II International workshop on
Theory of Hadronic Matter under Extreme conditions

Chiral perturbation theory in a chiral imbalance medium

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Based on **1908.09118 [hep-th]**

Outline of the talk

1. Chiral imbalance in QCD : how to measure it
   in central and non-central (CME) collisions

2. Chiral (Gasser-Leutwyler) lagrangian
   in chiral imbalanced hadron medium

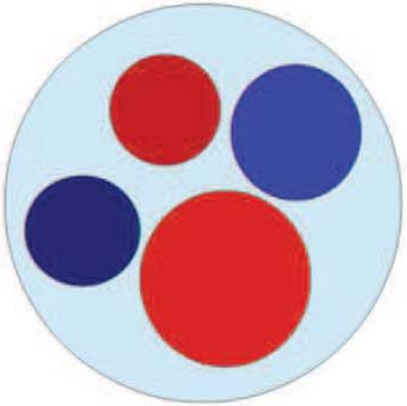
3. Mass-shell distortion for pions and their weak
   decay suppression, a chance to measure chiral
   imbalance

4. QCD inspired sigma model (SU(2) case)

5. Confronting chiral imbalance corrections
   for the chiral lagrangian and the linear sigma model

6. Perspectives to register CI

Topological charge



Bubbles with TC alternating in signs

$$< \; T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \mathrm{Tr} \left(G^j \partial^k G^l - i \frac{2}{3} G^j G^k G^l \right) \; >$$

it may survive for a sizeable lifetime in a heavy-ion fireball of finite volume

Typically $\langle \Delta T_5 \rangle \neq 0$ for $\Delta t \simeq \tau_{\text{fireball}} \simeq 5 \div 10$ fm/c;

Induced chiral imbalance

For the fireball lifetime one can control the value of $\langle \Delta T_5 \rangle$ by a topological chemical potential μ_θ via $\Delta\mathcal{L}_{top} = \mu_\theta \Delta T_5$

The partial conservation of axial current (broken by gluon anomaly)

$$\partial_\mu J_5^\mu - 2im_q J_5 = \frac{N_f}{2\pi^2}\partial_\mu K^\mu$$

predicts the induced chiral charge (in the chiral limit $m_q \simeq 0$)

$$\frac{d}{dt}(Q_5^q - 2N_f T_5) \simeq 0, \quad Q_5^q = \int_{\mathrm{vol.}} d^3x \bar{q}\gamma_0\gamma_5 q = \langle N_L - N_R \rangle$$

Chiral imbalance

to be conserved during $\tau_{fireball}$.

Thus QCD with a topological charge $\langle \Delta T_5 \rangle \neq 0$ can be described at the Lagrangian level equivalently by chiral chemical potential μ_5

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f}\langle Q_5^q \rangle; \qquad \Delta\mathcal{L}_{top} = \mu_\theta \Delta T_5 \iff \Delta\mathcal{L}_q = \mu_5 Q_5^q$$

A.A., V.A.Andrianov, D.Espriu, X.Planells, Phys.Lett. B 710 (2012) 230

Sizeable Chiral Imbalance: **how to measure?**

Initial Stage
After
HI collision
QGP

Hadronization:
Hadron vacuum with Chiral Imbalance
inherited from quark vacuum with CI

< 1 fm/c

1 fm/c < 7-10 fm/c

Quark-hadron continuity during hadronization through crossover (K. Fukushima et al)

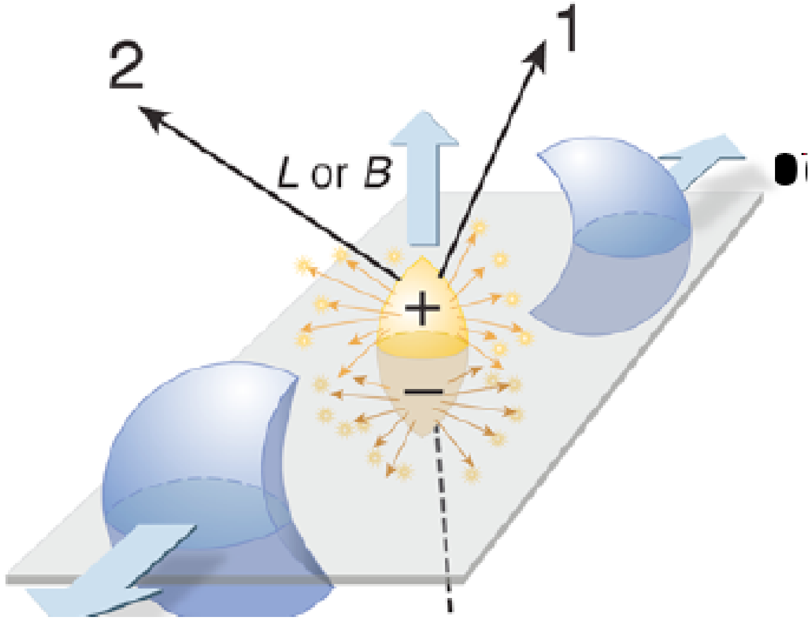
CHIRAL MAGNETIC EFFECT
Not possible to measure chiral imbalance or magnetic field separately

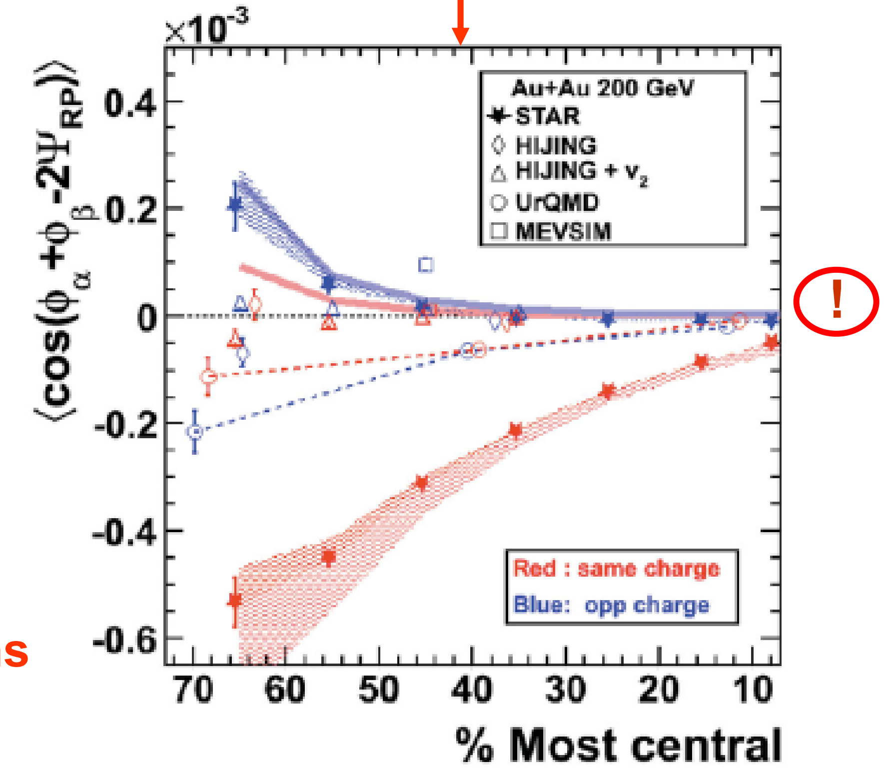
$$\vec{J} = \sigma_5 \mu_5 \vec{B}$$

Two inputs

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

(STAR Collaboration)





CME disappears in central collisions but chiral imbalance NOT!

Chiral Lagrangian with chiral chemical potential

The Chiral Lagrangian for pions with a chiral imbalance is implemented with properly constructed covariant derivative

$$D_\nu \Longrightarrow \bar{D}_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \star\} = \mathbf{I}_q \partial_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu}$$

In the large N_c approach the $SU(2)$ Chiral Lagrangian in the strong interaction sector reads,

$$\mathcal{L}_2 = \frac{F_0^2}{4} < -j_\mu j^\mu + \chi^\dagger U + \chi U^\dagger >,$$

where
$j_\mu \equiv U^\dagger \partial_\mu U$, the chiral field $U = \exp(i\hat{\pi}/F_0)$,
the bare pion decay constant $F_0 \simeq 92 MeV$, $\chi(x) = 2B_0 s(x)$
the tree-level pion mass $M_\pi^2 = 2B_0 \hat{m}_{u,d}$.
The constant $B_0 = -\langle\bar{q}q\rangle/F_0^2$ determines the chiral quark condensate

Gasser-Leutwyler lagrangian in a medium with chiral imbalance

For SU(2) one has a reduction of the dim=4 GL lagrangian with chiral imbalance,

$$\mathcal{L}_4 = \frac{1}{4} l_1 < j_\mu j^\mu >< j_\nu j^\nu > + \frac{1}{4} l_2 < j_\mu j_\nu >< j^\mu j^\nu > - \frac{1}{4} l_4 < j_\mu j^\mu (\chi^\dagger U + \chi U^\dagger) >$$

J. Gasser and H. Leutwyler, *Annals Phys.* 158, 142 (1984)

resulting in

$$\Delta \mathcal{L}_4(\mu_5) = -\mu_5^2 \{ 8(l_1 + l_2) < j^0 j^0 > -4(l_1 + l_2) < j_k j_k > - l_4 < \chi^\dagger U + \chi U^\dagger > \}$$

Distorted dispersion law

Dispersion law in energy p^0 and three-momentum $|\vec{p}|$ for the pion mass shell

$$\mathcal{D}^{-1}(\mu_5) = (F_0^2 + 32\mu_5^2(l_1 + l_2))p_0^2 - (F_0^2 + 16\mu_5^2(l_1 + l_2))|\vec{p}|^2 - (F_0^2 + 4 l_4 \mu_5^2)m_\pi^2 \to 0$$

In the pion rest frame

$$F_\pi^2(\mu_5^2) \simeq F_0^2 + 32\mu_5^2(l_1 + l_2); \quad m_\pi^2(\mu_5^2) \simeq \left(1 - 4\frac{\mu_5^2}{F_0^2}(8(l_1 + l_2) - l_4)\right) m_\pi^2(0), \quad (1)$$

i.e. the pion decay constant is growing and its mass is decreasing in the chiral media.

In the large N_c expansion the empirical values of the SU(2) GL constants are

J. Bijnens and G. Ecker, Annu. Rev. Nucl. Part. Sci., 64, 6.1 (2014)

$$l_1^r = (-0.4 \pm 0.6) \times 10^{-3}; \; l_2^r = (8.6 \pm 0.2) \times 10^{-3};$$
$$l_1^r + l_2^r = (8.2 \pm 0.8) \times 10^{-3}; \; l_4^r = (2{,}64 \pm 0.01) \times 10^{-2}$$

if neglecting the RG logarithm contributions $\log\left(m_\pi/\mu\right) \simeq 0$

Suppression of charged pion decay

The predicted distortion of the mass shell condition can be detected in decays of charged pions when the effective pion mass approaches to muon mass.
Let us find the threshold value for the $\pi^+ \to \mu^+ \nu$ decay. If a charged pion was generated in chiral medium its mass is lower than in the vacuum and the condition for its decay reads,

$$\left(1 - 16(l_1 + l_2)\frac{\mu_5^2}{F_0^2}\right)\left(|\vec{p}|^2 + m_{0,\pi}^2\right) \geq |\vec{p}|^2 + m_\mu^2, \quad \frac{m_a^2}{16\mu_5^2} \geq \frac{|\vec{p}|^2 + m_{0,\pi}^2}{m_{0,\pi}^2 - m_\mu^2},$$

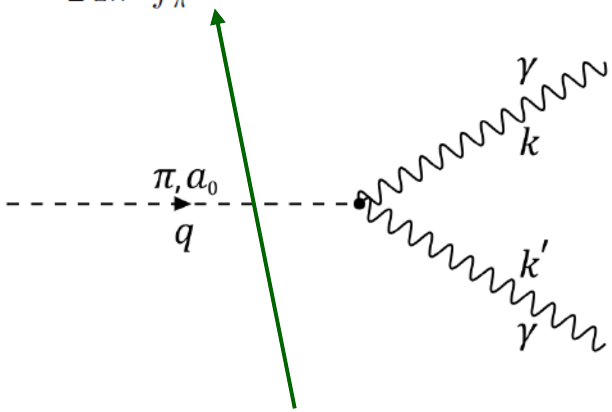
where we use the relations $4(l_1 + l_2) \simeq l_4$.
The decay channel is closed for $|\vec{p}|^2 \simeq 0$ if $\mu_5 \simeq 160 MeV$. For pions in flight the threshold value is lower $\mu_5 \leq 160 MeV$. The cutoff approaches to zero.
It must be detected as a substantial decrease of muon flow originated from pion decays in fireball.

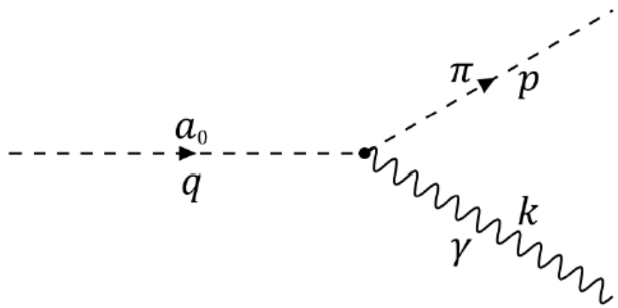
Wess-Zumino-Witten action

Describing of anomalous decay of strong interaction $\pi \to \gamma\gamma$
and other interaction: $\gamma\pi^- \to \pi^0\pi^-$ and $\gamma \to \pi\pi\pi$

$$-\frac{e^2 N_c}{24\pi^2 f_\pi}\epsilon^{\nu\sigma\lambda\rho}\partial_\sigma A_\lambda \partial_\nu A_\rho \pi^0 \quad (1)$$

$$-\frac{ie\mu_5 N_c}{6\pi^2 f_\pi^2}\epsilon_0^{\ \sigma\lambda\rho}A_\rho \partial_\sigma \pi^+ \partial_\lambda \pi^- \quad (2)$$





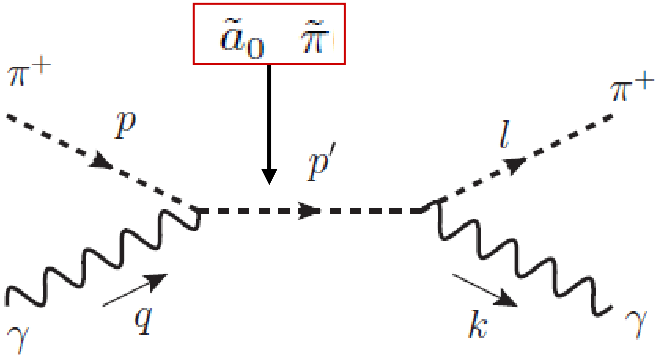
$\pi^0 \to \gamma\gamma, \quad a_0^0 \to \gamma\gamma$

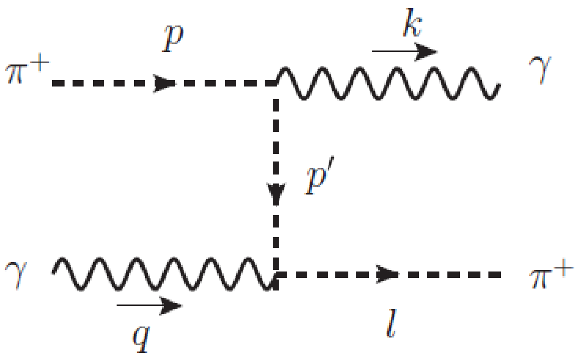
$a_0^\pm \to \pi^\pm\gamma$

K.Fukushima and K.Mameda, Phys.Rev. D86 (2012) 071501

M. Kawaguchi, M. Harada, S. Matsuzaki, R. Ouyang , PHYS. REV. C 95, 065204 (2017)

M. Kawaguchi, M. Harada, S. Matsuzaki, R. Ouyang , PHYS. REV. C 95, 065204 (2017)





Our prediction: Scalar resonance enhancement!

processes are parity conjugate:

$$\pi^{\pm}(\vec{p}) + \gamma(\vec{q}) \to \pi^{\pm}(\vec{l}) + \gamma_{+}(\vec{k}),$$
$$\pi^{\pm}(-\vec{p}) + \gamma(-\vec{q}) \to \pi^{\pm}(-\vec{l}) + \gamma_{-}(-\vec{k}),$$

where $\pm$ attached on photons in the final state denote photon helicities.

asymmetry ($\mathcal{A}$) can be evaluated as

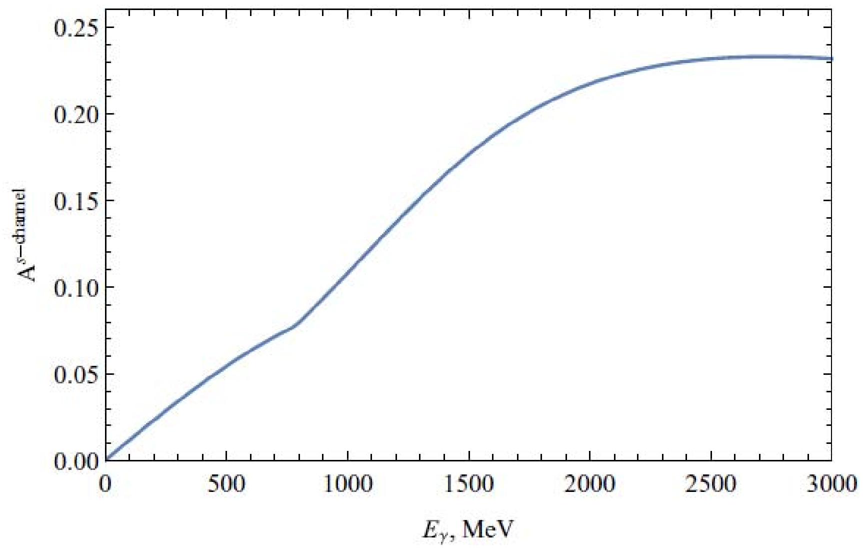
$$\mathcal{A} = \left| \frac{\mathcal{N}_{+} - \mathcal{P}[\mathcal{N}_{+}]}{\sum_{\lambda} \{\mathcal{N}_{\lambda} + \mathcal{P}[\mathcal{N}_{\lambda}]\}} \right|,$$

where $\mathcal{N}_{\lambda}$ stands for the number of events per the phase space, $d E_{\gamma}\, d\cos\theta\, d\phi$, for the parity conjugate processes with the helicity λ and the photon energy E_{γ} in the final state. The symbol $\mathcal{P}$ acts as the parity conjugation projection. The denominator represents the total number of the $\pi^{\pm}\gamma$ emission events with unpolarized photons per the phase space.

$$\mathcal{A}^{s\text{-channel}}\big|_{\max} = \frac{\mu_5 E_{\pi} N_c}{6\pi^2 f_{\pi}^2} \simeq 0.2 \times \left(\frac{\mu_5}{200\ \mathrm{MeV}}\right)\left(\frac{E_{\pi}}{1\ \mathrm{GeV}}\right)$$

Asymmetry in photon polarizations

$\pi^+\gamma \to a_0^{+*} \to \pi^+\gamma$



Asymmetry, $\mu_5 = 200$ MeV, $E_{\pi 2} = 1$ GeV

Extrapolation to higher masses: Linear Sigma Model with chiral imbalance for pions and scalar mesons

Let us compare the above constants with predictions of the Linear Sigma Model. It describes pions and extends chiral imbalance phenomenology on isosinglet and isotriplet scalar mesons.

A. A. Andrianov et al, Phys. Part. Nucl. Lett.15,357 (2018); EPJ Web of Conferences, 191, 05014 (2018). A. A. Andrianov, D. Espriu, and X. Planells, Eur. Phys. J. C, 73, 2294 (2013).

$$\begin{aligned} L &= N_c\Big\{\frac{1}{4} < (D_\mu H\,(D^\mu H)^\dagger > + \frac{B_0}{2} < m(H + H^\dagger > + \frac{M^2}{2} < HH^\dagger > \\ &- \frac{\lambda_1}{2} < (HH^\dagger)^2 > - \frac{\lambda_2}{4} < (HH^\dagger) >^2 + \frac{c}{2}(\det H + \det H^\dagger)\Big\}, \end{aligned}$$

where $H = \xi\,\Sigma\,\xi$ is an operator for meson fields, Σ includes the singlet scalar meson σ, its vacuum average v and the isotriplet of scalar mesons a_0^0, a_0^-, a_0^+, the covariant derivative of H contains the chiral chemical potential μ_5, ξ realizes a nonlinear representation of the chiral group $SU(2)_L \times SU(2)_R$, $\xi^2 = U$.

From spectral characteristics of scalar mesons in vacuum one fixes the Lagrangian parameters, $\lambda_1 = 16.4850$, $\lambda_2 = -13.1313$, $c = -4.46874 \times 10^4\,MeV^2$, $B_0 = 1.61594 \times 10^5\,MeV^2$

Mass spectrum

σ meson,

$$\tfrac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - \tfrac{1}{2}\,m_\sigma^2\sigma^2$$

Parity breaking mixture

Neutral meson sector,

$$\tfrac{1}{2}\,\partial_\mu a_0^0\,\partial^\mu a_0^0 + \tfrac{1}{2}\,\partial_\mu\pi^0\partial^\mu\pi^0 - \tfrac{1}{2}m_a^2\,(a_0^0)^2 - \tfrac{1}{2}m_\pi^2\,(\pi^0)^2 - 4\mu_5\,\dot\pi^0 a_0^0$$

Charged meson sector,

$$\partial_\mu a_0^-\,\partial^\mu a_0^+ + \partial_\mu\pi^-\partial^\mu\pi^+ - m_a^2 a_0^- a_0^+ - m_\pi^2\pi^-\pi^+ - 4\mu_5\,\dot\pi^+ a_0^- - 4\mu_5\,\dot\pi^- a_0^+$$

mass matrix
and
chiral condensate

$$\begin{cases} m_\sigma^2 = -2\left(M^2 - 6\,(\lambda_1+\lambda_2)v^2 + c + 2\mu_5^2\right) \\ m_a^2 = -2\left(M^2 - 2\,(3\lambda_1+\lambda_2)v^2 - c + 2\mu_5^2\right) \\ m_\pi^2 = \dfrac{2\,b\,m}{v} \\ v(\mu_5) = \sqrt{\dfrac{M^2+2\mu_5^2+c}{2(\lambda_1+\lambda_2)}} + \dfrac{b}{2(M^2+2\mu_5^2+c)}\,m \end{cases}$$

$$F_\pi^2(\mu_5) \approx \frac{M^2+c}{2(\lambda_1+\lambda_2)} + \frac{\mu_5^2}{(\lambda_1+\lambda_2)}$$

Comparison I

The change of pion coupling constant F_0 as compared to the ChPT definition,

$$\frac{\Delta F_\pi^2}{\mu_5^2} = \frac{1}{\lambda_1 + \lambda_2} \approx 0.3 \quad vs \quad 32(l_1 + l_2) \approx 0.26.$$

It is a quite satisfactory correspondence.
Analogously, in the rest frame using the pion mass correction, $m_\pi^2(\mu_5)F_\pi^2(\mu_5) \simeq 2m_q B_0 F_\pi(\mu_5)$ one finds the estimation for

$$l_4 \approx 2,64 \times 10^{-2} \quad vs \quad \frac{1}{8(\lambda_1 + \lambda_2)} \approx 3.8 \times 10^{-2},$$

wherefrom one guesses the relation $4(l_1 + l_2) \sim l_4$ following from the LSM.

Comparison II

For moving mesons with $|\vec{p}| \neq 0$ and the CP breaking mixing of scalar and pseudoscalar mesons the effective masses $m^2_{eff\mp}$ take the form,

$$m^2_{eff-} = \frac{1}{2}\left(16\,\mu_5^2 + m_a^2 + m_\pi^2 - \sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4\,(m_a^2\, m_\pi^2 - 16\mu_5^2\,|\vec{p}|^2)}\ \right),$$

$$m^2_{eff+} = \frac{1}{2}\left(16\,\mu_5^2 + m_a^2 + m_\pi^2 + \sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4\,(m_a^2\, m_\pi^2 - 16\mu_5^2\,|\vec{p}|^2)}\ \right).$$

For small $\mu_5^2, m_\pi^2 \ll m_a^2 \simeq 1 GeV^2$ one can approximate the dependence on the wave vector $\vec{p}$

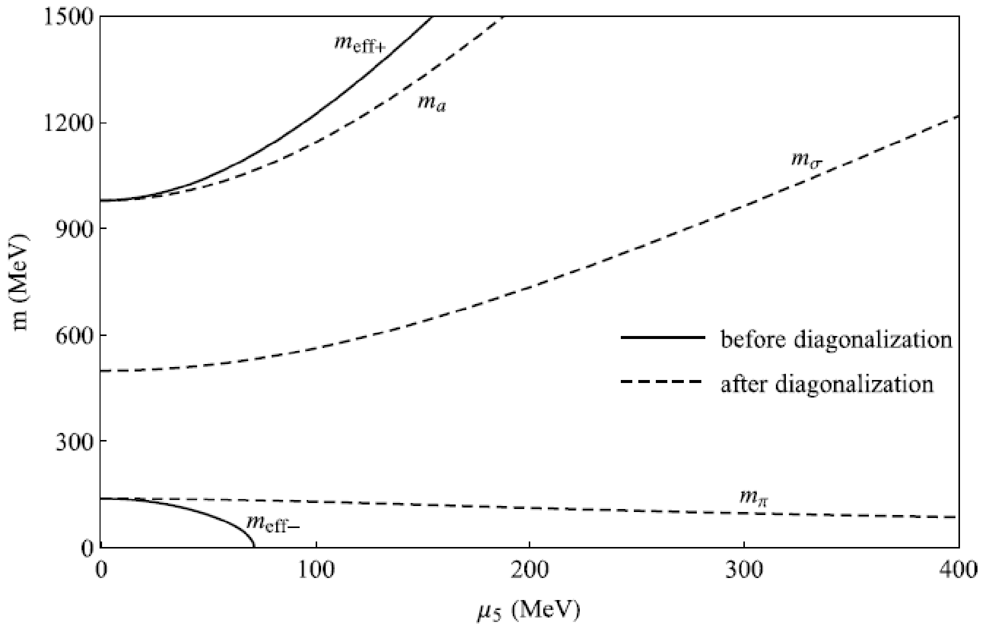
$$m^2_{eff-} \simeq m_\pi^2 - 16\mu_5^2\,\frac{|\vec{p}|^2}{m_a^2}.$$

Then the relationship of isotriplet scalar mass and GL constants is

$$m_a = \frac{F_0}{\sqrt{l_1 + l_2}} \simeq 1 GeV,$$

which confronts the PDG value with a remarkable precision.

Mass spectrum in chiral imbalanced medium



$$??\pi^{\pm} \to \mu^{\pm}\bar{\nu}$$

Conclusions and outlook

1. Topological charge fluctuations transmit their influence from QGP to hadron physics via chiral chemical potential: in this way local parity breaking (LPB) occurs in hadron sector

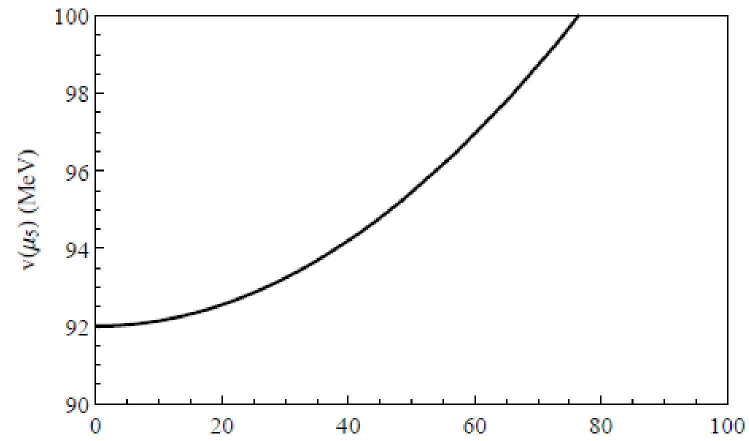
2. The constants of Chiral Perturbation Theory may enhance predictivity of low-energy pion dynamics in the chiral imbalanced medium

3. LPB modifies dispersion laws for scalar (and vector) mesons: lightest "pseudoscalar" mesons decrease masses in flight

4. <u>There exist observables unambiguously indicating LPB</u>
   (STAR, ALICE LHC, NICA, FAIR):
   suppression of charged pion decays into leptons,
   asymmetry in pion polarizabilities,
   exotic scalar/pseudoscalar meson decays breaking space parity
   etc.

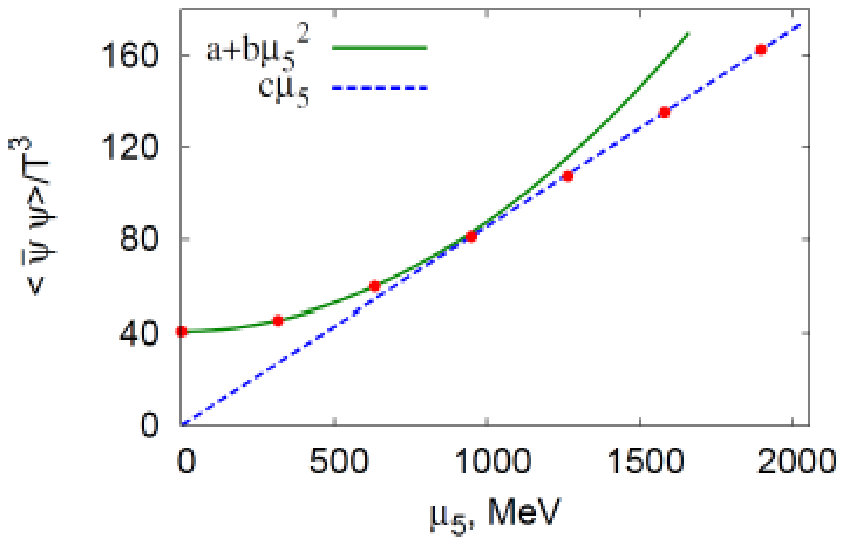
In the above model **quark condensate is governed by the decay constant v**

T = 0



SU(2) effective Lagrangian
this talk

T = 158 MeV



NJL model
V.Braguta, A.Kotov.
PRD, 93,105025(2016)

SU(2) QCD
V.Braguta et al
JHEP 06 (2015) 094)

Extended chiral lagrangian in the chiral imbalance background

ANNALS OF PHYSICS **158**, 142–210 (1984) J.Gasser, H. Leutwyler

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{tr}\left[d_\mu U^\dagger d^\mu U + \chi^\dagger U + \chi U^\dagger\right] + C\,\mathrm{tr}\left[Q_R U Q_L U^\dagger\right]$$

$$d_\mu U = \partial_\mu U - i(v_\mu + Q_R A_\mu + a_\mu)U + iU(v_\mu + Q_L A_\mu - a_\mu)$$

External e.m.charges

$$\mathcal{L}_{p^4} = \frac{l_1}{4}\langle d^\mu U^+ d_\mu U\rangle^2 + \frac{l_2}{4}\langle d^\mu U^+ d^\nu U\rangle\langle d_\mu U^+ d_\nu U\rangle$$
$$+\frac{l_3}{16}\langle \chi^+ U + U^+\chi\rangle^2 + \frac{l_4}{4}\langle d^\mu U^+ d_\mu \chi + d^\mu \chi^+ d_\mu U\rangle$$
$$+l_5\langle G^{\mathrm{R}}_{\mu\nu} U G^{\mathrm{L}\,\mu\nu} U^+\rangle + \frac{il_6}{2}\langle G^{\mathrm{R}}_{\mu\nu} d^\mu U d^\nu U^+ + G^{\mathrm{L}}_{\mu\nu} d^\mu U^+ d^\nu U\rangle$$
$$-\frac{l_7}{16}\langle \chi^+ U - U^+\chi\rangle^2 + \tfrac{1}{4}(h_1 + h_3)\langle \chi^+\chi\rangle$$
$$+\tfrac{1}{2}(h_1 - h_3)\,\mathrm{Re}\,(\det\chi) - h_2\langle G^{\mathrm{R}}_{\mu\nu} G^{\mathrm{R}\,\mu\nu} + G^{\mathrm{L}}_{\mu\nu} G^{\mathrm{L}\,\mu\nu}\rangle.$$

One loop renlormalization of chiral lagrangian of G-L

$$l_i = l_i^r + \gamma_i \lambda, \qquad i = 1, \ldots, 7$$

$$h_i = h_i^r + \delta_i \lambda, \qquad i = 1, 2, 3$$

$$\lambda = (4\pi)^{-2} \mu^{d-4} \left\{ \frac{1}{d-4} - \frac{1}{2} (\ln 4\pi + \Gamma'(1) + 1) \right\}$$

$$\gamma_1 = \tfrac{1}{3}, \quad \gamma_2 = \tfrac{2}{3}, \quad \gamma_3 = -\tfrac{1}{2}, \quad \gamma_4 = 2, \quad \gamma_5 = -\tfrac{1}{6}, \quad \gamma_6 = -\tfrac{1}{3}, \quad \gamma_7 = 0$$

$$\delta_1 = 2, \quad \delta_2 = \tfrac{1}{12}, \quad \delta_3 = 0.$$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i^r(\mu) - \ln \frac{M_\pi^2}{\mu^2}.$$

For the subtraction scale $\mu = 0.77$ GeV,

$$-\ln \frac{M_\pi^2}{\mu^2} = 3.42$$

Fits of constants

G. Colangelo, J. Gasser, H. Leutwyler   Nuclear Physics B 603 (2001) 125–179

$$\bar{\ell}_1 = -0.4 \pm 0.6, \qquad \bar{\ell}_2 = 4.3 \pm 0.1. \qquad \bar{\ell}_4 = 4.4 \pm 0.2.$$

Aoki S. et al FLAG working group arXiv:1607.00299 [hep-lat]
give similar results for $\bar{l}_4$

From resonance saturation G.~Ecker, J.~Gasser, A.~Pich and E.~de Rafael, Nucl.\ Phys.\ B {\bf 321}, 311 (1989).

$$\bar{\ell}_1 \simeq -0.7, \qquad \bar{\ell}_2 \simeq 5.0, \qquad \bar{\ell}_3 \simeq 1.9, \qquad \bar{\ell}_4 \simeq 3.7,$$

Effective meson theory in a medium with LPB

- Vector mesons

Low energy QCD can be described with the help of Vector Meson Dominance

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega \omega_\mu \mathbb{I} + \frac{1}{2}g_\rho \rho^0_\mu \tau_3,$$

$$(V_{\mu,a}) \equiv (A_\mu,\, \omega_\mu,\, \rho^0_\mu)$$

where $Q = \frac{\tau_3}{2} + \frac{1}{6}$, $g_\omega \simeq g_\rho \equiv g \simeq 6.$
In this framework, the following term is generated in the effective lagrangian for vector mesons

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma}\,\text{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]$$

with $\hat{\zeta}_\mu = \hat{\zeta}\delta_{\mu 0}$ for a spatially homogeneous and isotropic background ($\hat{}\ \equiv$ isospin content) and $\zeta \propto \mu_5$.

Vector Meson spectrum in PB medium

After diagonalization of mass matrix

$$m_{V,\epsilon}^2 = m_V^2 - \epsilon\zeta|\vec{k}| \quad \Longrightarrow \quad |\zeta|,$$

where $\epsilon = 0, \pm 1$ is the meson polarization.
The photon itself happens to be unaffected by a **singlet** $\hat{\zeta}$.
The position of the poles for $\pm$ polarized mesons is changing with wave vector $|\vec{k}|$.
Massive vector mesons split into three polarizations with masses $m_{V,+}^2 < m_{V,L}^2 < m_{V,-}^2$.

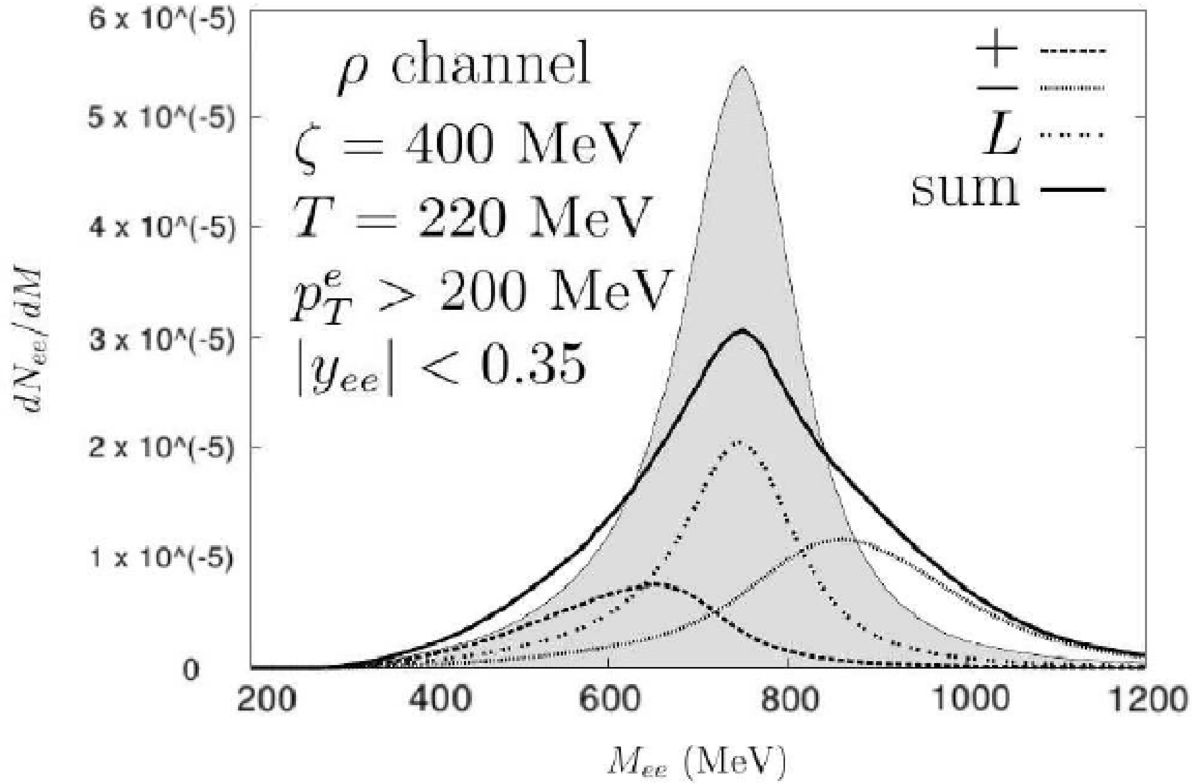
This splitting unambiguously signifies LPB. Can it be measured?

$\rightarrow$ dilepton production in HIC from the decays $\rho, \omega \rightarrow e^+e^-$

More details in
A.A., V.A. Andrianov's, D. Espriu and X.Planells, Phys. Lett. B 684 (2010) 101; B 710 (2012) 230,…

Manifestation of LPB in heavy ion collisions

ρ spectral function

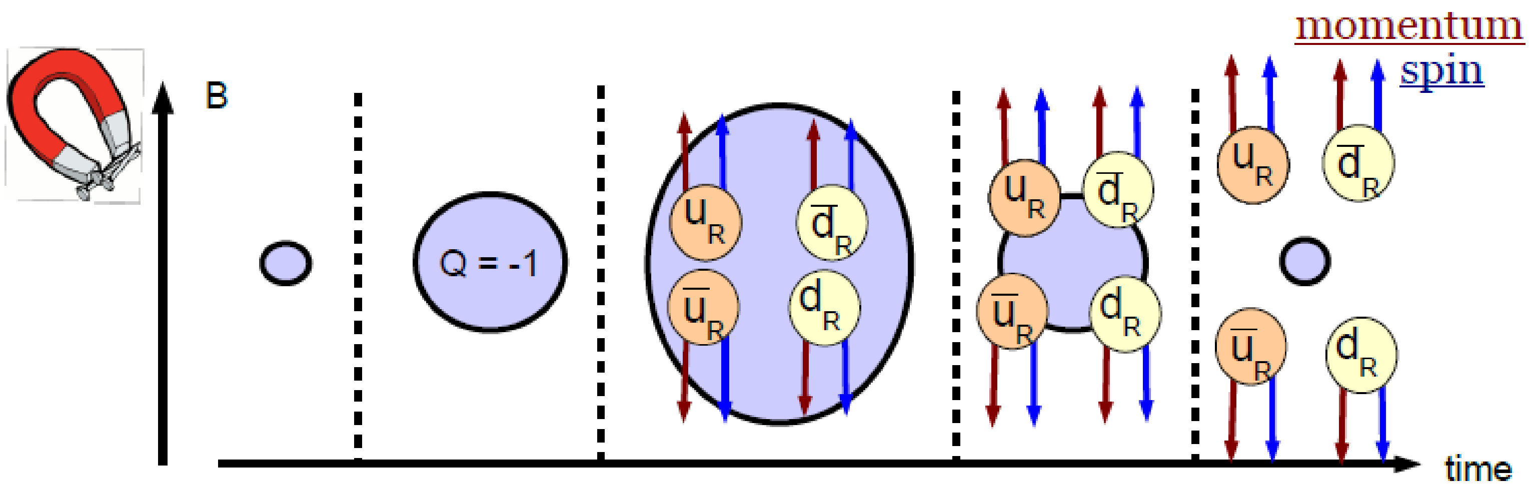


Polarization splitting in ρ spectral function for LPB $\zeta = 400$ MeV ($\mu_5 = 290$ MeV) compared with $\zeta = 0$ (shaded region).

Unfortunately the splitting is strongly contaminated by thermal effects

Chiral magnetic effect

Topological Charge + Magnetic field = Chirality + Polarization =



Q < -1: Positively charged particles move parallel to magnetic field, negatively charged antiparallel

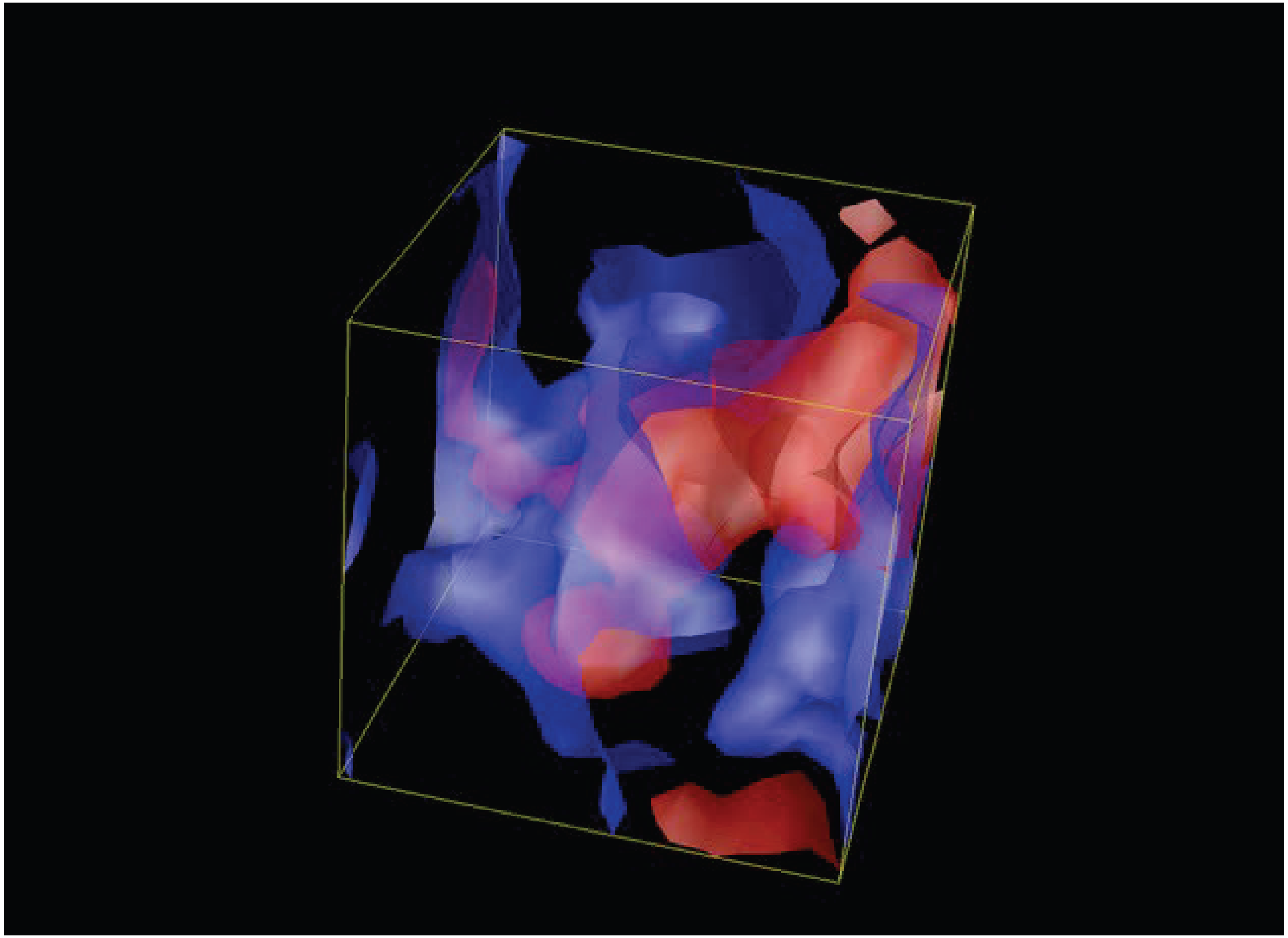
… = Electromagnetic Current

P- and CP-odd effect --> Chiral Magnetic Effect:

D.Kharzeev, L.McLerran, K.Fukushima, H.Warringa,…

Topological number fluctuations in QCD vacuum

ITEP Lattice Group



P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov

Extended chiral lagrangian with virtual photon loops

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$$\mathcal{L} = \mathcal{L}^0_{\mathrm{QCD}} + \mathcal{L}^0_\gamma + \bar{q}\gamma^\mu [v_\mu(x) + \gamma_5 a_\mu(x)] q - \bar{q}[s(x) - i\gamma_5 p(x)] q$$

$$+ A_\mu \bar{q}\gamma^\mu \left\{ Q_L(x) \left(\frac{1-\gamma_5}{2}\right) + Q_R(x) \left(\frac{1+\gamma_5}{2}\right) \right\} q.$$

$$
\begin{aligned}
\mathcal{L}_{e^2p^2} = F^2\{ & k_1\langle d^\mu U^+ d_\mu U\rangle\langle Q^2\rangle \\
& +k_2\langle d^\mu U^+ d_\mu U\rangle\langle QUQU^+\rangle \\
& +k_3\big(\langle d^\mu U^+ QU\rangle\langle d_\mu U^+ QU\rangle + \langle d^\mu UQU^+\rangle\langle d_\mu UQU^+\rangle\big) \\
& +k_4\langle d^\mu U^+ QU\rangle\langle d_\mu UQU^+\rangle \\
& +k_5\langle \chi^+ U + U^+\chi\rangle\langle Q^2\rangle \\
& +k_6\langle \chi^+ U + U^+\chi\rangle\langle QUQU^+\rangle
\end{aligned}
$$

$$
\begin{aligned}
& +k_7\langle(\chi U^+ + U\chi^+)Q + (\chi^+ U + U^+\chi)Q\rangle\langle Q\rangle \\
& +k_8\langle(\chi U^+ - U\chi^+)QUQU^+ + (\chi^+ U - U^+\chi)QU^+QU\rangle \\
& +k_9\langle d_\mu U^+[(c_R^\mu Q), Q]U + d_\mu U[(c_L^\mu Q), Q]U^+\rangle \\
& +k_{10}\langle(c_R^\mu Q)U(c_{L\mu}Q)U^+\rangle \\
& +k_{11}\langle(c_R Q)\cdot(c_R Q) + (c_L Q)\cdot(c_L Q)\rangle\},
\end{aligned}
$$

Type:	V	A	S	S_1	Total
$Z^R(M_\rho)$	−0.88	1.79	0	0	0.91$^{(*)}$
			Units: $\times 10^{-3}$		
$K_1^R(M_\rho)$	−4.6	−2.2	0.4	0	−6.4
$K_2^R(M_\rho)$	−4.9	2.2	−0.4	0	−3.1
$K_3^R(M_\rho)$	4.6	2.2	−0.1	−0.2	6.4
$K_4^R(M_\rho)$	−9.9	4.4	−0.2	−0.5	−6.2
$K_5^R(M_\rho)$	13.7	6.6	−0.4	0	19.9
$K_6^R(M_\rho)$	14.8	−6.6	0.4	0	8.6
$K_7^R \ldots K_{10}^R$	0	0	0	0	0
$K_{11}^R(M_\rho)$	2.8	−2.2	0	0	0.6
$K_{12}^R(M_\rho)$	−4.7	−4.4	0	0	−9.2
K_{13}^R	19.0$^{(*)}$	−4.7$^{(*)}$	0	0	14.2$^{(*)}$
K_{14}^R	0	2.4$^{(*)}$	0	0	2.4$^{(*)}$

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Large N_c in chiral perturbation theory*

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The covariant derivatives and the field strength te
are defined by

$$
\begin{aligned}
D_\mu U &= \partial_\mu U - \mathrm{i}(v_\mu + a_\mu)U + \mathrm{i}U(v_\mu - a_\mu),\\
D_\mu \psi &= \partial_\mu \psi - 2\langle a_\mu \rangle, \quad D_\mu \theta = \partial_\mu \theta + 2\langle a_\mu \rangle,\\
R_{\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - \mathrm{i}[r_\mu, r_\nu],\\
L_{\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - \mathrm{i}[l_\mu, l_\nu],
\end{aligned}
$$

where $r_\mu = v_\mu + a_\mu$ and $l_\mu = v_\mu - a_\mu$. The somewhat

$$U = \mathrm{e}^{(\mathrm{i}/3)\bar{\psi}}\bar{U}, \qquad \bar{\psi} = \psi + \theta, \qquad \det\bar{U} = \mathrm{e}^{-\mathrm{i}\theta}.$$

: convenient to define the covariant derivative of $\bar{U}$

$$
\begin{aligned}
D_\mu \bar{U} &= \partial_\mu \bar{U} - \mathrm{i}(v_\mu + \bar{a}_\mu)\bar{U} + \mathrm{i}\bar{U}(v_\mu - \bar{a}_\mu),\\
\bar{a}_\mu &= a_\mu - \frac{1}{3}\langle a_\mu \rangle - \frac{1}{6}\partial_\mu \theta = a_\mu - \frac{1}{6}D_\mu \theta,
\end{aligned}
$$

at $\langle \bar{U}^\dagger D_\mu \bar{U} \rangle = 0$. In this notation, the deriv
ıds

$$D_\mu U = \mathrm{e}^{(\mathrm{i}/3)\bar{\psi}} \left\{ D_\mu \bar{U} + \frac{\mathrm{i}}{3}(\partial_\mu \bar{\psi} - D_\mu \theta)\bar{U} \right\},$$

$$
\begin{aligned}
\mathcal{L}_B^{\mathrm{SU}_3} = & -\mathrm{i}L_{11}^{\mathrm{SU}_3} D_\mu\theta\langle \bar{U}^\dagger D^\mu \bar{U} D_\nu \bar{U}^\dagger D^\nu \bar{U}\rangle \\
& + L_{12}^{\mathrm{SU}_3} D_\mu\theta D^\mu\theta\langle D_\nu \bar{U}^\dagger D^\nu \bar{U}\rangle \\
& + L_{13}^{\mathrm{SU}_3} D_\mu\theta D_\nu\theta\langle D^\mu \bar{U}^\dagger D^\nu \bar{U}\rangle \\
& + L_{14}^{\mathrm{SU}_3} D_\mu\theta D^\mu\theta\langle \bar{U}^\dagger \chi + \chi^\dagger \bar{U}\rangle \\
& - \mathrm{i}L_{15}^{\mathrm{SU}_3} D_\mu\theta\langle D^\mu \bar{U}^\dagger \chi - D^\mu \bar{U}\chi^\dagger\rangle \\
& + \mathrm{i}L_{16}^{\mathrm{SU}_3} \partial_\mu D^\mu\theta\langle \bar{U}^\dagger \chi - \chi^\dagger \bar{U}\rangle \qquad (\\
& + \mathrm{i}L_{17}^{\mathrm{SU}_3} \epsilon^{\mu\nu\rho\sigma} D_\mu\theta\langle \bar{R}_{\nu\rho} D_\sigma \bar{U}\bar{U}^\dagger - \bar{L}_{\nu\rho}\bar{U}^\dagger D_\sigma \bar{U}\rangle
\end{aligned}
$$

$$
\begin{aligned}
L_{11}^{\mathrm{SU}_3} &= -4\left(L_2 + \frac{1}{3}L_3\right) + O(1), \\
L_{12}^{\mathrm{SU}_3} &= \frac{2}{3}\left(L_1 + \frac{1}{2}L_2 + \frac{1}{3}L_3\right) + O(1), \\
L_{13}^{\mathrm{SU}_3} &= \frac{4}{3}\left(L_2 + \frac{1}{3}L_3\right) + O(1), \\
L_{14}^{\mathrm{SU}_3} &= \frac{1}{3}(L_4 + 3L_5 + L_{18}) + O(1), \\
L_{15}^{\mathrm{SU}_3} &= \frac{1}{3}(2L_5 + 3L_{18}) + O(1), \\
L_{16}^{\mathrm{SU}_3} &= -F^4(1+\Lambda_1)(1+\Lambda_2)(72\tau)^{-1} + O(1), \\
L_{17}^{\mathrm{SU}_3} &= N_c(288\pi^2)^{-1} + \frac{1}{2}\tilde{L}_4 + O(N_c^{-1}),
\end{aligned}
$$

ant part $\bar{r}, \bar{l}$ and a remainder reads $r = \bar{r} + (1/6)D\theta$, $l = \bar{l} - (1/6)D\theta$. Using the identity $\langle \mathrm{d}\bar{U}\bar{U}^\dagger \rangle = -\mathrm{i}\mathrm{d}\theta$, which follows from $\det \bar{U} = \mathrm{e}^{-\mathrm{i}\theta}$, we then obtain

$$S_{\mathrm{WZW}}\{U, v, a\} = S_{\mathrm{WZW}}\{\bar{U}, v, \bar{a}\} + \int (A + B + P_1 + P_2),$$

$$A = -\frac{N_c}{144\pi^2}\bar{\psi}\{\langle \mathrm{i}\bar{F}_{\mathrm{R}} D\bar{U} D\bar{U}^\dagger + \bar{F}_{\mathrm{R}}\bar{U}\bar{F}_{\mathrm{L}}\bar{U}^\dagger + 2\bar{F}_{\mathrm{R}}^2 + (\mathrm{R} \leftrightarrow \mathrm{L})\rangle + \frac{1}{2}\langle F_{\mathrm{R}} + F_{\mathrm{L}}\rangle^2 + \frac{1}{6}\langle F_{\mathrm{R}} - F_{\mathrm{L}}\rangle^2\}.$$

$$B = \frac{N_c}{144\pi^2}\mathrm{i}D\theta\langle \bar{F}_{\mathrm{R}} D\bar{U}\bar{U}^\dagger - \bar{F}_{\mathrm{L}}\bar{U}^\dagger D\bar{U}\rangle,$$

$$\bar{F}_{\mathrm{R}} = F_{\mathrm{R}} - \frac{1}{3}\langle F_{\mathrm{R}}\rangle, \qquad \bar{F}_{\mathrm{L}} = F_{\mathrm{L}} - \frac{1}{3}\langle F_{\mathrm{L}}\rangle.$$

Note that terms proportional to $D\theta\langle (D\bar{U}\bar{U}^\dagger)^3\rangle$ cancel out on account of charge conjugation invariance. The term

$$\frac{N_c \epsilon^{\mu\nu\rho\sigma}}{288\pi^2}\left\{ -\mathrm{i}D_\mu\theta\langle \bar{R}_{\nu\rho} D_\sigma \bar{U}\bar{U}^\dagger - \bar{L}_{\nu\rho} D_\sigma \bar{U}^\dagger \bar{U}\rangle \right\}.$$

$\partial_\mu\theta. \longrightarrow \mu_5 \delta_{\mu 0}$

ref. [1]. In particular, the seven low-energy constants $l_1, \ldots, l_7$ and the three high-energy constants h_1, h_2 and h_3 which specify the general effective lagrangian of $SU(2)_L \times SU(2)_R$ at order p^4 can be expressed in terms of the parameters $L_1, \ldots, L_{10}$, H_1 and H_2 [2]:

$$
\begin{aligned}
l_1^r &= 4L_1^r + 2L_3 - \tfrac{1}{24}\nu_K\,, \\
l_2^r &= 4L_2^r - \tfrac{1}{12}\nu_K\,, \\
l_3^r &= -8L_4^r - 4L_5^r + 16L_6^r + 8L_8^r - \tfrac{1}{18}\nu_\eta\,, \\
l_4^r &= 8L_4^r + 4L_5^r - \tfrac{1}{2}\nu_K\,, \\
l_5^r &= L_{10}^r + \tfrac{1}{12}\nu_K\,, \\
l_6^r &= -2L_9^r + \tfrac{1}{6}\nu_K\,, \\
l_7 &= \frac{f^2}{8B_0 m_s}\left(1 + \tfrac{10}{3}\bar{\mu}_\eta\right) + 4\left(L_4^r - L_6^r - 9L_7 - 3L_8^r + \tfrac{1}{8}\nu_K\right), \\
h_1^r &= 8L_4^r + 4L_5^r - 4L_8^r + 2H_2^r - \tfrac{1}{2}\nu_K\,, \\
h_2^r &= -\tfrac{1}{4}L_{10}^r - \tfrac{1}{2}H_1^r - \tfrac{1}{24}\nu_K\,, \\
h_3^r &= 4L_8^r + 2H_2^r - \tfrac{1}{2}\nu_K - \tfrac{1}{3}\nu_\eta + \frac{1}{96\pi^2}\,,
\end{aligned}
\tag{B.1}
$$

$$\nu_P = \frac{1}{32\pi^2}\left(\ln\frac{\overline{M}_P^2}{\mu^2}+1\right), \qquad P = K, \eta,$$

$$\bar{\mu}_\eta = \frac{1}{32\pi^2}\frac{\overline{M}_\eta^2}{f^2}\ln\frac{\overline{M}_\eta^2}{\mu^2},$$

$$\overline{M}_K^2 = B_0 m_s, \qquad \overline{M}_\eta^2 = \tfrac{4}{3}\overline{M}_K^2.$$

TABLE 4

Values of low-energy constants $l_1, \ldots, l_7$ and total resonance contributions for $SU(2)_L \times SU(2)_R$. We did not work out an error for l_7. The individual resonance contributions are listed in table 5. The barred quantities are defined in eqs. (B.3) and (B.5).

	$10^3 \times l_i^r(M_\rho)$	[0.5 GeV, 1 GeV]	$10^3 \times \Sigma_P l_i^P$	$\bar{l}_i$	$\Sigma_P \bar{l}_i^P - \ln(M^2/M_\rho^2)$
l_1	-6.1 ± 3.9	[−5.2, −6.7]	−3.6	-2.3 ± 3.7	0.04
l_2	5.3 ± 2.7	[7.1, 4.2]	4.7	6.0 ± 1.3	5.7
l_3	0.9 ± 3.8	[−0.5, 1.7]	1.4	2.9 ± 2.4	2.6
l_4	3.4 ± 5.7	[8.8, 0.1]	5.5	4.3 ± 0.9	4.4
l_5	-5.2 ± 0.3	[−5.7, −5.0]	−6.0	13.4 ± 0.5	14.8
l_6	-13.7 ± 0.3	[−14.7, −13.2]	−13.7	16.5 ± 1.1	16.5
l_7	7.1		4.5		

Table 5

Resonance contributions l_i^P and $\bar{l}_i^P$ evaluated from table 3 and eq. (B.1).

l_i	η, K	V	A	S	S_1	η_1
$10^3 \times l_1^P$	~0	−4.7	0	0.4	0.7	0
$\bar{l}_1^P$	~0	−4.5	0	0.5	0.7	0
$10^3 \times l_2^P$	~0	4.7	0	0	0	0
$\bar{l}_2^P$	~0	2.2	0	0	0	0
$10^3 \times l_3^P$	−0.1	0	0	0.5	1.0	0
$\bar{l}_3^P$	~0	0	0	−0.3	−0.6	0
$10^3 \times l_4^P$	−0.1	0	0	1.9	3.7	0
$\bar{l}_4^P$	~0	0	0	0.3	0.6	0
$10^3 \times l_5^P$	~0	−10.0	4.0	0	0	0
$\bar{l}_5^P$	~0	18.9	−7.6	0	0	0
$10^3 \times l_6^P$	~0	−13.8	0	0	0	0
$\bar{l}_6^P$	~0	13.0	0	0	0	0
$10^3 \times l_7$	3.7	0	0	−11.2	0.7	11.3