QCD phase diagram and its dualities

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in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
Prof. V. Ch. Zhukovsky

strong connections with
prof. D. Ebert, Humboldt University of Berlin

details can be found in

Phys.Rev. D100 (2019) no.3, 034009 arXiv:1907.04151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

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БАЗИС
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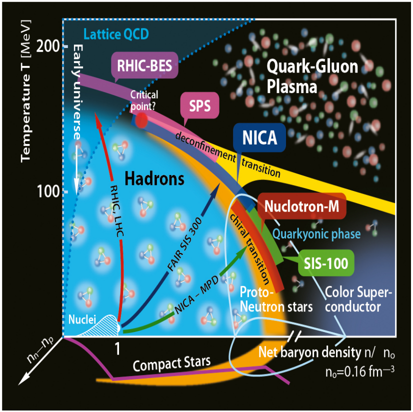
QCD at T and μ
(QCD at extreme conditions)

- neutron stars
- heavy ion collisions
- Early Universe

Methods of dealing with QCD

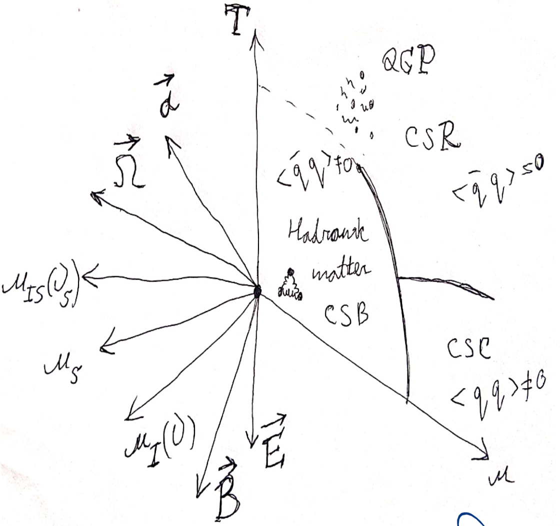
- First principle calcultion – lattice QCD
- Effective models
- .....



More external conditions to QCD

More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields *(see talk of A. Kotov)*
- rotation of the system $\vec{\Omega}$ *(see talk of Prof. Fukushima)*
- acceleration $\vec{a}$ *(see talk of Prof. Zakharov and G. Prohorov/Prof. Teryaev)*
- finite size effects (finite volume and boundary conditions)



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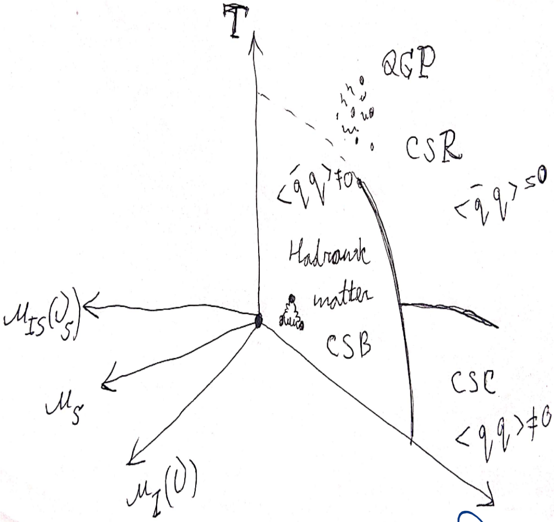


Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

see talk of F. Cuteri

chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

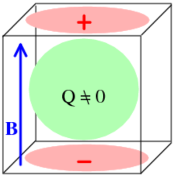
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

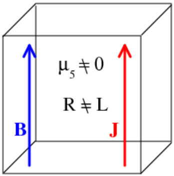
The corresponding term in the Lagrangian is

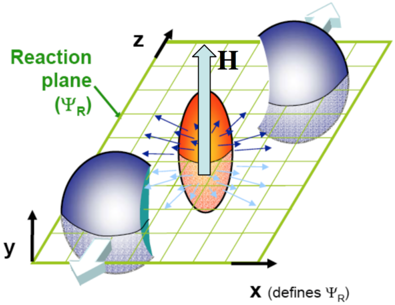
$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

See talk by A. V. Andrianov

Chiral magnetic effect

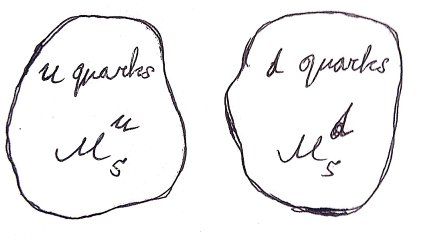






$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

Chiral imbalance n_5 and hence μ_5 can be generated in parallel magnetic and electric fileds

$$\vec{E}\,||\,\vec{B}$$

M. Ruggieri, M. Chernodub, H. Warringa et al

Chiral isospin imbalance n_{I5} and hence μ_{I5} can be generated in parallel magnetic and electric fileds $\vec{E}\,||\,\vec{B}$

μ_{I5} and μ_5 are generated by $\vec{E}\,||\,\vec{B}$

Generation of Chiral imbalance in dense quark matter

Chiral imbalance could appear in dense matter

- Chiral separation effect
  (Thanks to Igor Shovkovy)
- Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k $(k = 1, 2, 3)$ are Pauli matrices.

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[\mathrm{i}\partial\!\!\!/ + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi_a^2\Big].$$

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

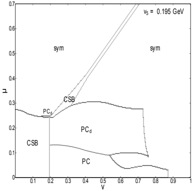
Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates

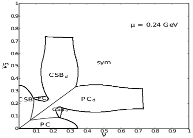
$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0.$$

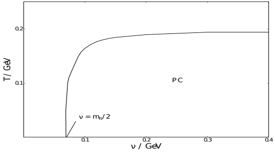
where M and Δ are already constant quantities.

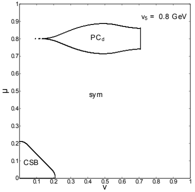
Phase diagram, lots of plots

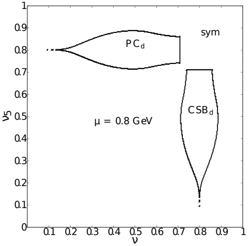
Symmetry 2019, 11(6), 778

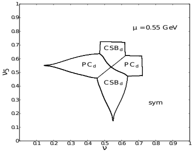












Chiral imbalance lead to the generation of PC in dense quark matter (PC$_d$)

Dualities

Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...)$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \mu_i, ..., M, \Delta, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...) \qquad \Omega(T, \mu, \mu_i, ..., M, \Delta, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \Delta, \mu_I, \mu_{I5})$$

$$M \longleftrightarrow \Delta, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \Delta, \mu_I, \mu_{I5}) = \Omega(\Delta, M, \mu_{I5}, \mu_I)$$

Duality in the phase portrait

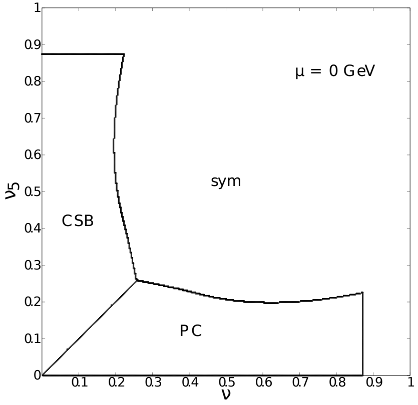


Figure: NJL model results

$$\mathcal{D}: \; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

Duality, Chiral limit

$$\mathcal{L}_{\rm QCD} = \sum_{f=u,d} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a .$$

$$\mathcal{L}_{\rm NJL} = \sum_{f=u,d} \bar{q}_f \Big[\mathrm{i}\gamma^\nu \partial_\nu - m_f \Big] q_f + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2 \Big]$$

m_f is current quark masses

In the chiral limit $m_f = 0$ the Duality $\mathcal{D}$ is exact

m_f : $\frac{m_u+m_d}{2} \approx 3.5$MeV
In our case typical values of $\mu, \nu, ..., T, ..\sim 10 - 100s$ MeV, for example, 200-400 MeV
One can work in the chiral limit $m_f \to 0$
$m_f = 0 \quad \to \quad m_\pi = 0$
physical m_f a few MeV $\quad \to \quad$ physical $m_\pi \sim 140$ MeV

Duality between CSB and PC is **approximate** in **physical point**

duality is approximate

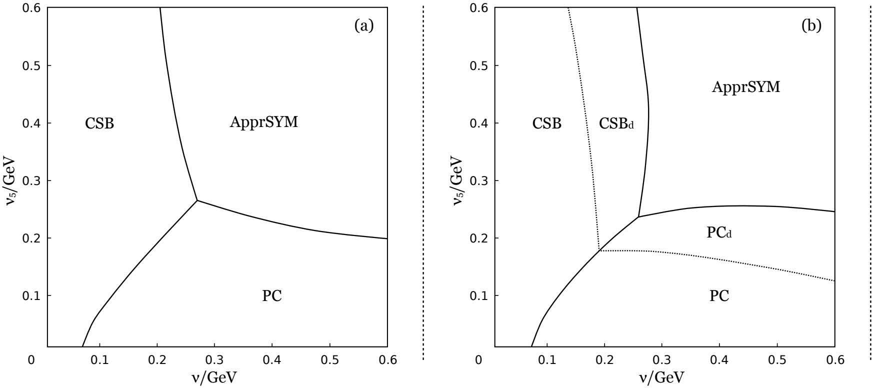


Figure: (ν, ν_5) phase diagram

Other Dualities

They are not that strong but still...

They could still be usefull

The TDP

$$\Omega(T, \mu, \nu, \nu_5, \mu_5;\ M, \Delta)$$

Let us assume that there is no PC

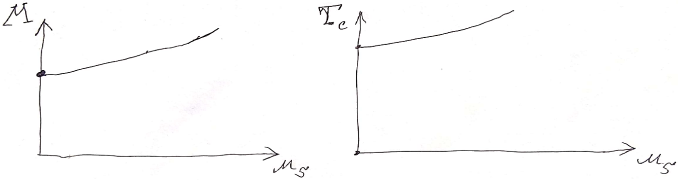
$$\Delta = 0$$

The TDP (phase diagram) is invariant under

$$\mu_5 \longleftrightarrow \nu_5$$

Chiral symmetry breaking phenomenon does not feel the difference between μ_5 and ν_5

QCD at non-zero μ_5



catalysis of CSB by chiral imbalance:

- increase of $\langle \bar{q}q \rangle$ as μ_5 increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

M(ν_5) the same as M(μ_5)

all the results can be obtained by duality only (no PC)

catalysis of CSB by chiral isospin imbalance:

- increase of $\langle \bar{q}q \rangle$ as μ_{I5} increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_{I5} increases

The TDP

$$\Omega(T, \mu, \nu, \nu_5, \mu_5;\ M, \Delta)$$

Let us assume that there is no CSB

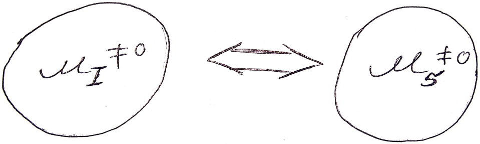
$$M = 0$$

The TDP (phase diagram) is invariant under

$$\mu_5 \longleftrightarrow \nu$$

Pion condensation phenomenon does not feel the difference between ν and μ_5

Two completely different systems



It was shown that chiral imbalance generates pion condensation in dense matter $n_B \neq 0$

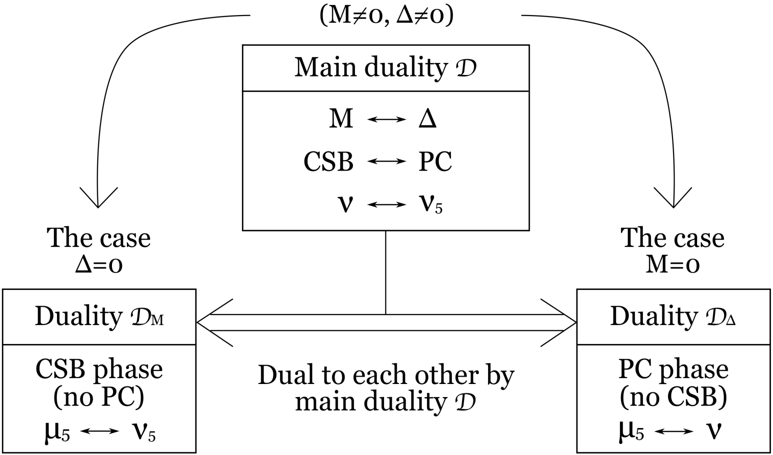
$$\mu_5 \rightarrow \textbf{PC with } n_B \neq 0$$

- ν **and** μ_5 **has the same effect on PC**
- $\nu \rightarrow$ **PC**

So it can be

$$\mu_5 \rightarrow \textbf{PC and } \mu_5 \rightarrow \textbf{PC with } n_B \neq 0$$

Dualities of the phase diagram



Dualities on the lattice

Dualities on the lattice

$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

- **QCD at** μ_5 —- (μ_5, T)

  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at** μ_I —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()

No lattice calculations at μ_{I5}

QCD at μ_{I5}

But

there is duality $\mathcal{D}_M : \mu_5 \longleftrightarrow \nu_5$ if $\Delta = 0$

at $\mu_I = 0$ there is no PC $(\Delta = 0)$

- $M(\mu_5) = M(\nu_5)$
- $T_c(\mu_5) = T_c(\nu_5)$

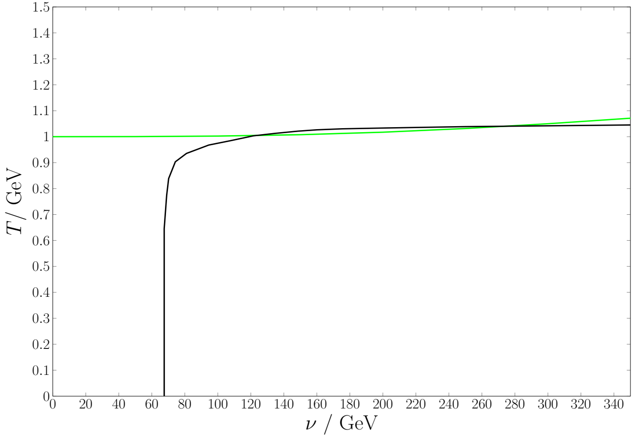
$$M \longleftrightarrow \Delta, \qquad \nu_5 \longleftrightarrow \nu$$

$$M \longleftrightarrow M \longleftrightarrow \Delta, \qquad \mu_5 \longleftrightarrow \nu_5 \longleftrightarrow \nu$$

So in this particular case you have a duality

$$M \longleftrightarrow \Delta, \qquad \mu_5 \longleftrightarrow \nu$$

- $M(\mu_5) = \Delta(\nu)$
- $T_c^M(\mu_5) = T_c^\Delta(\nu_5)$



T_c^M as a function of μ_5 (green line) and $T_c^\Delta(\nu)$ (black)

Use of Dualities: catalysis of ChSB

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

lattice results show the **catalysis**

(ITEP lattice group, V. Braguta, A. Kotov, et al)

But unphysically large pion mass

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

- **Large N_c orbifold equivalences** connect gauge theories with different gauge groups and **matter content** in the large N_c limit.

M. Hanada and N. Yamamoto, JHEP 1202 (2012) 138, arXiv:1103.5480 [hep-ph], PoS LATTICE **2011** (2011), arXiv:1111.3391 [hep-lat]

- two gauge theories with gauge groups G_1 and G_2

- but with different μ_1 and μ_2

Duality

$$G_1 \longleftrightarrow G_2, \qquad \mu_1 \longleftrightarrow \mu_2$$

or

Phase structure (G_1 at μ_1) $\longleftrightarrow$ Phase structure (G_2 at μ_2)

Duality

$$\text{QCD at } \mu_1 \longleftrightarrow \text{QCD at } \mu_2$$

- QCD with μ_2 —- sign problem free,
- QCD with μ_1 —- sign problem (no lattice)

Investigations of $(\text{QCD with } \mu_2)_{\text{on lattice}} \Longrightarrow (\text{QCD with } \mu_1)$

Inhomogeneous phases (case)

Homogeneous case

$\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_\pm(x)\rangle = \Delta, \quad \langle\pi_3(x)\rangle = 0.$$

In vacuum the quantities $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x.

in a dense medium the ground state expectation values of bosonic fields might depend on spatial coordinates

CDW ansatz for CSB
the single-plane-wave LOFF ansatz for PC

$$\langle\sigma(x)\rangle = M\cos(2kx^1), \quad \langle\pi_3(x)\rangle = M\sin(2kx^1),$$
$$\langle\pi_1(x)\rangle = \Delta\cos(2k'x^1), \quad \langle\pi_2(x)\rangle = \Delta\sin(2k'x^1)$$

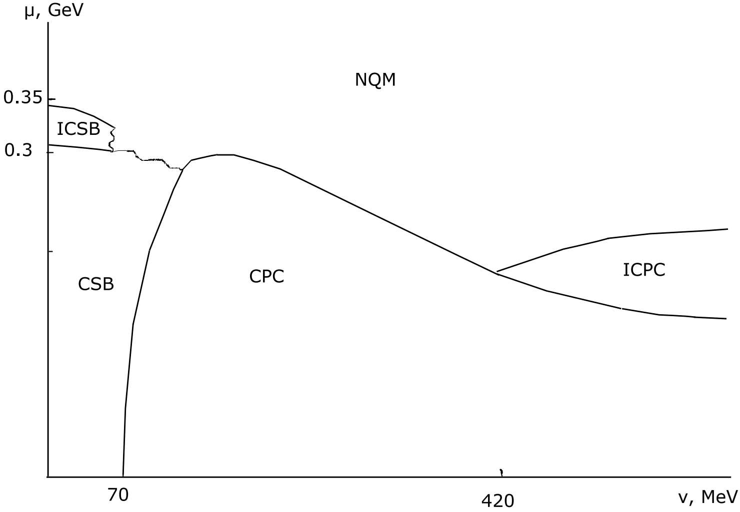
equivalently

$$\langle\pi_\pm(x)\rangle = \Delta e^{\pm 2k'x^1}$$

Duality in inhomogeneous case is shown

$$\mathcal{D}_I: \quad M \longleftrightarrow \Delta, \ \ \nu \longleftrightarrow \nu_5, \ \ k \longleftrightarrow k'$$

schematic (ν, μ)-phase diagram



Combined schematic (ν, μ)-phase diagram.

schematic (ν_5, μ)-phase diagram

- exchange axis ν to the axis ν_5,
- rename the phases ICSB $\leftrightarrow$ ICPC, CSB $\leftrightarrow$ CPC, and NQM phase stays intact here

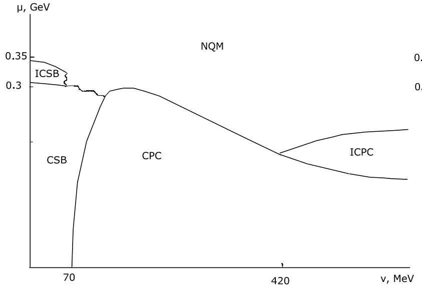


Figure: (ν, μ)-phase diagram

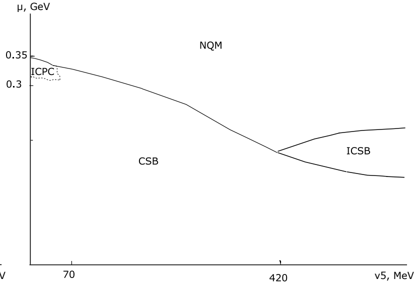


Figure: (ν_5, μ)-phase diagram

Duality between CSB and PC was found in

- In the framework of NJL model
- In the large N_c approximation (or mean field)
- In the chiral limit

QCD Lagrangian is

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \bar{\psi}\Big[\mu\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]\psi$$

$$\mathcal{D}: \qquad \psi_R \to i\tau_1\psi_R$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$$\bar{\psi}\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_1\psi$$

$$M \longleftrightarrow \Delta, \qquad \nu \longleftrightarrow \nu_5, \quad \mu_I \longleftrightarrow \mu_{I5}$$

$$i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi \leftrightarrow i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi, \qquad \bar{\psi}^C\sigma_2\tau_2\psi \leftrightarrow \bar{\psi}^C\sigma_2\tau_2\psi$$

$$\bar{\psi}\tau_2\psi \leftrightarrow \bar{\psi}\tau_3\psi, \qquad \bar{\psi}\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \qquad i\bar{\psi}\gamma^5\tau_2\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_3\psi$$

$$\mathcal{D} \in \quad SU_R(2) \quad \in SU_L(2) \times SU_R(2)$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$M \neq 0$ breaks the chiral symmetry

Duality $\mathcal{D}$ is a remnant of chiral symmetry

$$\tilde{\mathcal{D}} \in \quad SU_R(2) \times U_A(1)$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$U_A(1)$ is anomalous

$$\tilde{\mathcal{D}} \in \quad SU_R(2) \times U_A(1)$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$U_A(1)$ is anomalous

The NJL Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + G_1\Big\{(\bar{\psi}\psi)^2 + (i\bar{\psi}\vec{\tau}\gamma^5\psi)^2\Big\} + G_2\Big\{(i\bar{\psi}\gamma^5\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2\Big\}$$

$$\bar{\psi}\psi \leftrightarrow \bar{\psi}\tau_1\psi, \quad \bar{\psi}\tau_1\psi \leftrightarrow \bar{\psi}\psi, \quad i\bar{\psi}\gamma^5\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \quad \bar{\psi}\tau_3\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_2\psi$$

The transformation should be

$$\mu_I \leftrightarrow \mu_{I5}, \qquad G_1 \leftrightarrow G_2$$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + G_1\Big\{(\bar{\psi}\psi)^2 + (i\bar{\psi}\vec{\tau}\gamma^5\psi)^2\Big\} + G_2\Big\{(i\bar{\psi}\gamma^5\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2\Big\}$$

$$+H_1(i\bar{\psi}\sigma_2\lambda_2\gamma^5\psi^C)(i\bar{\psi}^C\sigma_2\lambda_2\gamma^5\psi) + H_2(\bar{\psi}\sigma_2\lambda_2\psi^C)(\bar{\psi}^C\sigma_2\lambda_2\psi)$$

$$\bar{\psi}\psi \leftrightarrow \bar{\psi}\tau_1\psi, \quad \bar{\psi}\tau_1\psi \leftrightarrow \bar{\psi}\psi, \quad i\bar{\psi}\gamma^5\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \quad \bar{\psi}\tau_3\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_2\psi$$

$$|\bar{\psi}^C\sigma_2\lambda_2\psi|^2 \leftrightarrow |i\bar{\psi}^C\sigma_2\lambda_2\gamma^5\psi|^2$$

The transformation should be

$$\mu_I \leftrightarrow \mu_{I5}, \quad G_1 \leftrightarrow G_2, \quad H_1 \leftrightarrow H_2$$

QCD: $SU_L(2) \times SU_R(2)$

QC$_2$D: $SU(4)$

$$\mathcal{D} \in SU(4)$$

$$\mu \leftrightarrow \nu, \qquad \Delta \leftrightarrow \Delta_{CSC}$$

$$\bar\psi\psi \to \bar\psi\psi,\ i\bar\psi\gamma^5\tau_3\psi \to i\bar\psi\gamma^5\tau_3\psi,\ i\bar\psi\gamma^5\psi \to i\bar\psi\gamma^5\psi,\ \bar\psi\tau_3\psi \to \bar\psi\tau_3\psi$$

$$i\bar\psi\gamma^5\tau_1\psi \to \frac{-i\bar\psi\sigma_2\gamma^5\tau_2\psi^C - i\bar{\psi^C}\sigma_2\gamma^5\tau_2\psi}{2}$$

$$i\bar\psi\gamma^5\tau_2\psi \to \frac{-\bar\psi\sigma_2\gamma^5\tau_2\psi^C + \bar{\psi^C}\sigma_2\gamma^5\tau_2\psi}{2}$$

$$\mathcal{D} \in SU(4)$$

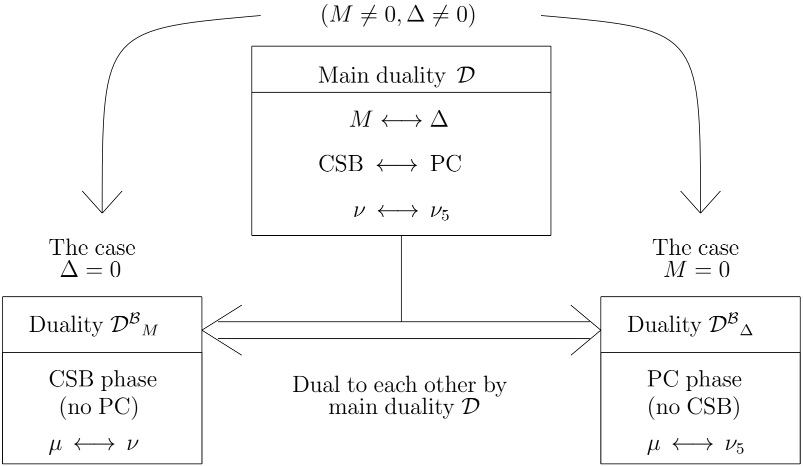
$$\mu \leftrightarrow \nu_5, \qquad M \leftrightarrow \Delta_{CSC}$$

$$\bar{\psi}\psi \to \frac{-i\bar{\psi}\sigma_2\gamma^5\tau_2\psi^C - i\bar{\psi^C}\sigma_2\gamma^5\tau_2\psi}{2},\ i\bar{\psi}\gamma^5\tau_3\psi \to \frac{-\bar{\psi}\sigma_2\gamma^5\tau_2\psi^C + \bar{\psi^C}\sigma_2\gamma^5\tau_2\psi}{2}$$

Dualities concerning baryon density

They could be even more usefull

Dualities with baryon density



CSB phenomenon
does not feel the difference between μ and ν
if there is no pion condensation phenomenon

in finite volume there cannot be a breaking of continuous symmetry

on lattice to get pion condensation (breaking of $U_{\tau_3}(1)$) one adds the pion source λ

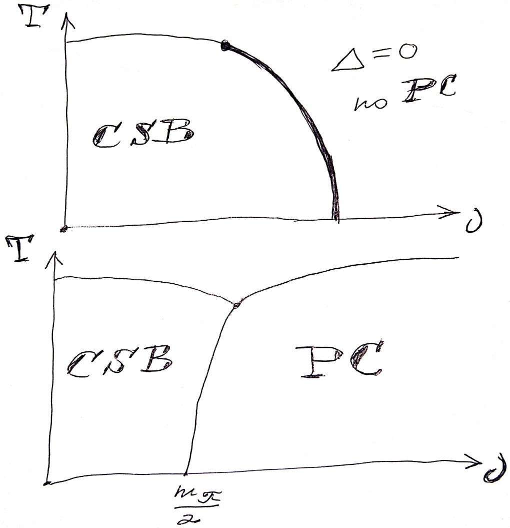
CSB phenomenon
does not feel the difference between μ and ν
if there is no pion condensation phenomenon

probably

if $\lambda = 0$ then there is no symmetry breaking and pion condensation

it is unphysical and there should be (M, Δ)
but artificially one can probe the local minimum $(M, 0)$

Duality in CSB phenomenon



Let us assume that $\nu \neq 0$
as a rule there is PC and there is no CSB

$$M = 0, \qquad \mu \longleftrightarrow \nu_5$$

$$\text{PC at non-zero } \mu \qquad \longleftrightarrow \qquad \text{PC at non-zero } \nu_5$$

Conclusions

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied
  PC *in dense matter with chiral imbalance*

- It was shown that there exist dualities

- There have been shown ideas how dualities can be used

- Duality is **not just entertaining mathematical property** but an **instrument with very high predictivity power**

- (μ_B, ν_5) phase diagram is **quite rich** and contains various **inhomogeneous phases**