Study of mesonic correlation functions at finite baryon chemical potential

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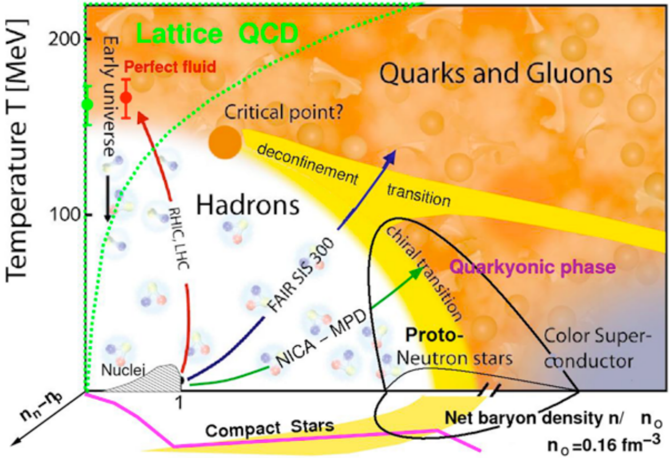
HMEC, Dubna, 16.09.2019

QCD phase diagram



- Thermal transition is well understood nowadays
- Small μ_B/T region is accessible via Taylor series expansion

The picture is from The Phases of Dense Matter, INT, 11.07 - 12.08, 2016

Taylor expansion

- Direct LQCD simulations at finite chemical potential are inaccessible
- Taylor expansion in μ_B/T is widely used to probe $\mu_B > 0$ region:
  - thermodynamics
  - higher-order susceptibilities
  - curvature of the crossover line
- Expansion may be applied to hadronic correlators to calculate finite μ_B corrections to the spectrum [pioneering paper on this is by QCD-TARO Collaboration, hep-lat/0107002 (2001)]
- Mesons: corrections are $O(\mu_B^2)$
- Baryons: corrections are $O(\mu_B)$

Taylor expansion of the meson correlator

Let's consider $\vec{p} = 0$ non-singlet meson correlator:

$$G(\tau) = \frac{1}{V} \int d^3\vec{x} \left\langle J(\tau, \vec{x}) \bar{J}(0, \vec{0}) \right\rangle ,$$

where $J = \bar{\psi}_u \Gamma \psi_d$, $\langle \ldots \rangle$ – thermodynamic average.

The idea is to perform the Taylor expansion of the correlator itself:

$$\frac{1}{T} G = \frac{1}{T} G\Big|_{\mu=0} + \frac{\mu}{T} \frac{\partial G}{\partial \mu}\Big|_{\mu=0} + \frac{T}{2} \left(\frac{\mu}{T}\right)^2 \frac{\partial^2 G}{\partial \mu^2}\Big|_{\mu=0} + O\left(\frac{\mu^3}{T^3}\right)$$

- Expansion of both determinant and correlator
- $\dot{G}_\mu$ term does not contribute at $\mu = 0$, thus $G(\tau, \mu) = G(\tau)\big|_{\mu=0} + (\mu^2/2)\, \ddot{G}_\mu(\tau)\big|_{\mu=0}$
- $\ddot{G}_\mu$ has to be computed
- $\ddot{G}_\mu$ contains several noisy disconnected terms

Taylor expansion of the meson correlator

- Meson correlator is $G(x) = \langle g(x) \rangle$, where
  $g(x) = \mathrm{Tr}\left[S^{(u)}_{x;0} \, \Gamma \, S^{(d)}_{0;x} \, \Gamma^{\dagger} \right]$ (after the integration over $d\psi \, d\overline{\psi}$)
  $S^{(f)}_{x;0} = D^{-1}_{x,0}(\mu_f, \, m_f)$
- $\langle g \rangle = (1/Z) \int dU \, g \, \det D_{u,d} \, \det D_s \exp[-S_G]$
- We consider symmetric μ set: $\mu_u = \mu_d$

First derivative in chemical potential:

$$\dot{G}_\mu(x) = \left\langle g'_\mu(x) \right\rangle + \left\langle g(x) \, \mathrm{det}'_\mu \right\rangle - \left\langle \mathrm{det}'_\mu \right\rangle \langle g(x) \rangle$$

- sum of the first two terms here is purely imaginary
- $\left\langle \mathrm{det}'_\mu \right\rangle$ is quark number density, it is zero at $\mu = 0$

Taylor expansion of the meson correlator

Second derivative of $G(x)$ in chemical potential:

$$
\begin{aligned}
\ddot{G}_\mu(x) = \quad & \left\langle g''_\mu(x) \right\rangle \\
& + \left\langle \frac{\det''_\mu}{\det} g(x) \right\rangle - \left\langle \frac{\det''_\mu}{\det} \right\rangle \langle g(x) \rangle \\
& + 2 \left\langle \frac{\det'_\mu}{\det} g'_\mu(x) \right\rangle - 2 \left\langle \frac{\det'_\mu}{\det} \right\rangle \langle g'_\mu(x) \rangle \\
& - 2 \left(\left\langle \frac{\det'_\mu}{\det} g(x) \right\rangle - \left\langle \frac{\det'_\mu}{\det} \right\rangle \langle g(x) \rangle \right) \left\langle \frac{\det'_\mu}{\det} \right\rangle
\end{aligned}
$$

$\ddot{G}$ structure: connected and disconnected terms

$$
\begin{aligned}
\ddot{G}_\mu\Big|_{\mu=0} = & -2\mathrm{Re}\left\langle \mathrm{Tr}\left[\gamma_5\Gamma^\dagger\left(D^{-1}\dot{D}D^{-1}\right)_{n,0}\Gamma\gamma_5\left(D^{-1}\dot{D}D^{-1}\right)^\dagger_{n,0}\right]\right\rangle \\
& +4\mathrm{Re}\left\langle \mathrm{Tr}\left[\gamma_5\Gamma^\dagger\left(D^{-1}\dot{D}D^{-1}\dot{D}D^{-1}\right)_{n,0}\Gamma\gamma_5\left(D^{-1}\right)^\dagger_{n,0}\right]\right\rangle \\
& -2\mathrm{Re}\left\langle \mathrm{Tr}\left[\gamma_5\Gamma^\dagger\left(D^{-1}\ddot{D}D^{-1}\right)_{n,0}\Gamma\gamma_5\left(D^{-1}\right)^\dagger_{n,0}\right]\right\rangle \\
& -8\left\langle \mathrm{Im}\ \mathrm{Tr}\left[\gamma_5\Gamma^\dagger\left(D^{-1}\dot{D}D^{-1}\right)_{n,0}\Gamma\gamma_5\left(D^{-1}\right)^\dagger_{n,0}\right]\mathrm{Im}\ \mathrm{Tr}\left[\dot{D}D^{-1}\right]\right\rangle \\
& +2\left\langle \mathrm{Tr}\left[\gamma_5\Gamma^\dagger\left(D^{-1}\right)_{n,0}\Gamma\gamma_5\left(D^{-1}\right)^\dagger_{n,0}\right]\right. \\
& \qquad \left.\times\left(2\mathrm{Tr}\left[\dot{D}D^{-1}\right]^2+\mathrm{Tr}\ \mathrm{Tr}\left[\left(\dot{D}D^{-1}\right)^2\right]\right)\right\rangle \\
& +2\left\langle \mathrm{Tr}\left[\gamma_5\Gamma^\dagger\left(D^{-1}\right)_{n,0}\Gamma\gamma_5\left(D^{-1}\right)^\dagger_{n,0}\right]\right\rangle \\
& \qquad \times\left\langle\left(2\mathrm{Tr}\left[\dot{D}D^{-1}\right]^2+\mathrm{Tr}\left[\ddot{D}D^{-1}\right]-\mathrm{Tr}\left[\left(\dot{D}D^{-1}\right)^2\right]\right)\right\rangle
\end{aligned}
$$

Free case

For free massless quarks μ^2-correction may be written analytically:

$$T^2 \ddot{G}_\Gamma(\tau)\Big|_{\mu=0} = \frac{N_c T^3}{\pi^2}\left[a_\Gamma^{(1)} + a_\Gamma^{(2)} - \frac{1}{12}\left(a_\Gamma^{(1)} - a_\Gamma^{(2)}\right) h(u)\right]$$

where

$$h(u) = \big[3u(\pi^2 - u^2 - 2) + u(\pi^2 - u^2 + 6)\cos(2u) - 2(\pi^2 - 3u^2)\sin(2u)\big] / \sin^3(u)$$

and $u = 2\pi T(\tau - 1/2T)$, $-\pi < u < \pi$

- τ-dependence is contained in $h(u)$
- Coefficients $a_\Gamma^{(j)}$ depend on the channel

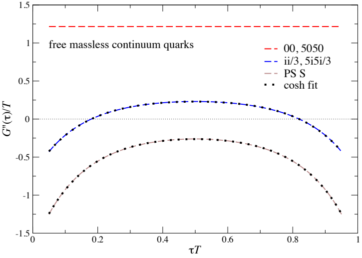
Free analytical expression above provides qualitative check

Free case

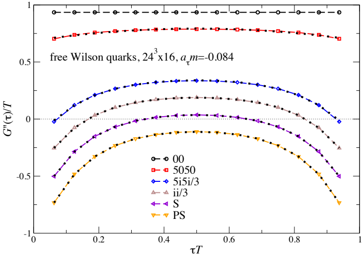
- $\ddot{G}$ is well described by one-cosh fit

$$\ddot{G}(\tau)/T = c_0 + c_1 \cosh[c_2(\tau T - 1/2)]$$

both in continuum and on lattice



continuum, $c_2 \sim 7$



lattice, $c_2 \sim 7.4$

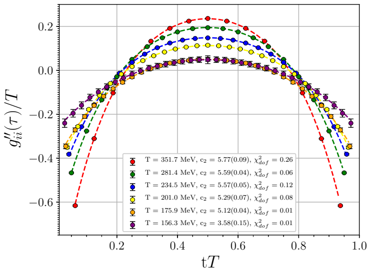
Lattice parameters (Gen2 ensemble)

- $N_f = 2+1$ clover-improved Wilson fermions
  tree-level improved anisotropic gauge action, mean-field improved Wilson clover fermion action with stout-smeared links
- Anisotropic lattice:
  $a_s = 0.1227(8)$ fm, $a_\tau = 0.0350(2)$ fm, $\xi = a_s/a_\tau = 3.5$
- $L_s = 24$, $m_\pi = 384(4)$ MeV, $m_\pi L_s = 5.7$
- L_τ is varied to vary temperature (see the table below)
- Tuning and ensembles at the lowest temperatures have been provided by HadSpec

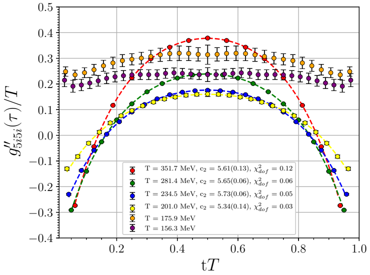
L_τ	128	40	36	32	28	24	20	16
T [MeV]	44	141	156	176	201	235	281	352
T/T_c	0.24	0.76	0.84	0.95	1.09	1.27	1.52	1.90
$N_{\rm cfg}$	139	501	501	1000	1001	1001	1000	1001

$\ddot{G}$: connected part only

- For $\gamma_5\gamma_i$ one may see drastic change in behaviour at $T < 180$ MeV



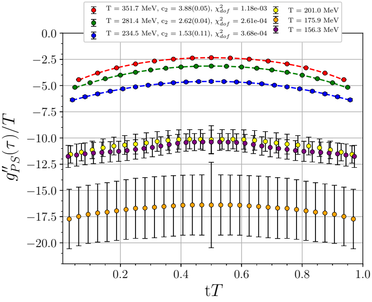
vector



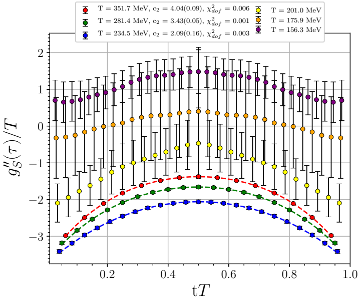
axial-vector

$\ddot{G}$: connected part only

- For γ_5 and I behaviour changes at $T \propto 200$ MeV



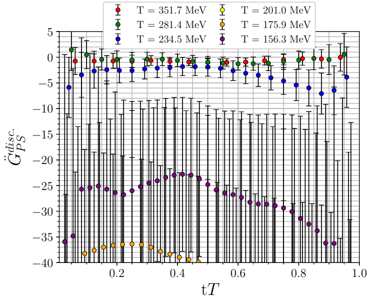
pseudo-scalar



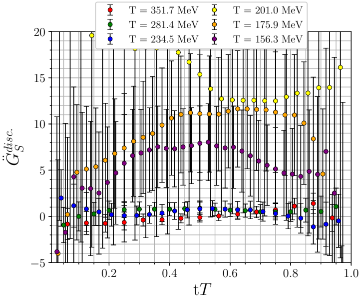
scalar

$\ddot{G}$: disconnected part

- Disconnected parts vanish in the absence of interactions
- Very noisy despite $N_{st.} = 10000$ Gaussian random vectors



pseudo-scalar

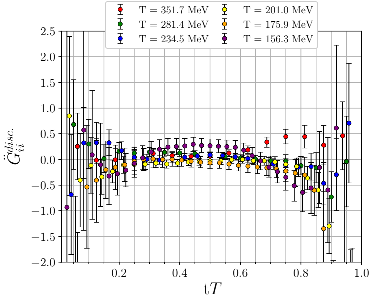


scalar

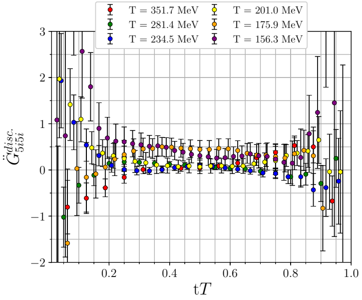
No signal at low T for pseudoscalar and scalar mesons

$\ddot{G}$: disconnected part

- Disconnected parts vanish in absence of interactions
- Very noisy despite $N_{st.} = 10000$ Gaussian random vectors



vector



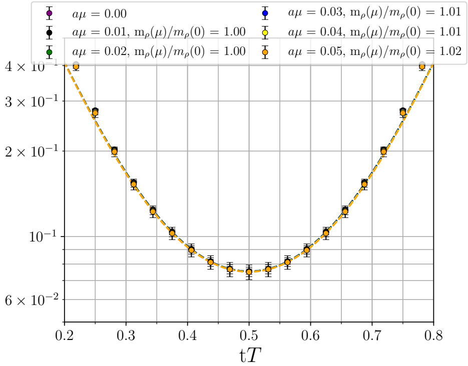
axial-vector

For ρ-meson the signal is the best compared to other channels

$G(\tau,\ \mu)$ for ρ-meson

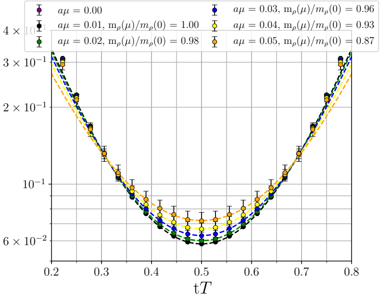
$$\frac{1}{T}G(\tau,\mu) = \frac{1}{T}G(\tau)\Big|_{\mu=0} + \frac{T}{2}\left(\frac{\mu}{T}\right)^2 \ddot{G}_{\mu}(\tau)\Big|_{\mu=0}$$ is plotted below

- At T > 170 MeV μ^2-correction to the correlator has no effect

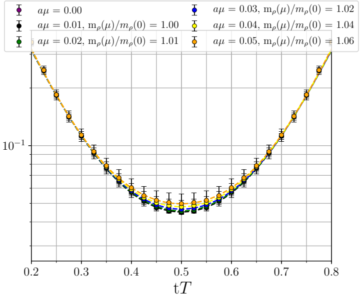


$G(\tau, \mu)$ for ρ-meson

- At T $<$ 160 MeV μ^2-correction to the correlator is noticeable
- Mass may be extracted from single cosh fit



T = 156 MeV



T = 141 MeV

The mass of ρ-meson decreases by $\propto$10% at $\mu_q = 280$ MeV.

Conclusions

- High temperatures: good agreement with non-interacting theory
- Low temperatures: results are very noisy
- $\ddot{G}_\mu$ in some channels seems to be sensitive to confinement-deconfinement transition
- Noise reduction techniques for disconnected $\ddot{G}_\mu$ contributions are needed
- Probably the spectral function for $G + \mu^2 \ddot{G}$ will provide more insight