Gluon Propagators in QC_2D at High Baryon Density

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- Definitions and details of simulation
- Zero-momentum gluon propagators versus μ_B and screening masses
- Comparison with perturbation theory
- Gribov-Stingl fit for the dressing function
- $D_L - D_T$ as the infrared sensitive quantity
- D_L and D_T at $p_4 \neq 0$
- Conclusions

We study QCD with $N_c = 2$, $N_f = 2$, improved gauge field action and standard staggered fermion action; $a = 0.044$ fm, $m_\pi = 740$ MeV,

$N_s = 32, N_t = 32$; Lattice size $L = 1.4$ fm.

String tension σ: $\sqrt{\sigma} = 0.476$ GeV.

Landau gauge fixing condition is

$$(\partial \mathcal{A})_x = \sum_{\mu=1}^{4} \left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu} \right) = 0 \ , \tag{1}$$

which is equivalent to finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V} \sum_{x\mu} \frac{1}{2} \ \mathrm{Tr} \ U^g_{x\mu} \ , \tag{2}$$

with respect to gauge transformations g_x .

Find the global maximum of the gauge fixing functional
- **absolute Landau gauge**.

- efficient optimization algorithm - simulated annealing.
- Gribov copy effects are small (from 5 to 30 copies per confugurations were used)

$$D_{\mu\nu}(p) = \frac{1}{3a^4 N_s^3 N_t} \sum_{b=1}^{3} \langle \tilde{A}_\mu^b(p) \tilde{A}_\nu^b(-p) \rangle = D_L(p) P_{\mu\nu}^L + D_T(p) P_{\mu\nu}^T + \alpha \frac{p_\mu p_\nu}{p^4}$$

$$P_{\mu\nu}^T = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \dfrac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P_{\mu\nu}^L = \begin{pmatrix} \dfrac{|\vec{p}|^2}{p^2} & -\dfrac{p_4 p_i}{p^2} \\ -\dfrac{p_4 p_j}{p^2} & -\dfrac{p_i p_j p_4^2}{|\vec{p}|^2 p^2} \end{pmatrix}$$

$p_4 \neq 0, \vec{p} \neq 0 \qquad D_L \sim \frac{p^2}{|\vec{p}|^2}\langle A_4 A_4\rangle; D_T \sim \frac{1}{2}\left(\sum_{i=1}^{3}\langle A_i A_i\rangle - \frac{p_4^2}{|\vec{p}|^2}\langle A_4 A_4\rangle\right)$

$p_4 = 0, \vec{p} \neq 0 \qquad D_L \sim \langle A_4 A_4\rangle; D_T \sim \frac{1}{2}\sum_{i=1}^{3}\langle A_i A_i\rangle$

$p_4 = 0, \vec{p} = 0 \qquad D_L \sim \langle A_4 A_4\rangle; D_T \sim \frac{1}{3}\sum_{i=1}^{3}\langle A_i A_i\rangle$

$\frac{1}{3a^4 N_s^3 N_t}$ is omitted, $p^2 = p_4^2 + |\vec{p}|^2$, $\sum_{b=1}^{3}\langle \tilde{A}_\mu^b(p)\tilde{A}_\nu^b(-p)\rangle = \langle A_\mu A_\nu\rangle$

If $p_4 \neq 0, \vec{p} = 0$ then we arrive at $D_T = D_L \sim \frac{1}{3}\sum_{i=1}^{3}\langle A_i A_i\rangle$

Screening mass can be defined in either of the two ways:

$$m^2_{Linde} \qquad m^2_{Linde} = G^{-1}(0, \vec{p} \to 0)$$

$$m^2_{proper} \qquad G^{-1}(0, |\vec{p}|) = \frac{1}{Z_G}(m^2_{proper} + |\vec{p}|^2 + \underline{O}(|\vec{p}|^4)) \tag{3}$$

Strictly speaking, only m_{proper} is related to the far-distant behavior

$$\sum_{x_4} G(x_4, |\vec{x}|) \sim \exp(-m_{proper}|\vec{x}|) \quad \text{when} \quad |\vec{x}| \to \infty$$

If the Maclaurin expansion of $G^{-1}(0, |\vec{p}|)$ in $|\vec{p}|^2$ works well, then these definitions coincide up to the normalization constant $\sqrt{Z_G}$.
Otherwise it may turn out that m_{Linde} exists, whereas m_{proper} — NOT.
An example may be provided by finite-temperature theories if ultrasoft ($0 < |\vec{p}| < 100$ MeV) gluon-field modes are not taken into account.

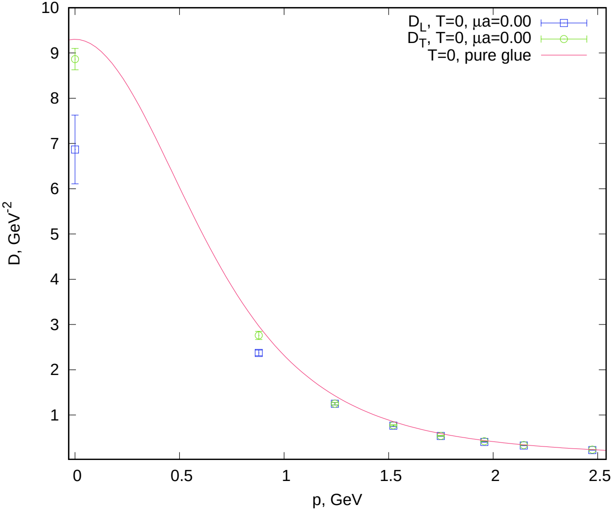
We consider propagators mainly for *soft modes* $p_4 = 0$, where

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \dfrac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

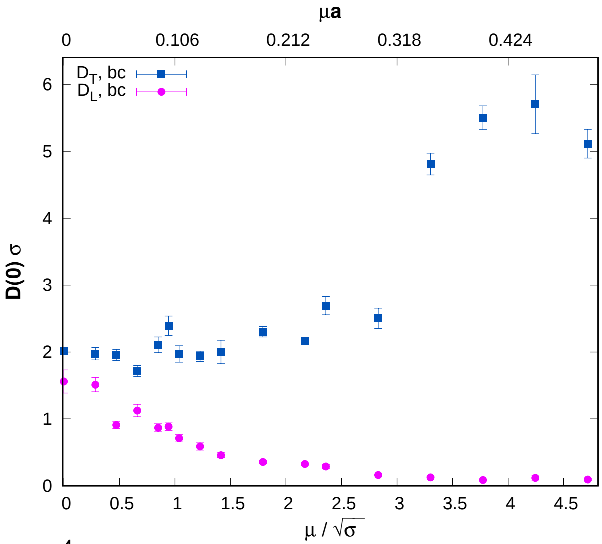
$$D_L(p) = \frac{1}{p^2 + F(p)},$$

$D_L(0) \simeq \dfrac{1}{m_e^2} \simeq r_e^2$ — chromoelectric forces

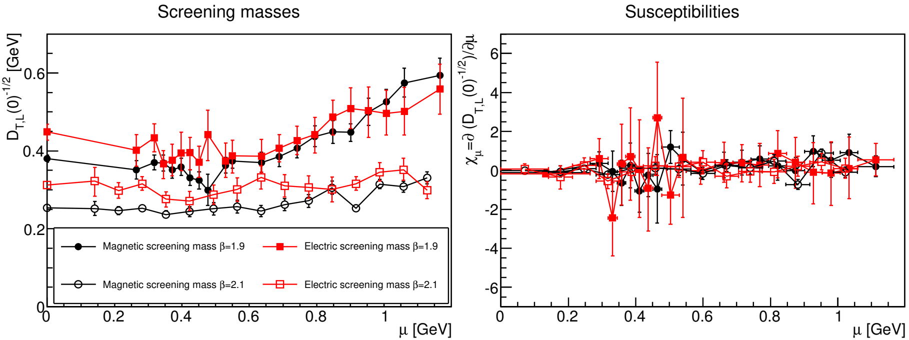
- To analyze far-distant behavior of $D_T(\vec{x})$, its values at ultrasoft nonzero momenta are needed. ($D_T(0)$ only is not sufficient, in contract to the longitudinal case)



$SU(2)$ theory with fermions versus pure glue theory at $T = 0$
In contrast to the $SU(3)$ case, fermions have only a little effect on the propagator


μa
0
0.106
0.212
0.318
0.424
D_T, bc
D_L, bc
6
5
4
3
2
1
0
D(0) σ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
μ / √σ

QC_2D; 32^4



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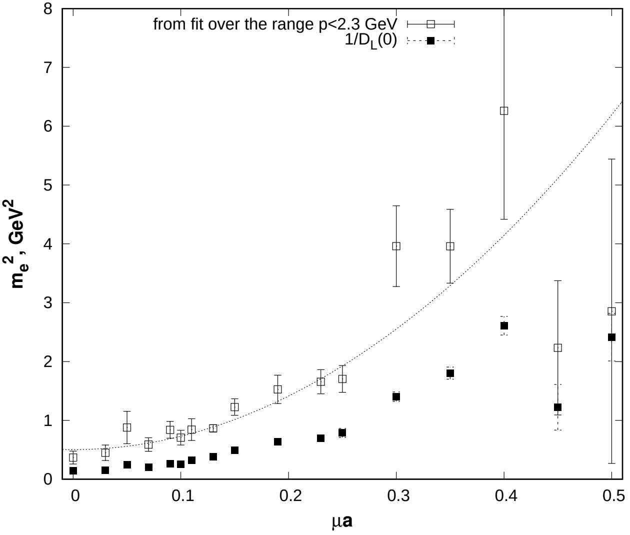
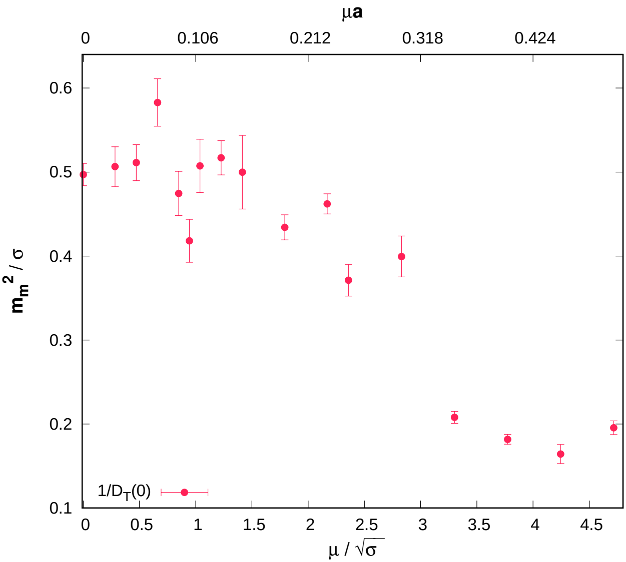
unimproved Wilson gauge action with 2 flavors of unimproved Wilson quarks $m_\pi = 717(25)$ MeV, $a = 0.186 \div 0.138$ fm

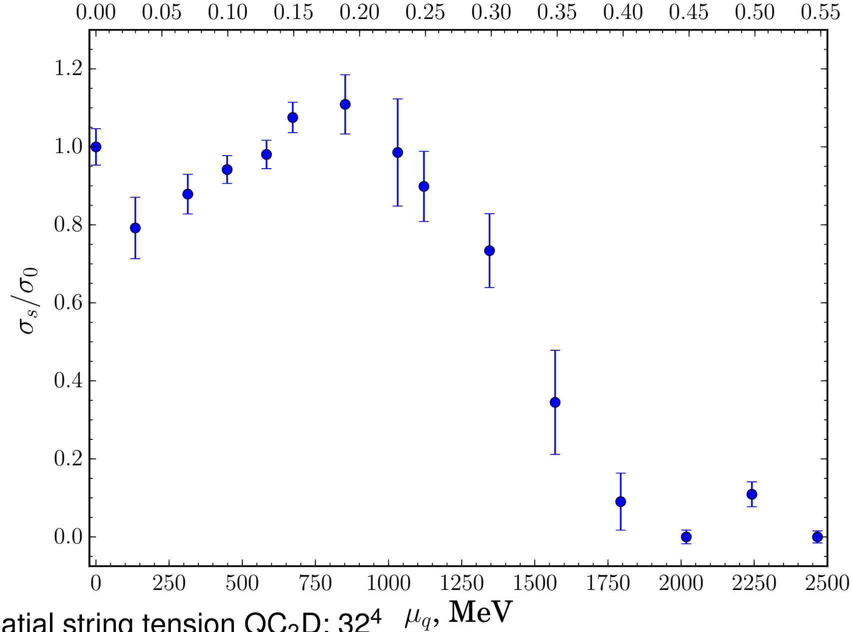
versus our parameters:

improved gauge field action and standard staggered fermion action $m_\pi = 740$ MeV, $a = 0.044$ fm



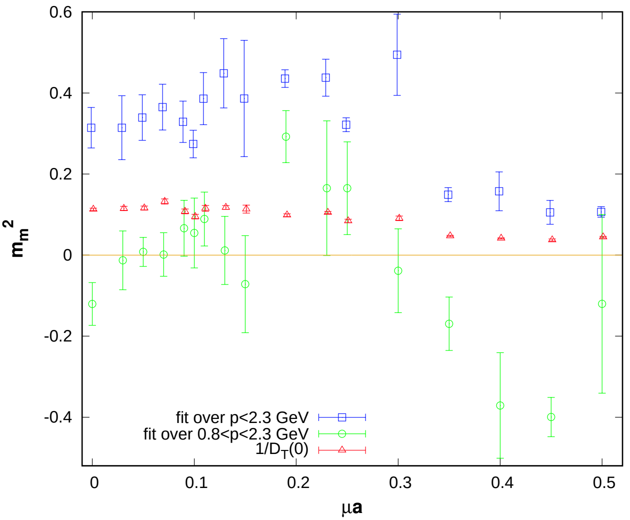
from fit over the range p<2.3 GeV
$1/D_L(0)$
m_e^2, GeV^2
μa
0
1
2
3
4
5
6
7
8
0
0.1
0.2
0.3
0.4
0.5

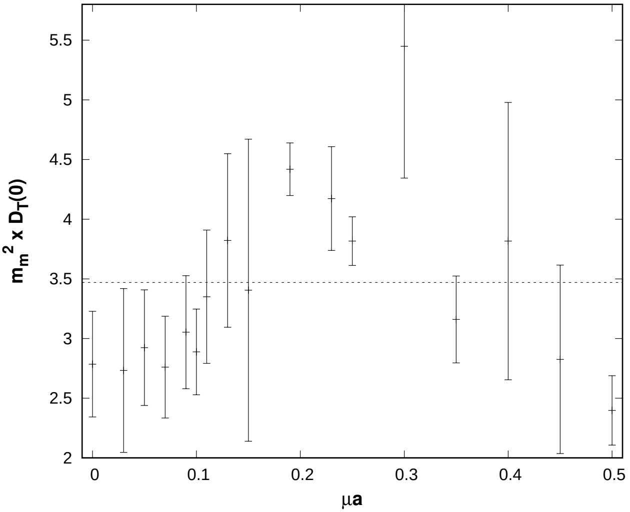

μa
0
0.106
0.212
0.318
0.424
0.6
0.5
0.4
0.3
0.2
0.1
m_m^2 / σ
1/D_T(0)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
$\mu / \sqrt{\sigma}$



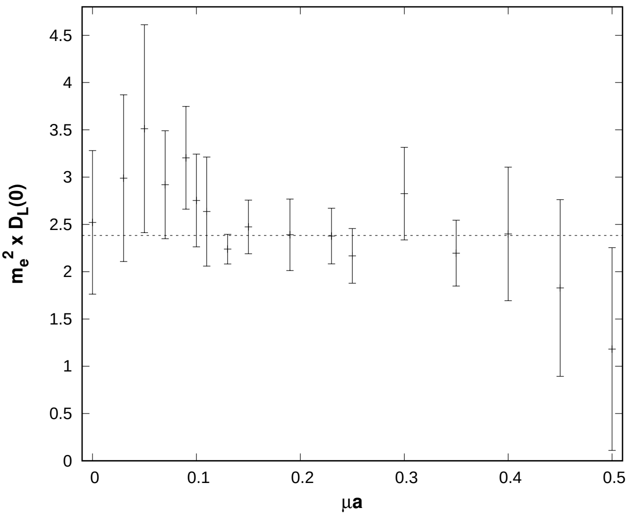
spatial string tension QC_2D; 32^4

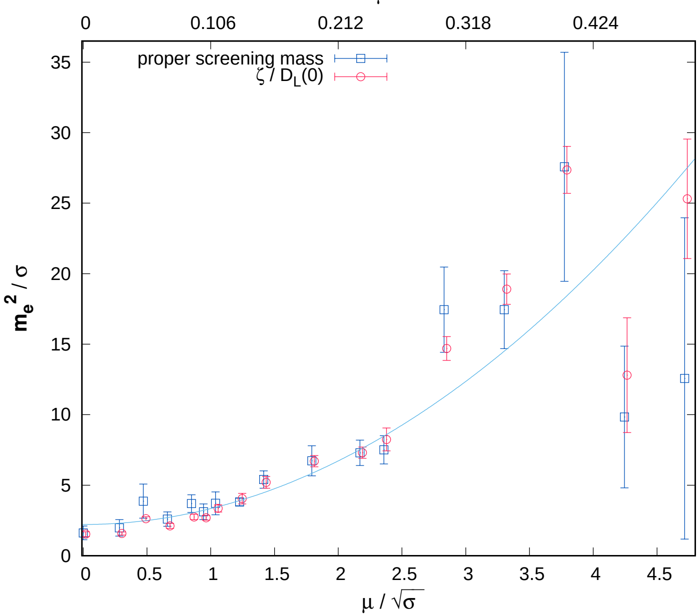
steep decreasing at 800 MeV


0.6
0.4
0.2
0
-0.2
-0.4
m_m^2
0
0.1
0.2
0.3
0.4
0.5
μa
fit over p<2.3 GeV
fit over 0.8<p<2.3 GeV
$1/D_T(0)$



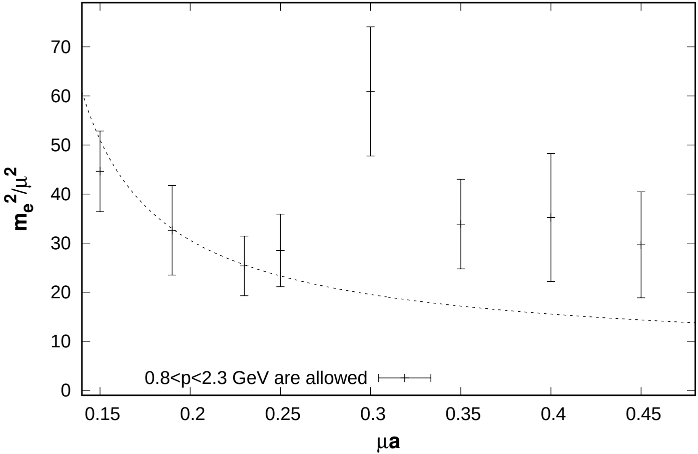
5.5
5
4.5
4
3.5
3
2.5
2
0
0.1
0.2
0.3
0.4
0.5
μa
$m_m^2 \times D_T(0)$


m_e^2 x $D_L(0)$
μa
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
0.1
0.2
0.3
0.4
0.5



0
0.106
0.212
0.318
0.424
proper screening mass
ζ / D_L(0)
35
30
25
20
15
10
5
0
m_e^2 / σ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
μ / √σ

QC_2D; 32^4


70
60
50
40
30
20
10
0
0.15
0.2
0.25
0.3
0.35
0.4
0.45
m_e^2/μ^2
μa
0.8<p<2.3 GeV are allowed

QC_2D; 32^4

The Debye screening mass in the one-loop appriximation at $T \to \infty$ and/or $\mu \to \infty$:

$$m_e^2 = \frac{g^2 N_c T^2}{3} + \sum_f \frac{g^2 \mu_f^2}{2\pi^2} . \qquad \text{where} \qquad g^2 \simeq \frac{24\pi^2}{11 \ln \frac{\mu^2}{\Lambda^2}} .$$

Our data can be fitted by such function at $p > p_{cut}$ ($T = 0$)

$\Lambda = 439(45)$ MeV

$\frac{\chi^2}{N_{d.o.f.}} = 2.66$, $p = 0.009$ $\quad p_{cut} = 630$ MeV

Perturbalively motivated fits

In the one loop approximation,
asymptotic behavior of the gluon propagator has the form

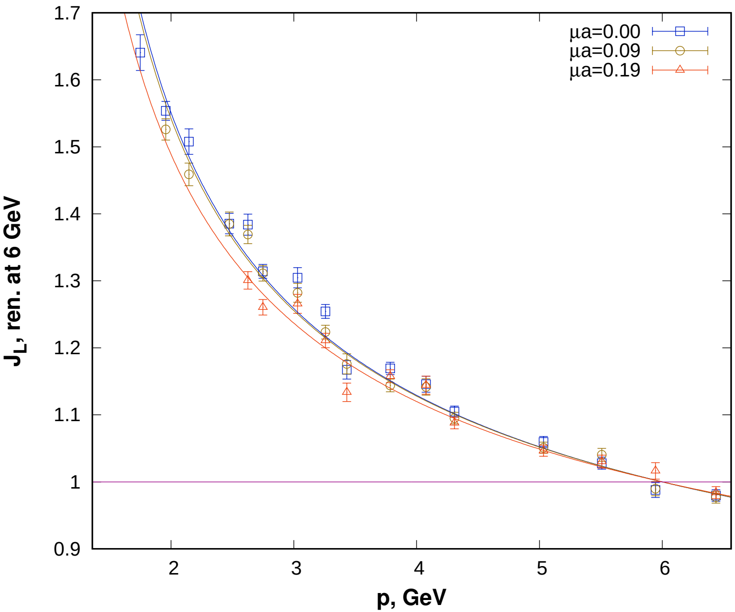
$$\lim_{p\to\infty; g=const} J(p;g) \simeq \left[\frac{\ln\left(\frac{p^2}{\Lambda^2}\right)}{\ln\left(\frac{\kappa^2}{\Lambda^2}\right)}\right]^{-\ c/(2b)} , \tag{4}$$

c and b are the coefficients of the RG functions,

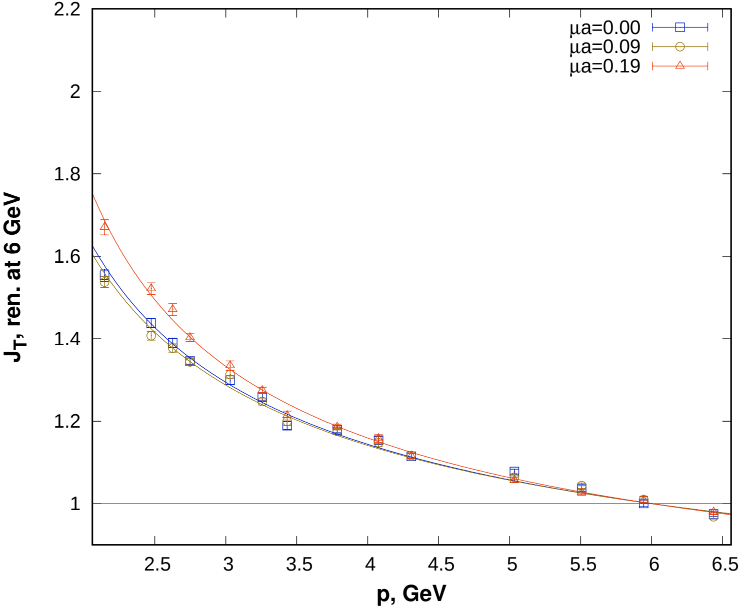
$$\beta(g) \simeq -bg^3, \qquad \gamma(g) \simeq -cg^2 .$$

In the Landau-gauge $SU(N_c)$ theories

$$\frac{c}{2b} = \frac{13N_c - 4N_F}{2(11N_c - 2N_F)} = \frac{1}{2} ; \tag{5}$$



Perturbalive fit to the longitudinal dressing functions



Perturbalive fit to the transverse dressing function

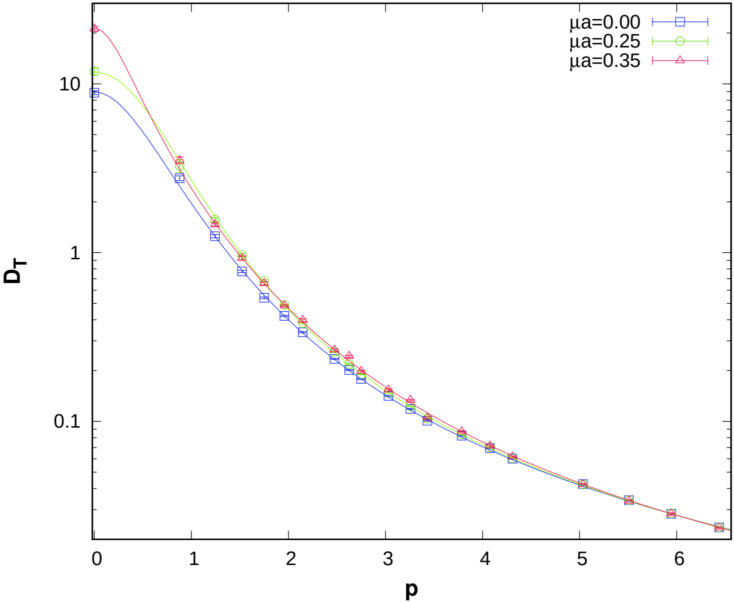
Perturbalive domain

- D_L: $p_{cut} = 1.75$ GeV $+ 0.6$ GeVμa;
- D_T: $p_{cut} = 2.7$ GeV

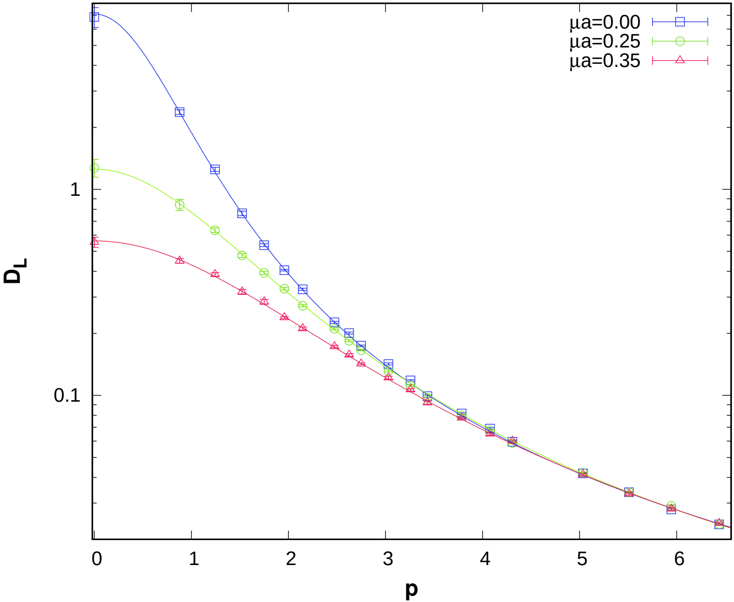
Behavior of Λ as μ increases from 0 to 2 GeV

- D_L: Λ decreases from 1.1 to 0.2 GeV
- D_T: Λ increases from 1.1 to 1.7 GeV

PRELIMINARY RESULTS



Transverse gluon propagator normalized at $p = 6$ GeV; Gribov-Stingl fit is shown.



Longitudinal gluon propagator normalized at $p = 6$ GeV; Gribov-Stingl fit is shown.

Gribov-Stingl fit was used for the renormalized propagators:

$$D(p^2) \simeq \frac{(p^2+d^2)}{(\kappa^2+d^2)} \, \frac{((\kappa^2+a^2)^2+b^4)}{((p^2+a^2)^2+b^4)} \tag{6}$$

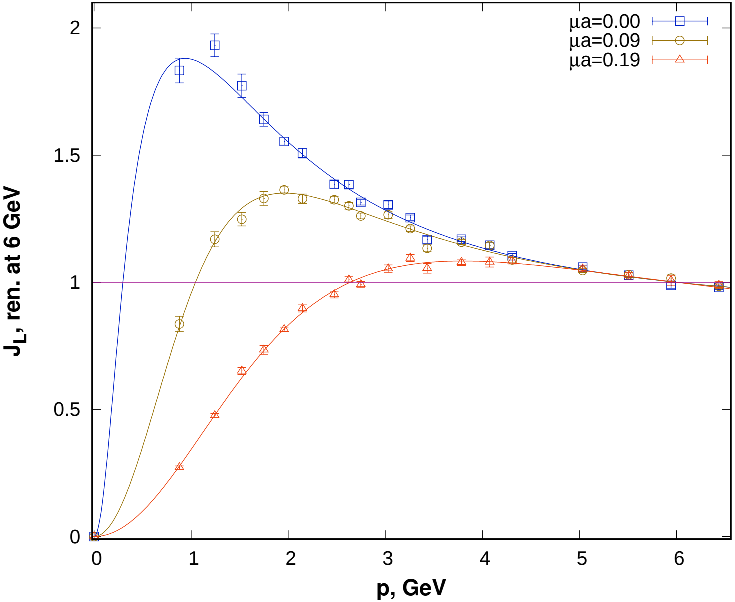
$\kappa = 6$ GeV
instead of

$$D(p^2) \simeq c \, \frac{p^2+d^2}{(p^2+a^2)^2+b^4} \tag{7}$$

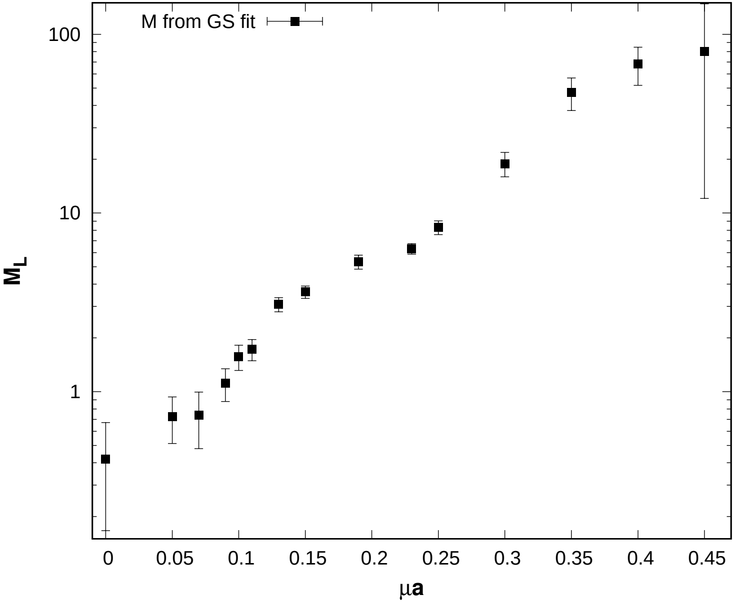
The best choice:

$$D(p^2) \simeq \frac{(\delta p^2+1)}{(\delta \kappa^2+1)} \, \frac{(\kappa^4+2r\kappa^2+M)}{(p^4+2rp^2+M)} \tag{8}$$

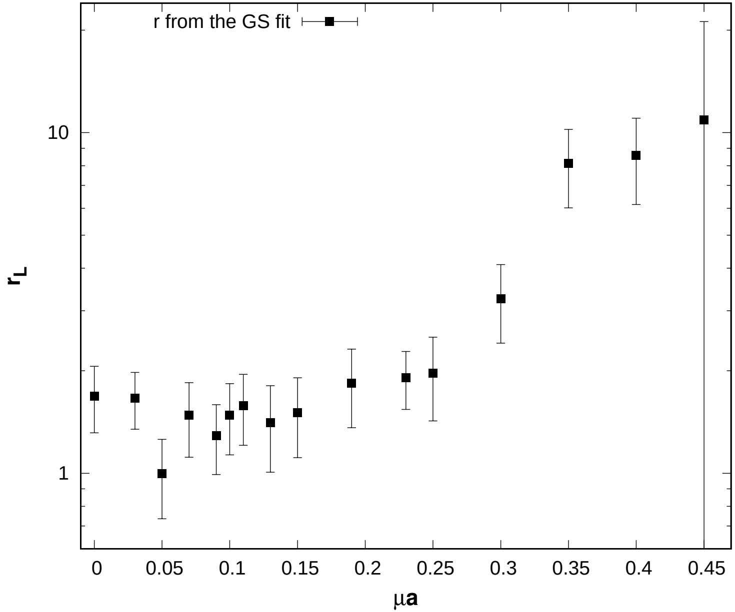
Fit parameters: M, r, δ



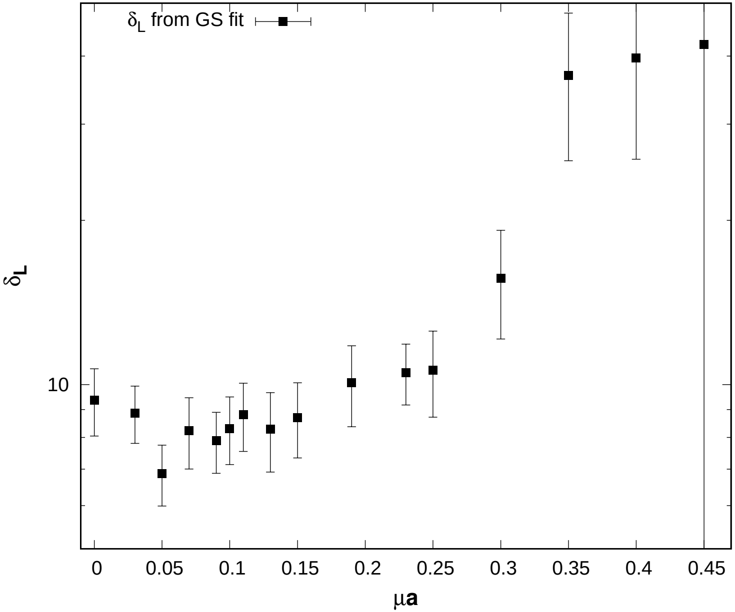
"Electric antiscreening" disappears with an increase of μ



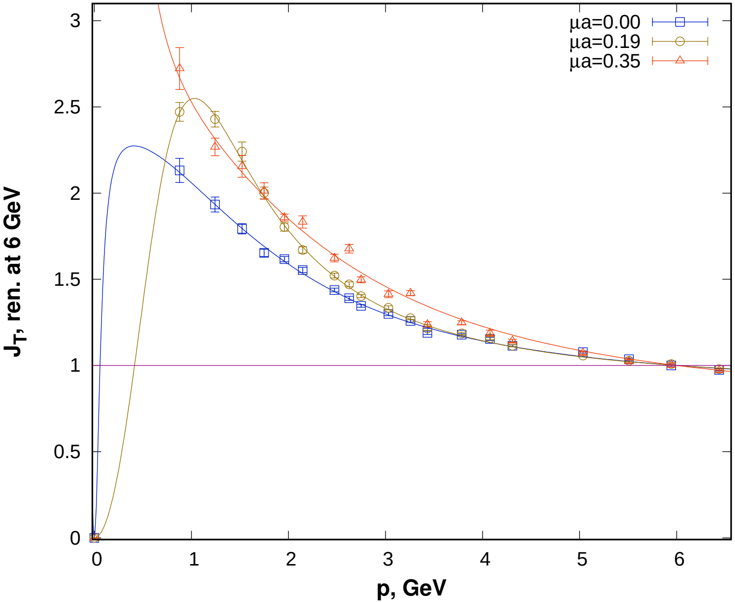
GS fit parameter $M_L = a^4 + b^4$ associated with "screening"



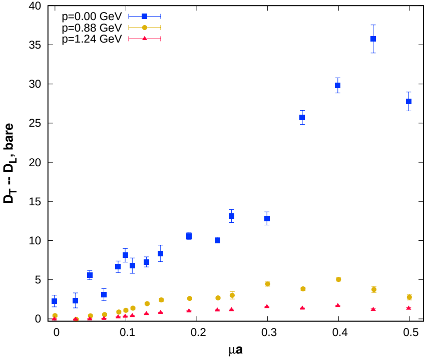
GS fit parameter $r_L = 2a^2$



GS fit parameter $\delta_L = d^2$ shows a similar behavior to r_L

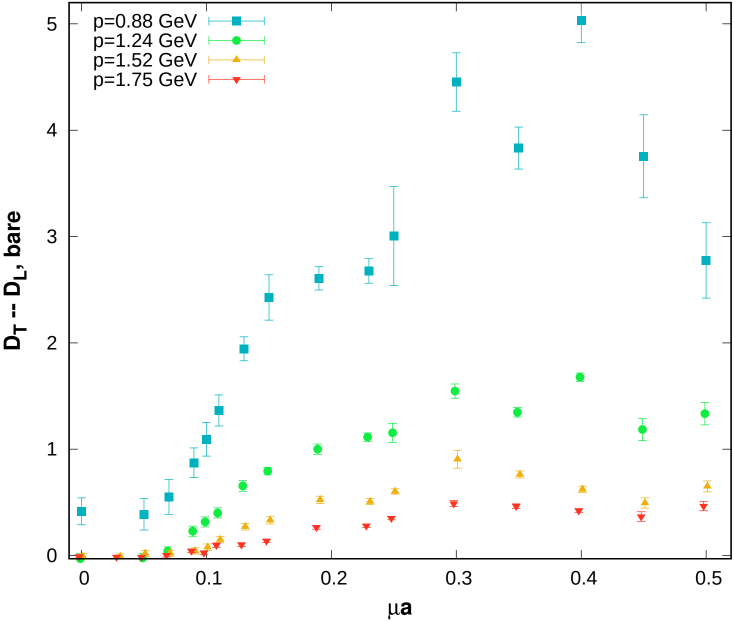


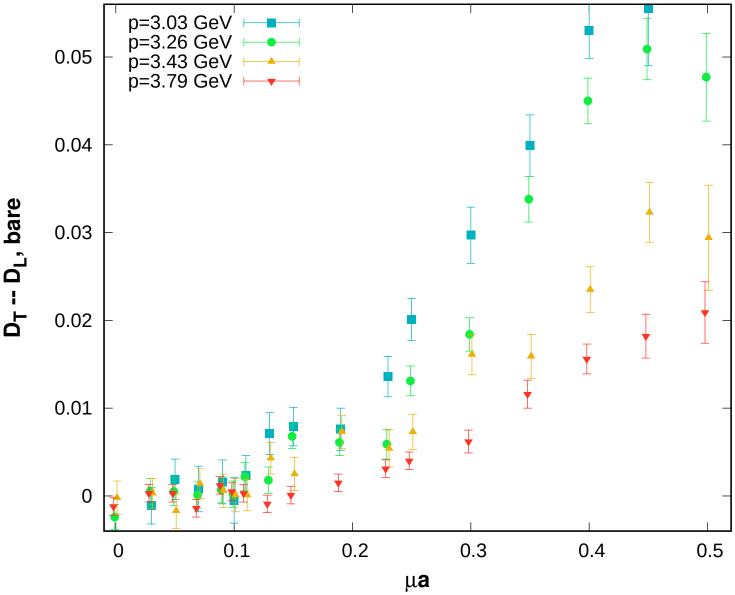
Magnetic dressing function may be consistent with the concept of massive particle only at $0.8 < \mu < 1.4$ GeV

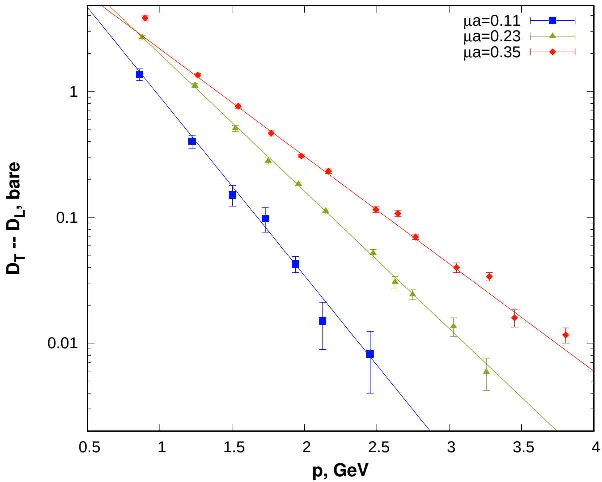


The Gribov-Stingl fit for D_T
works only partially:
$p < 10^{-2}$ at several values of μ,
parameters are
poorly determined.

Instead of D_L and D_T,
we suggest to study D_L and $\Delta = D_T - D_L$,
because the latter quantity
shows an interesting behavior.
At nonzero momenta
it differs from zero only
at $\mu > 350 \div 400$ MeV,
and then increases
with μ

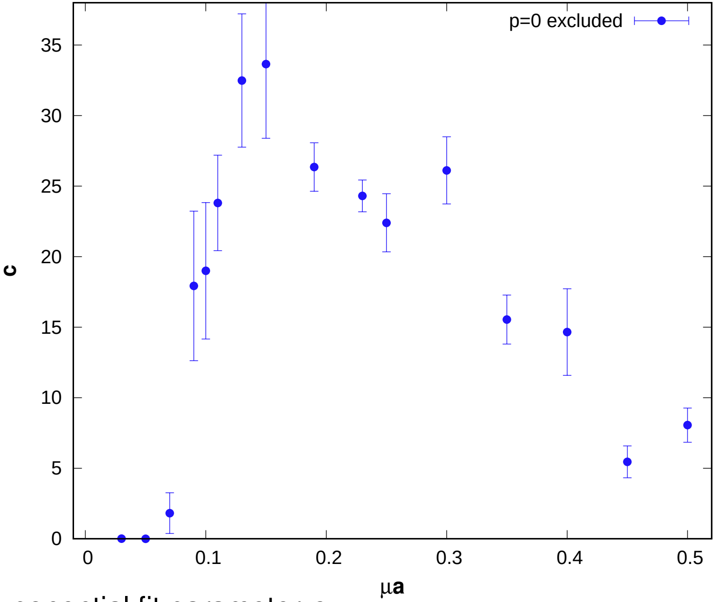

p=0.88 GeV
p=1.24 GeV
p=1.52 GeV
p=1.75 GeV
D_T -- D_L, bare
μa
0
1
2
3
4
5
0
0.1
0.2
0.3
0.4
0.5


p=3.03 GeV
p=3.26 GeV
p=3.43 GeV
p=3.79 GeV
0.05
0.04
0.03
0.02
0.01
0
0
0.1
0.2
0.3
0.4
0.5
D_T -- D_L, bare
μa

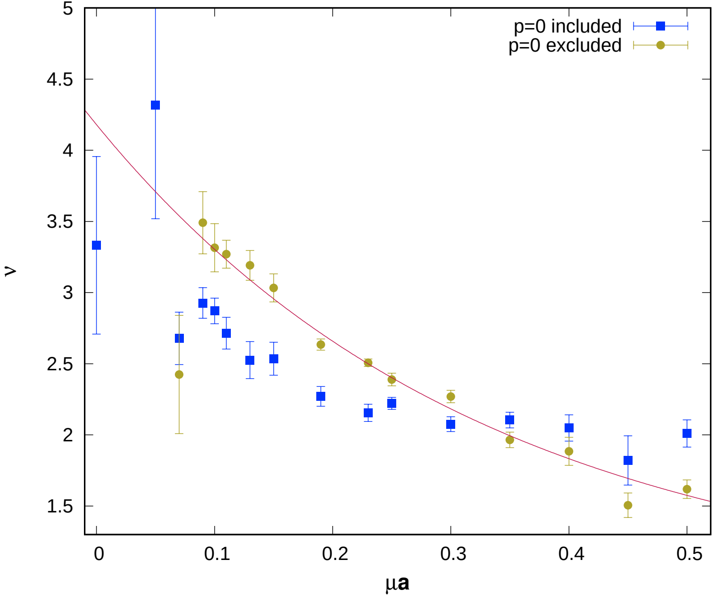


The difference between the propagators decreases exponentially:

$$(D_T(p) - D_L(p))|_{p_4=0} \simeq c\,\exp(-\nu|\vec{p}|)$$



The exponential fit parameter c



The exponential fit parameter ν

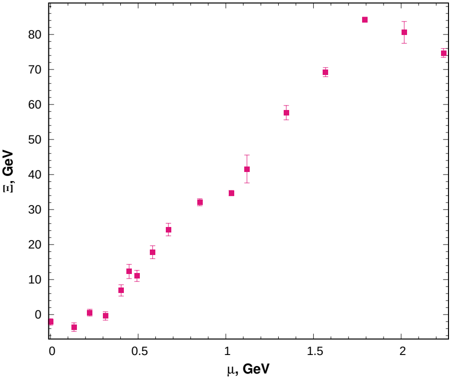
We also study the quantity

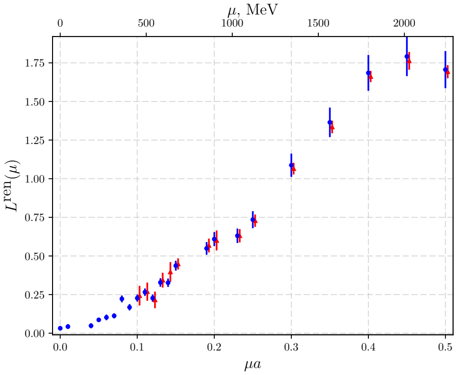
$$\Xi = \left(\frac{2\pi}{N_s a}\right)^3 \sum_{\vec{p}} (D_T(p_4 = 0, \vec{p}) - D_L(p_4 = 0, \vec{p})) \sim$$

$$\sim \sum_{\vec{x}} \sum_{x_4, y_4} \left(- \langle A_4(x_4, \vec{x}) A_4(y_4, \vec{x}) \rangle + \frac{1}{2} \sum_{i=1}^{3} \langle A_i(x_4, \vec{x}) A_i(y_4, \vec{x}) \rangle \right)$$

which is sensitive to infrared dynamics of gluon degrees of freedom.

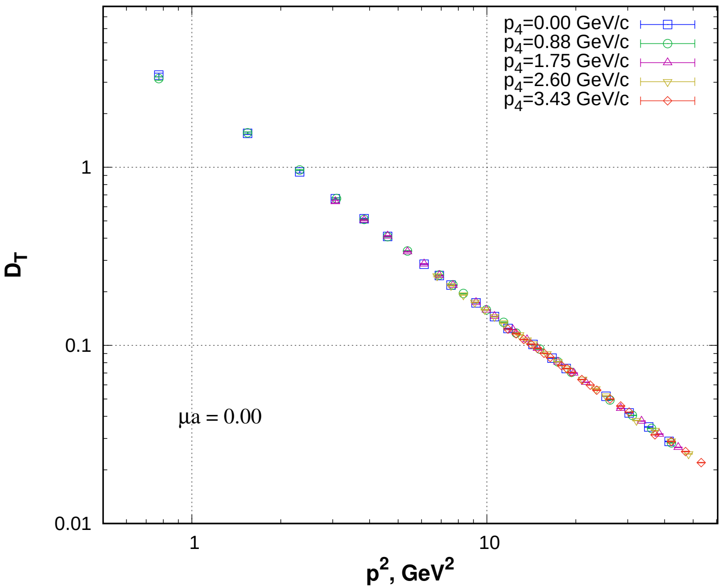
$$\Xi \simeq \int d\vec{p} (D_T(p_4 = 0, \vec{p}) - D_L(p_4 = 0, \vec{p}))$$



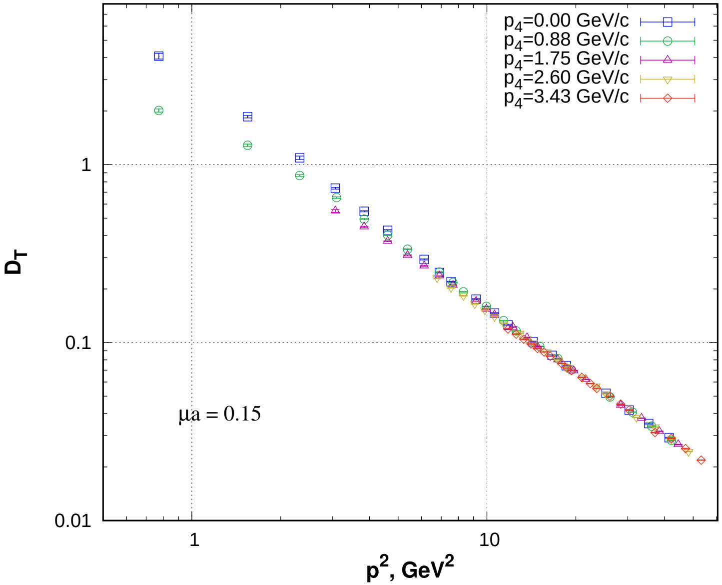


$$\Xi \simeq \int d\vec{p}(D_T(0,\vec{p}) - D_L(0,\vec{p}))$$

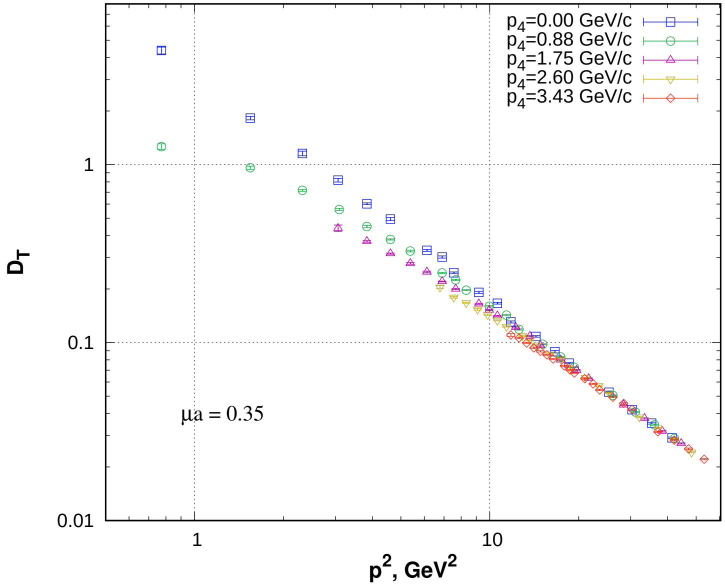
Polyakov loop



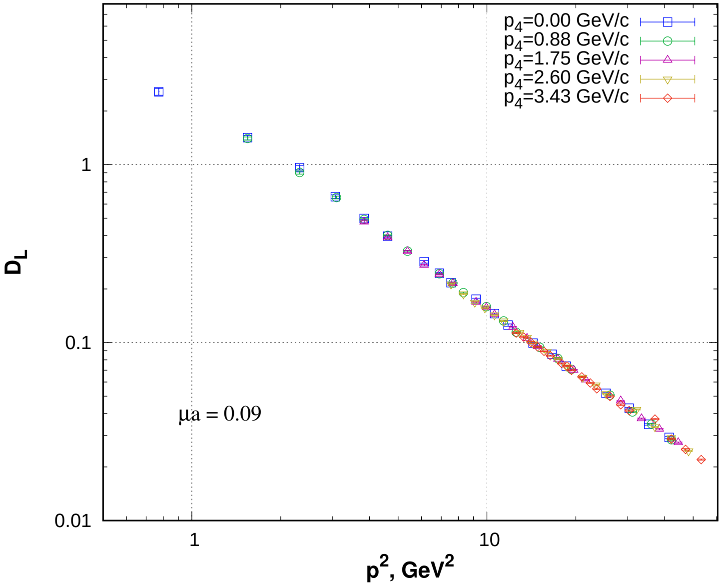
$D_T(\vec{p}^2 + p_4^2)$ at various values of p_4



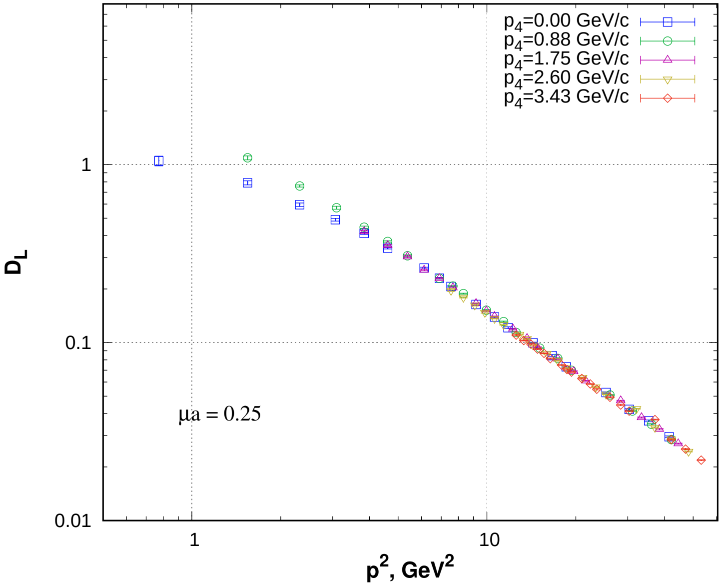
$D_T(\vec{p}^2 + p_4^2)$ at various values of p_4



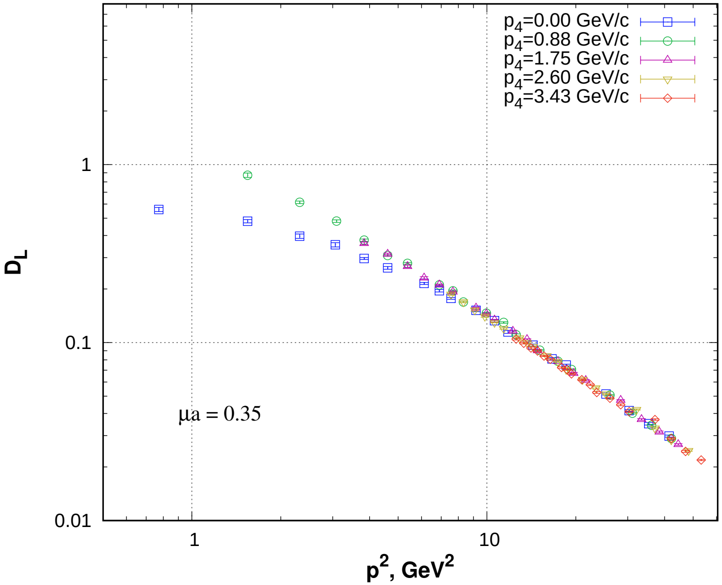
$D_T(\vec{p}^2 + p_4^2)$ at various values of p_4



$D_L(\vec{p}^2 + p_4^2)$ at various values of p_4



$D_L(\vec{p}^2 + p_4^2)$ at various values of p_4



$D_L(\vec{p}^2 + p_4^2)$ at various values of p_4

Conclusions

- Debye screening mass is consistent with perturbation theory at $\mu > 650$ MeV, where it increases as $g^2(\mu)\mu^2$.
- Magnetic Linde mass decreases approximately twice as μ varies over the range under study. To determine magnetic screening mass, substantially larger lattices are needed.
- The behavior of the gluon masses is in sharp disagreement with ealier findings in simulations with Wilson fermions and large a
- At large $|\vec{p}|$ gluon propagators agree well with RG-improved PT
- μ-dependence and T-dependence of D_L are similar, whereas μ-dependence and T-dependence of D_T are completely different.
- Gribov-Stingl fit parameters depend strongly on the chemical potential.
- The difference $D_T - D_L$ decreases exponentially with $|\vec{p}|$, its integral over momenta depends on μ much like the Polyakov loop.

The work is in progress