Fluctuations
and
the QCD Phase Structure

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Germany

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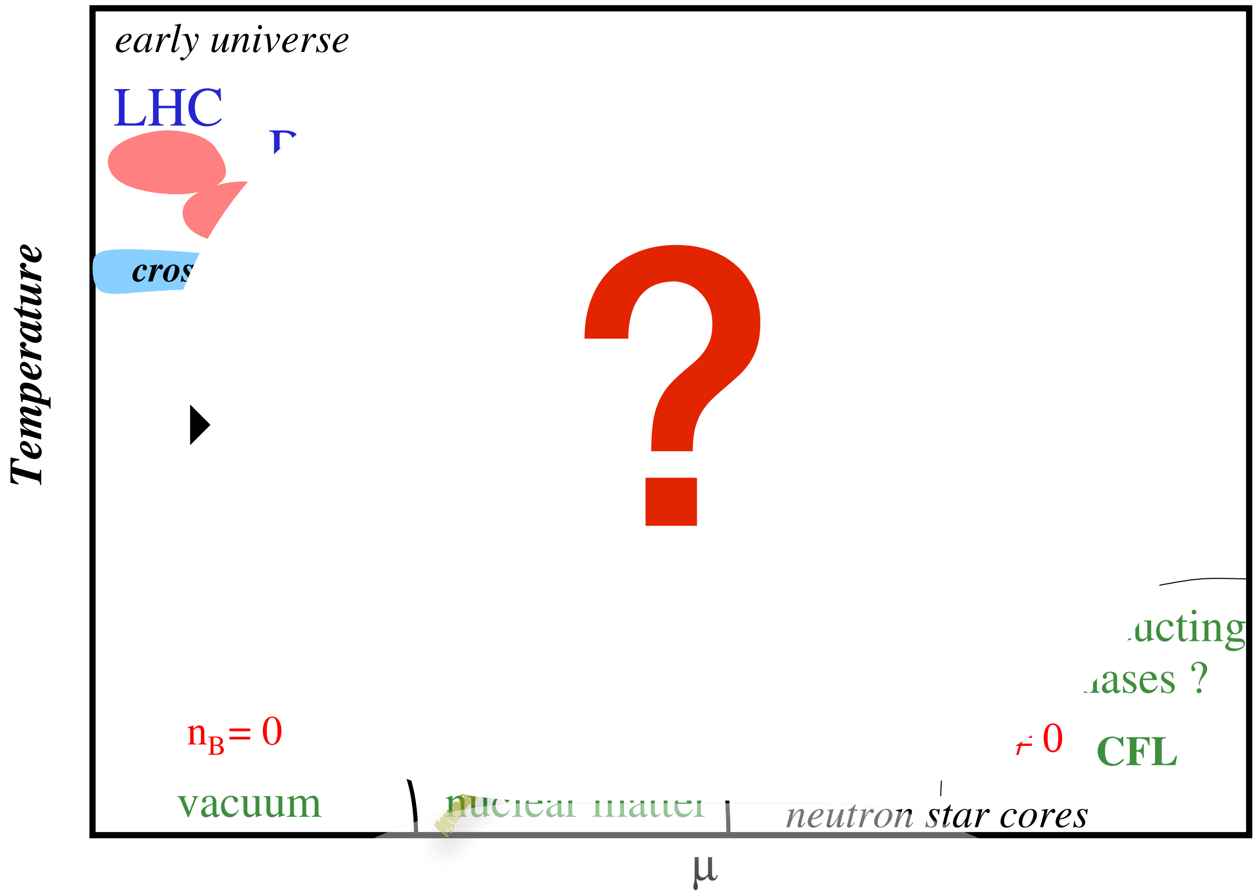
QC_3D phase structure

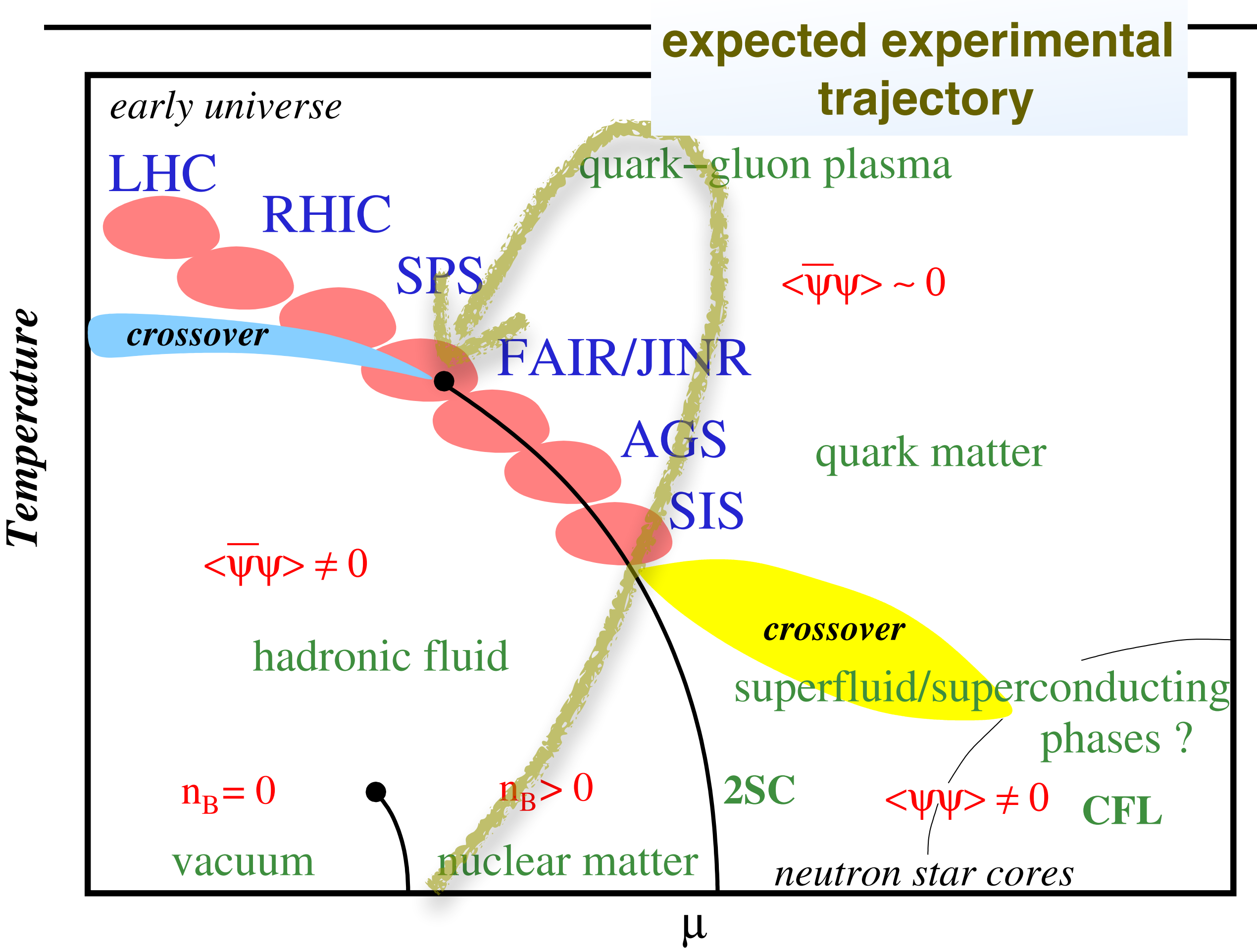


vacuum/nuclear matter/transition &

only corners of the phase diagram are known

from “first principles”

conjectured QC$_3$D phase structure



knowledge so far

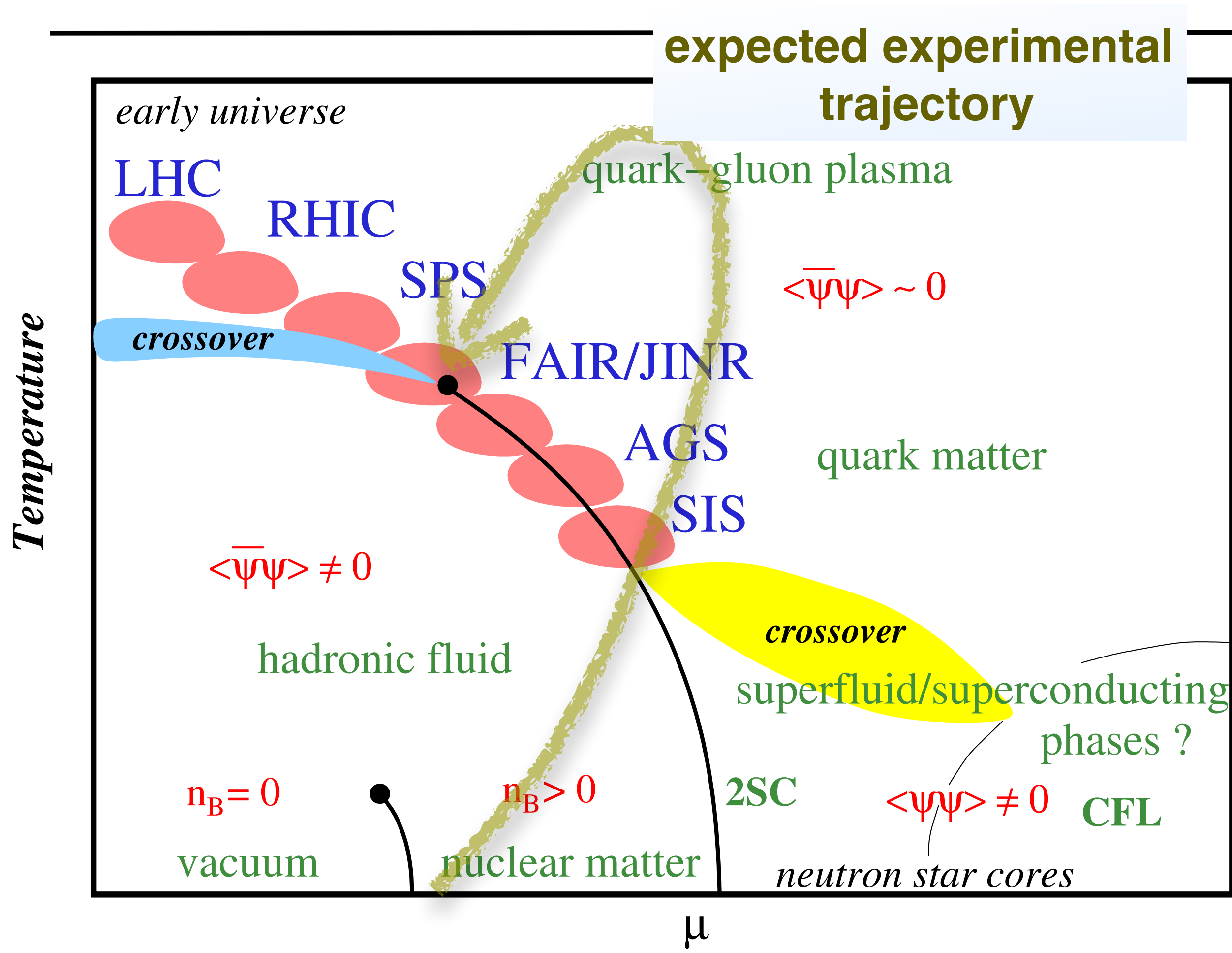
mostly based on model calculations

assumptions:

equilibrium, homogeneous phases,

infinite volume, ...

conjectured QC$_3$D phase structure



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Open/unclear questions:

- **CEP:** existence/location/number
- relation between chiral & deconfinement?

  chiral ⇔ deconfinement CEP?
- **Quarkyonic phase:** coincidence of both transitions at μ=0 & μ>0?
- **inhomogeneous phases?** ➜ more favored?
- **axial anomaly restoration** around chiral transition?
- **finite volume effects?** ➜ lattice comparison/ influence boundary conditions
- **role of fluctuations?** so far mostly Mean-Field results

  ➜ **effects of fluctuations important**

  examples: size of crit reg. around CEP
- **What are good experimental signatures?**

  ➜ cumulants?

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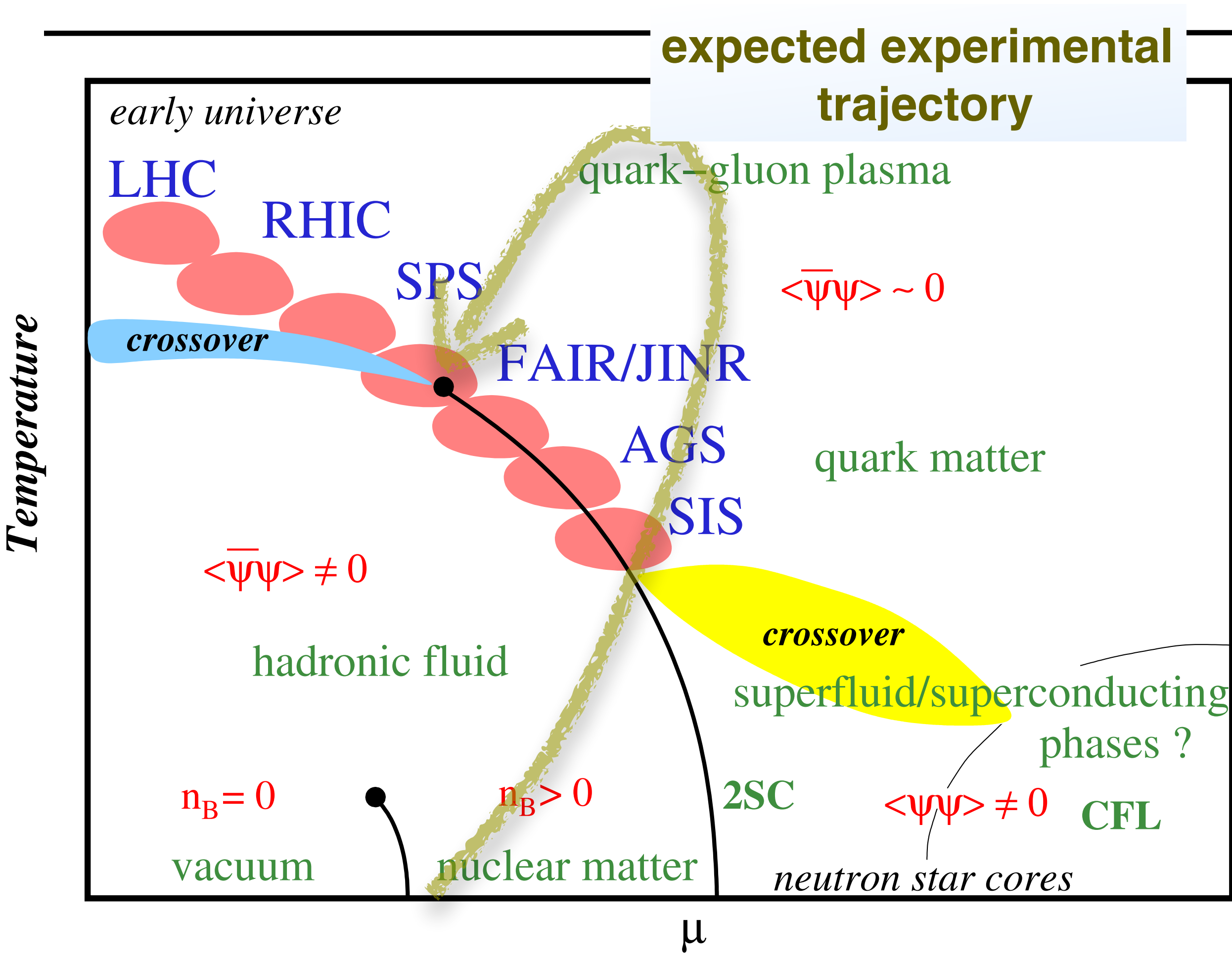
conjectured QC$_3$D phase structure



Theory:

➜ Lattice: but simulations restricted to small μ

➜ Models: effective theories parameter dependency

➜ **Functional QFT methods: FRG,DSE, nPI**

knowledge so far

mostly based on model calculations

assumptions:

equilibrium, homogeneous phases,

infinite volume, ...

Theoretical aim:

deeper understanding & more realistic HIC description

➜ existence of critical end point(s)?

Agenda

- **Role of (quantum and thermal) Fluctuations for QCD phases**

  from mean-field approximations (MFA)

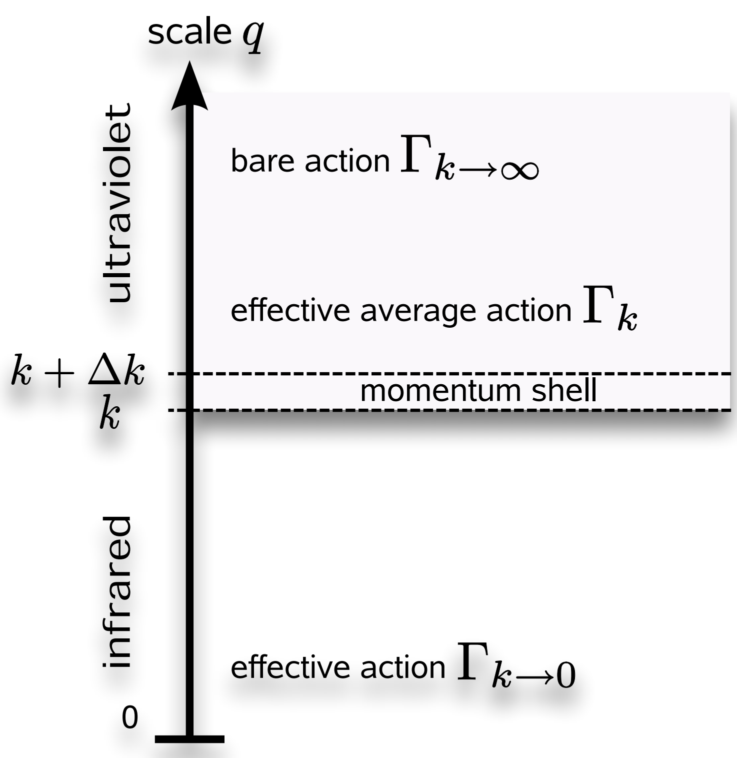
  to the Functional Renormalization Group (FRG)

- **Impact of truncations**

- **Columbia plot**

Functional Renormalization Group

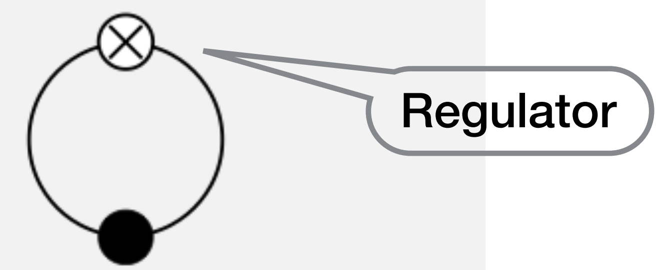
- $\Gamma_k[\phi]$ scale dependent effective action $\quad t = \ln(k/\Lambda) \quad R_k$ regulators $\quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$



FRG (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

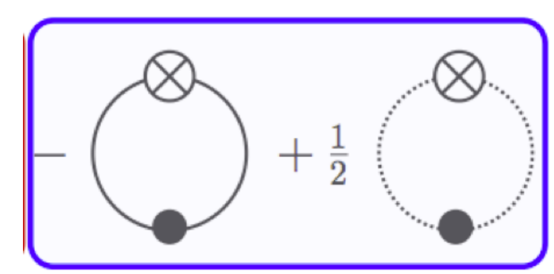
$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



[Wetterich 1993]

- Ansatz for Γ_k: **Example: Leading order derivative expansion** **arbitrary potential**

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V_k(\phi^2)$$

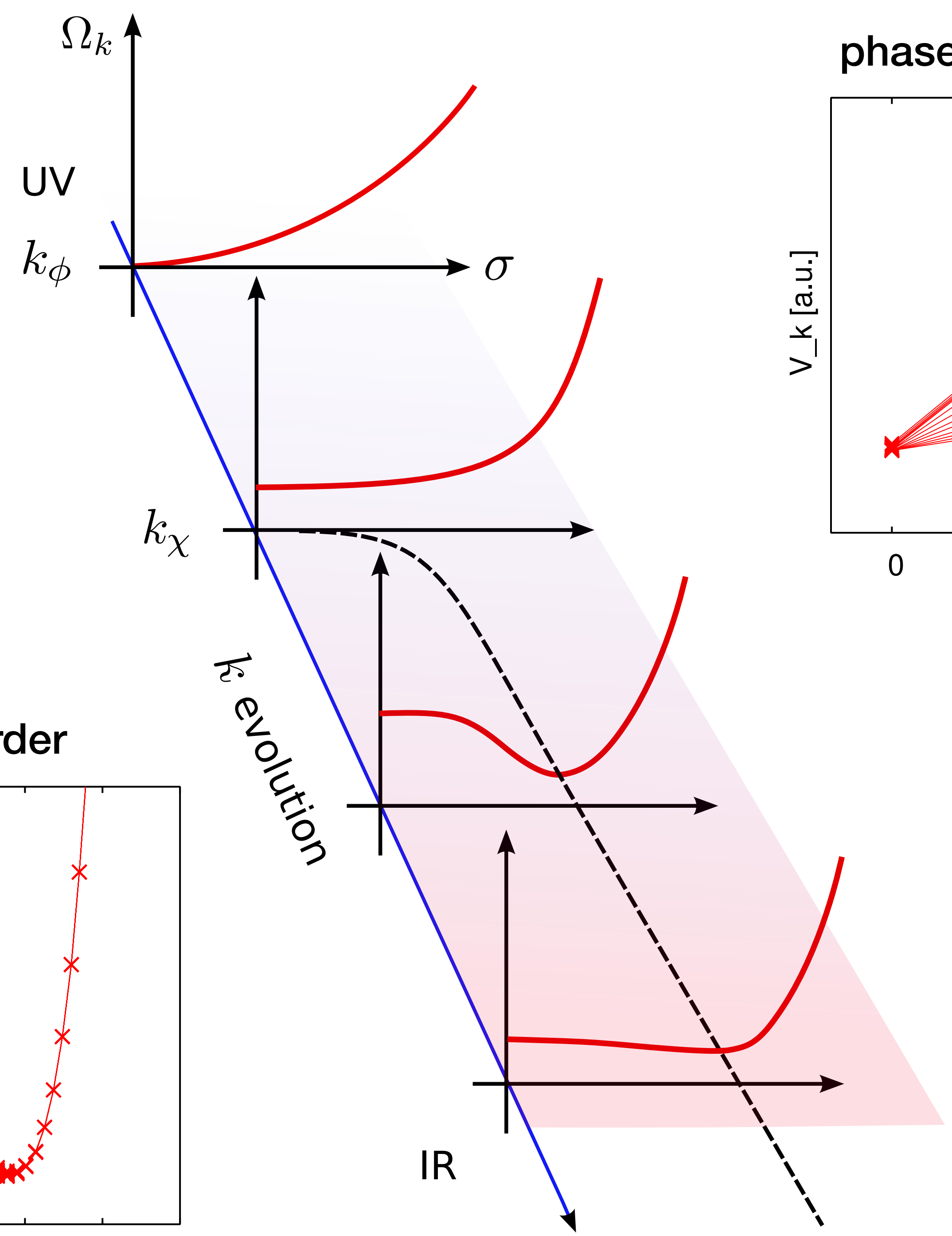


$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

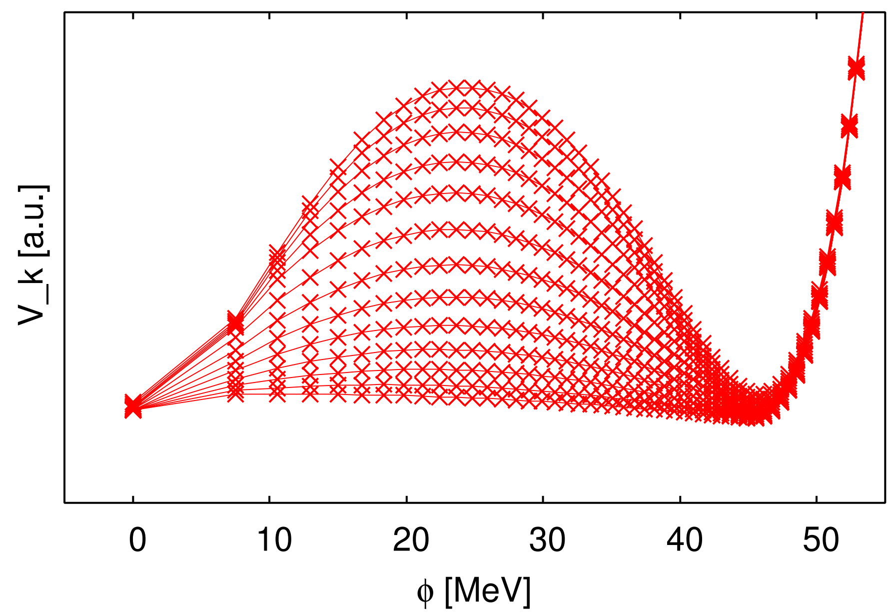
Example: grid technique

Scale evolution

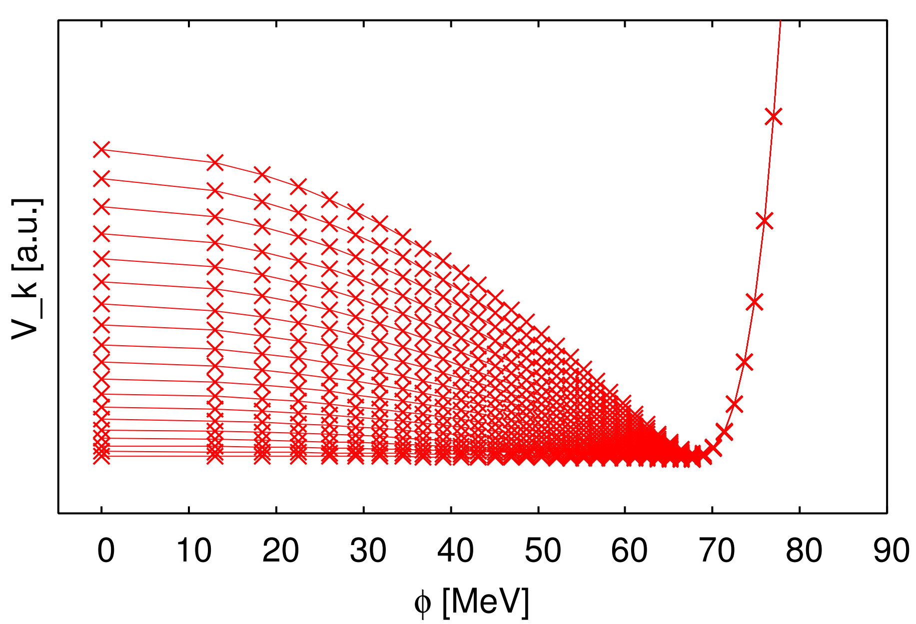
of meson potential:



phase transition: First order



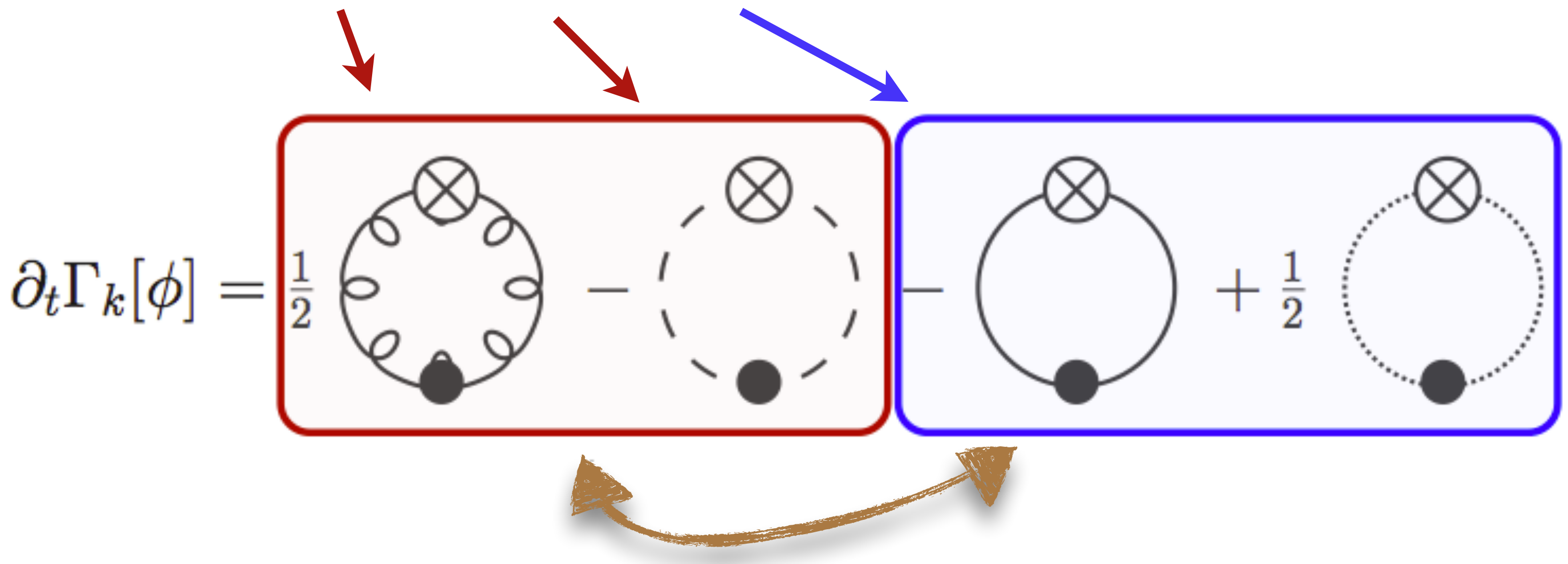
phase transition: Second Order



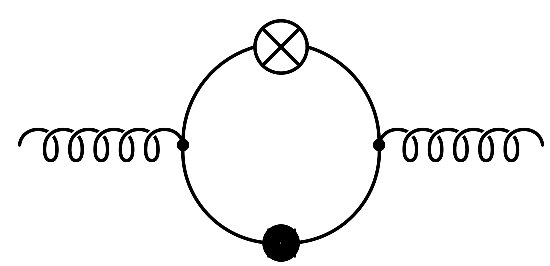
FRG and QCD

- **full dynamical QCD FRG flow:** [fQCD: Pawlowski, Rennecke, Mitter, et al. since 2009]

  fluctuations of **gluon**, **ghost**, **quark** and (via hadronization) **meson**



in presence of **dynamical quarks**:
gluon propagator is modified

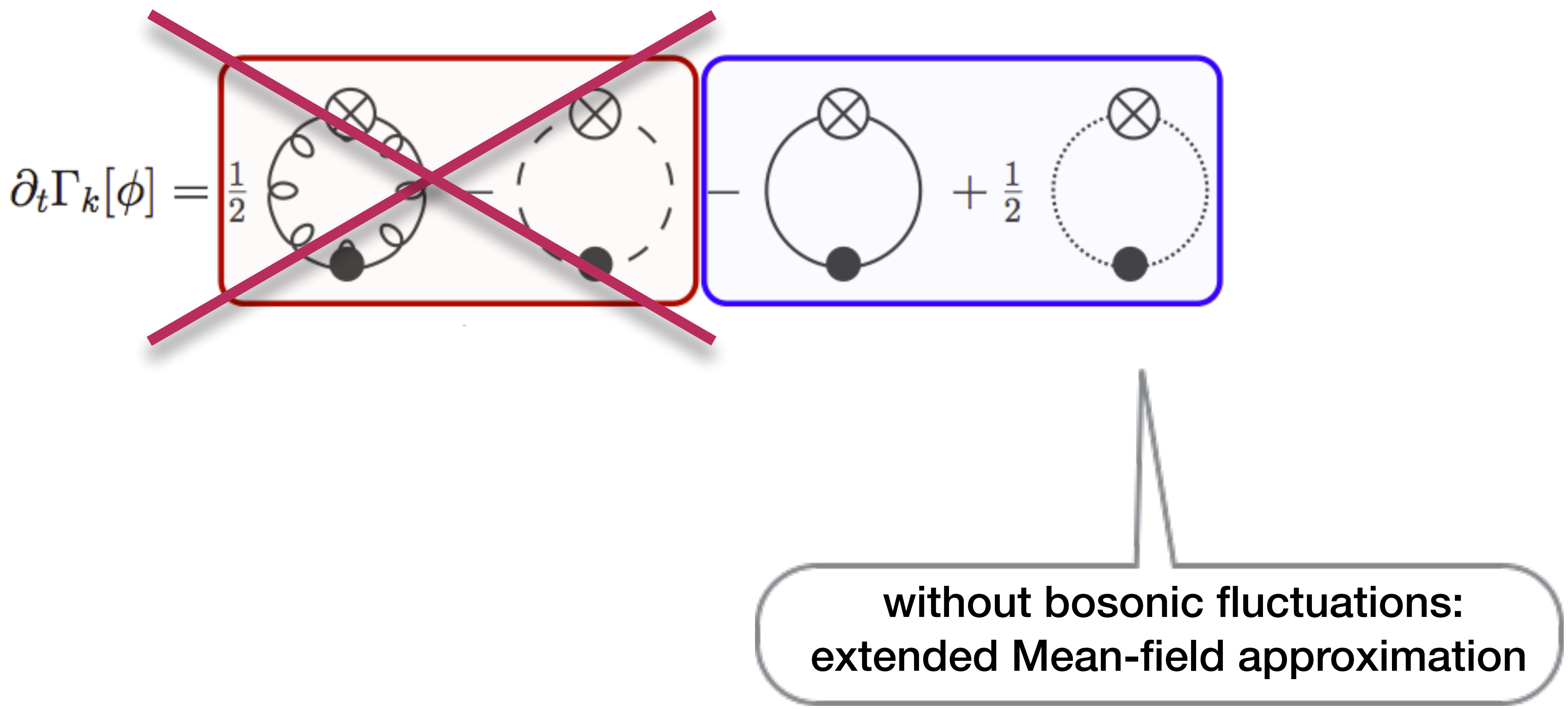


pure Yang Mills flow + matter back-coupling

FRG: quark-meson truncation

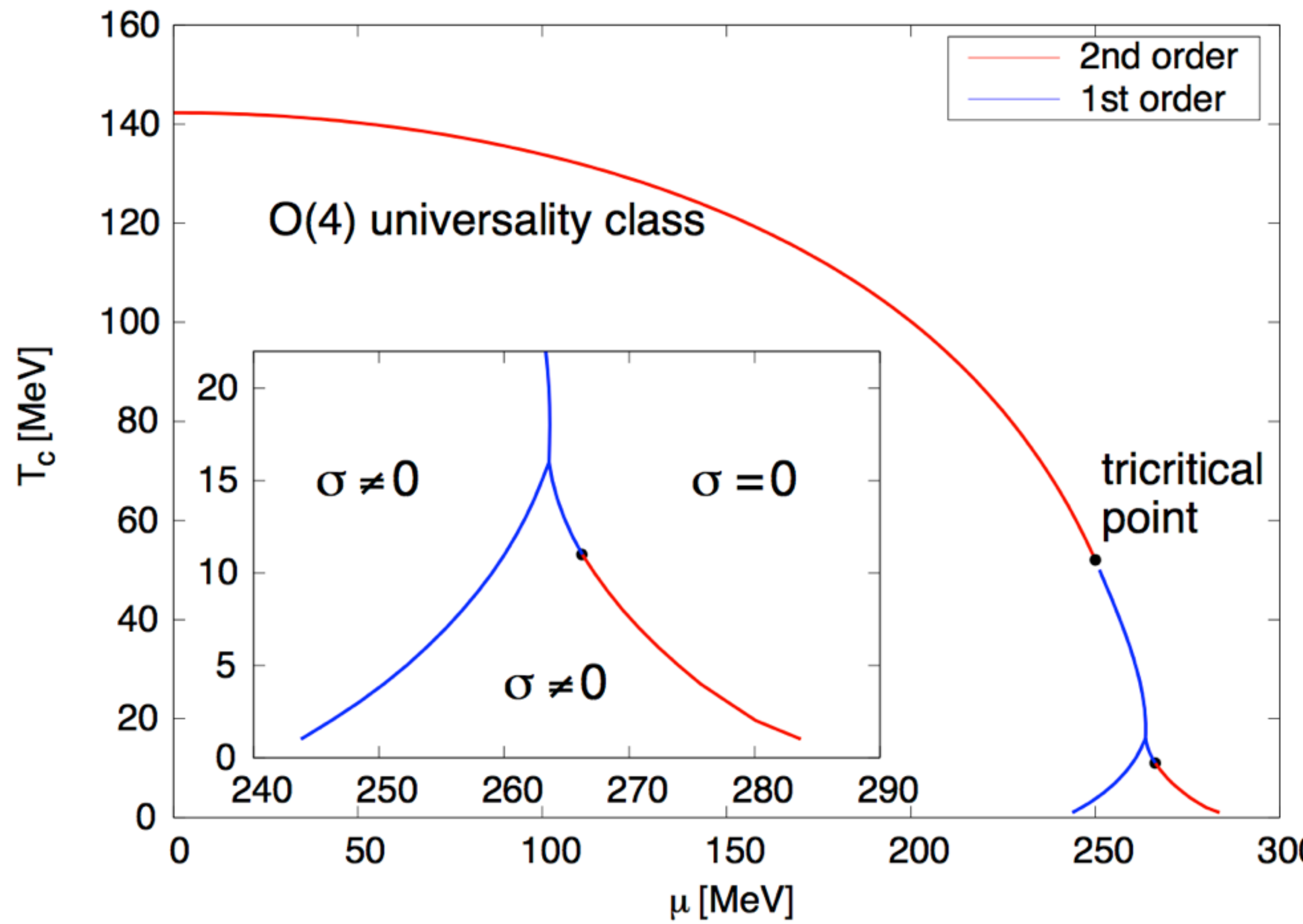
chiral phase transition:

flow for **quark-meson** model truncation:
neglect
YM contributions and bosonic fluctuations

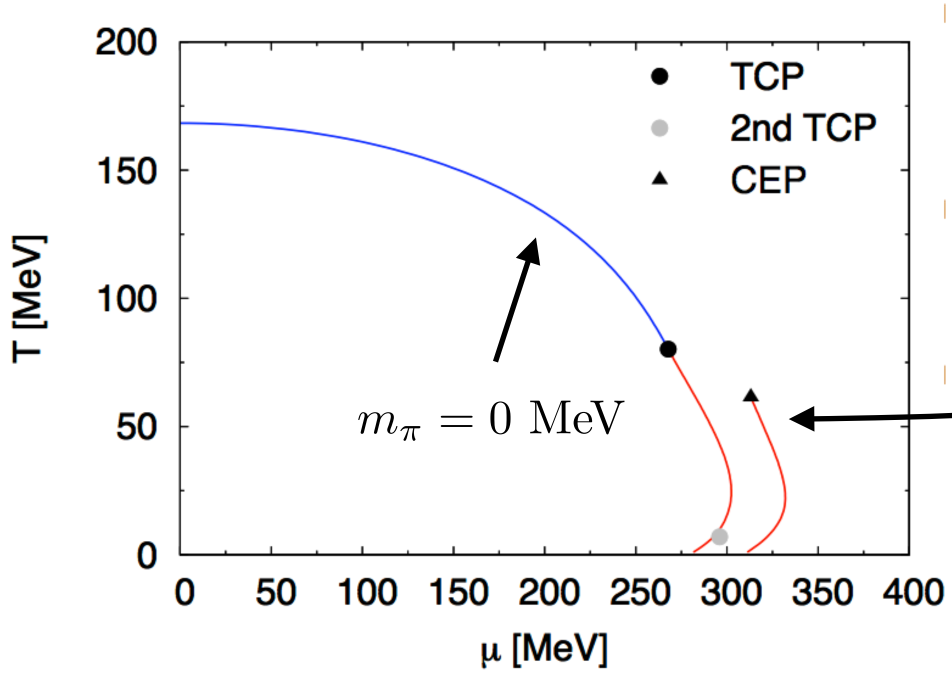


Phase diagram N_f=2

$O(4) \sim SU(2) \times SU(2)$ chiral limit no spinodal lines!



[BJS, Wambach 2005]

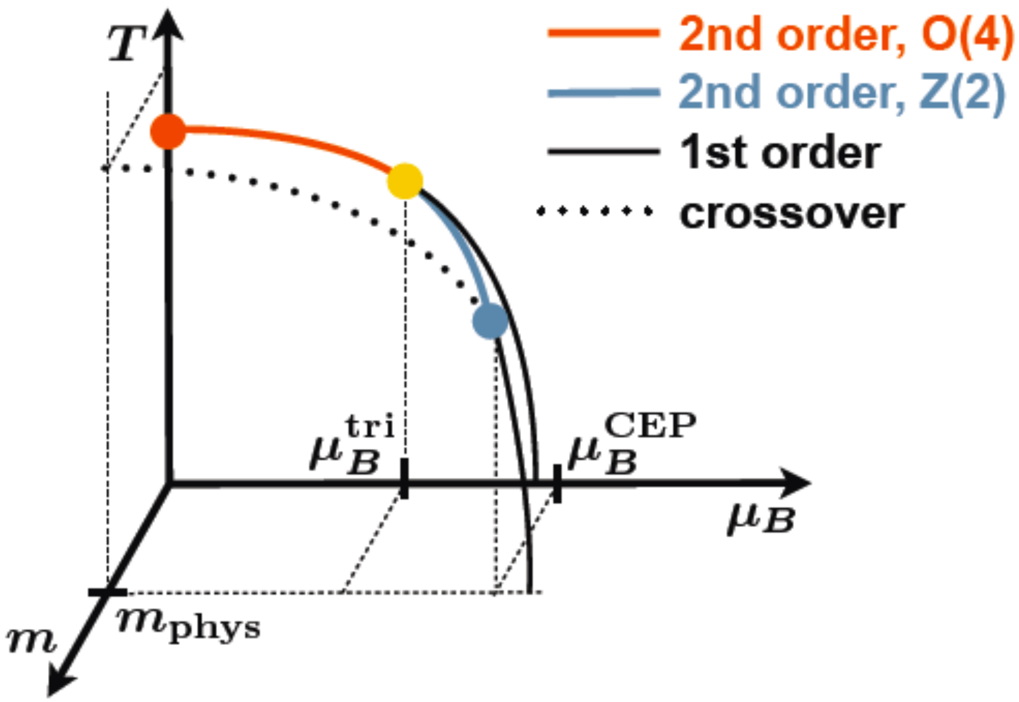


$m_\pi = 0$ MeV

$m_\pi = 138$ MeV

⬅ two phase diagrams

Phase diagram N_f=2



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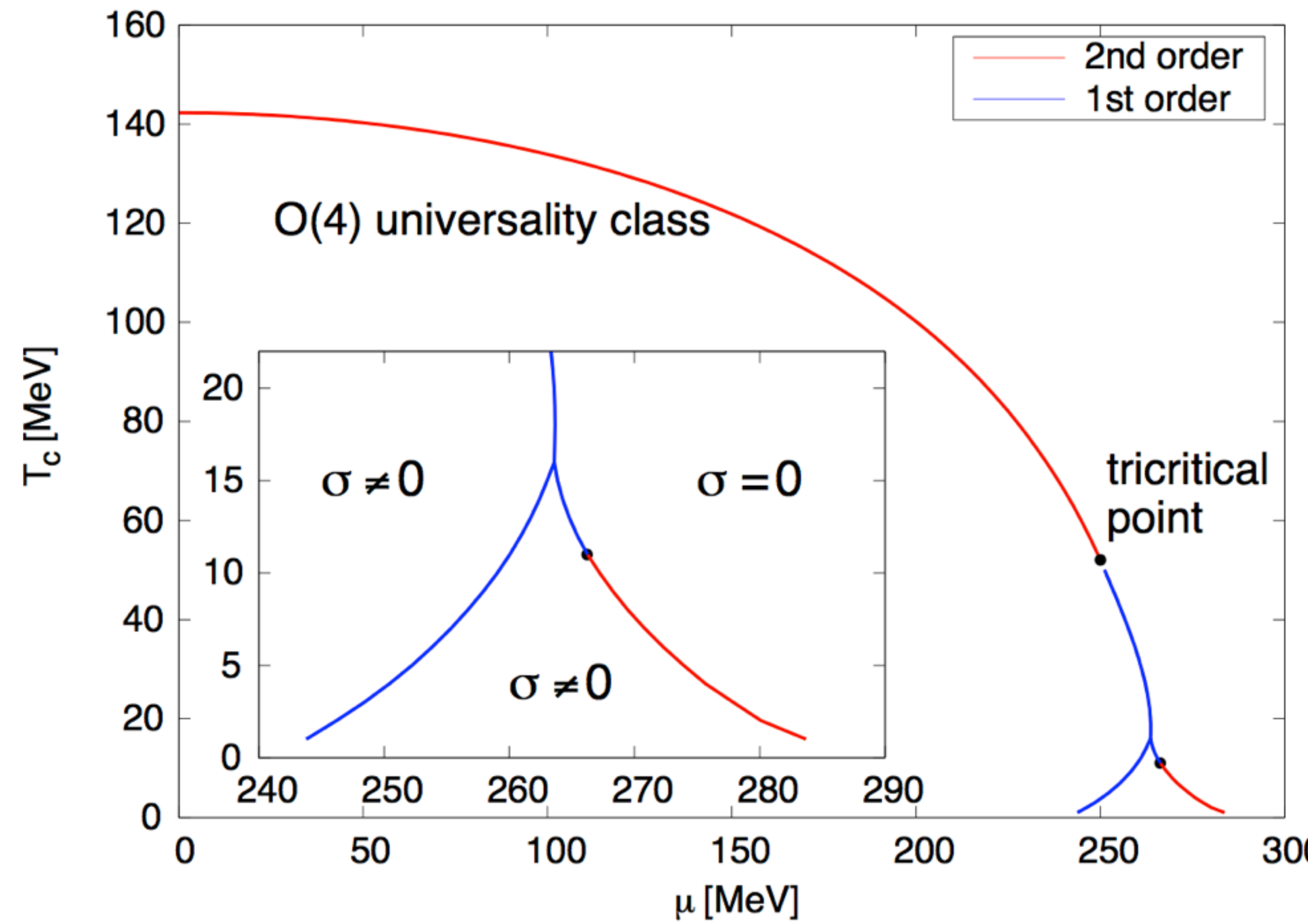

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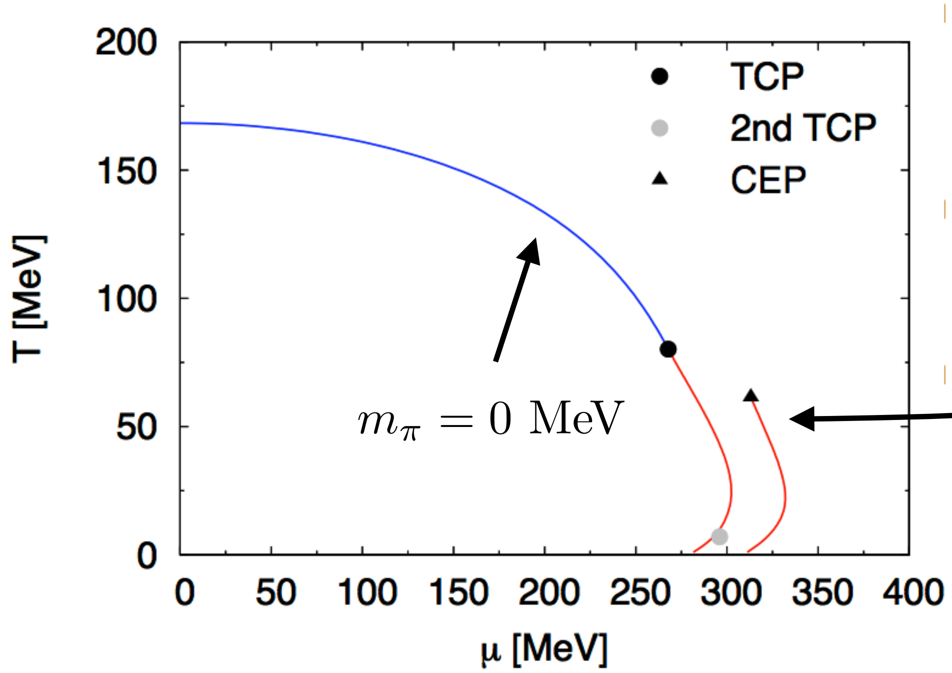

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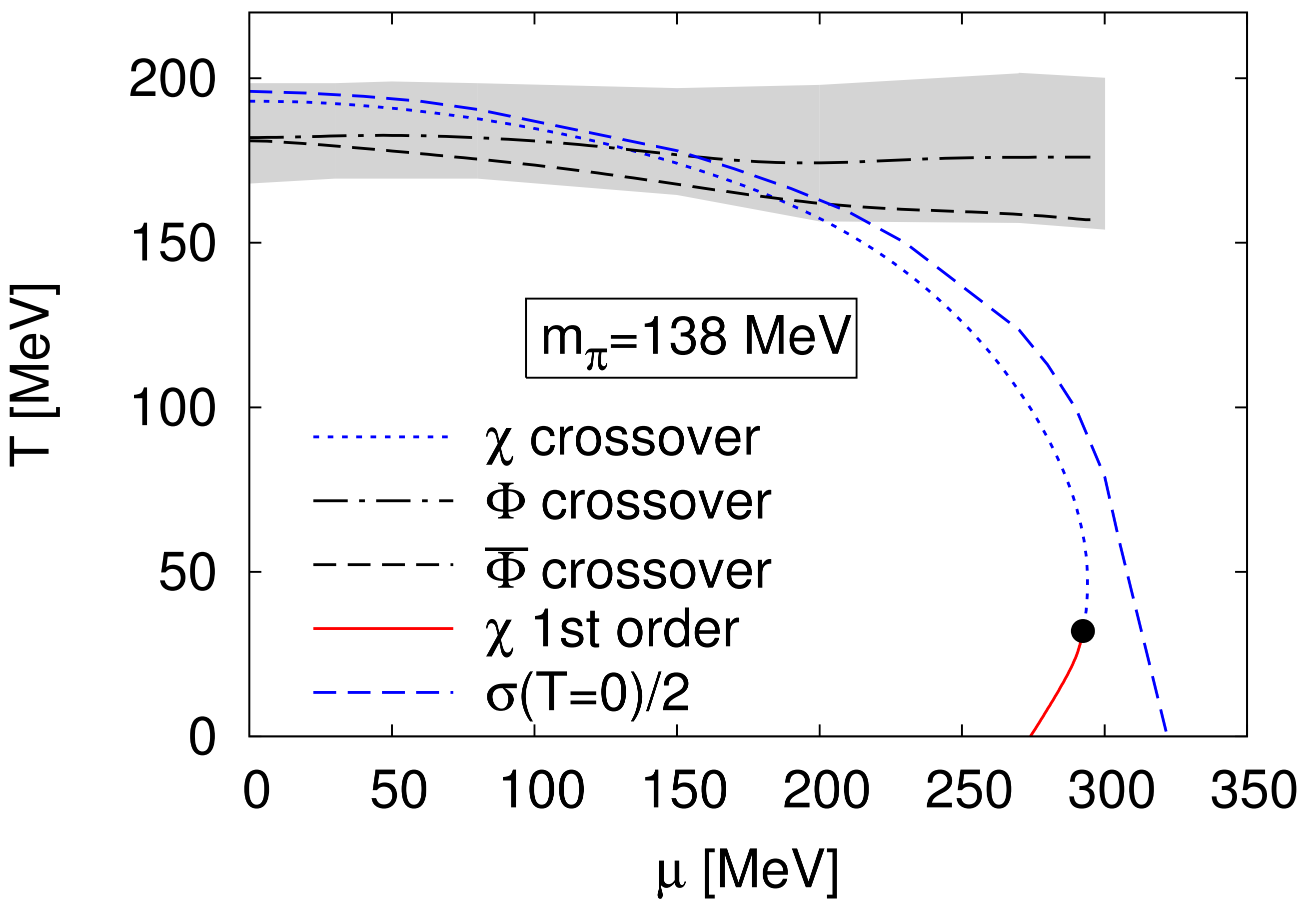
⬅ two phase diagrams

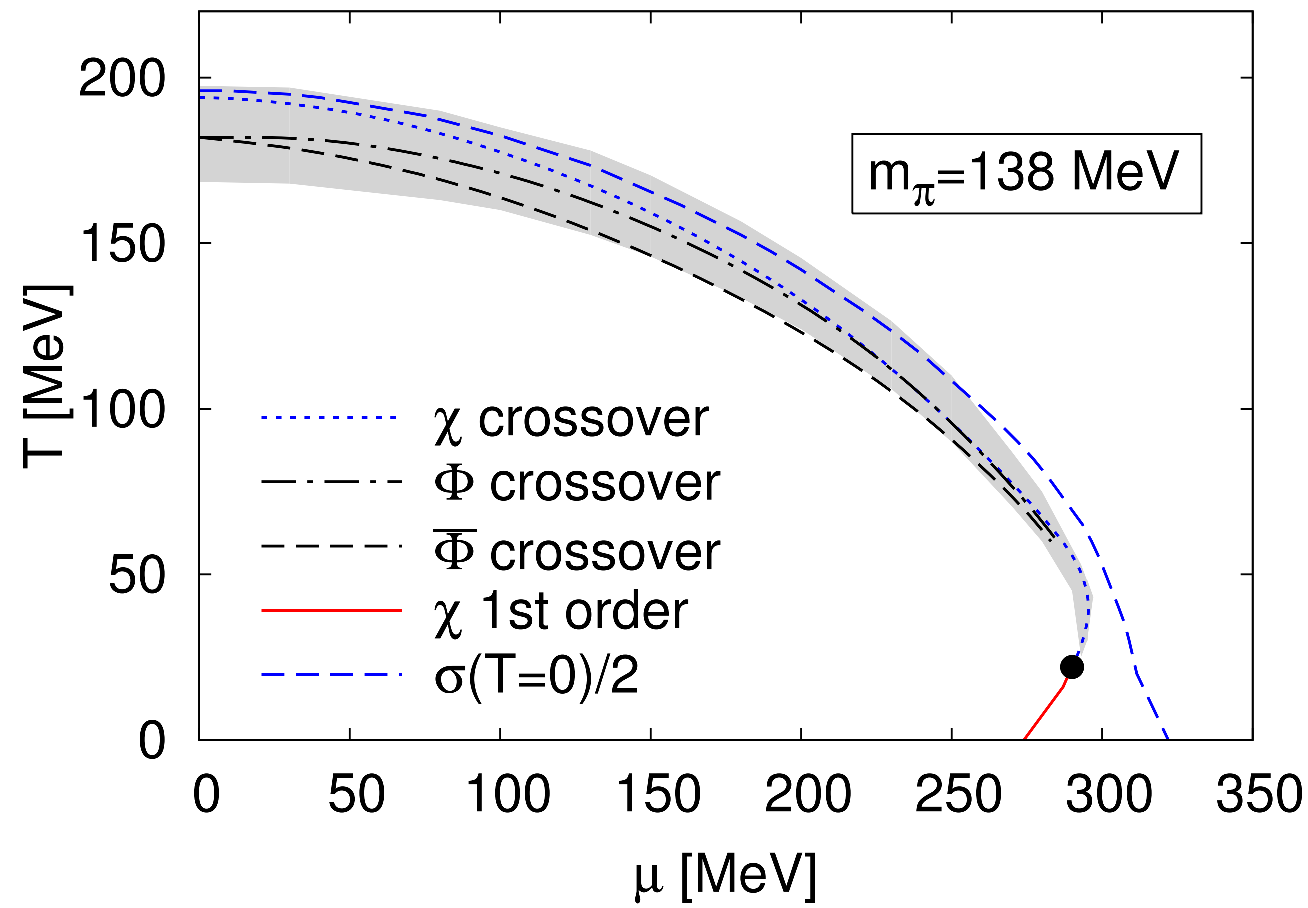
FRG: Quark-Meson with Polyakov

N_f =2 quark flavor

without back reaction $(T_0(\mu) = \text{const})$



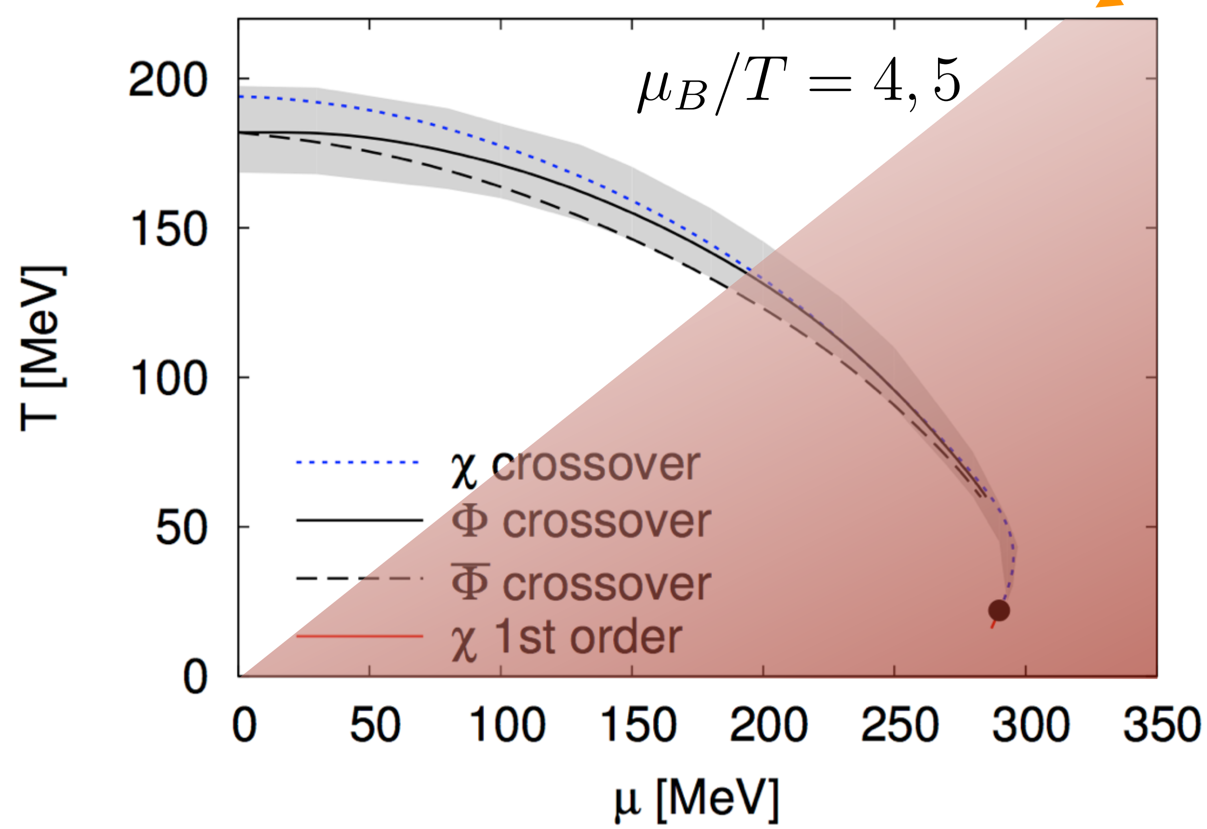
with back reaction $(T_0(\mu))$



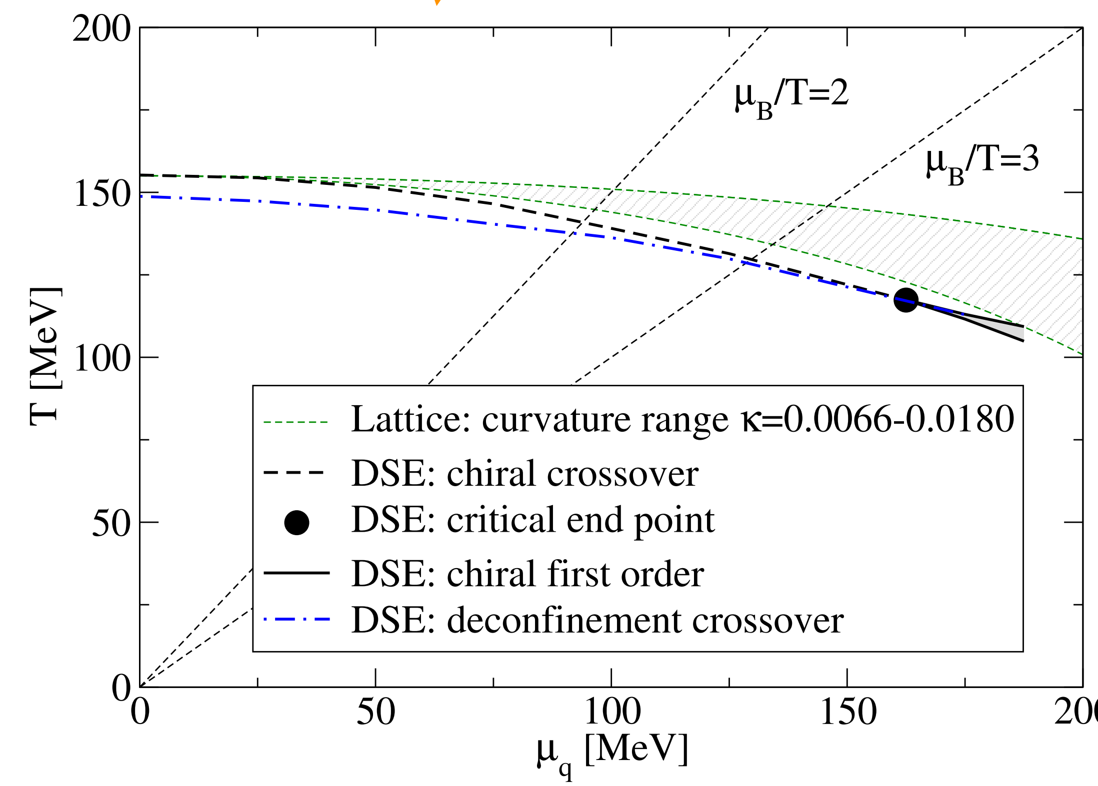
[Herbst, Pawlowski, BJS 2010,2013]

Critical Endpoint ?

Exact location of CEP **not** (**yet**) accessible with lattice, FRG & DSE



[Herbst, Pawlowski, BJS 2011]



[Fischer, Luecker, Welzbacher 2014]

so far:

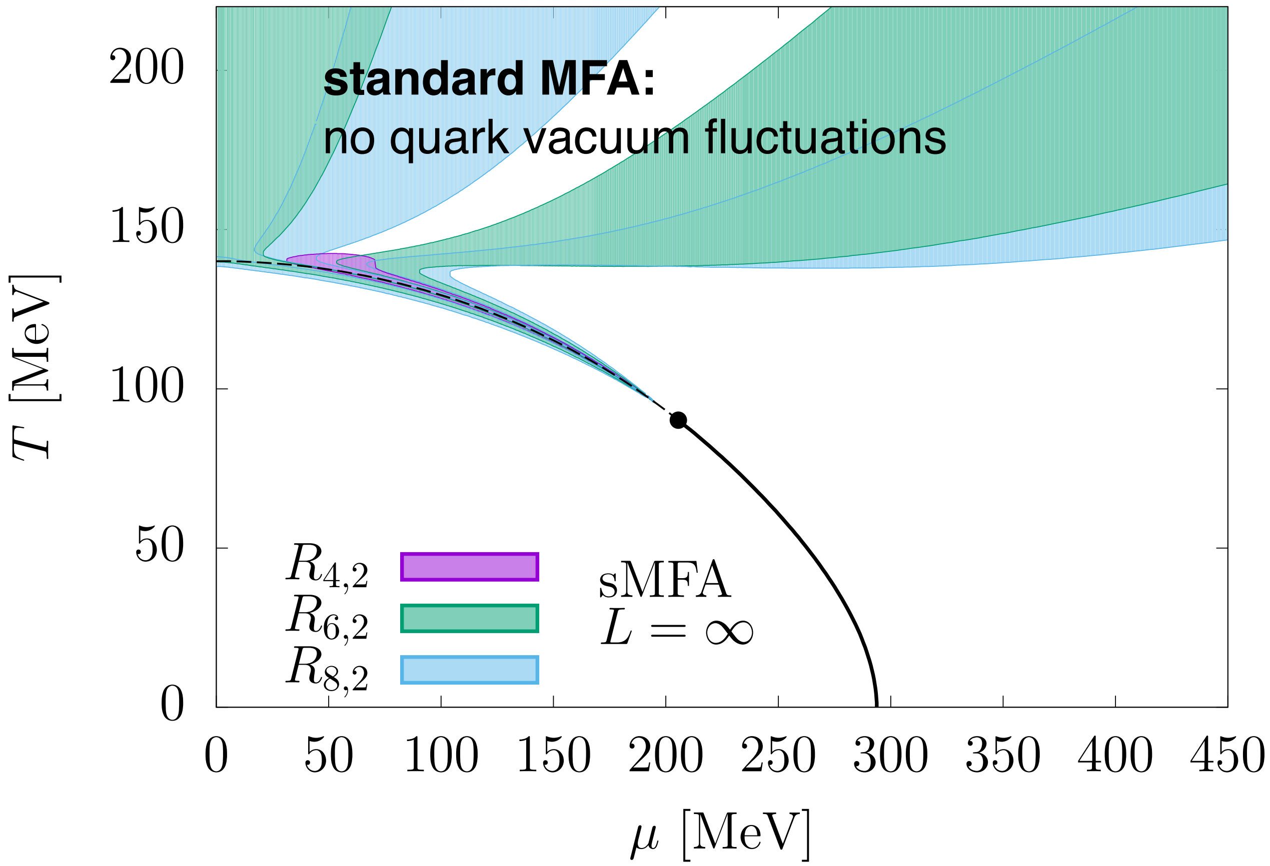
we can exclude CEP for small densities: $\mu_B/T < 2 \dots 3$

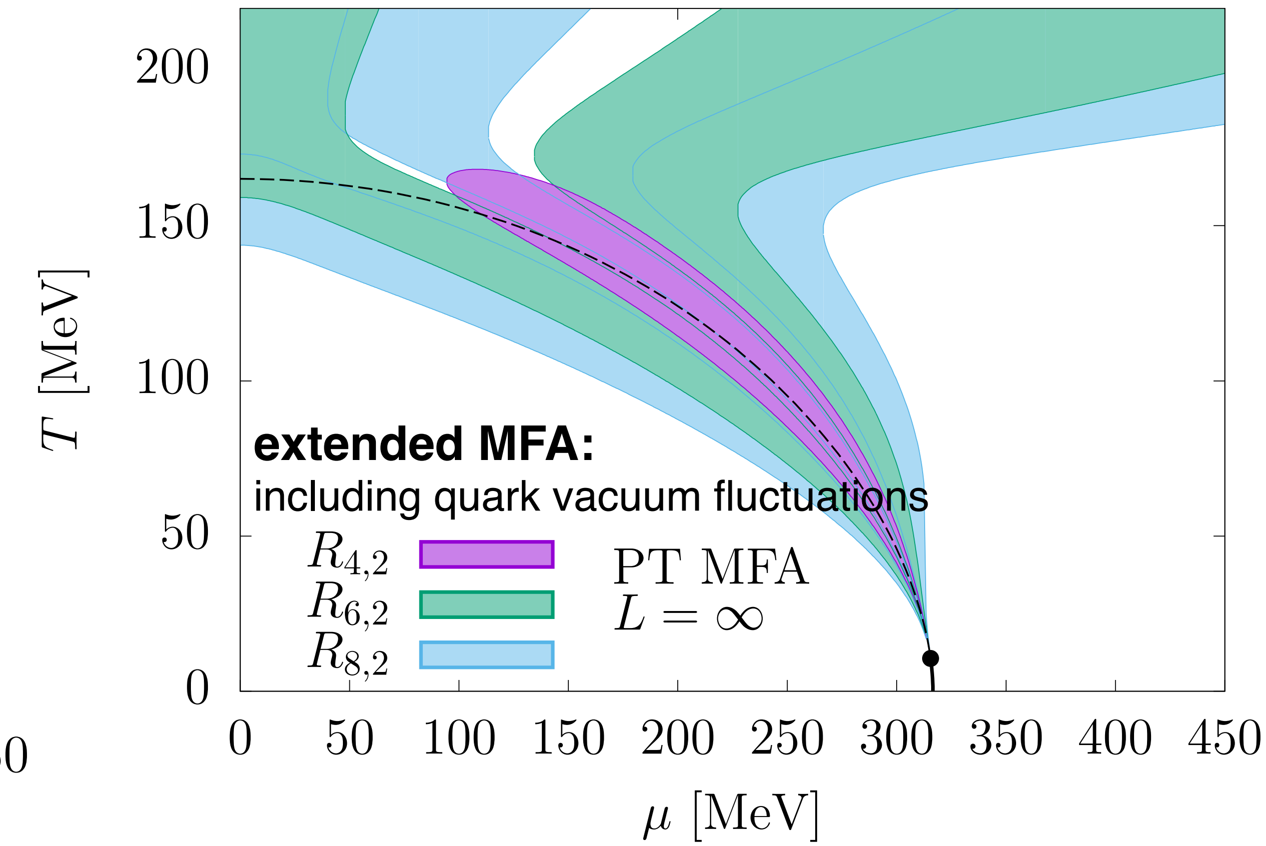
Higher densities: dynamical baryons needed!

Higher cumulants

[S. Resch, BJS to be published]

infinite volume: influence of fluctuations





findings:

(quark) fluctuations pushes CEP to smaller T and bigger µ

Fluctuations wash out phase transition ➜ broader negative regions

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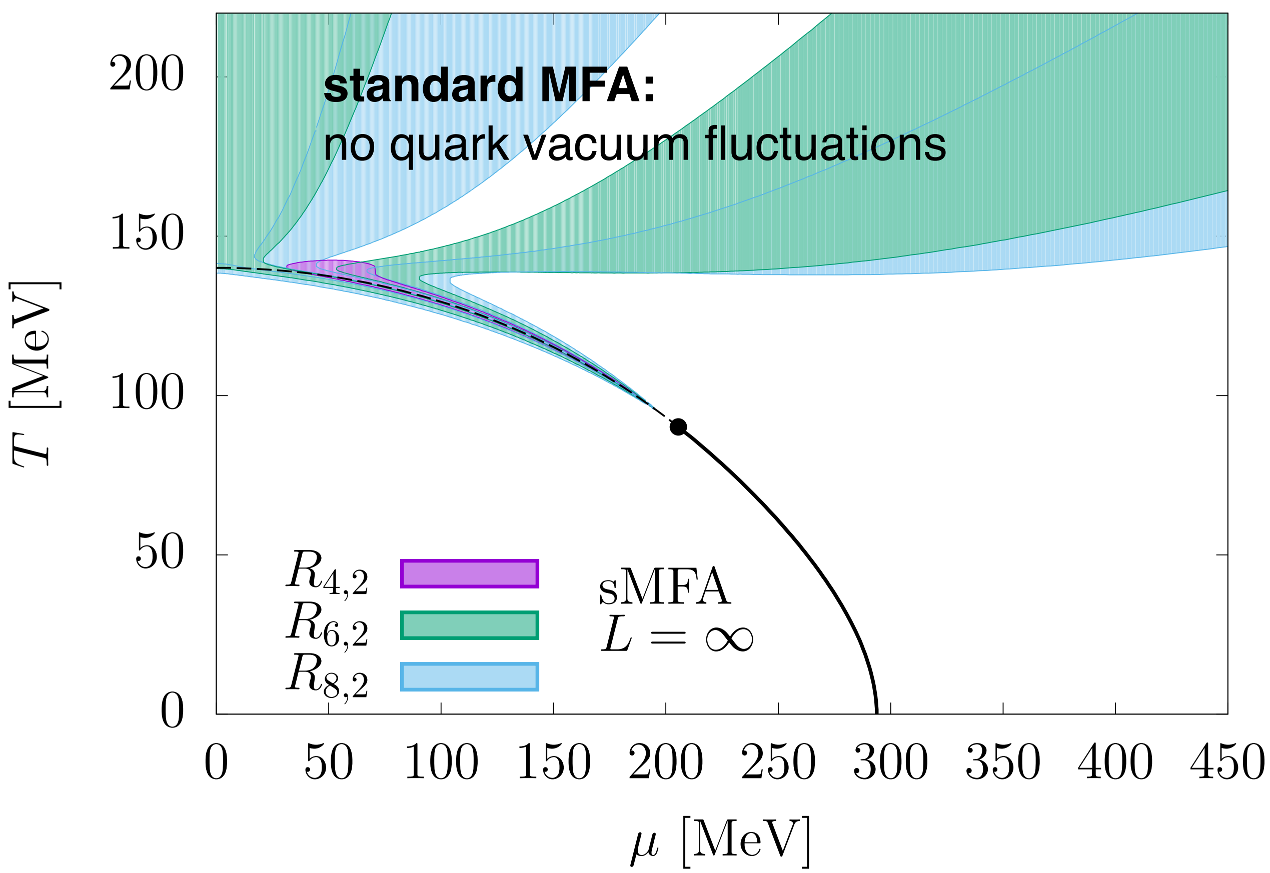

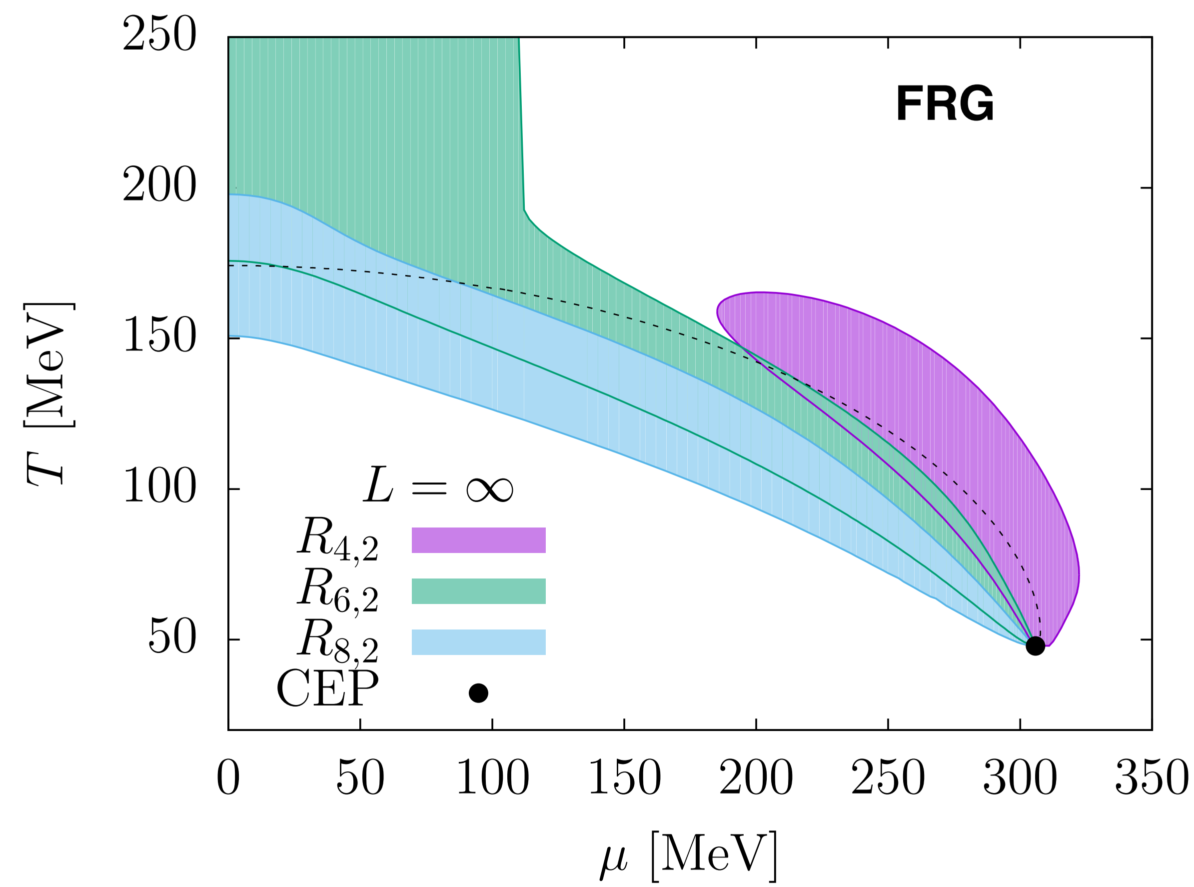

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- **Impact of truncations**

- Columbia plot

FRG: N_f = 2+1 quark flavor

[Rennecke, BJS 2018]

$$\Gamma_k = \int_x \Big\{ \bar{q}\, Z_{q,k} \big(\gamma_\mu \partial_\mu + \gamma_0 \mu\big) q + \bar{q} h_k \cdot \Sigma_5 q + \mathrm{tr}\big(Z_{\Sigma,k}\, \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger\big) + \tilde{U}_k(\Sigma) \Big\}$$

$$q^T = (l, l, s) \qquad \Sigma_5 = T_a(\sigma_a + i\gamma_5 \pi_a)$$

- effective potential $\tilde{U}_k = U_k(\rho_1, \tilde{\rho}_2) - j_l \sigma_l - j_s \sigma_s - c_k\, \xi$ $\qquad \xi = \det \Sigma + \det \Sigma^\dagger$

U(3) x U(3) sym. potential
(two chiral invariants)
$\rho_i = \mathrm{tr}(\Sigma \cdot \Sigma^\dagger)^i$

explicit chiral symmetry breaking:
finite light &
strange current quark masses

anomalous U(1)A breaking
via 't Hooft determinant

- wave function renormalizations $p^2 + m^2 \;\to\; Z_k\, p^2 + m^2$

$$Z_{q,k} = \begin{pmatrix} Z_{l,k} & 0 & 0 \\ 0 & Z_{l,k} & 0 \\ 0 & 0 & Z_{s,k} \end{pmatrix}$$

$$Z_{\Sigma,k} = \begin{pmatrix} Z_{\phi,k} & 0 & 0 \\ 0 & Z_{\phi,k} & 0 \\ 0 & 0 & Z_{\phi,k} \end{pmatrix}$$

- Yukawa couplings

$$h_k = \begin{pmatrix} h_{l,k} & h_{l,k} & h_{ls,k} \\ h_{l,k} & h_{l,k} & h_{ls,k} \\ h_{sl,k} & h_{sl,k} & h_{s,k} \end{pmatrix}$$

Different FRG truncations

[Rennecke, BJS 2018]

- different truncations:

$$\Gamma_k = \int_x \Big\{ \bar{q}\, Z_{q,k} \big(\gamma_\mu \partial_\mu + \gamma_0 \mu\big) q + \bar{q} h_k \cdot \Sigma_5 q + \mathrm{tr}\big(Z_{\Sigma,k}\, \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger\big) + \tilde{U}_k(\Sigma) \Big\}$$

truncation	running couplings
LPA′+Y	$\bar{\tilde{U}}_k,\ \bar{h}_{l,k},\ \bar{h}_{s,k},\ Z_{l,k},\ Z_{s,k},\ Z_{\phi,k}$
LPA+Y	$\bar{\tilde{U}}_k,\ \bar{h}_{l,k},\ \bar{h}_{s,k}$
LPA	$\bar{\tilde{U}}_k$

LPA = local potential approximation = leading order derivative expansion

Y = Yukawa coupling running

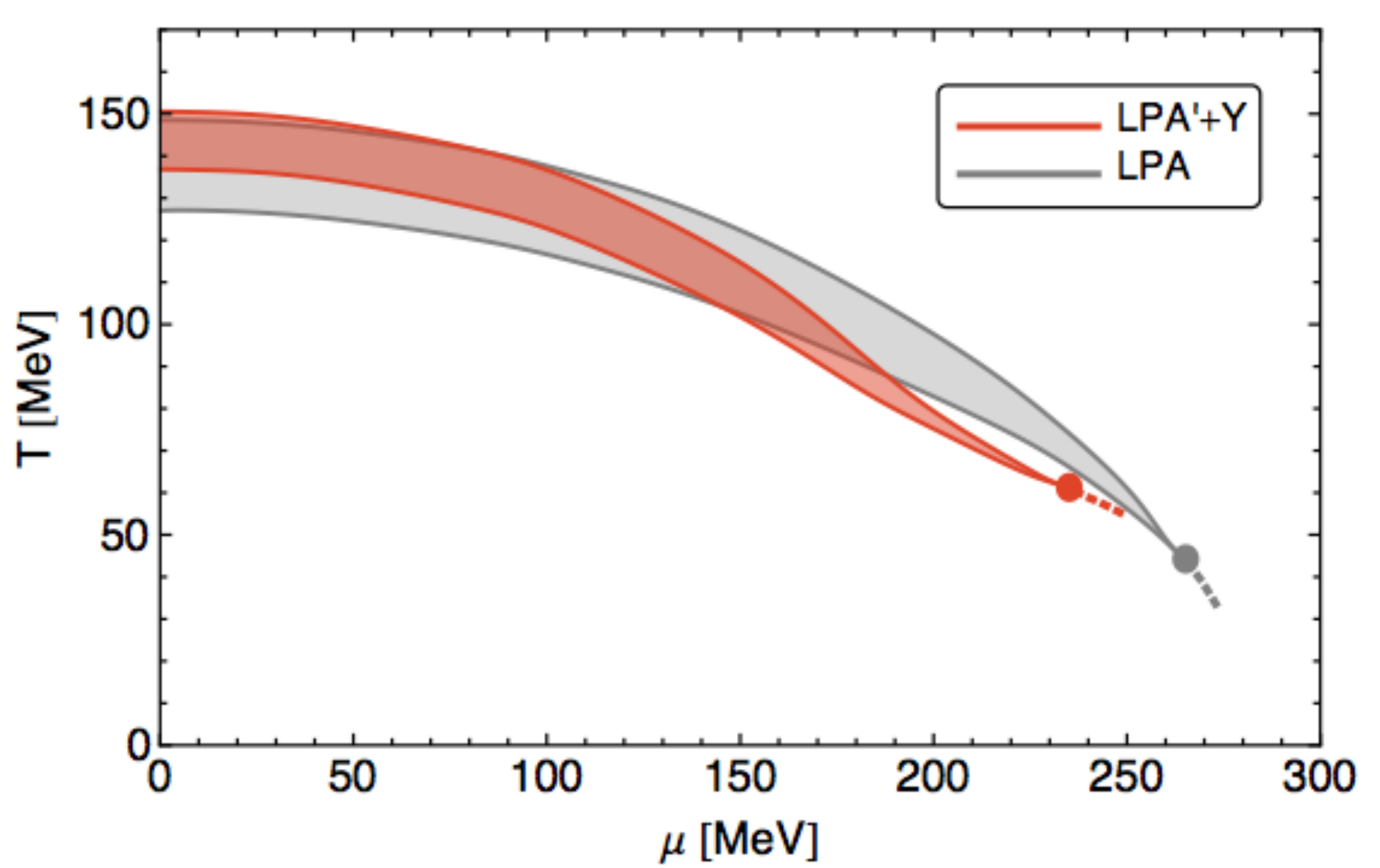
LPA’ = beyond local potential approximation include wave function renormalization

Chiral Phase Diagram

[Rennecke, BJS 2018]

- Critical Endpoint for different truncations:

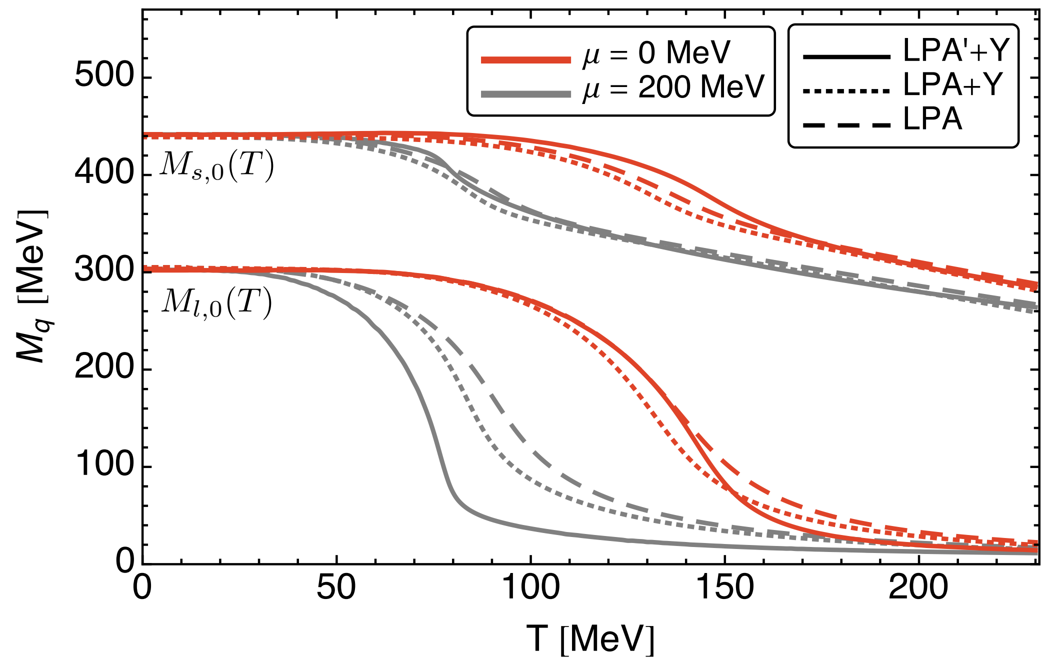
truncation	running couplings	$(T_{\rm CEP}, \mu_{\rm CEP})$ [MeV]
LPA′+Y	$\tilde{\bar{U}}_k,\ \bar{h}_{l,k},\ \bar{h}_{s,k},\ Z_{l,k},\ Z_{s,k},\ Z_{\phi,k}$	(61,235)
LPA+Y	$\tilde{\bar{U}}_k,\ \bar{h}_{l,k},\ \bar{h}_{s,k}$	(46,255)
LPA	$\tilde{\bar{U}}_k$	(44,265)



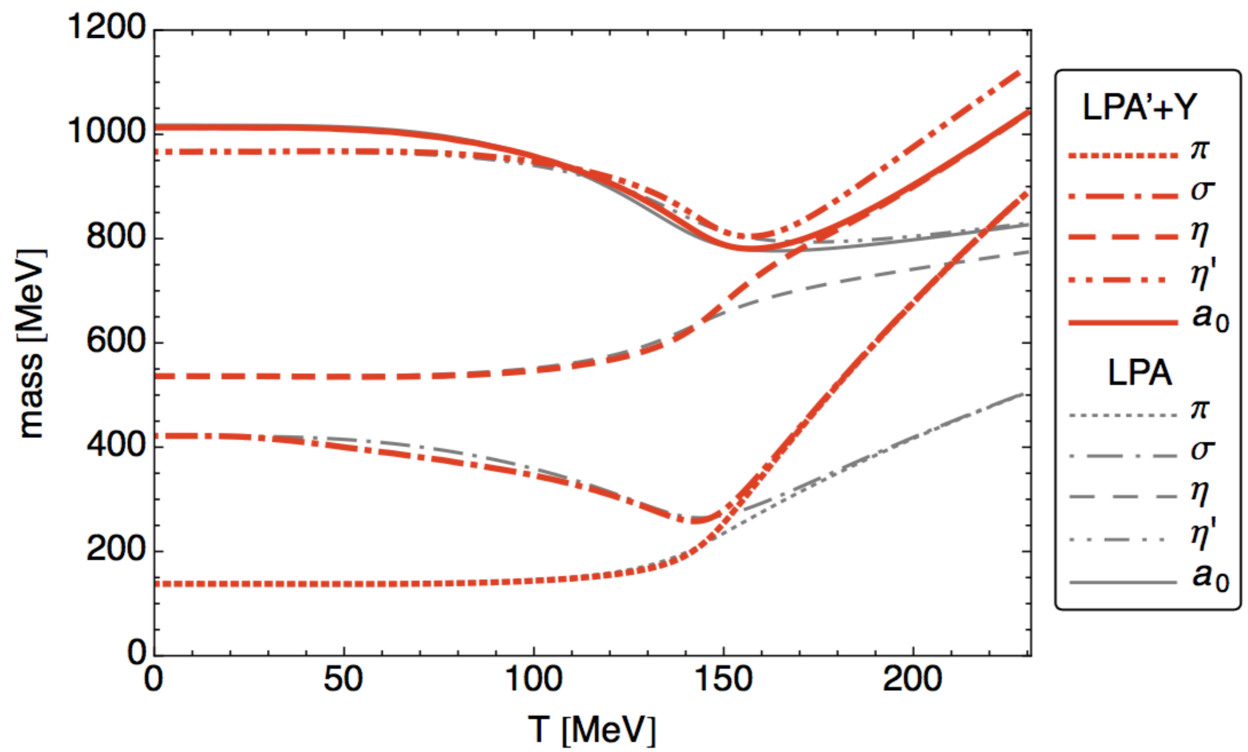
Masses

[Rennecke, BJS 2018]

- quark masses



- meson masses



driven by the mesonic wave-function renormalization

mesons decouple more rapidly beyond LPA

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Masses

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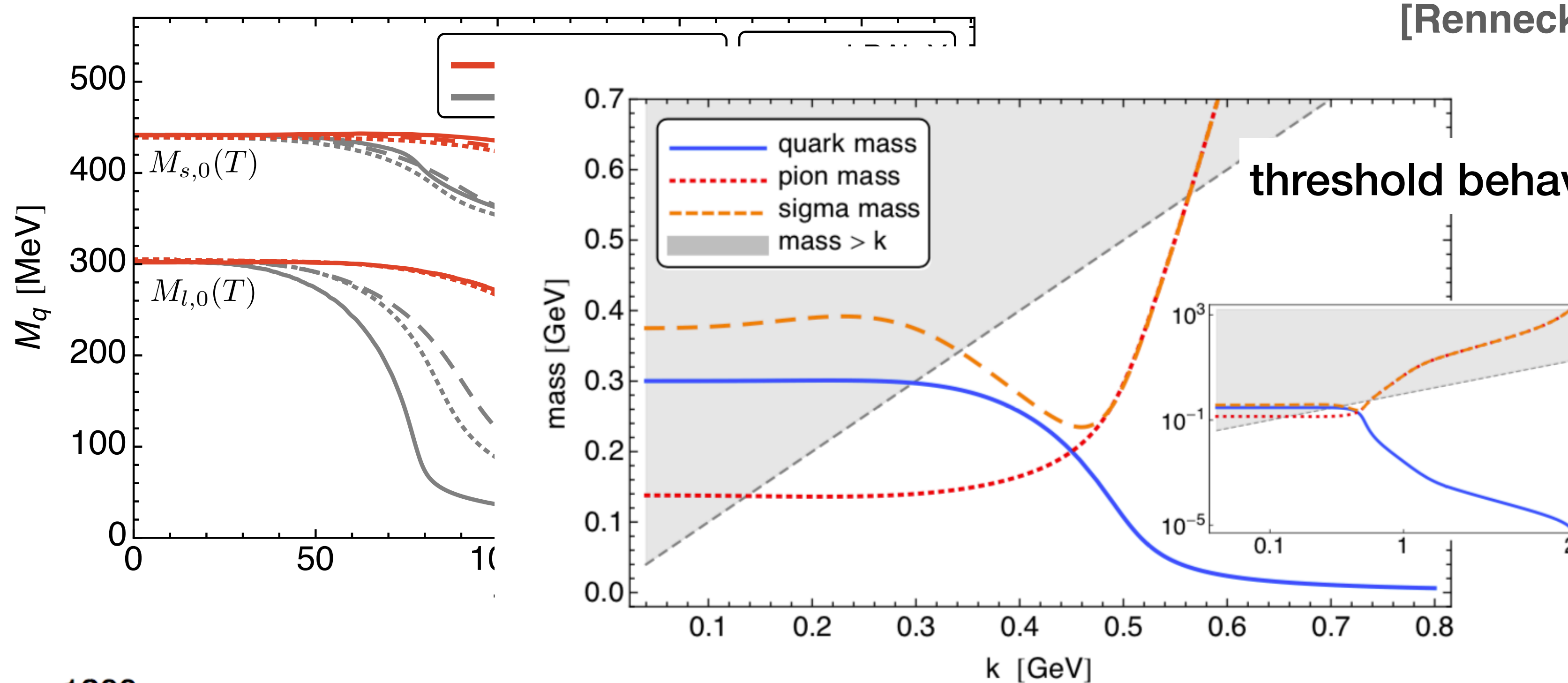
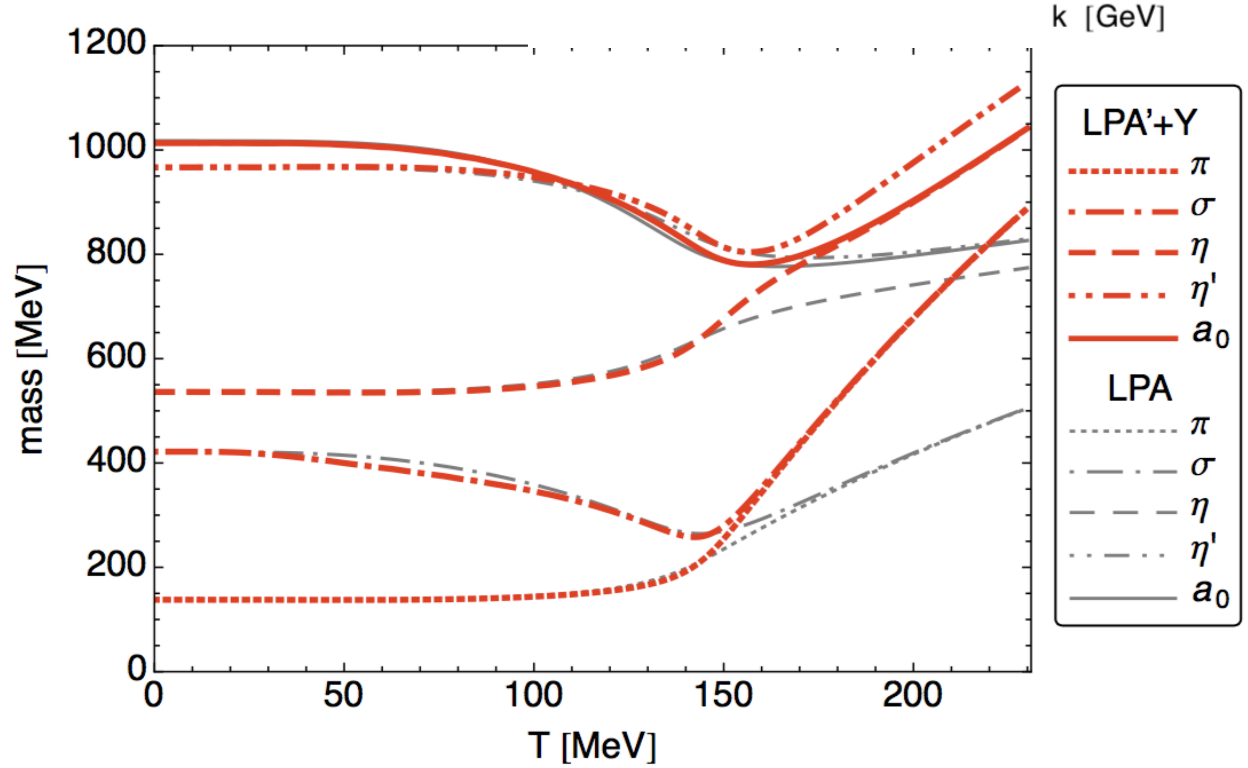


threshold behavior

- meson masses



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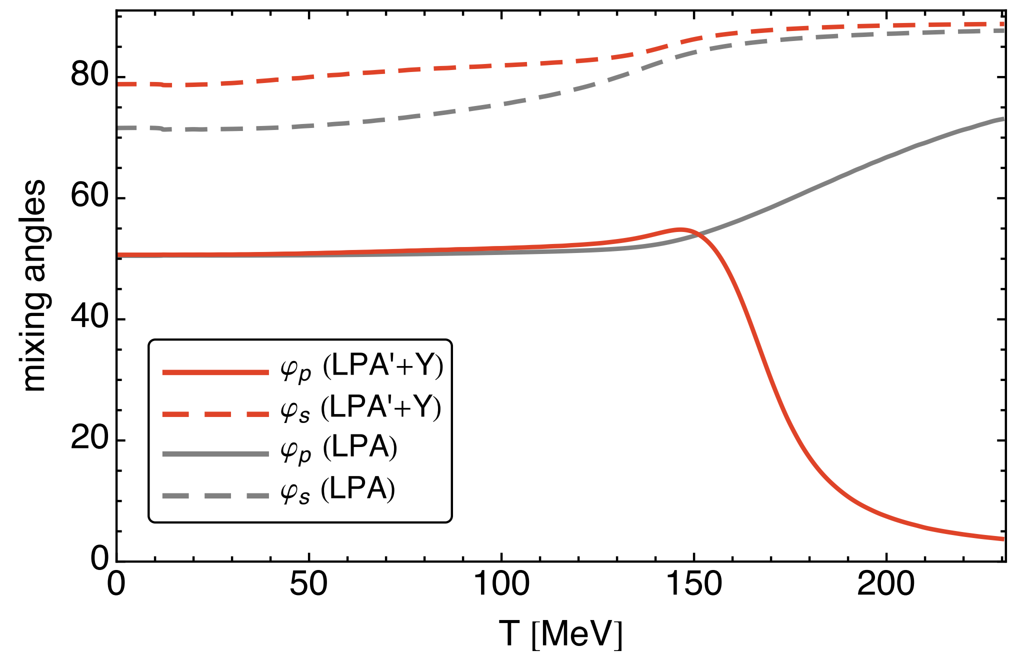
Mixing angles

[Rennecke, BJS 2018]

- mixing angles determine light and strange quark content of σ, f0, η, η’ mesons



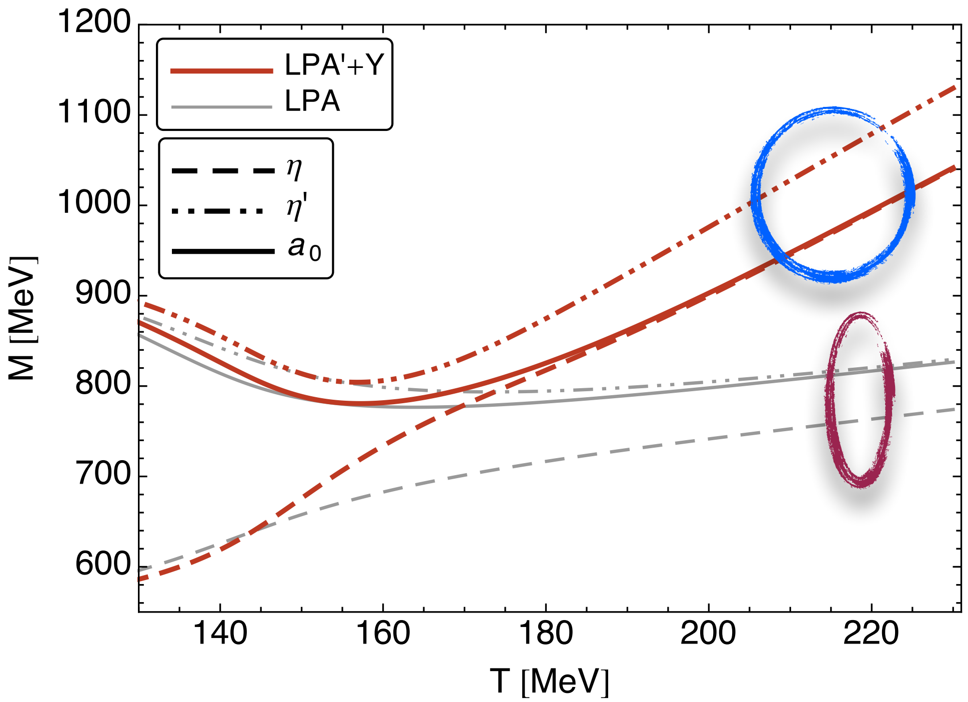
$$\begin{pmatrix} f_0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \cos\varphi_s & -\sin\varphi_s \\ \sin\varphi_s & \cos\varphi_s \end{pmatrix} \begin{pmatrix} \sigma_l \\ \sigma_s \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\varphi_p & -\sin\varphi_p \\ \sin\varphi_p & \cos\varphi_p \end{pmatrix} \begin{pmatrix} \eta_l \\ \eta_s \end{pmatrix}$$

significant effects on pseudoscalar mixing beyond LPA!

consequence: chiral partners of η and η’ change!

mean-field/LPA	LPA′ + Y
(η, f_0)	(η, a_0)
(η', a_0)	(η', f_0)



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Quark mass sensitivity: Columbia plot

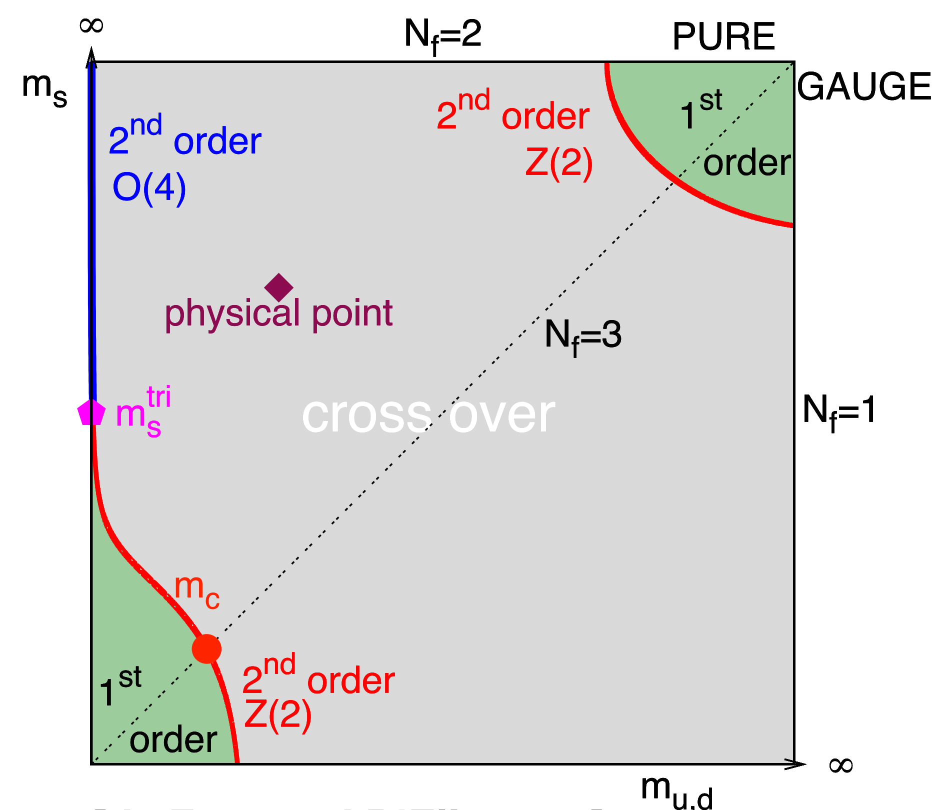
For physical quark masses: smooth phase transitions ➜ deconfinement: analytic change of d.o.f.

➜ associated global QCD symmetries only exact in two mass limits

1.infinite quark masses (center symmetry) Order parameter: VEV of traced Polyakov loop(s)

2. massless quarks (chiral symmetry) Order parameter: chiral condensate(s)

for finite quark masses:
both symmetries
explicitly broken



[de Forcrand D'Elia 2017]

Role of axial anomaly?

For physical quark masses: smooth phase transitions ➜ deconfinement: analytic change of d.o.f.

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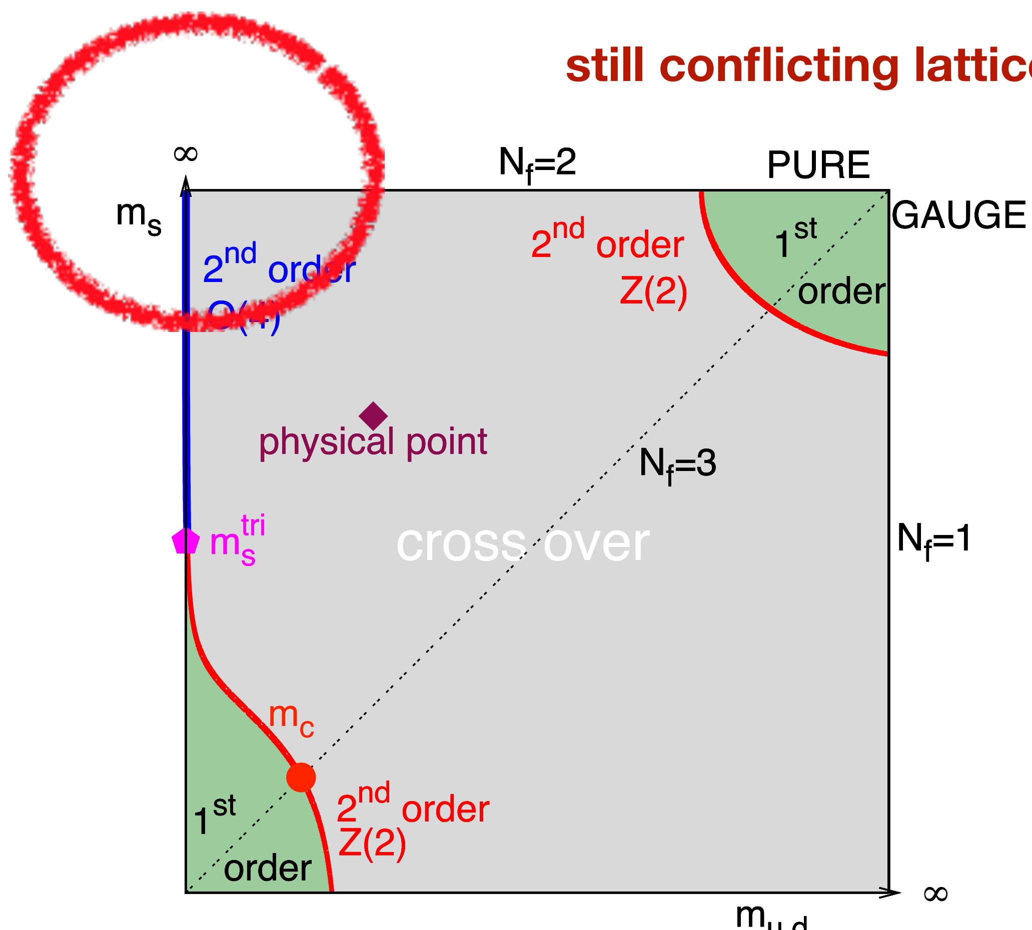
still conflicting lattice results!



[de Forcrand D'Elia 2017]

open issue: Nf=2: O(4)? $U(2)_L \times U(2)_R / U(2)_V$?

➜ similar crit. exponents

or even 1st order?

➜ dep. on strength of axial anomaly!

- **example:**

if $U_A(1)$ broken @T_c ➜ O(4) universality

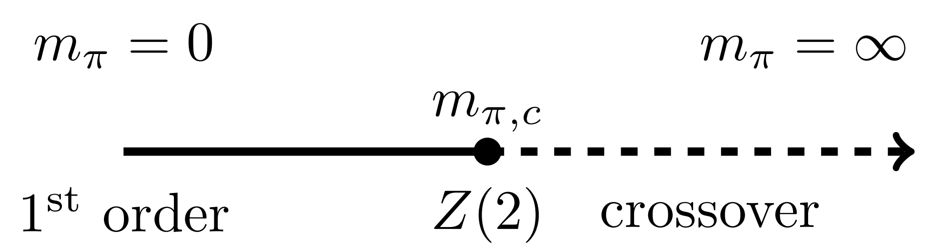
➜ there exist tricritical m^{tri}_s

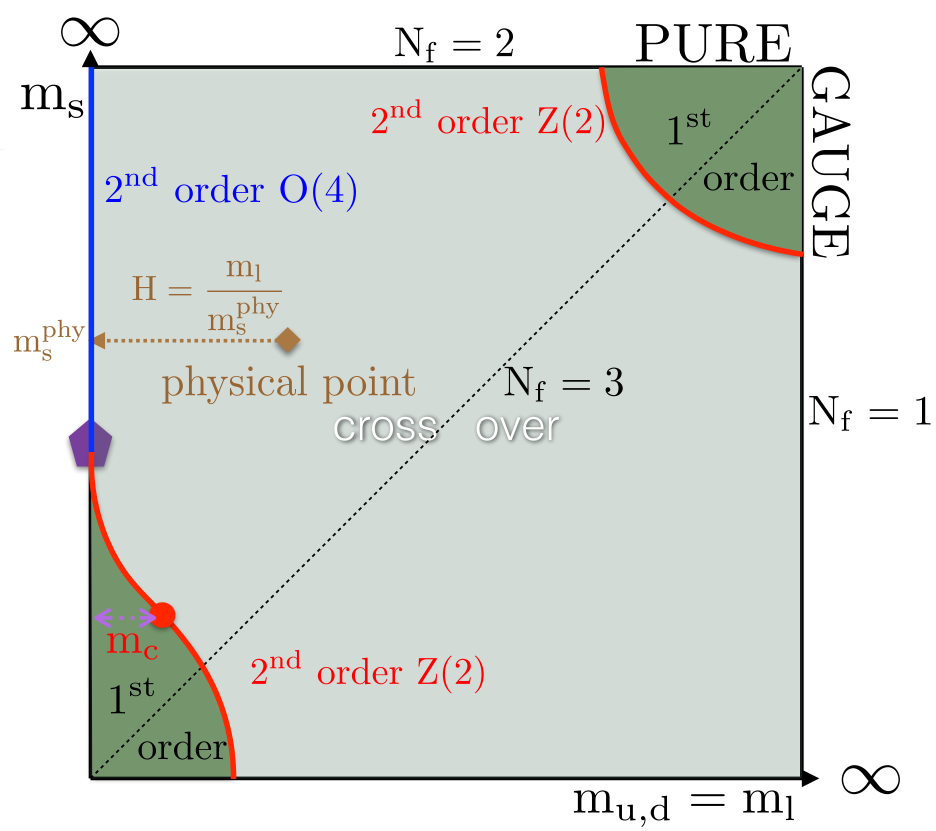
Role of axial anomaly?

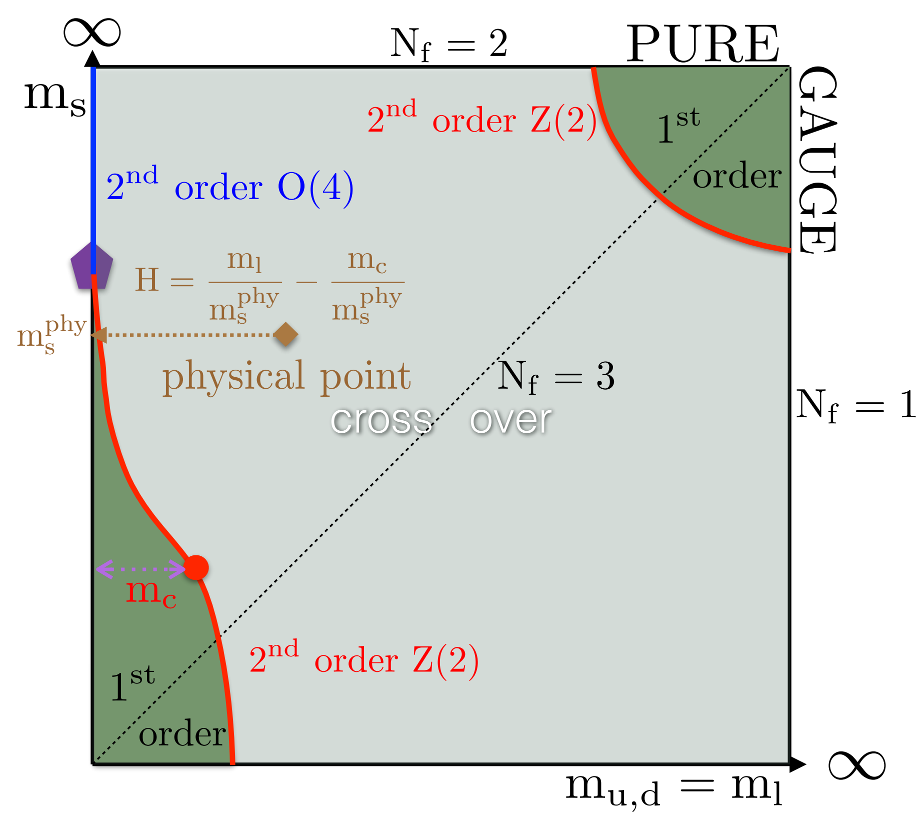
location of tri-critical point still an open question (maybe shifts to infinite strange quark mass)

below physical point

above physical point







[HotQCD 2019]

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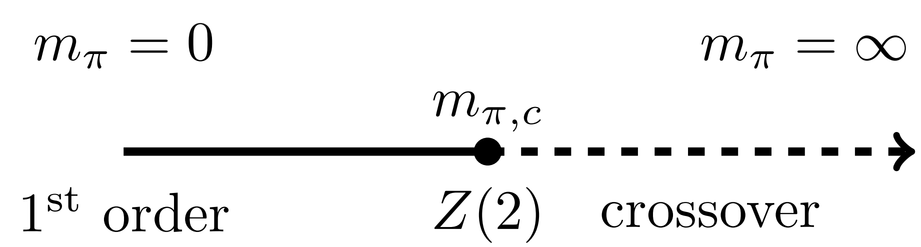
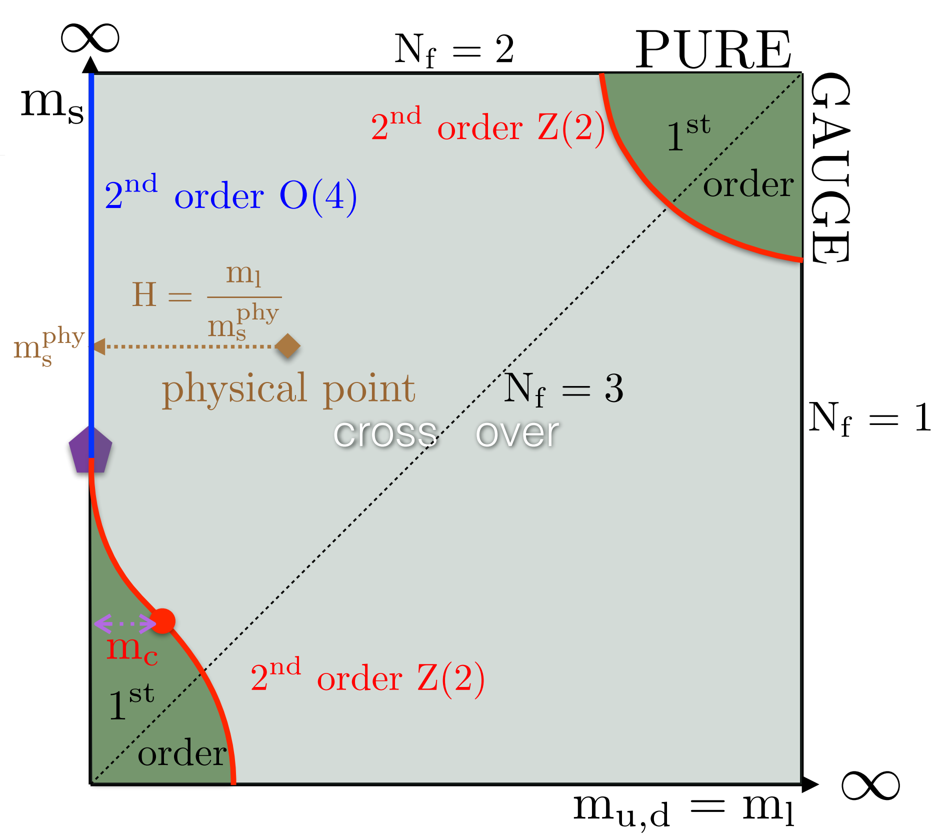
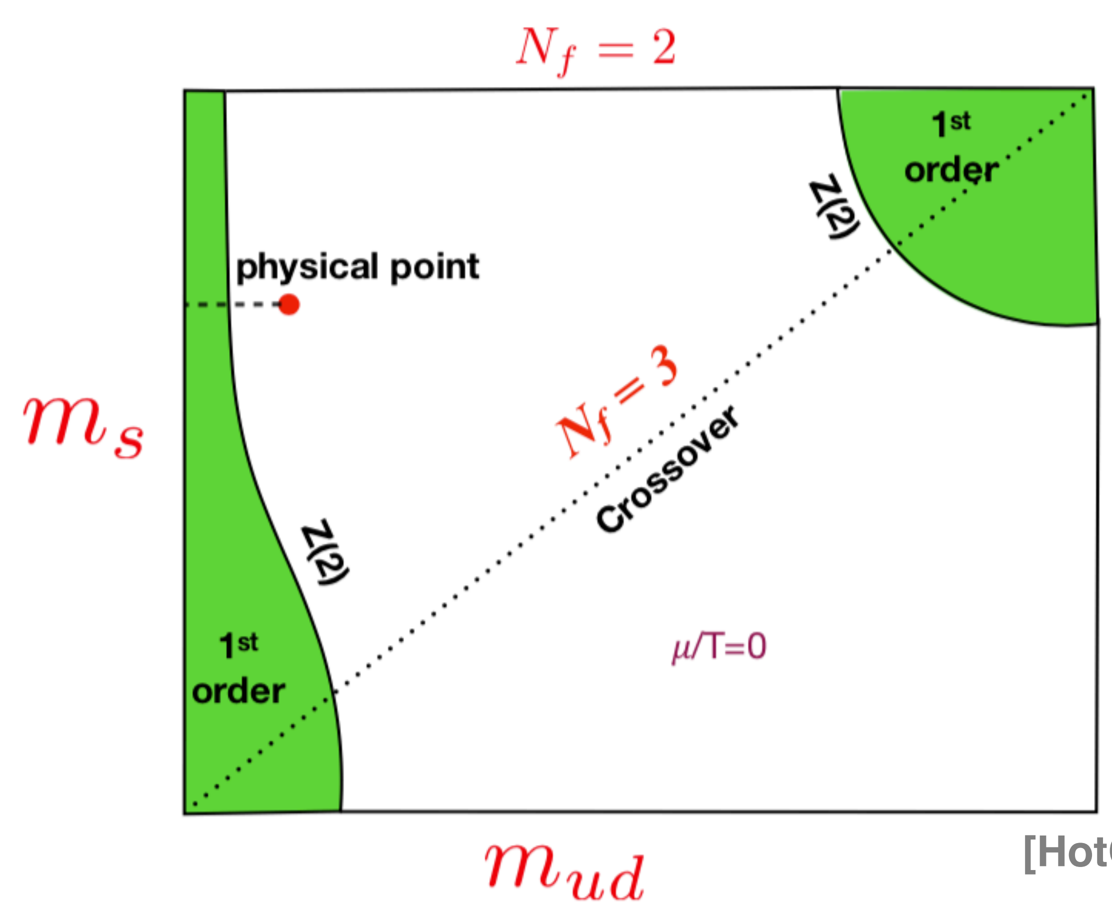


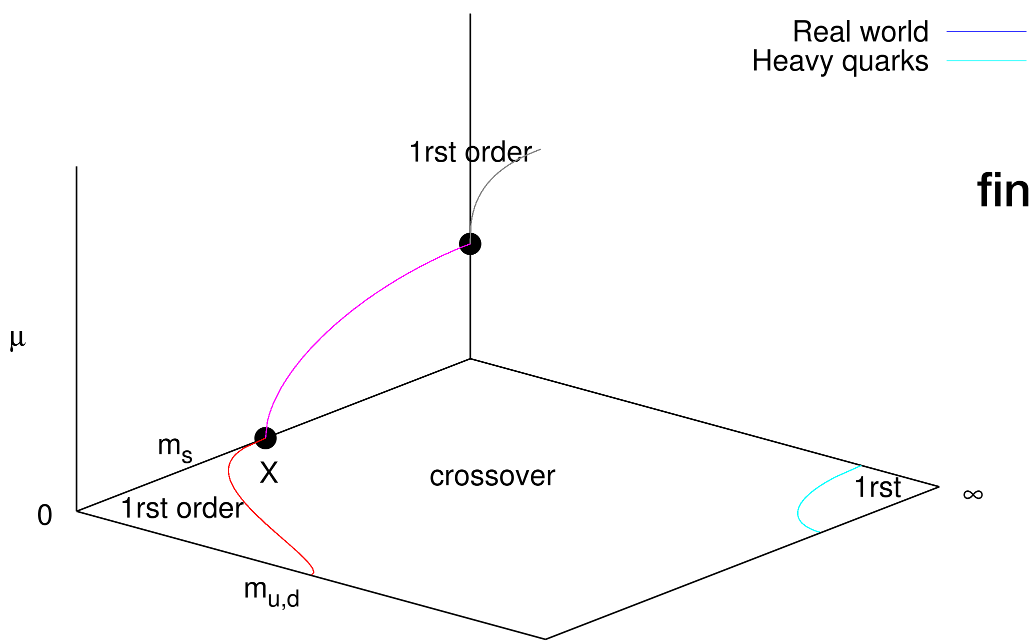




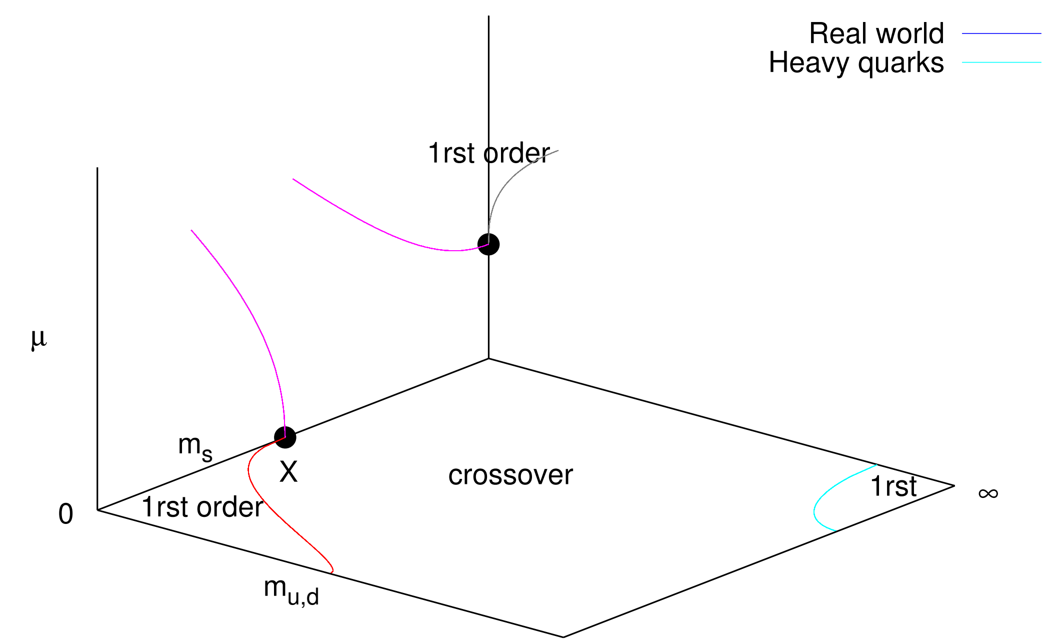
[HotQCD 2019]

Mean-field analysis

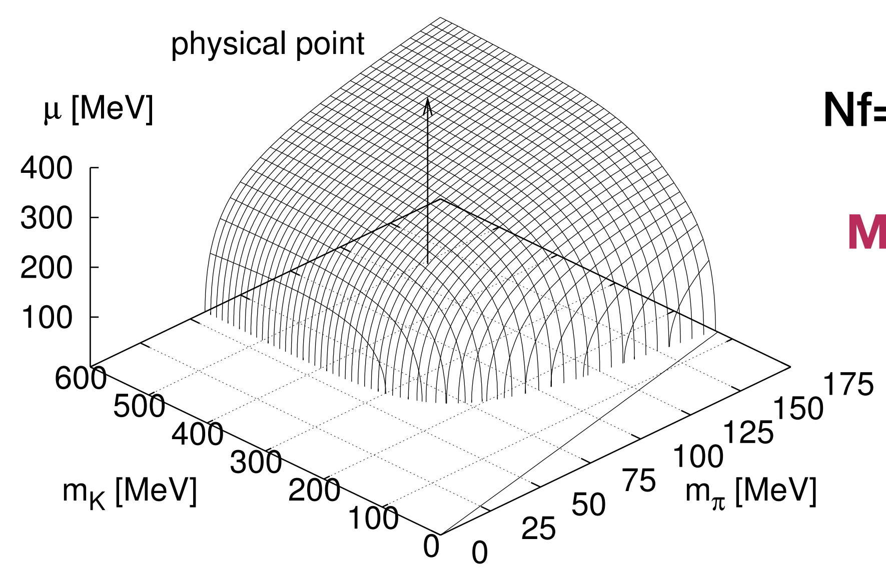
chiral critical surface
standard scenario



chiral critical surface
non-standard scenario



finite chemical potential

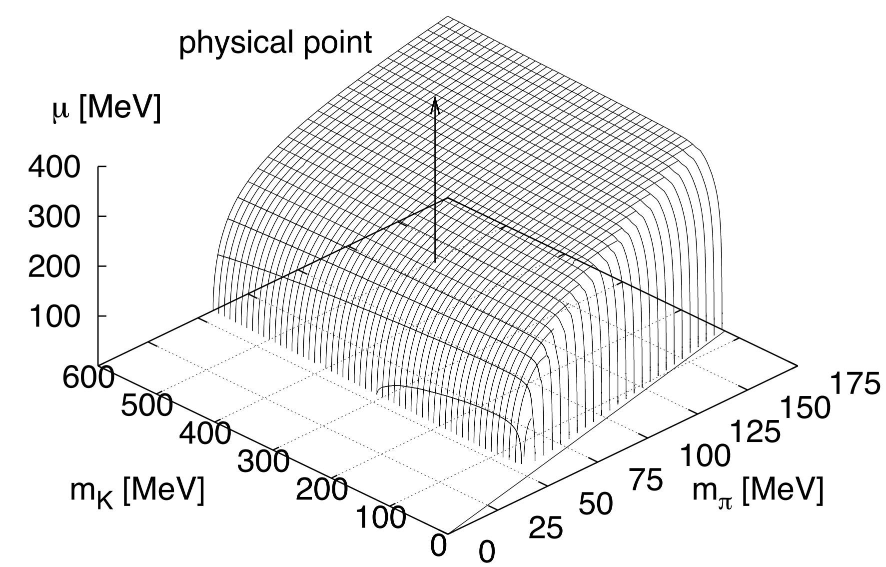


with $U_A(1)$-anomaly

Nf=2+1 quark-meson model

Mean-field approximation

[BJS, M. Wagner 2009]



without $U_A(1)$-anomaly

FRG analysis

Nf=2+1 FRG QM truncation:

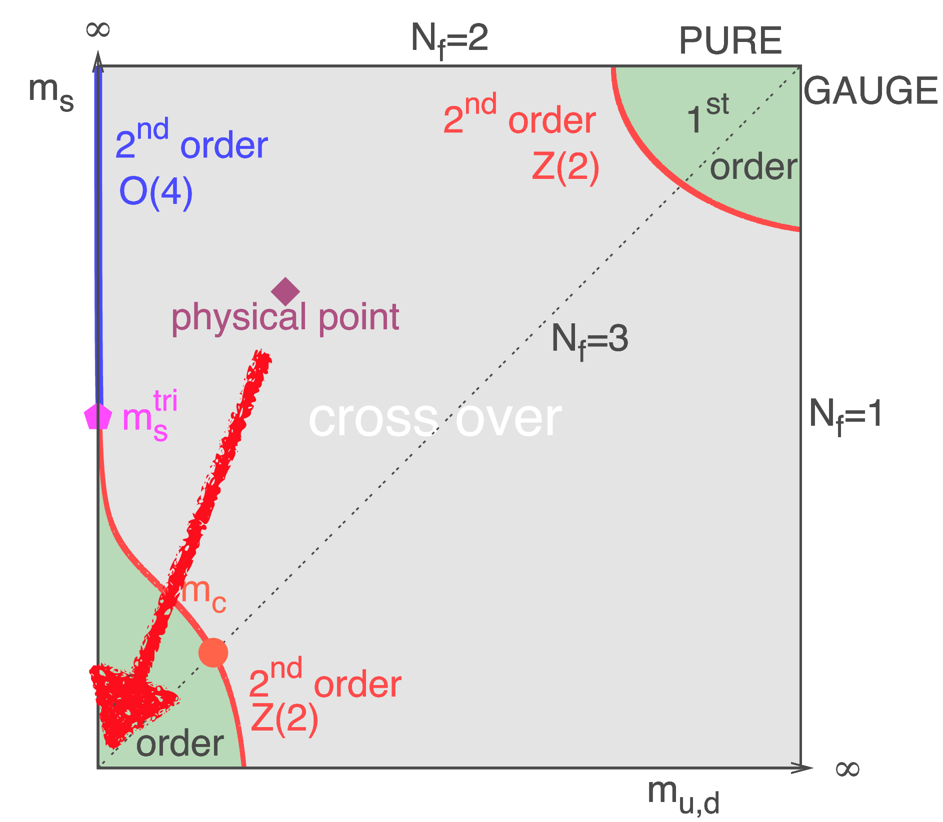
initial action in the UV: 7 parameters, (4 couplings, 2 explicit symmetry breaking, 1 ‘t Hooft determinant)
axial $U_A(1)$ symmetry: on or off

How to fix initial action in the UV away from the physical mass point?

1.) **fixed UV -scenario**:
vary only explicit SB
remaining parameter
adjusted @ physical
point

α=1 physical mass point

α=0 chiral limit



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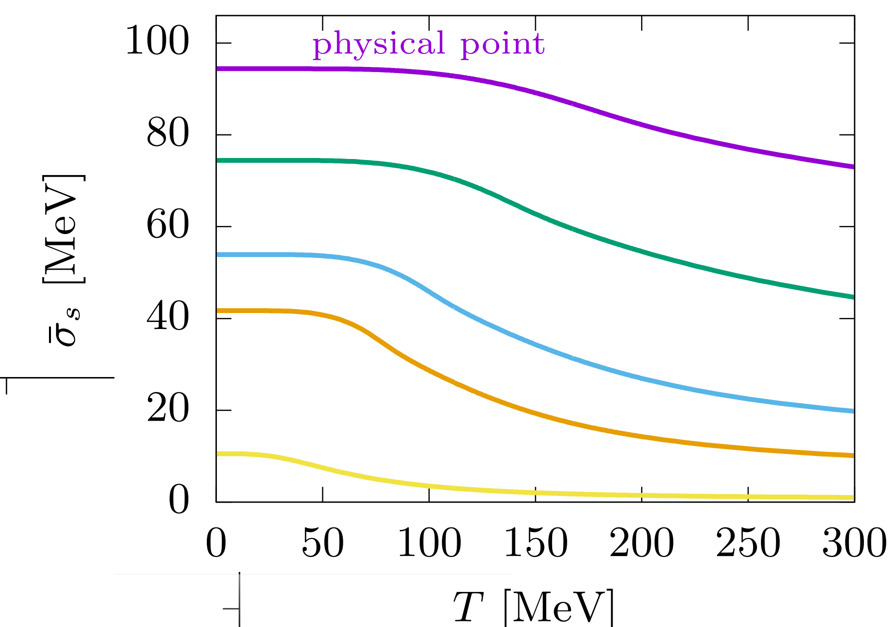
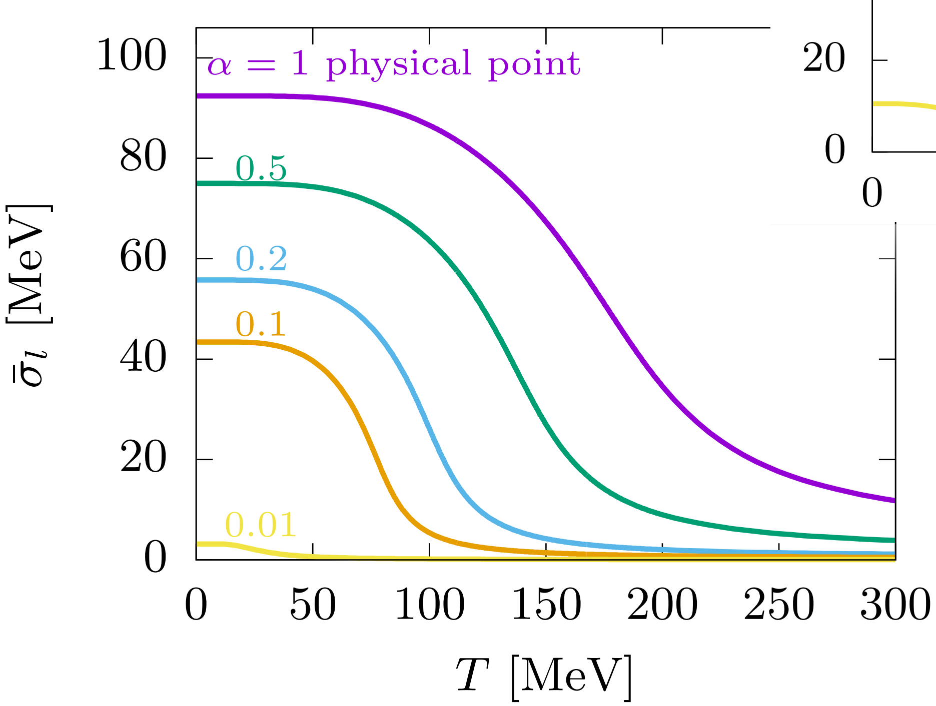
vary only explicit SB remaining parameter adjusted @ physical point

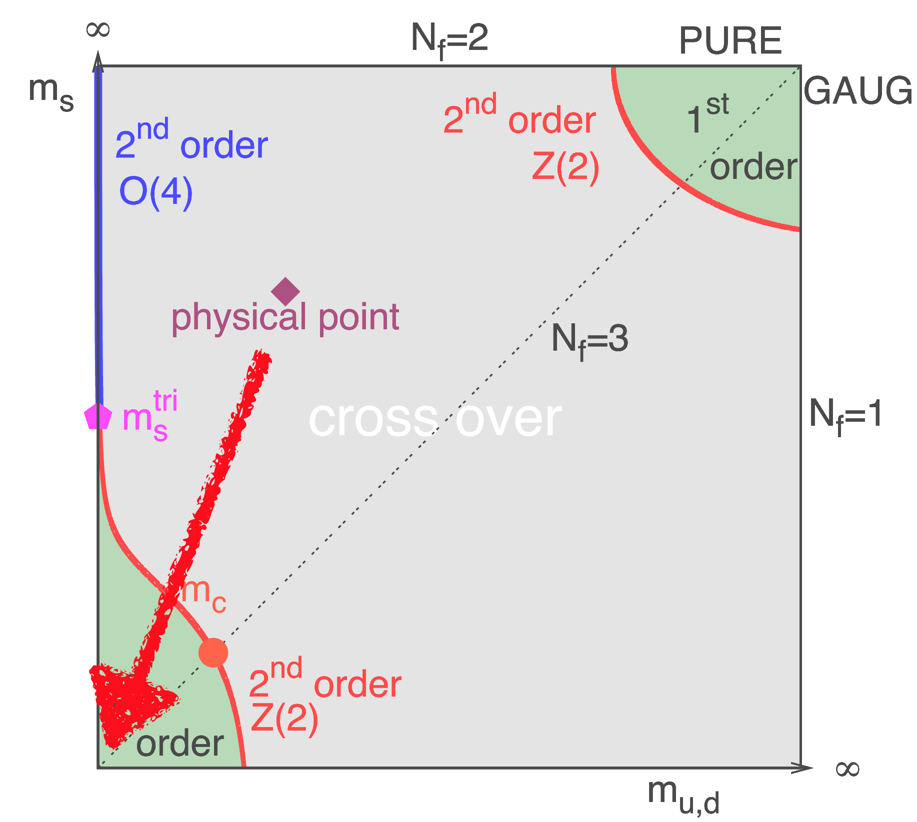




No symmetry breaking in the chiral limit!

α=1 physical mass point

α=0 chiral limit



[Resch, Rennecke, BJS 2019]

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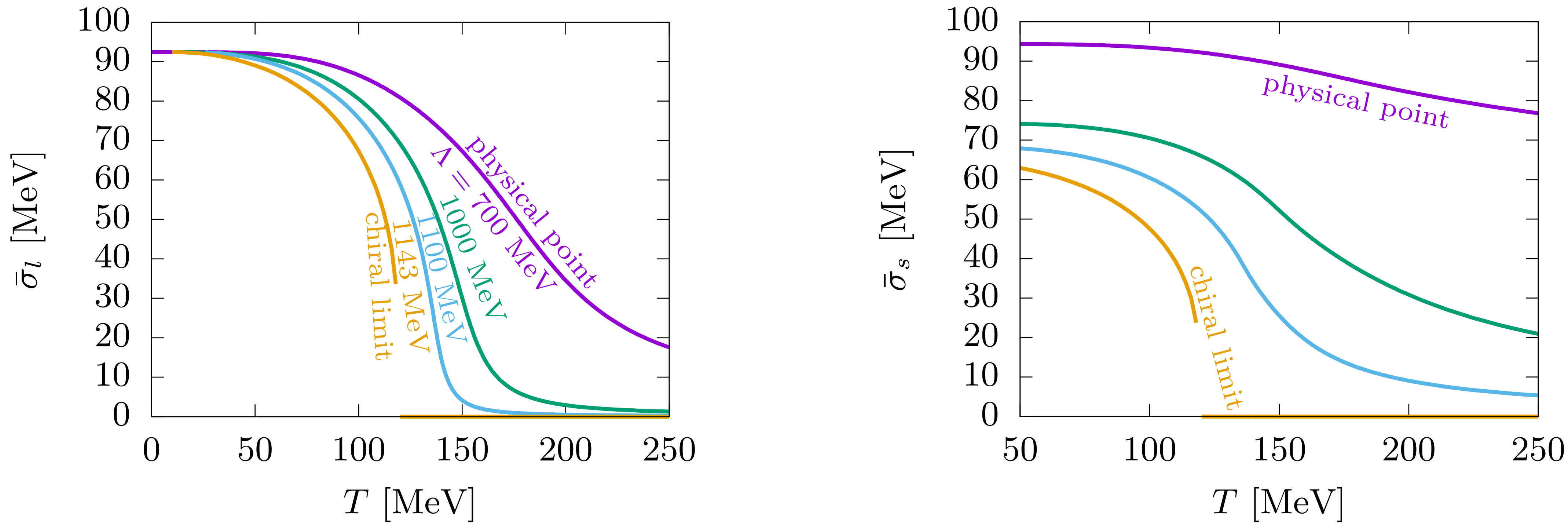
FRG analysis

Nf=2+1 FRG QM truncation:

initial action in the UV: 7 parameters, (4 couplings, 2 explicit symmetry breaking, 1 ‘t Hooft determinant)
axial $U_A(1)$ symmetry: on or off

How to fix initial action in the UV away from the physical mass point?

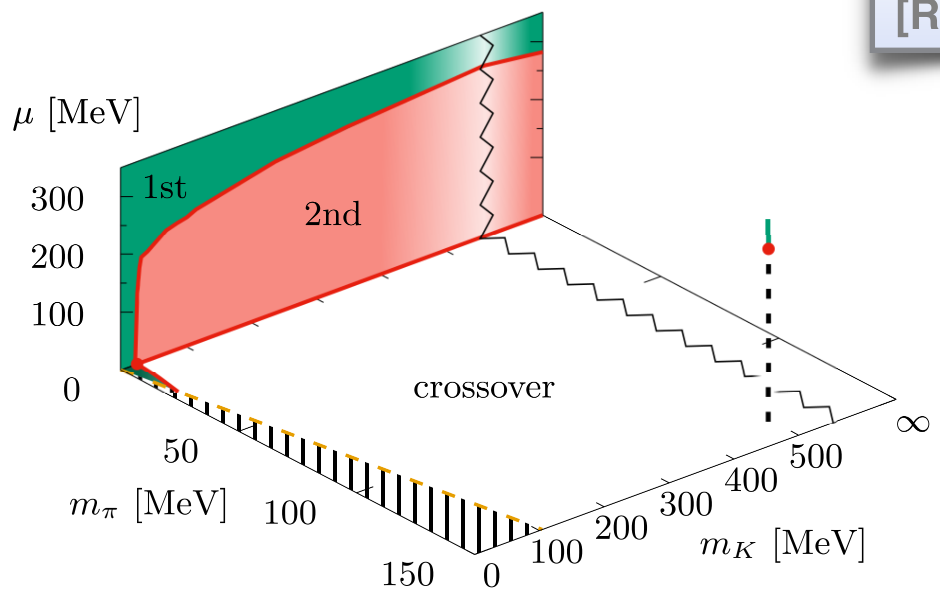
2.) **fixed f_π-scenario**: (motivated by chiral perturbation theory)
fix f_π in the infrared when explicit SB is varied
trick: vary only UV cutoff while initial action is fixed



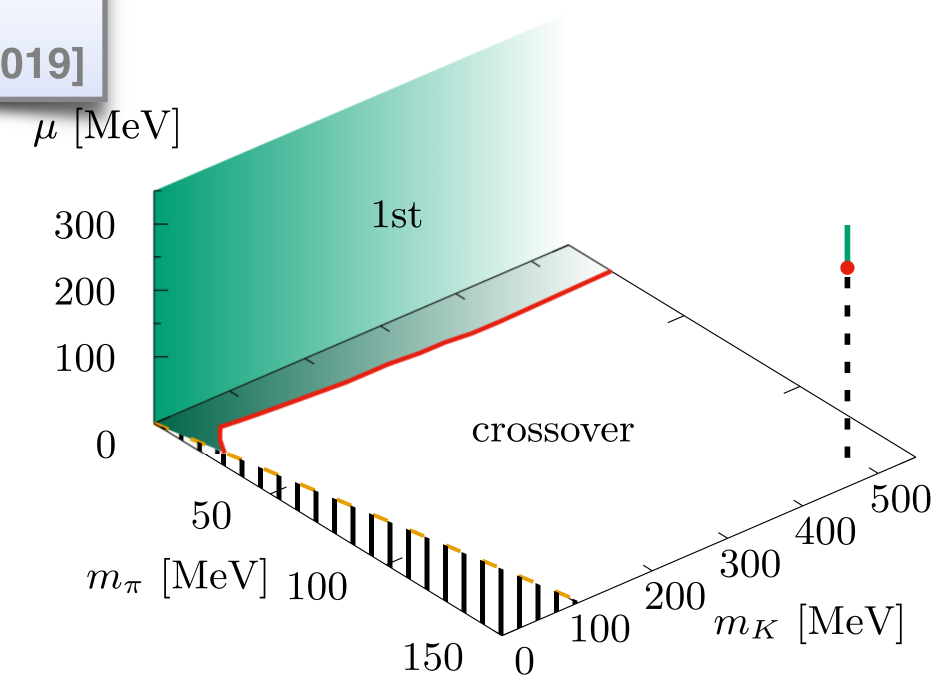
[Resch, Rennecke, BJS 2019]

Columbia plots

FRG
[Resch, Rennecke, BJS 2019]



all fluctuations
quarks & mesons



with $U_A(1)$-anomaly

without $U_A(1)$-anomaly

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Columbia plots

FRG
[Resch, Rennecke, BJS 2019]

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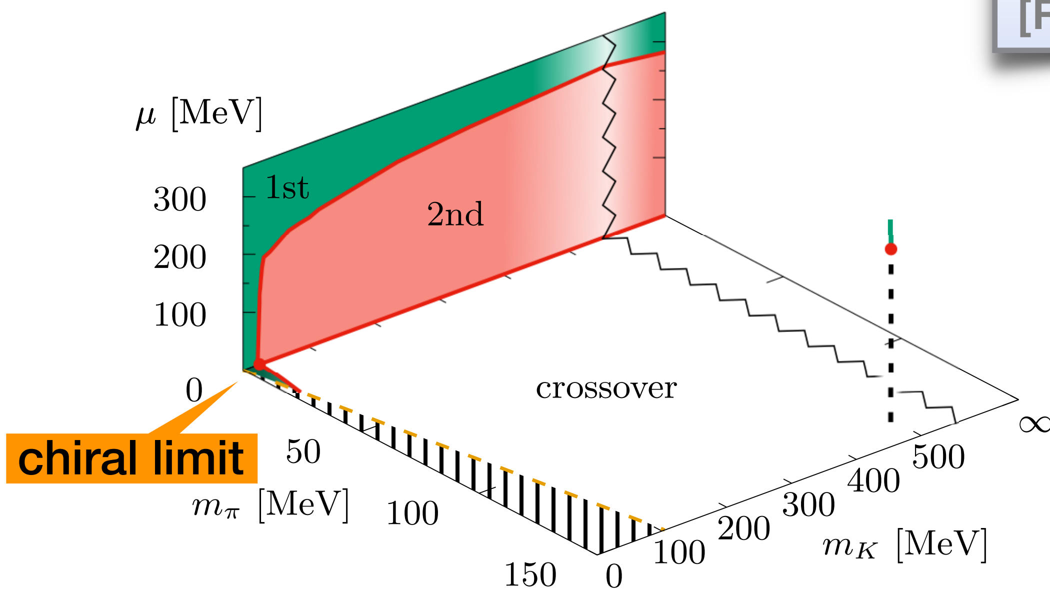
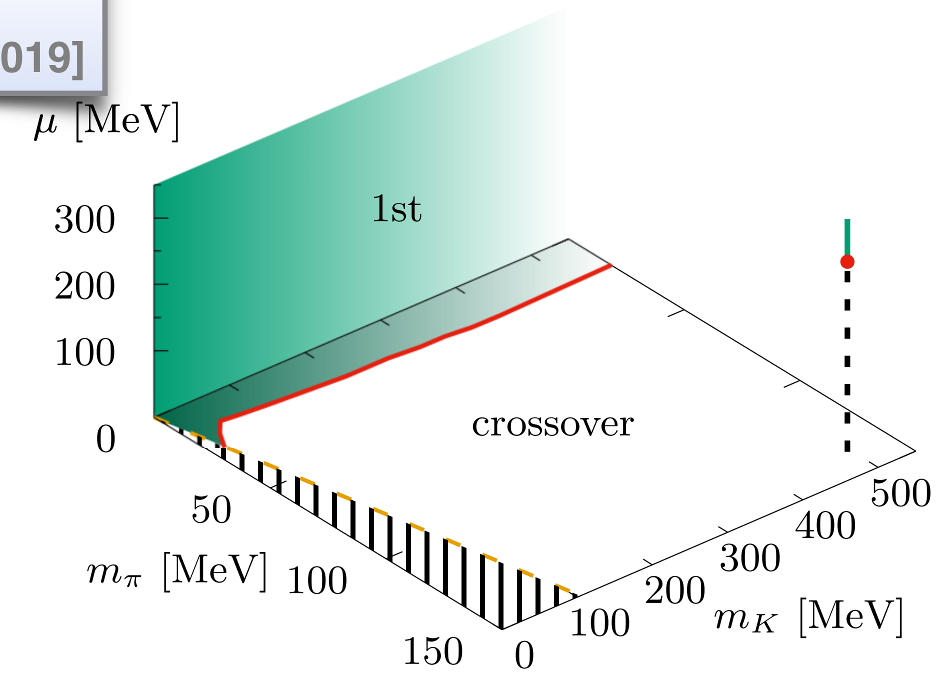
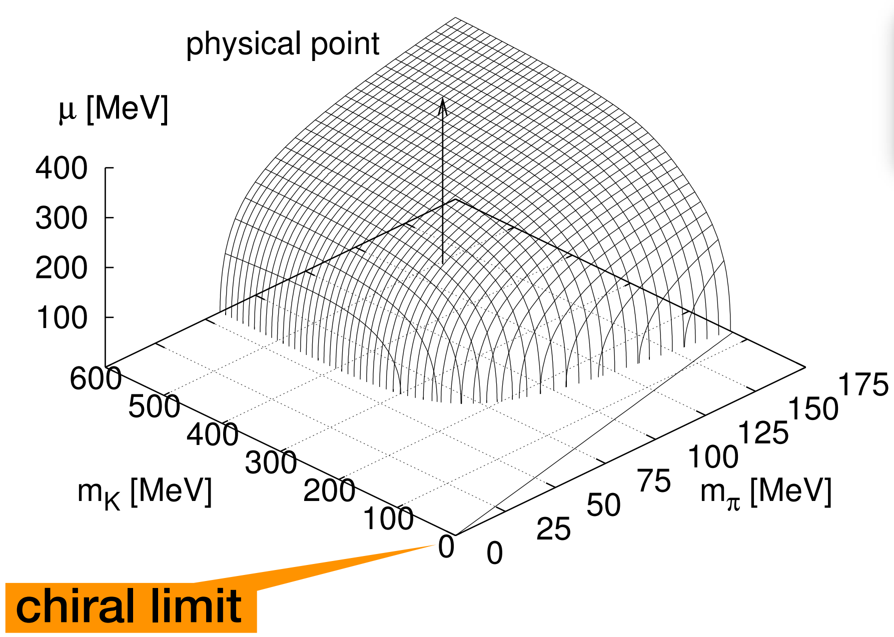


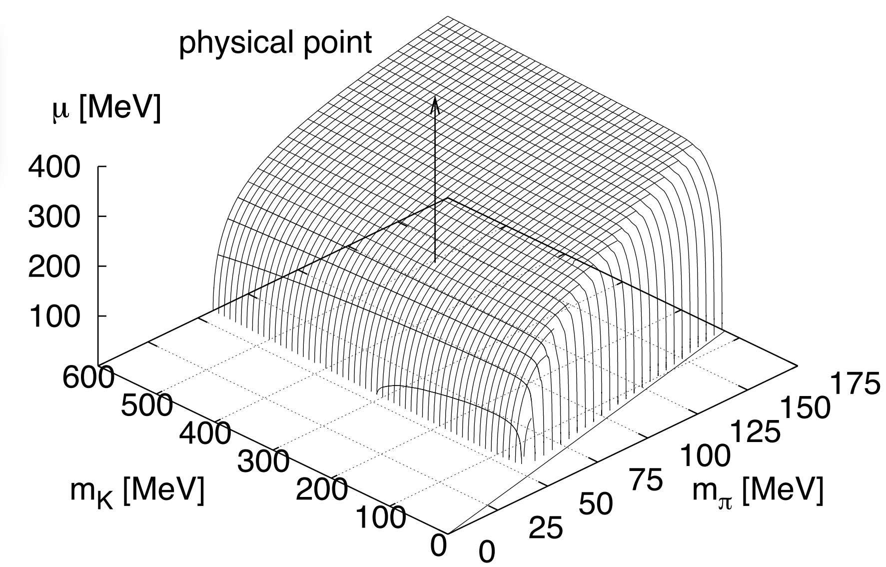


with $U_A(1)$-anomaly

without $U_A(1)$-anomaly

Mean-field analysis
[BJS, Wagner 2009]





Columbia plots

FRG
[Resch, Rennecke, BJS 2019]

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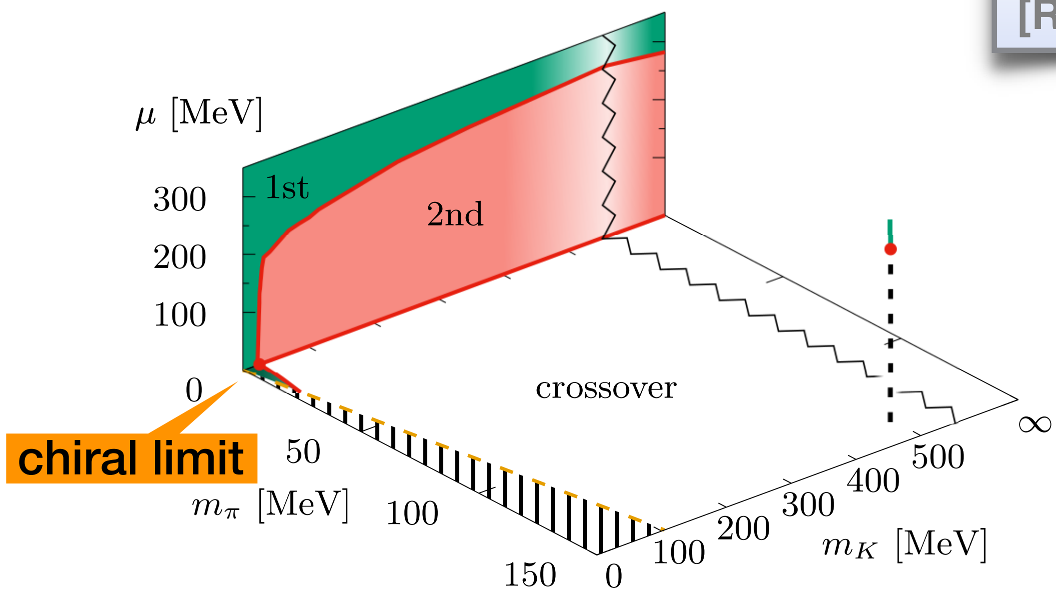
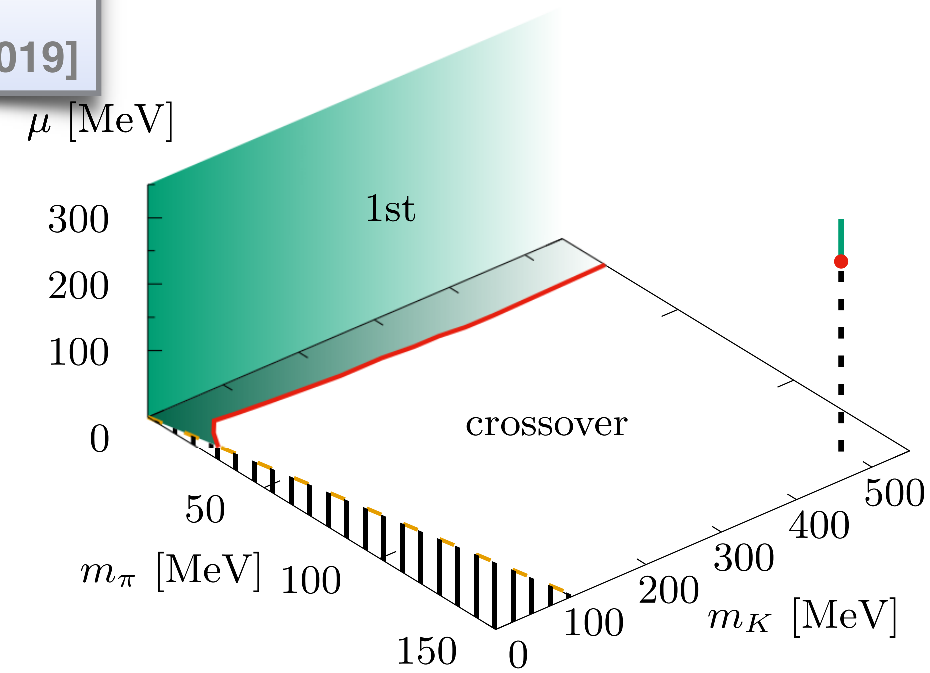


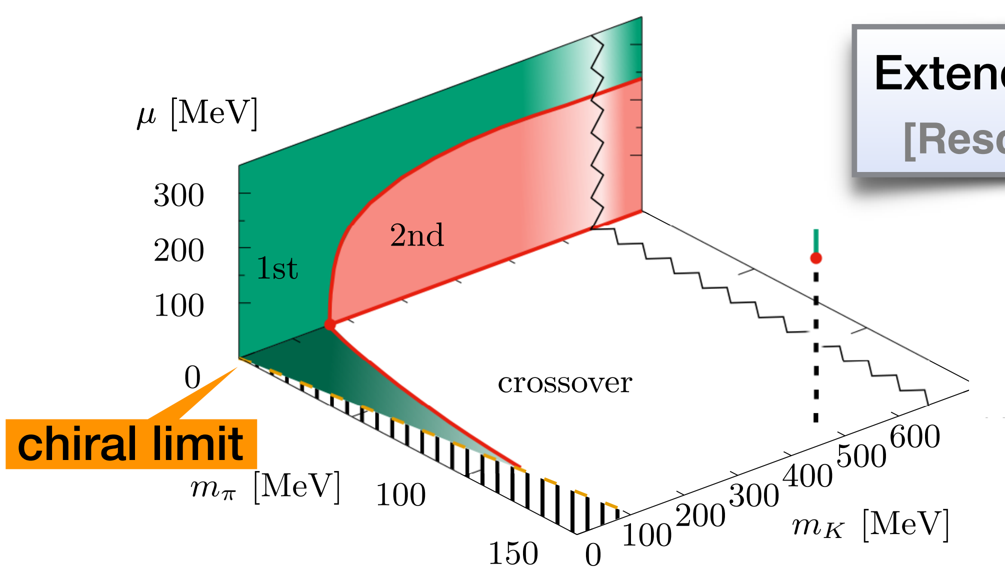


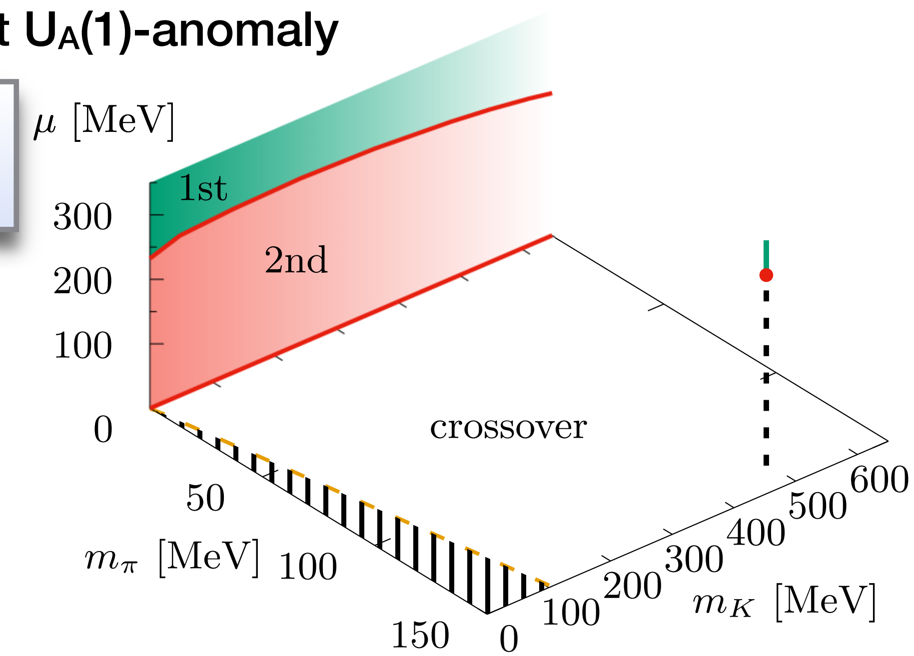
with $U_A(1)$-anomaly

without $U_A(1)$-anomaly

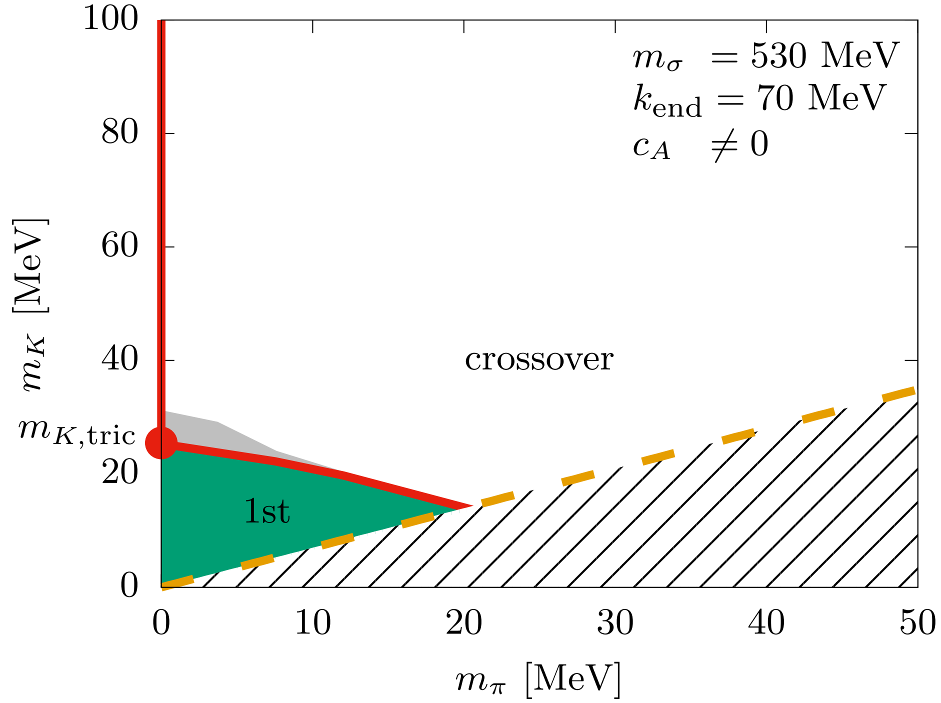
Extended Mean-field analysis
[Resch, Rennecke, BJS 2019]

influence of vacuum
fluctuations of quarks





Columbia plots



findings:

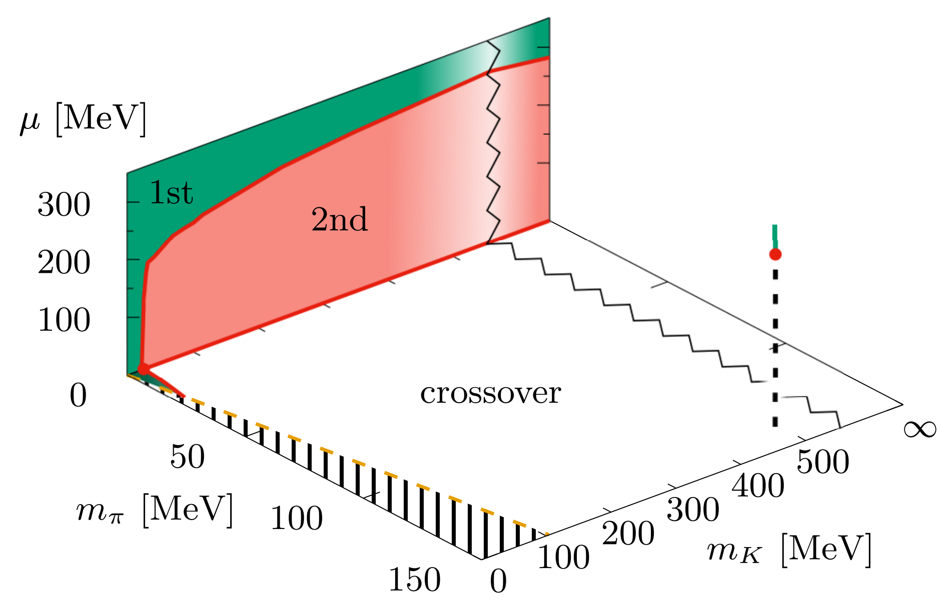
conventional bending of chiral critical surface
➜ critical endpoint @ physical mass point

tricritical strange quark mass far away from light chiral limit

First-order region around chiral limit very small
➜ in agreement with lattice
[Enrödi, et al. 2007, de Forcrane et al. 2017]

Nf=2+1 ➜ Nf=2 analytically connected two-flavor chiral limit

influence of axial anomaly on chiral critical line



with $U_A(1)$-anomaly

all fluctuations
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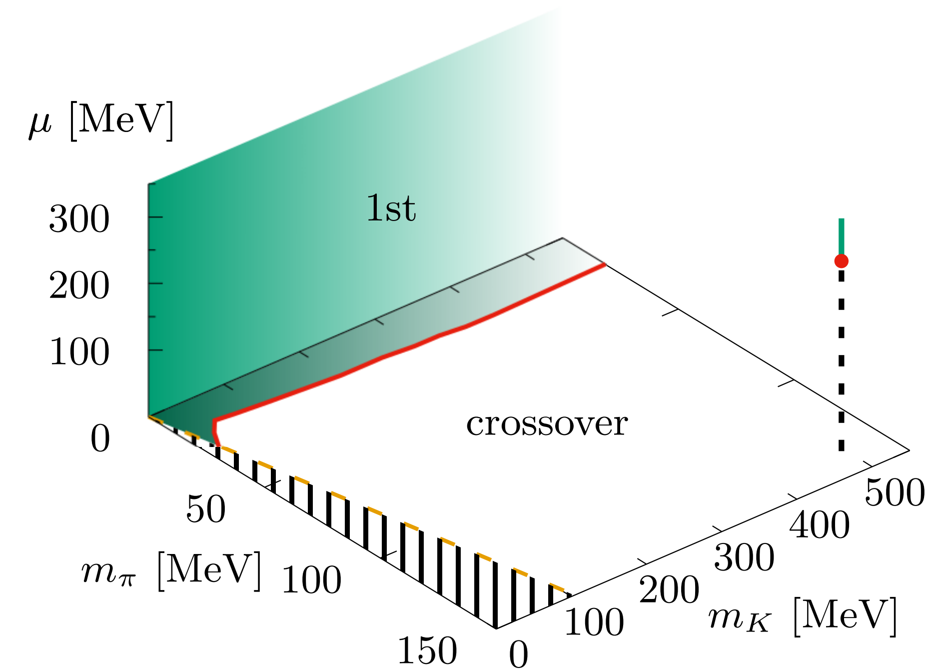

without $U_A(1)$-anomaly

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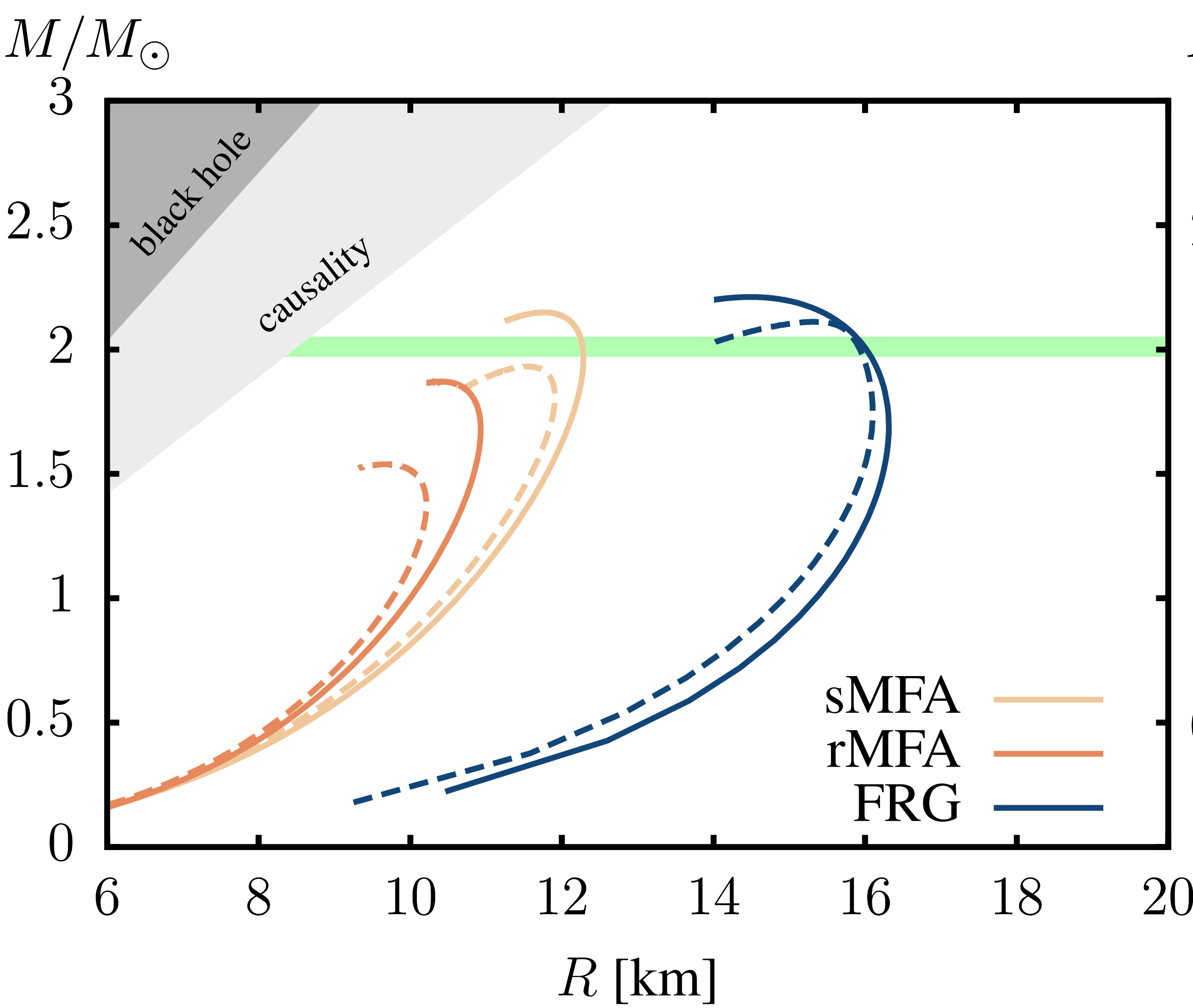
Outlook

EoS for high-density and vanishing temperature

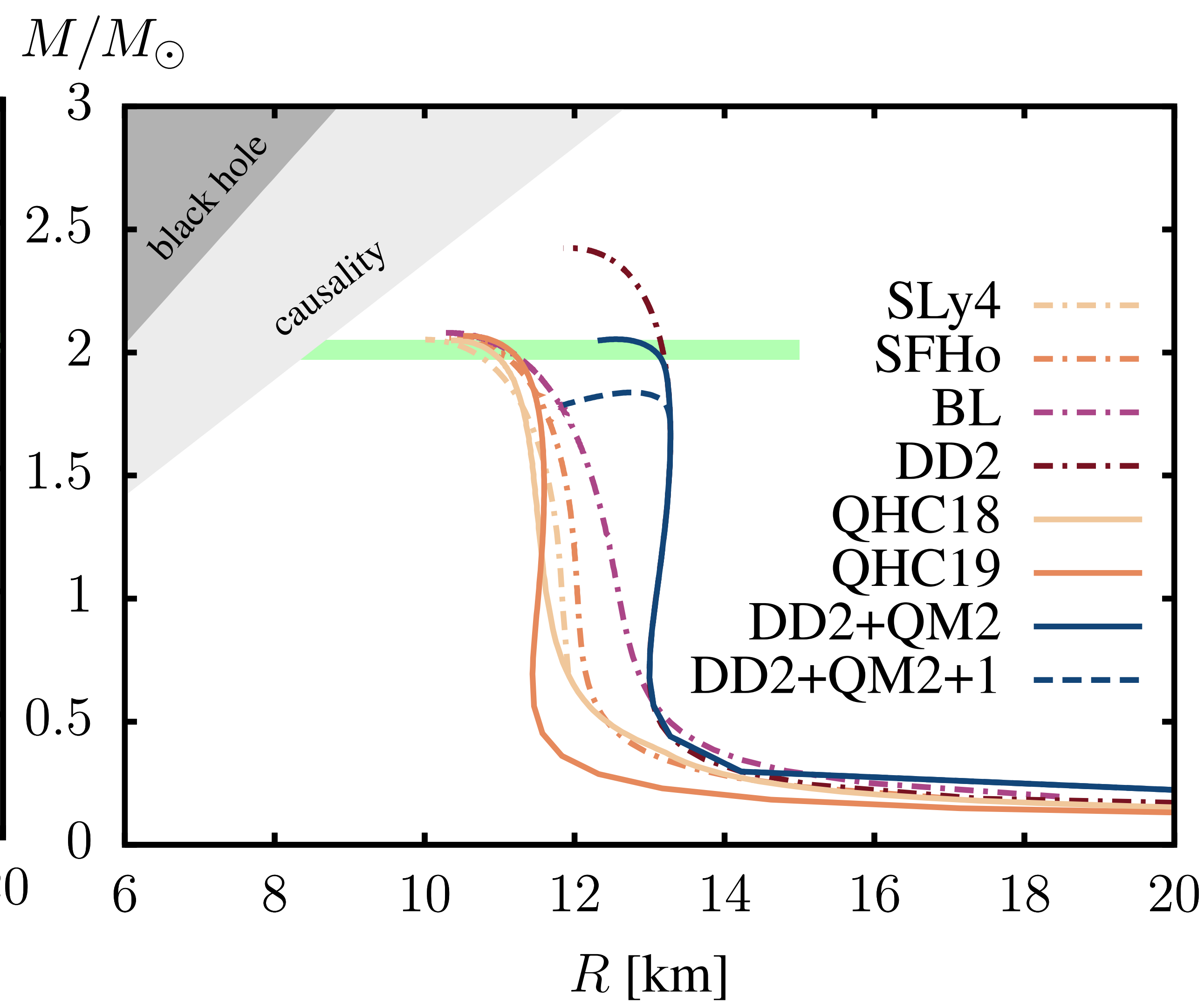
➜ Neutron Stars physics

Mass-Radius relation

[Otto, Oertel, BJS to be publish]



pure quark matter



hadronic EoSs

hadronic and quark matter

Summary & Conclusions

- quantum and thermal fluctuations on QCD phase diagram via

  FRG investigation with different truncations: LPA, LPA', LPA'+Y

  ➜ fluctuations are important (beyond LPA)

  ➜ mass sensitivity of the chiral phase structure (Columbia plot)

  small first order region with $U_A(1)$ breaking (tricritical mass)
  first-order band without $U_A(1)$ breaking