu \sim 1200\,\mathrm{MeV}$

The Polyakov line at finite isospin density



It seems it rises but uncertainties are quite large

Tentative phase diagram of SU(3) QCD at finite isospin density



- Low densities were studied in Phys.Rev. D97 (2018) no.5, 054514;
- Deconfinement transition at $\mu \sim 1\,\text{GeV}$;
- Relevant degrees of freedom in the region $\mu \sim 1\,\text{GeV}$: color-singlet Cooper pairs (pions) and quarks with $\Delta(\mu)$;

Conclusion:

- observation of deconfinement in dense SU(2) QCD and isospin dense $SU(3)$ QCD;
- deconfinement in both theories takes place at $\mu \sim 1\,\mathrm{GeV}$;
- relevant degrees of freedom are quarks with mass gap $\Delta(\mu)$ and color-singlet Cooper pairs;
- in $SU(2)$ QCD spatial string tension disappears $\mu \geqslant 1800\,\mathrm{MeV}$;