

In-medium spectral functions of vector and axialvector mesons from aFRG flow equations

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- Intro / Motivation, Dileptons in HIC
- Spectral functions from analytically continued (aFRG) flows
- Vector and axial-vector spectral functions
- Fluctuating (axial-)vectors
- Summary and Outlook









courtesy H. van Hees



T. Galatyuk et al., Physik Journal 17 (2018) no. 10

- from all stages of the collision
- measure temperature in QGP, lifetime of fireball...





Dilepton Spectra





dilepton rate (local thermal equilibrium):

$$\frac{dN_{ll}}{d^4x d^4q} = -\frac{\alpha_{\rm em}^2}{\pi^3 M^2} \frac{1}{3} g_{\mu\nu} \operatorname{Im} \Pi_{\rm em}^{\mu\nu} (M, |\vec{q}|; \mu, T)$$

electromagnetic correlator:

$$\Pi_{\rm em}^{\mu\nu}(q;\mu,T) = -i \int d^4x \ e^{iqx} \theta(x^0) \left\langle [j_{\rm em}^{\mu}(x), j_{\rm em}^{\nu}(0)] \right\rangle$$

vector meson dominance & quark counting:

$$\mathrm{Im}\,\Pi^{\mu\nu}_{\mathrm{em}} \left(M \le 1 \mathrm{GeV} \right) \, \sim \, \mathrm{Im}\, D^{\mu\nu}_{\rho} + \frac{1}{9} \mathrm{Im}\, D^{\mu\nu}_{\omega} + \frac{2}{9} \mathrm{Im}\, D^{\mu\nu}_{\phi}$$









deduce medium modifications to the vector spectral function, find signatures of chiral symmetry restoration

Spectral Functions

commutator of interacting fields:

free fields $\left< \left[\phi(x), \phi(0) \right] \right> = \int_0^\infty dm^2 \ \rho(m^2) \ i \Delta(x; m^2)$

> $\rho(p^2) = (2\pi)^3 \sum \delta^4(p - q_{\psi}) \left| \langle \Omega | \phi(0) | \psi \rangle \right|^2 , \quad p_0 > 0$ TECHNISCHE UNIVERSITÄT DARMSTAD

free fields (stable pion):

spectral function:

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Spectral Functions

two-particle thresholds:

(inverse Laplace, try e.g. MEM, but ill-posed numerical problem)

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• e.g. quark-meson model:

$$\Gamma_k = \int d^4x \left\{ \bar{\psi} \left(\partial \!\!\!/ + g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi + \frac{1}{2} (\partial_\mu \vec{\phi})^2 + U_k(\phi^2) - c\sigma \right\}, \quad \vec{\phi} = (\sigma, \vec{\pi})$$

(leading order derivative expansion)

• flow of Landau free energy density:

$$\begin{aligned} \partial_k \Omega_k(T,\mu;\phi^2) &= \\ \frac{k^4}{12\pi^2} \Biggl\{ \frac{1}{E_k^{\sigma}} \coth\left(\frac{E_k^{\sigma}}{2T}\right) + \frac{3}{E_k^{\pi}} \coth\left(\frac{E_k^{\pi}}{2T}\right) \\ -\frac{2N_c N_f}{E_k^q} \Biggl[\tanh\left(\frac{E_k^q - \mu}{2T}\right) + \tanh\left(\frac{E_k^q + \mu}{2T}\right) \end{aligned}$$

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• extract mass parameters & running 3- and 4-point vertices

(thermodynamically consistent & symmetry preserving)

Euclidean Mass Parameters

HIC FAIR for FAIR

CRC-TR 211

17 September 2019 | Lorenz von Smekal | p. 11

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Analytically Continued aFRG Flows

• e.g. O(4) linear sigma model:

• continue to real time:

 $p_0 = -i(\omega + i\varepsilon)$ (retarded)

solve analytically continued flow equation

 $T=\mu=0:$

CRC-TR 211

K. Kamikado, N. Strodthoff, L.v.S. & J. Wambach, EPJC 74 (2014) 2806

• compare:

Lattice: J. Engels & O. Vogt, NPB 832 (2010) 538

2-PI: D. Röder, J. Ruppert & D.H. Rischke, NPA 775 (2006) 127

Classical-statistical: S. Schlichting, D. Smith & L.v.S., arXiv:1908.00912

• large boson occupancies, universal critical behavior

S. Schlichting, D. Smith & L.v.S., arXiv:1908.00912

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aFRG Flow of Pion and Sigma

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• quark-meson model, $T = \mu = 0$:

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• quark-meson model:

A. Tripolt, N. Strodthoff, L.v.S. & J. Wambach, PRD 89 (2014) 34010

pion SF $ho(\omega, \vec{p})$ below T_c

ightarrow transport coefficients

A. Tripolt, L.v.S. & J. Wambach, PRD 90 (2014) 074031

• gauged linear-sigma model with quarks:

$$\Gamma_{k} = \int d^{4}x \left\{ \bar{\psi} \left(\partial - \mu \gamma_{0} + h_{S} \left(\sigma + i \vec{\tau} \vec{\pi} \gamma_{5} \right) + i h_{V} \left(\gamma_{\mu} \vec{\tau} \vec{\rho}_{\mu} + \gamma_{\mu} \gamma_{5} \vec{\tau} \vec{a}_{1\mu} \right) \right) \psi + U_{k}(\phi^{2}) - c\sigma + \frac{1}{2} \left(D_{\mu} \phi \right)^{\dagger} \left(D_{\mu} \phi \right) + \frac{1}{8} \left(V_{\mu\nu} V_{\mu\nu} \right) + \frac{1}{4} m_{V,k}^{2} \left(V_{\mu} V_{\mu} \right) + \frac{1}{4} \lambda_{k} \left(\partial_{\mu} V_{\mu} \right)^{2} \right\}$$

• aFRG flow equations for (axial-)vector mesons from:

Vector and Axial-Vector SFs

Ch. Jung, F. Rennecke, A. Tripolt, L.v.S. & J. Wambach, PRD95 (2017) 036020

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 $\begin{array}{c} \textbf{3} \ a_1^* + \pi \to \sigma \\ \textbf{4} \ a_1^* \to \pi + \sigma \\ \textbf{5} \ a_1^* \to \bar{\psi} + \psi \end{array}$

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 $\textcircled{1} \rho^* \to \pi + \pi$ (2) $\rho^* \to \bar{\psi} + \psi$

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• spectral representation of conserved current:

$$\langle T_{\rm cov} j_{\mu}(x) j_{\nu}(0) \rangle =$$

 $-i \int_{0}^{\infty} ds \, \frac{\rho(s)}{s} \int \frac{d^4 p}{(2\pi)^4} \, \mathrm{e}^{-\mathrm{i}px} \, \frac{p^2 g_{\mu\nu} - p_{\mu} p_{\nu}}{p^2 - s + \mathrm{i}\epsilon}$

• current-field identity, transverse vector propagator:

$$D_{\mu\nu}^{T,V}(p) = -i \int_0^\infty ds \, \frac{\rho_v(s)}{s} \int \frac{d^4p}{(2\pi)^4} \, e^{-ipx} \, \frac{p^2 g_{\mu\nu} - p_\mu p_\nu}{p^2 - s + i\epsilon}$$
$$= -i \frac{Z}{m_v^2} \, \frac{p^2 g_{\mu\nu} - p_\mu p_\nu}{p^2 - m_v^2 + i\epsilon} + \dots$$

• Euclidean two-point function, single-particle contribution:

$$\Gamma^{(2)T}_{\mu\nu}(p) = -\frac{m_0^2}{p^4} (p^2 + m_v^2) \left(p^2 \delta_{\mu\nu} - p_\mu p_\nu \right) \qquad m_{0,k}^2 = m_{v,k}^2 / Z_k$$

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new processes/imaginary parts contribute to SFs:

 $T = 150 \text{ MeV}, \mu = 0 \text{ MeV}$

rho meson

a₁ meson

Fluctuating (Axial-)Vectors

Fluctuating (Axial-)Vectors

Summary and Outlook

- Spectral functions from analytically contd. aFRG flows effective theories (chiral, linear)
- Vector and axial-vector SFs at finite T and μ

melting-rho scenario

• Fermionic spectral functions

R.-A. Tripolt, J. Weyrich , L. v. S. & J. Wambach, Phys. Rev. D98 (2018) 094002

• Electromagnetic spectral function

R.-A. Tripolt, Ch. Jung, N. Tanji, L. v. S. & J. Wambach, Nucl. Phys. A982 (2019) 775

• SFs in nuclear matter, parity doubling

Ch. Jung, PhD thesis, JLU 2019

Summary and Outlook

• SFs in nuclear matter, parity doubling

Ch. Jung, PhD thesis, JLU 2019

Self-consistent spectral functions

O(4)-model, Ch. Jung, N. Wink, J. Pawlowski, & L.v.S., in preparation

Thank you for your attention!

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