Lattice study of QCD phase diagram in (B, T, μ) space

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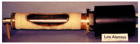
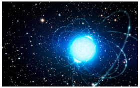
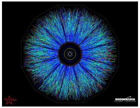
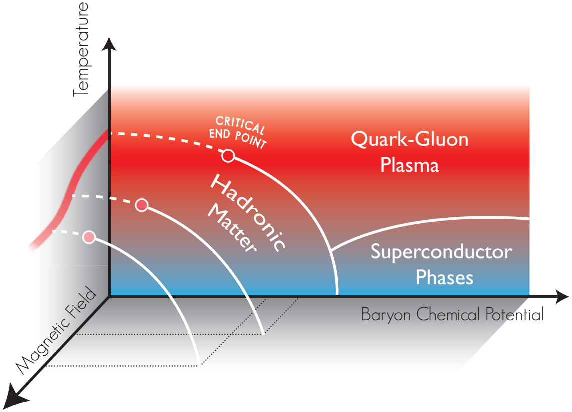
Magnetic fields in nature

	Souvenir magnet	5×10^{-3} T
	Max permanent magnet	1.25 T
	Magnetic field to levitate a frog	16 T
	Human produced pulsed magnetic field	2.8×10^{3} T
	Magnetars	$10^{8} - 10^{11}$ T
	Heavy ion collisions	$10^{14}\text{T} \sim m_\pi^2$

Wikipedia

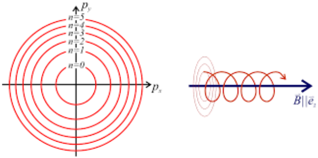
QCD and Magnetic Field



[P.Costa, M.Ferreira, C. Providência, 2018]

Can be studied in effective models
What can LQCD tell us about the phase diagram in μ, B, T?

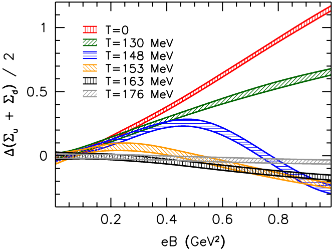
LQCD in $T - B$ plane



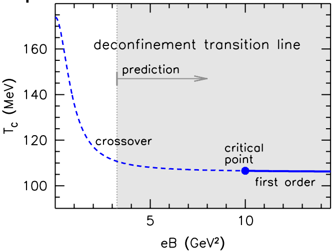
- Magnetic Catalysis

  [Gusynin V., Miransky V., Shovkovy I., 1994]

- Inverse Magnetic Catalysis
- CEP?
- Exotic phases?

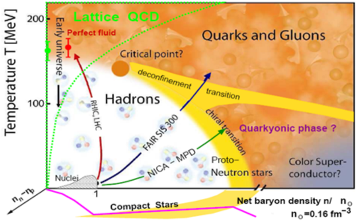


[F.Bruckmann et al., 2013]



[G.Endrodi, 2015]

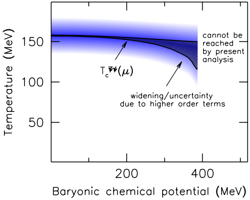
LQCD in $T - \mu$ plane

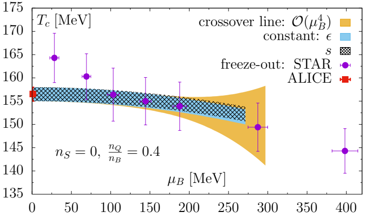


Sign problem!

- Curvature of pseudocritical line

$$T_c(\mu_B) = T_c(0) - A_2\mu_B^2 + A_4\mu_B^4 + O(\mu_B^6)$$





[R. Bellwied et al., 2015] [HotQCD, 2018]

Lattice setup

- Physical masses of u, d, s quarks
- $\mu_s = 0$, $\mu_u = \mu_d = \mu_l \to i\mu_l$

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- Fix $eB = \frac{6\pi n_B}{(N_s a)^2} \in [0.1, 1.5]$ GeV^2
- Fix $\frac{\mu_I}{\pi T} \in [0.00, 0.275]$
- T-scan (lattice step a is varied)
- $n_B \in \mathbb{Z} \longrightarrow$ discrete values of $T = \frac{1}{N_t a}$

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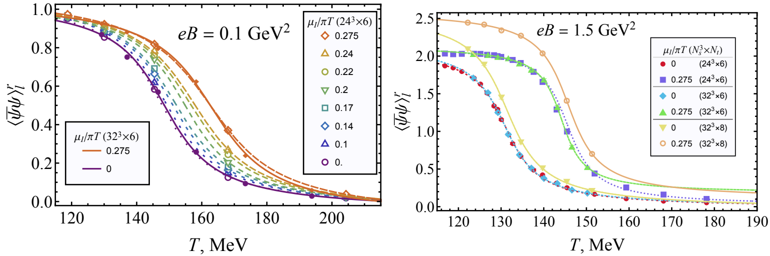
Renormalized observables:

- Light quark chiral condensate $\langle\bar{\psi}\psi\rangle^r$
- Polyakov loop L^r

Chiral condensate

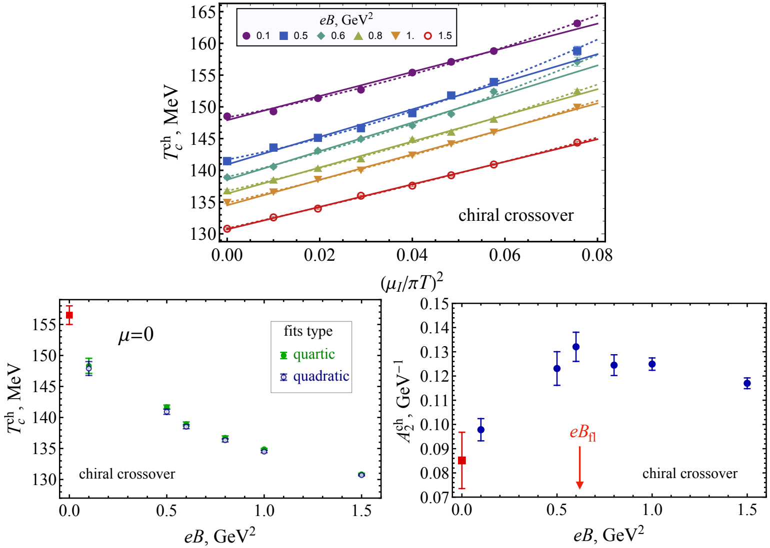


Inflection point:

$$\langle\bar\psi\psi\rangle_l^r(T) = A_1 + B_1 \arctan\left(\frac{T - T_c}{\delta T_c}\right)$$

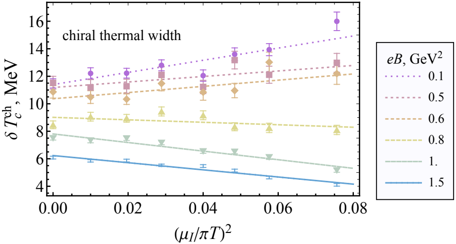
- eB grows, T_c decreases
- Large eB - transition is sharper, but still a crossover
- Finite volume effects are under control
- Finite lattice step has small effect on T_c, δT_c

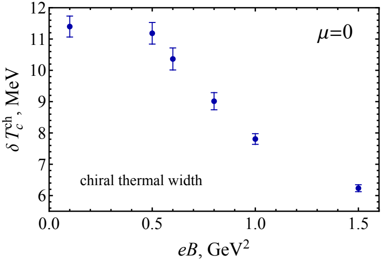
Critical temperature vs $\mu_I/\pi T$

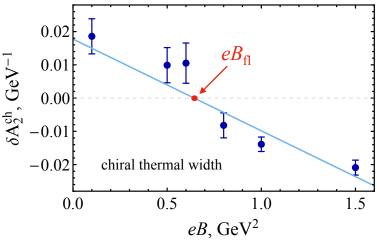


$$T_c(\mu_B, B) = T_c(0, B) - A_2(B)\mu_B^2 + O(\mu_B^4)$$

Chiral thermal width

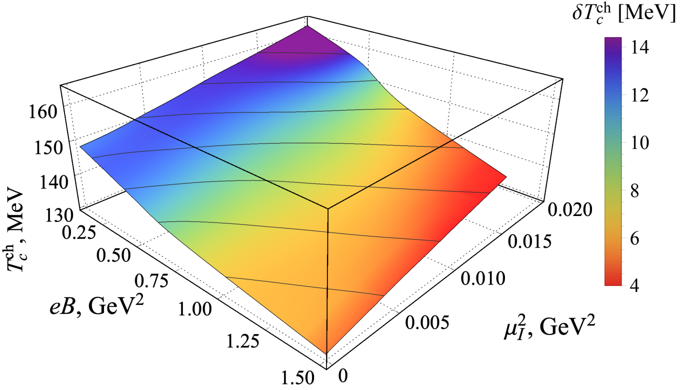




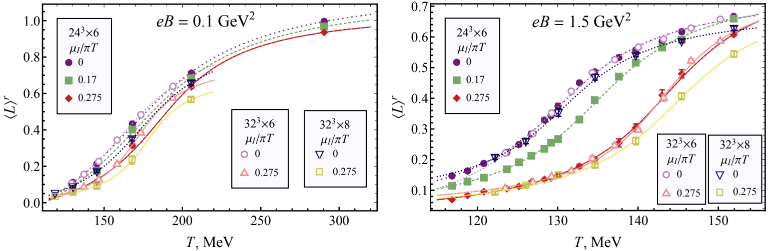


$$\delta T_c^{\rm ch}(\mu_B, B) = \delta T_c^{\rm ch}(0, B) - \delta A_2^{\rm ch}(B)\mu_B^2 + O(\mu_B^4)$$

Critical temperature and width of the chiral crossover

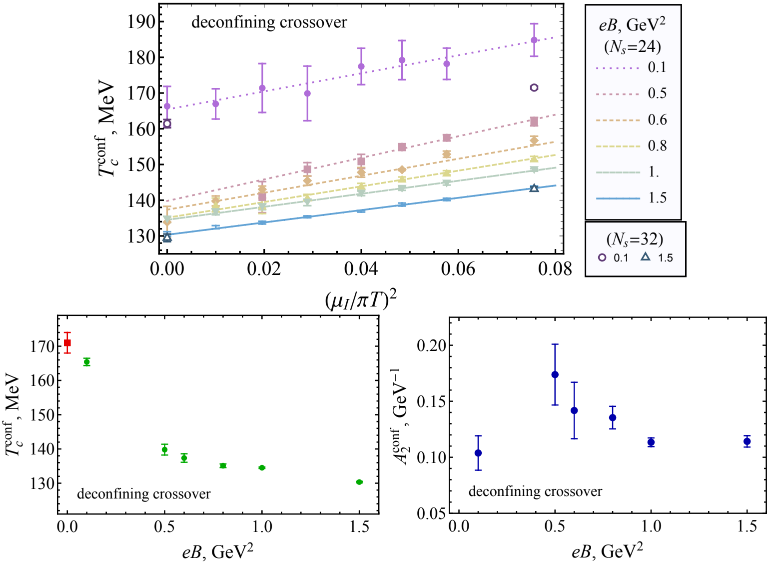


Polyakov loop



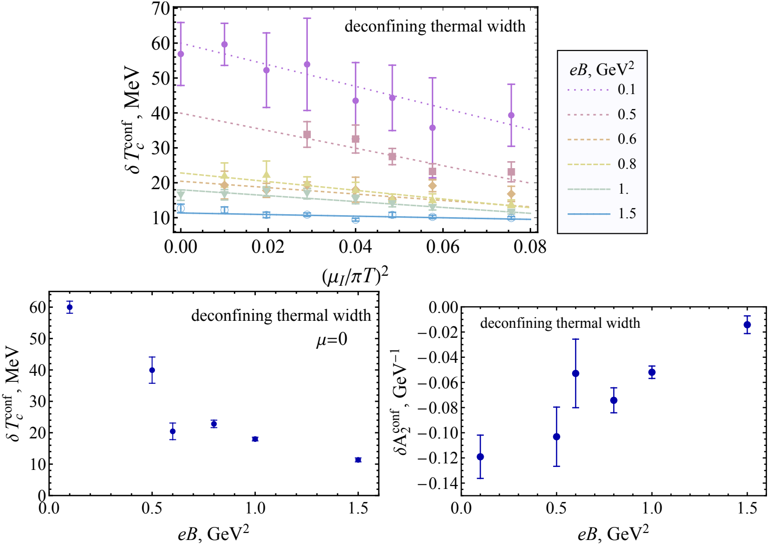
$$\langle L\rangle^r(T) = A_2 - B_2 \arctan\left(\frac{T - T_c}{\delta T_c}\right)$$

Confining crossover, critical temperature T_c



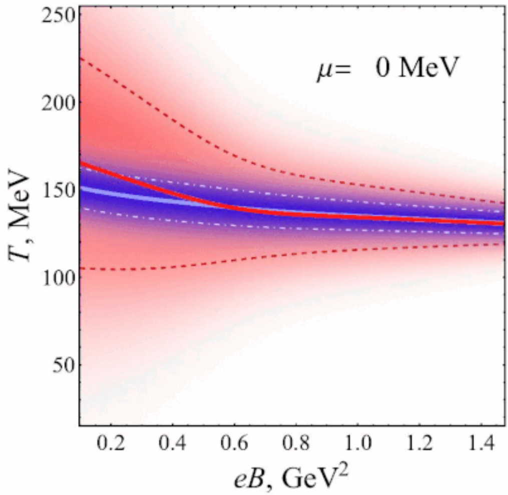
$$T_c(\mu_B, B) = T_c(0, B) - A_2(B)\mu_B^2 + O(\mu_B^4)$$

Confining crossover, width δT_c



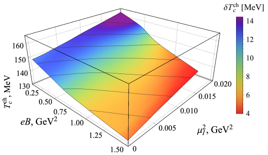
$$\delta T_c^{\rm conf}(\mu_B, B) = \delta T_c^{\rm conf}(0, B) - \delta A_2^{\rm conf}(B)\mu_B^2 + O(\mu_B^4)$$

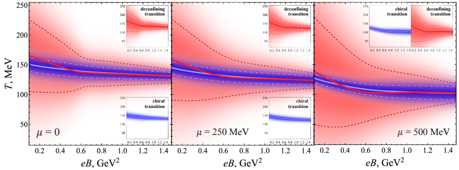
Chiral and deconfining crossover at real μ_B



Results and conclusions

- QCD phase diagram with nonzero T, eB, μ
- Simulations with imaginary $\mu_u = \mu_d = i\mu_I$, $\mu_s = 0$
- Critical temperature: mild interplay between eB and μ_B:
  - Inverse Magnetic Catalysis
  - Mild dependence of curvature A_2 on eB, peak $eB \approx 0.6\text{GeV}^2$
- Width of the transition:
  - Magnetic field makes the transition sharper (chiral and deconfining transitions merge at large eB) at $\mu_I \neq 0$
  - No signatures of CEP or first order transition
  - Chiral thermal width (Behaviour changes at $eB_c \approx 0.6\text{GeV}^2$):
    - $eB < eB_c$: δT_c^{ch} slightly decreases with μ_B
    - $eB > eB_c$: δT_c^{ch} increases with μ_B
  - Baryonic matter always weakens the deconfining crossover





Backup

Details of lattice setup

- Lattice size: $\underline{6 \times 24^3}$, 6×32^3, 8×32^3
- Stout improved staggered $N_f = 2 + 1$ fermions
- Tree level Symanzik gauge action
- Physical masses of u, d, s quarks
- $q_u = 2/3e$, $q_d = q_s = -1/3e$
- Data for $eB = 0$ are taken from [C. Bonati et al., 2014]
- $\mu_s = 0$, $\mu_u = \mu_d = \mu_I \rightarrow i\mu_I$
- $O(100)$ configurations per each T, μ_I, eB

Obserbables and renormalization

Light quark chiral condensate:

$$\langle\bar{\psi}\psi\rangle_l = \frac{T}{V}\frac{\partial \log Z}{\partial m_l} = \langle\bar{u}u\rangle + \langle\bar{d}d\rangle$$

$$\langle\bar{\psi}\psi\rangle_r = \frac{\left[\langle\bar{\psi}\psi\rangle_l - \frac{2m_l}{m_s}\langle\bar{s}s\rangle\right](T, eB, \mu)}{\left[\langle\bar{\psi}\psi\rangle_l - \frac{2m_l}{m_s}\langle\bar{s}s\rangle\right](0, 0, 0)}$$

Polyakov loop ($\langle L\rangle = exp(-F_Q/T)$):

$$L(\vec{x}) = \frac{1}{3V}\,\mathrm{tr}\prod_{\tau=1}^{N_\tau} U_4(\vec{x}, \tau)$$

Renormalized with GF [P.Petreczky and H.-P. Schadler, 2016]

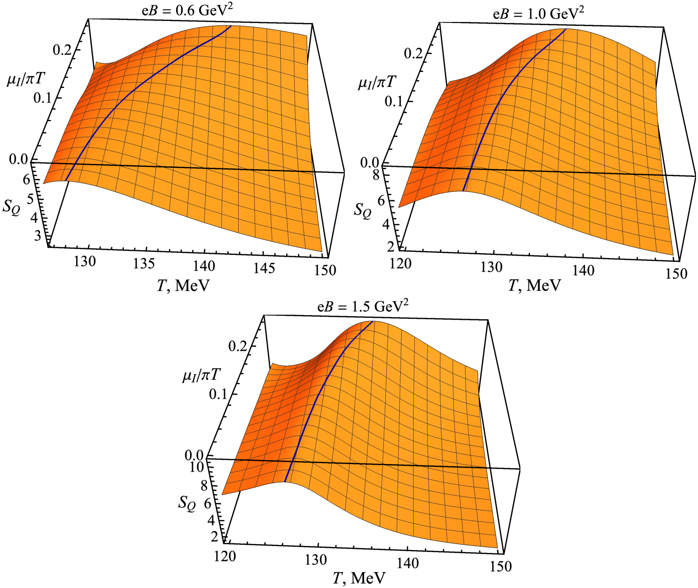
$$L^r(\vec{x}) = L(\vec{x})[V_{t=f}(x, \mu)]$$

Thermodynamic properties of heavy quarks

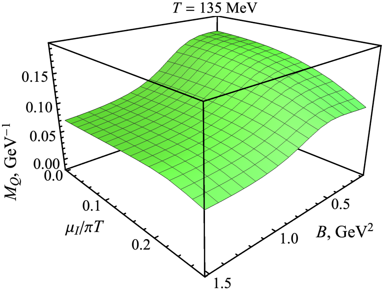
$$|\langle P \rangle| = e^{-\Omega_Q/T}$$

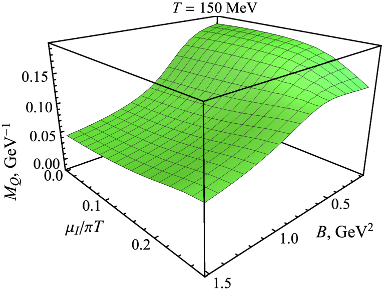
$$d\Omega_Q = -S_Q dT - N_Q d\mu - M_Q dB$$

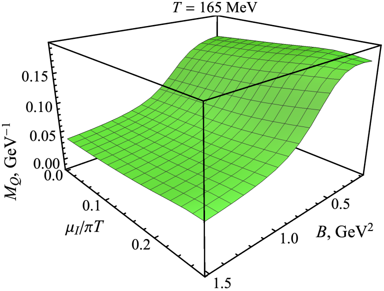
Single-quark entropy



Single-quark magnetization







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