Improvements of vortex detection and influence of fermions on vortices in SU(2)-QCD

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in coop. with

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why interests in vacuum?

vacuum should explain the non-perturbative properties of QCD

- confinement
- symmetry breaking
  - breaking of chiral symmetry
  - breaking of center symmetry
  - breaking of scale invariance

$\rightarrow$ Vortex model of QCD-vacuum

Vortex properties

- thick quantised magnetic flux tubes propagating in time, form closed surfaces in dual space,
- vortices have a thick core,
- percolating in all directions,
- deconfinement transition a de-percolation transition,
- in deconfinement: percolation in spatial directions only,
- scaling of the P-vortex density.

Vortex model of QCD-vacuum explains

- non-trivial vacuum $\rightarrow$ gluon condensate
- area law of Wilson loops $\rightarrow$ confinement
- Casimir scaling of heavy-quark potential
- finite temperature phase transition $\rightarrow$ loss of center symmetry
  behaviour of Polyakov loops
- area law for spatial Wilson loops
- orders of phase transitions in SU(2) and SU(3)
- topological charge of field configurations
  vortex intersections, color structure $\rightarrow$ instantons
- chiral symmetry breaking $\rightarrow$ quark condensate
- breaking of scale symmetry
- monopole picture of confinement $\rightarrow$ dual superconductor model
- behaviour of double winding Wilson loops

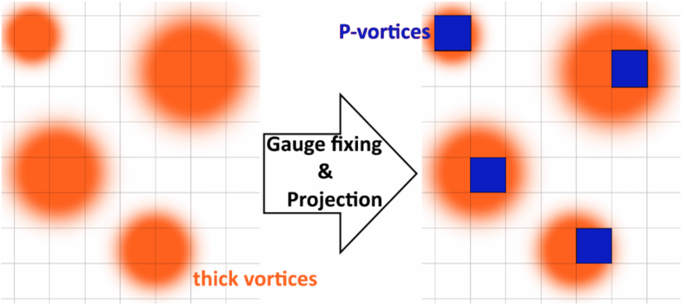
But there is a dark cloud !

Detecting center vortices

(Direct) Maximal Center Gauge:

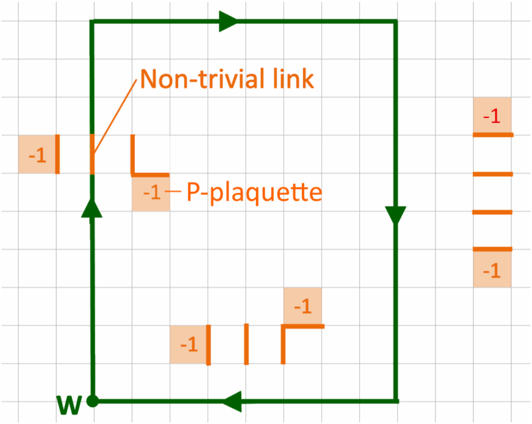
$$R^2 = \sum_x \sum_\mu |\,\mathrm{Tr}[U_\mu(x)]\,|^2 \rightarrow \text{ maximized using simulated annealing}$$

Followed by center projection: $U_\mu(x) \rightarrow Z_\mu(x) = \mathrm{sign}\ \mathrm{Tr}[U_\mu(x)]$



Finds a fit of the given gauge field by a thin vortex configuration [1, 2].

P-vortices and the string tension



Each P-plaquette contributes a factor -1 to Wilson loops surrounding it.

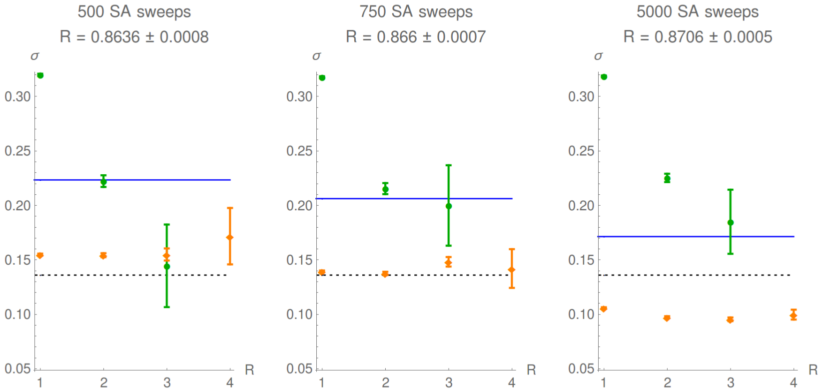
With the vortex density ρ_{vortex}, that is, the number of uncorrelated P-plaquettes per unit volume, the string tension σ can be calculated.

$$\langle \frac{1}{2}\, Tr(\boldsymbol{W}(R,T))\rangle = (-1\ \rho_{vortex} + 1\ (1-\rho_{vortex}))^{R\times T} \;=\; e^{ln(1-2\ \rho_{vortex})\ R\times T}$$

$$\Longrightarrow \sigma \;=\; -ln(1-2\ \rho_{vortex}) \;\approx\; 2\ \rho_{vortex}$$

Problems concerning the string tension

Different local maxima can lead to different values of the string tension predicted by the center vortex model, see [3, 4].



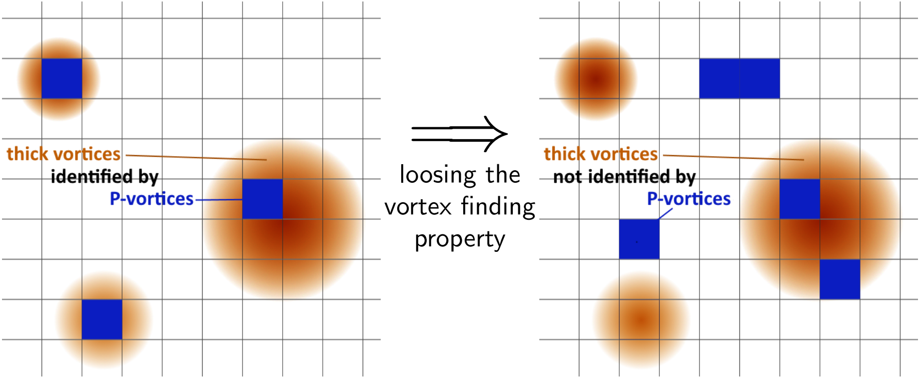
<u>String tension calculated via:</u> ● $\chi(R)_{SU(2)}$ ◆ $\chi(R)_{Z2}$ — Vortexdensity -- Literature

Respective lattice sizes 12^4, 12^4, 14^4, with 300 Wilson configurations

Better value of gauge functional leads to an underestimation of the string tension? ⇔ loss of the *vortex finding property*?

Possible explanation: loss of vortex finding property

P-vortices fail to locate thick vortices



→ high vortex density, but P-vortices do not cover the long range effects.

Resolvable by usage of the non-Abelian Stokes theorem?

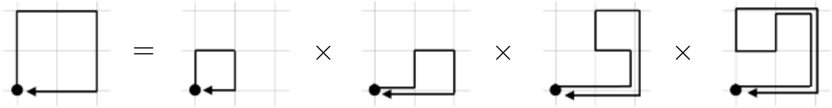
The non-Abelian Stokes theorem and center regions

$$P \exp\left(i \oint_{\partial S} A_\mu(x)\, dx^\mu\right) = \mathcal{P} \exp\left(\frac{i}{2} \int_S \mathcal{F}_{\mu\nu}(x)\, dx^\mu\, dx^\nu\right)$$

$$\mathcal{F}_{\mu\nu}(x) = U^{-1}(x,O)\, F_{\mu\nu}(x)\, U(x,O) \qquad U(x,O) = P \exp\left(i \int_l A_\eta(y)\, dy^\eta\right)$$

with P denoting path ordering, $\mathcal{P}$ "surface ordering" and l being a path from the base O of ∂S to x, see [5].

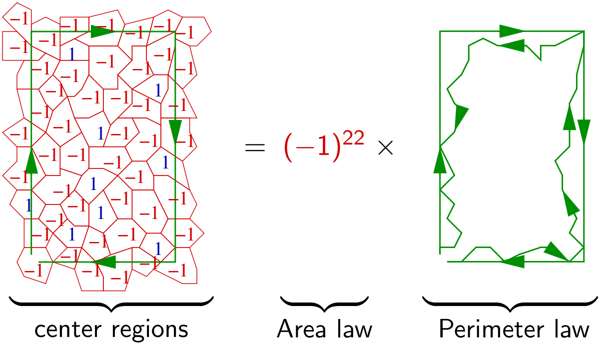
On the lattice:



Regions within ∂S evaluating to a center element commute with all other factors and can be collected into a single factor.

$\rightarrow$ Stokes theorem becomes abelian for center regions.

Center regions and the behaviour of Wilson loops



Regions, whose boundaries evaluate to center elements can be used to factorize a Wilson loop into two parts:

- an area factor collecting the fully enclosed non-trivial regions
  $\rightarrow$ linear rising potential,
- a perimeter factor from non-center contributions due to partially enclosed center regions
  $\rightarrow$ Coulombic behaviour.

Using center regions as guidance for gauge fixing

The assumption: The trace of Center regions or arbitrary Wilson loops evaluating to centre elements ("*center loops*") should neither be modified by gauge fixing nor by center projection $\iff$ **Vortex finding property**

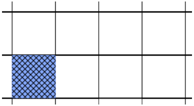
The idea: Modify the simulated annealing procedure so that

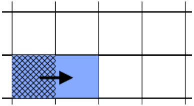
- gauge transformations resulting in a non-trivial center region projecting to the trivial center are prohibited,
- gauge transformations resulting in a non-trivial center region, previously projecting to the trivial center, now projecting to a non-trivial center are enforced.

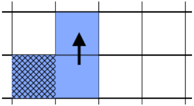
Requirement: identifying non-trivial centre regions.

Identifying non-trivial center regions

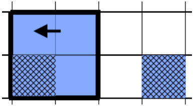
Start with a plaquette and enlarge so that the bigger regions evaluation is nearer the non-trivial center (1-3).

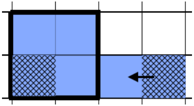
1) 

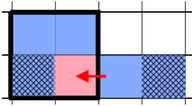
2) 

3) 

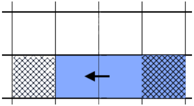
When no more enlargment results in an improved evaluation, store the region and start with a new plaquette (4-5)

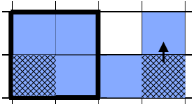
4) 

5) 

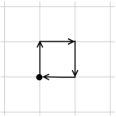
6) 

When a new region grows into an older one, the better one survives (6-7).

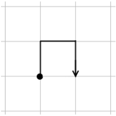
7a) 

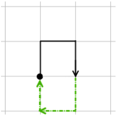
7b) 

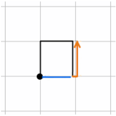
The enlargement procedure

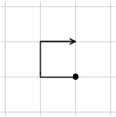
1) 

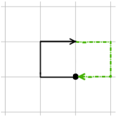
A double connected list is generated and evaluated with one link missing. The evaluation is complemented around one neighbouring plaquette after another to identify the best neighbour.

2) 

3) 

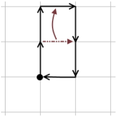
4) 

5) 

6) 

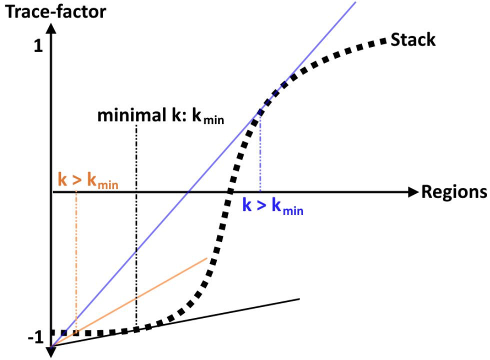
n) ...

With the best neighbour identified, the relevant link within the connected list is pushed outwards. The procedure is repeated until no further enlargement leads to an improvement of the regions evaluation.

n+1) 

Selecting non-trivial center regions for gauge fixing

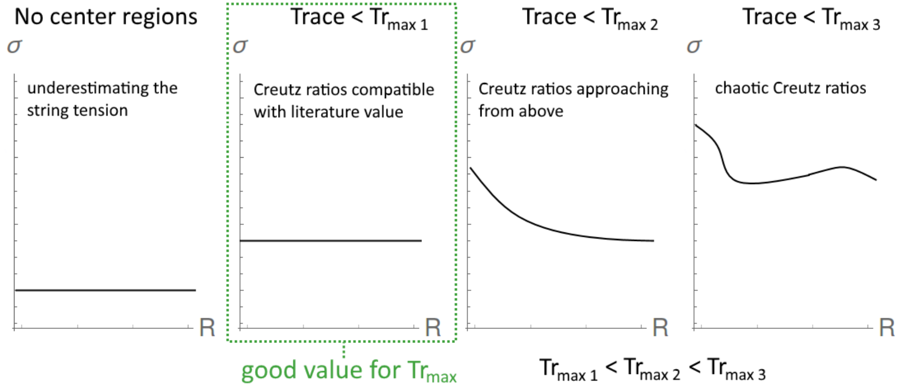
A stack is generated via sorting the regions identified within a single configuration by rasing trace. Only regions within this stack with an index below a maximal index are taken into account. From the criteria below the one giving a smaller number of non-trivial center regions is chosen:



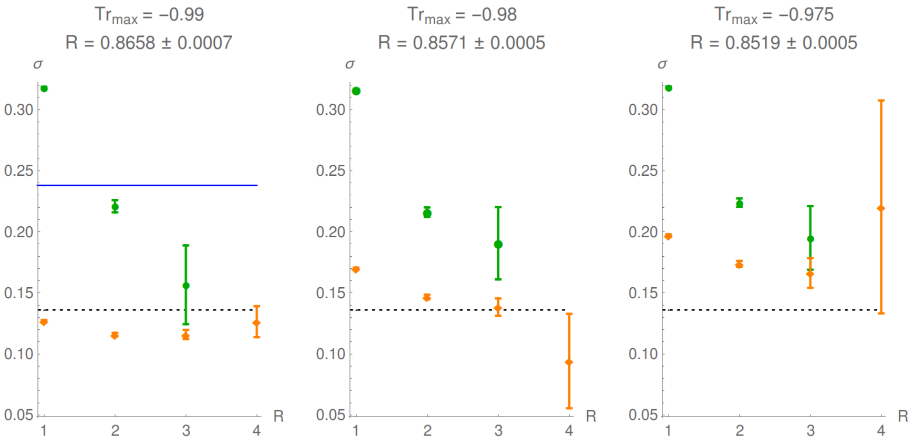
- A tangent through $(0,\ 1.1 * Tr(0))$ is identified and defines a maximal index.
- Only regions with a trace factor below an arbitrary upper limit Tr_{max} are taken into account.

Fine tuning the parameter Tr_{max}

By taking a look at the behaviour of $\mathcal{Z}_2$ Creutz ratios, the value for Tr_{max} can be optimized:



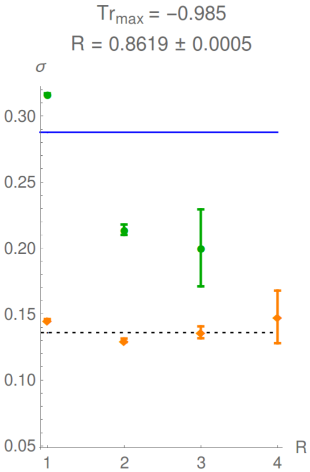
Optimizing Tr_{max} for $\beta = 2.3$



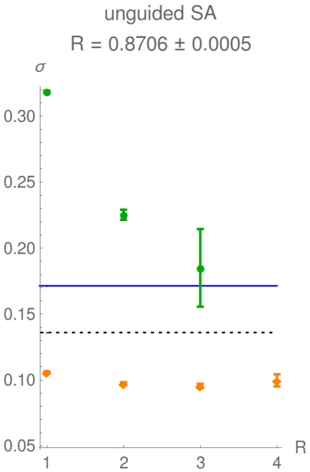
<u>String tension calculated via:</u> ● $\chi(R)_{SU(2)}$ ◆ $\chi(R)_{Z2}$ — Vortexdensity -- Literature

Respective lattice sizes 12^4, 12^4, 14^4, with 300 Wilson configurations

Resulting Creutz ratios at $\beta = 2.3$ for $Tr_{max} = -0.985$



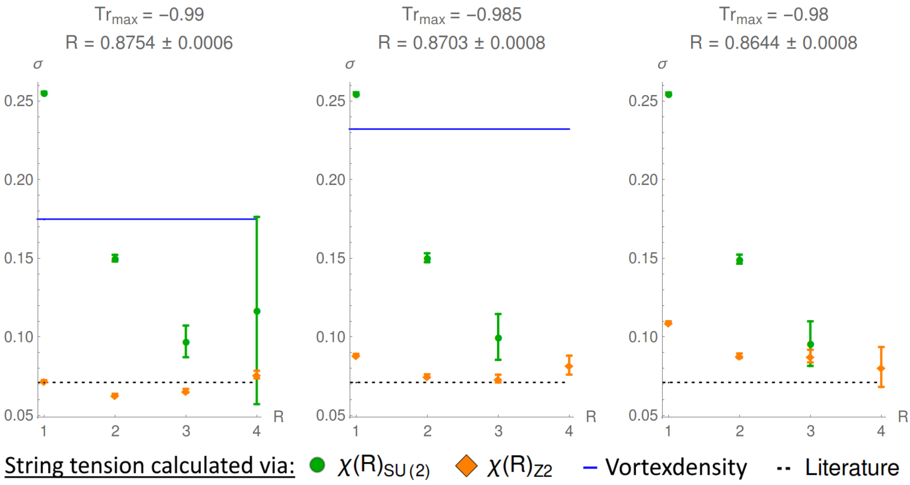
versus



String tension calculated via: ● $\chi(R)_{SU(2)}$ ◆ $\chi(R)_{Z2}$ — Vortexdensity -- Literature

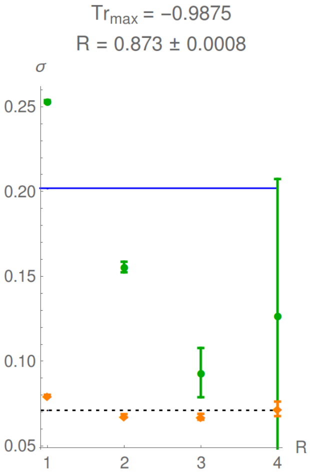
Lattice size 14^4, with 300 Wilson configurations

Optimizing Tr_{max} at $\beta = 2.4$

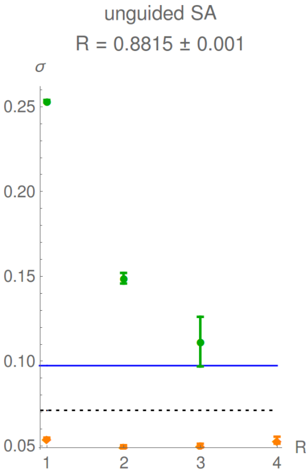


Respective lattice sizes 14^4, 12^4, 12^4, with 300 Wilson configurations

Resulting Creutz ratios at $\beta = 2.4$ for $Tr_{max} = -0.9875$



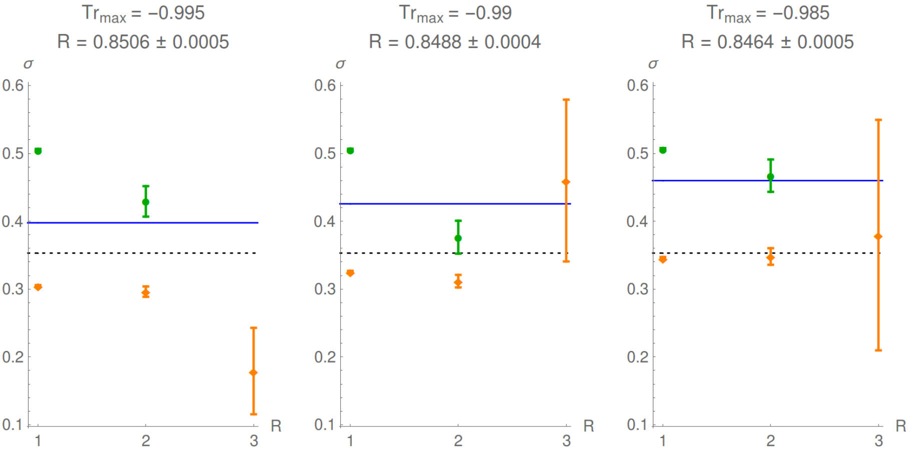
versus



<u>String tension calculated via:</u> ● $\chi(\mathrm{R})_{SU(2)}$ ◆ $\chi(\mathrm{R})_{Z2}$ — Vortexdensity -- Literature

Lattice size 12^4, with 300 Wilson configurations

Difficulties at $\beta = 2.1$: big errors



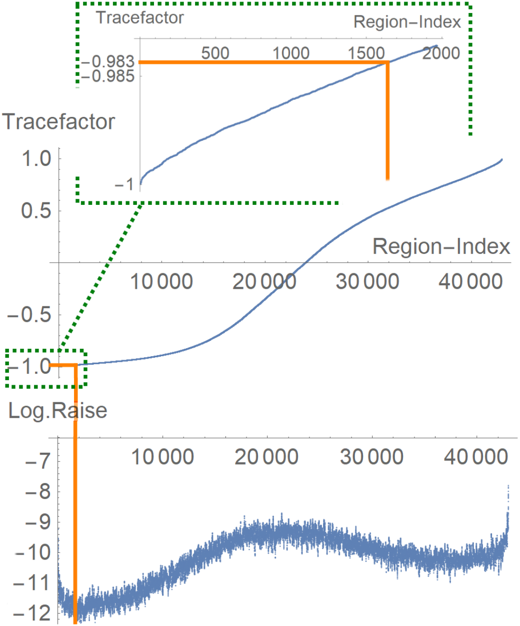
<u>String tension calculated via:</u> ● $\chi(R)_{SU(2)}$ ◆ $\chi(R)_{Z2}$ — Vortexdensity -- Literature

Lattice size 12^4 with 300 Wilson configurations

Center regions can be used to recover the full string tension in center projected configurations, but reduce the value of the gauge functional.

The choice in the value of Tr_{max} might seem arbitrary, but the development of a parameter free selection of non-trivial center regions is in progress:

Ongoing work: improve selection of non-trivial center regions

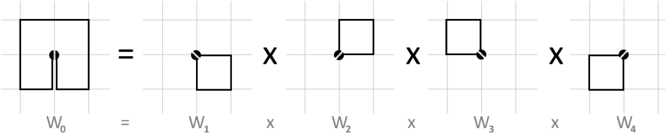


The upper limit Tr_{max} of the tracefactors within a list of the identified center regions of a single configuration sorted by their tracefactors seems to coincide with the RegionIndex of the global Minimum of

$$\log \frac{\Delta \text{Tracefactor}}{\Delta \text{RegionIndex}}.$$

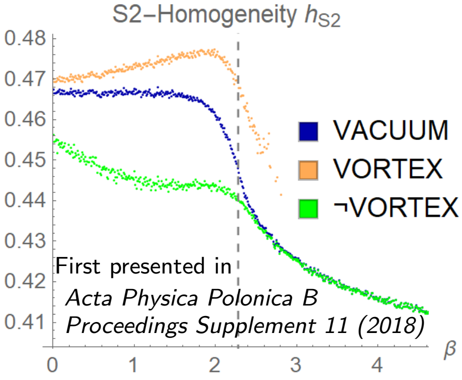
Tr_{max}	$\beta = 2.3$	$\beta = 2.4$
manual:	-0.985	-0.9875
automatic:	-0.983	-0.982

Some interesting data: S2-Homogeneity and P-plaquettes



$\sigma_0 = \mathbb{1}_2$
Pauli matrices $\boldsymbol{\sigma}_k$
$\boldsymbol{n}_j \in \mathbb{S}^2, \ | \boldsymbol{n}_j | = 1$

$$\boldsymbol{W}_j = \cos(\alpha_j)\, \boldsymbol{\sigma}_0 + \mathrm{i} \sum_{k=1}^{3} \sin(\alpha_j)\, (\boldsymbol{n}_j)_k\, \boldsymbol{\sigma}_k \quad : \qquad h_{S2} := \frac{1}{4} \left| \sum_{j=1}^{4} \boldsymbol{n}_j \right|.$$



- Color fluctuations are reduced in the vicinity of P-plaquettes.
- Phase transition at $\beta \approx 2.28$ also in symmetric lattices?

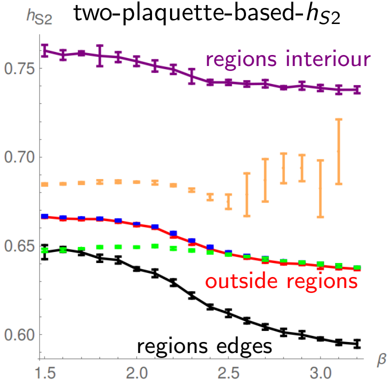
What about non-trivial center regions?

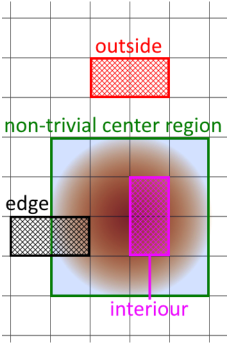
Some interesting data: S2-Homogeneity and center regions

$\sigma_0 = \mathbb{1}_2$, Pauli matrices $\boldsymbol{\sigma}_k$, $\boldsymbol{n}_j \in \mathbb{S}^2$, $\mid \boldsymbol{n}_j \mid = 1$

Instead of four, we now use two neighbouring plaquettes:

$$\boldsymbol{W}_j = \cos(\alpha_j)\ \boldsymbol{\sigma}_0 + \mathrm{i}\ \sum_{k=1}^{3}\ \sin(\alpha_j)\ (\boldsymbol{n}_j)_k\ \boldsymbol{\sigma}_k \quad : \quad h_{S2} := \frac{1}{2} \mid \sum_{j=1}^{2} \boldsymbol{n}_j \mid .$$

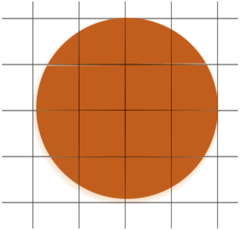




Non-trivial center regions are:

- S2-homogeneous regions
- sharply seperated from the surrounding vacuum

With respect to the color-structure: Are thick vortices seperated sharply or smoothly from the surrounding vacuum?



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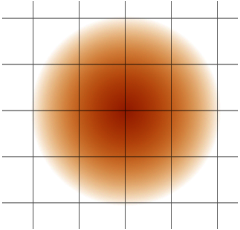
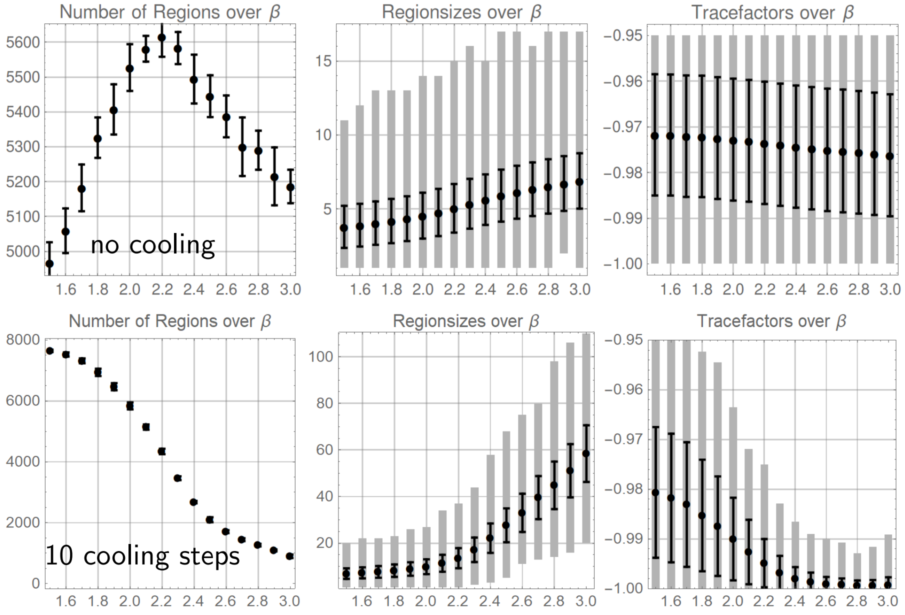


S2-homogeneity indicates a sharp seperation

More interesting data: 12^4 lattice, $Tr_{max} = -0.95$

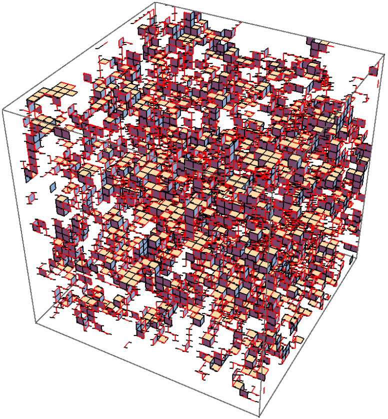


First results on Vortices and Fermions

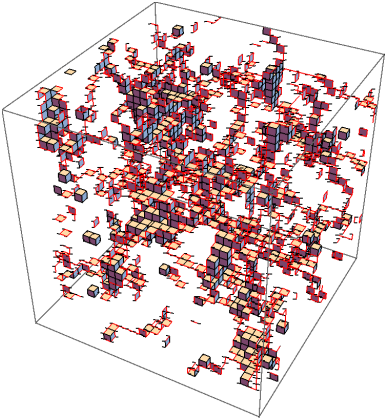
thanks to Aleksandr Nikolaev for providing us with 1000 field configurations on 32^4 lattices
with Symanzik improved action:

- gluodynamics $\beta = 1.8$
- $\beta = 1.8$ rooted staggered fermions with diquark source term
- gluodynamics $\beta = 2.55$

one time-slice of the 2D-vortex surfaces in 4D



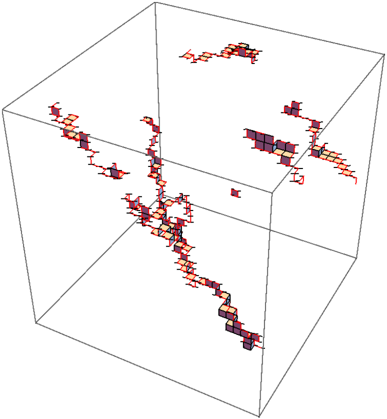
gluonic, $\beta = 1.8$
277000 P-plaquettes
largest cluster: 249000



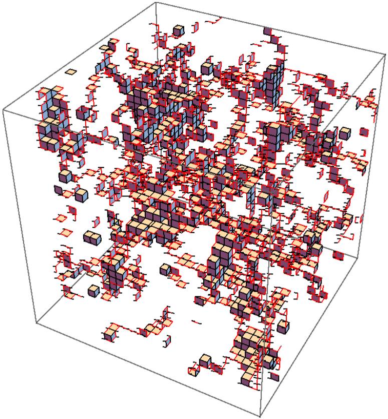
with fermions, $\beta = 1.8$
124000 P-plaquettes
largest cluster: 105000

fermions reduce the string tension to 42%.

one time-slice of the Vortex surfaces

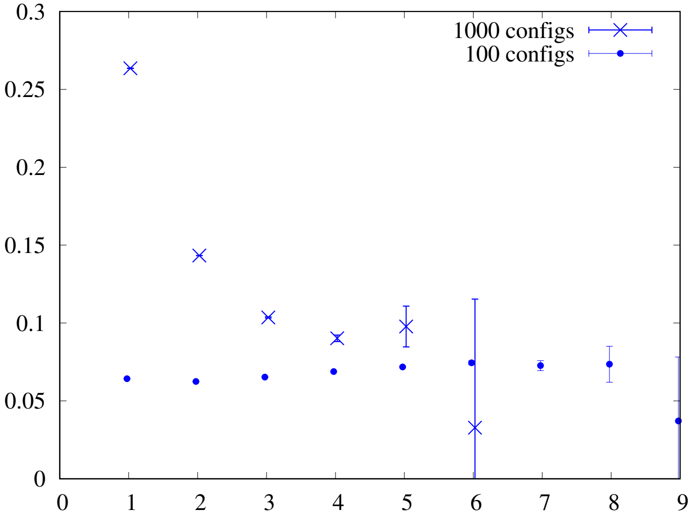


gluonic, $\beta = 2.55$
14300 P-plaquettes
largest cluster: 14140



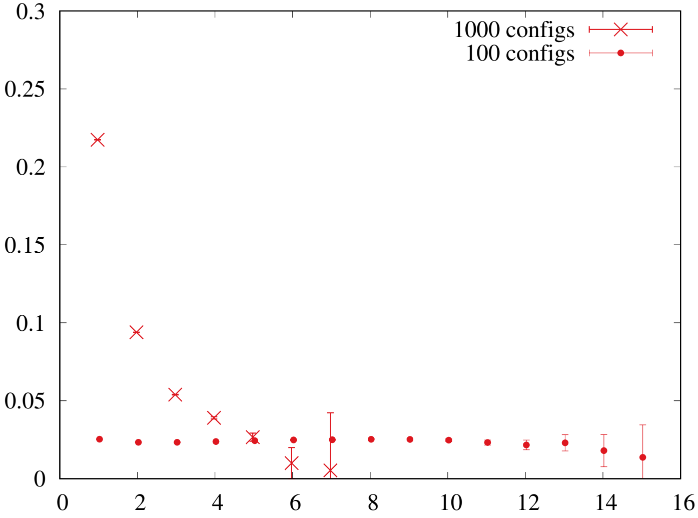
with fermions, $\beta = 1.8$
124000 P-plaquettes
largest cluster: 105000

gluonic Creutz ratios, $\beta = 1.8$

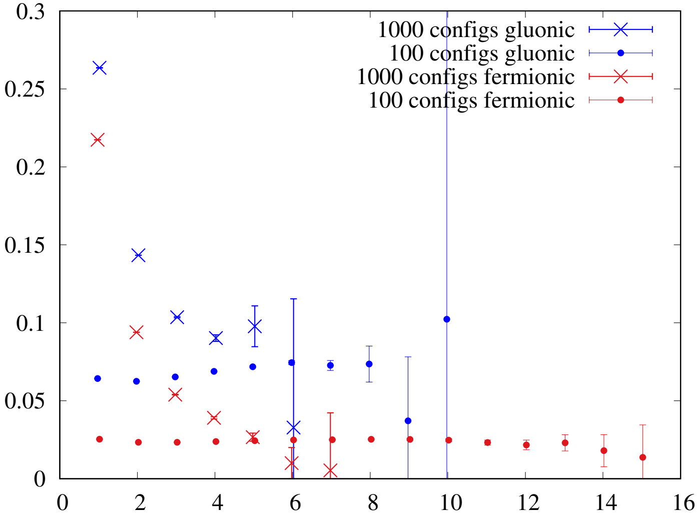


Vortices do not see the Coulomb contribution

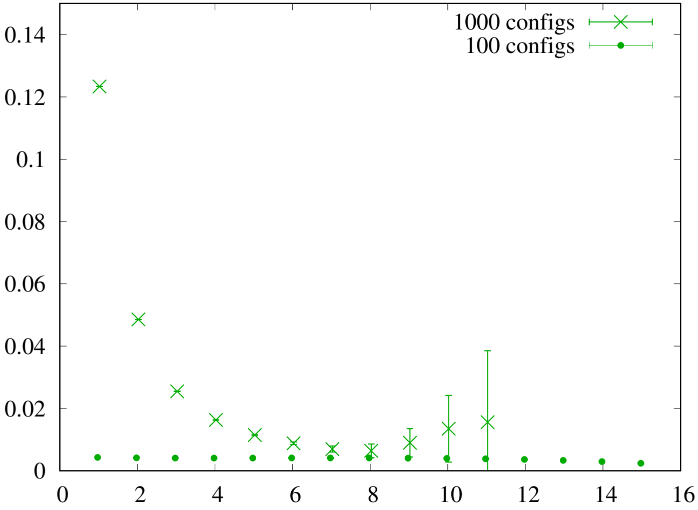
fermionic Creutz ratios, $\beta = 1.8$



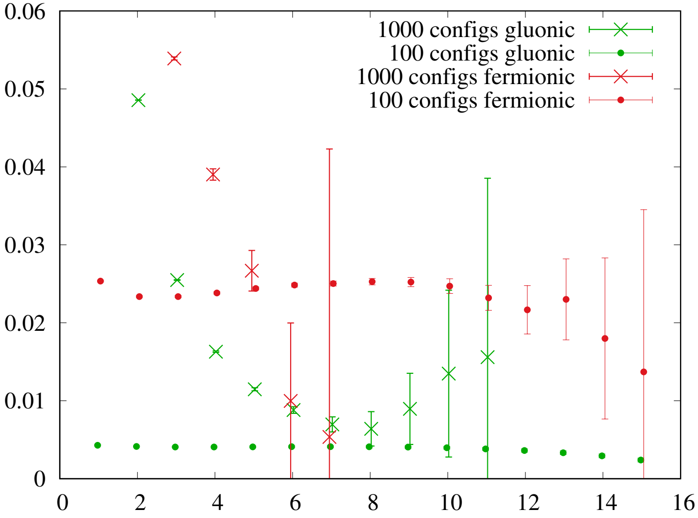
comparison of Creutz ratios, $\beta = 1.8$



gluonic Creutz ratios, $\beta = 2.55$



gluonic Creutz ratios, $\beta = 2.55$



lattice constants from Creutz ratios

$\hbar c = 197$ MeV fm

$$\sigma = \frac{(440 \text{ MeV})^2}{\hbar c} = \hbar c \left(\frac{2,23}{\text{fm}}\right)^2 = \hbar c \frac{\chi}{a^2} \quad \Leftrightarrow \quad a = \frac{\sqrt{\chi}}{2.23} \text{ fm}$$

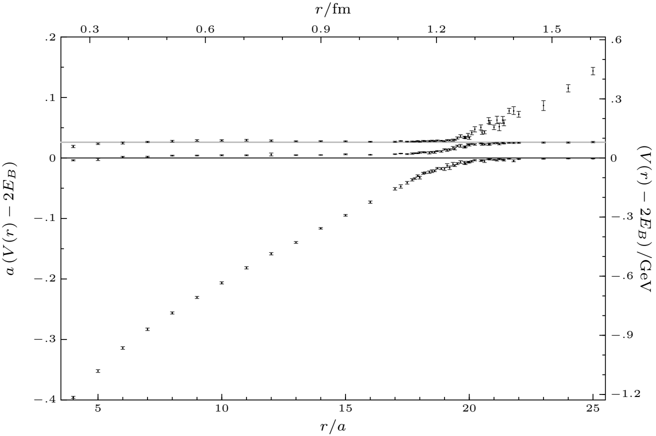
Creutz ratios from P-vortices

- $\chi_{1.8} = 0.0745 \quad \Rightarrow \quad a_{1.8} = 0.112$
- $\chi_{\text{ferm}} = 0.0253 \quad \Rightarrow \quad a_{\text{ferm}} = 0.071 \text{ fm} \quad \overset{?}{\leftrightarrow} \quad 0.044 \text{ fm}$
- $\chi_{2.55} = 0.00412 \quad \Rightarrow \quad a_{2.55} = 0.029 \text{ fm}$

For fermionic configurations on 32^4 lattice: $16 * 0.071 \text{ fm} = 1.14 \text{ fm}$

Have P-vortices a structure of double sheets?
Could this explain string breaking?

String breaking from mixing matrix



From: arXiv:1902.04006v2:
John Bulava, Ben Hörz, Francesco Knechtli, Vanessa Koch, Graham Moir, Colin Morningstar, Mike Peardon

Do Wilson loops force the string not to break?

Final conclusio:

Identification of non-trivial center regions improves center vortex detection.

Fermions reduce P-vortex density drastically.

No signal yet for string breaking from vortices.

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