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The brachistochrone problem as a minimum path on the graph

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The brachistochrone problem was posed by Johann Bernoulli in Acta Eruditorum in June 1696. He introduced the problem as follows [1].

Given two points A and B in a vertical plane. What is the curve traced out by a point acted on only by gravity, which starts at A and reaches B in the shortest time.

The modern method to solve this problem is the method of calculus of variations exclusively [2].

The proposed report presents a graphical method of solution of the brachistochrone problem. The method consists of constructing a number of curves connecting points A and B. The minimal time of sliding along every of curve is calculated, and the minimal time curve is taken as the solution of the problem. The curves are searching as the broken lines. The minimum time broken line is searched as the minimal path on a special graph. The nodes of this graph are the nodes of a rectangular grid superimposed on the domain of the plane. The domain contains the points A and B. This graph is similar to graph from [3]. But now the weight of the arc is the time to move from one node to another. The special rule determines for each node a set of adjacent nodes. This rule uses the mutually simple numbers. The power of set may be 8, 16, 32, 48, 80, The minimal path is searched by means of the Dijkstra algorithm [4], adapted to this task. If the power of that set tends to infinity, then the minimal path tends to a smooth curve. If the power of that set tends to infinity, then the minimal path tends to a smooth curve. The paper [5] gives formula to calculate the proximity in case the curve is straight line and power of set is finite. This procedure replaces the continuous variational problem with the problem of discrete optimization. The closeness of the discrete problem solution to the solution of the continuous problem is defined. The proximity of the discrete problem solution to the continuous problem solution is determined. by means of the concept of proximity k-th order for curves and $y = [2]$. The proximity of the zero-order is determined through the distance from points of one curve to another curve. The proximity of the first order, is determined through that distance and at the same time through the difference in the length of these curves.

References

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