Presentation: “Mirror QCD and Cosmological Constant”

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In collaboration with

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Structure

- *Vacuum catastrophe, history of the problem.*
- *QCD ground state and cosmology, general review.*
- *Mirror QCD (mQCD).*

} *Background*

- *Vacuum compensation from positive beta-function.*
- *mQCD as the basis for vacuum compensation scenario. Discussion.*
- *Effective Yang-Mills theory in the expanding Universe: exact quasiclassical solution with positive beta-function.*
- *Main Results*

Vacuum catastrophe

Problem

"Astronomical observations indicate that the cosmological constant is many orders of magnitude smaller than estimated in modern theories of elementary particles." (Steven Weinberg, Rev.Mod.Phys. 61 (1989) 1-23 UTTG-12-88).

Illustration

Observation

$$|\rho_V| \lesssim 10^{-29}\ \mathrm{g/cm^3} \approx 10^{-47}\ \mathrm{GeV^4}$$

1. **Supernovae Ia** *(S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [astro-ph/9812133];)* .
2. **Cosmic microwave background anisotropies** *(E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]])* .
3. **Large scale structure** *(U. Seljak et al. [SDSS Collaboration], Phys. Rev. D 71, 103515 (2005) [astro-ph/0407372])* .

Theoretical prediction

Scalar field

$$\langle\rho\rangle = \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3}\cdot\frac{1}{2}(k^2+m^2)^{1/2} \approx \frac{\Lambda^4}{16\pi^2}$$

Plank scale

$$\Lambda \approx (8\pi G)^{-1/2}$$

$$\langle\rho\rangle \approx 2^{-10}\pi^{-4}G^{-2} = 2\times10^{71}\ \mathrm{GeV^4}$$

QCD vacuum contribution

$$\epsilon^{\rm QCD} \equiv \frac{1}{4}\langle 0|T_\mu^{\mu,\rm QCD}|0\rangle \simeq -(5\pm1)\times 10^9\ \mathrm{MeV^4}$$

$$\epsilon_{\rm CC} > 0\,, \qquad \left|\frac{\epsilon_{\rm CC}}{\epsilon^{\rm QCD}}\right| \simeq 10^{-44}$$

10^{118}, 10^{44} times difference between theory and observation and opposite sign in QCD case

Solutions

1. Additional fine tuned λ – term. → *0 vacuum energy in unbroken phase and huge positive contribution in the broken one*
2. Compensation in Supersymmetric theories (due to boson-fermion symmetry). → *V=0 is non-stationary point*
3. Supergravity (due to special set of scalar fields V=0 in broken phase). → *Conservation in higher perturbation orders*
4. Superstrings (Kahler potentials appear → stabilise V=0 point).
5. Modified gravity (Constant of integration). → *Needs fine-tuning*
6. $\Delta\epsilon_{\rm vac} \equiv \epsilon_{\rm FLRW} - \epsilon_{\rm Mink}$ J. Sola, J. Phys. Conf. Ser. **453**, 012015 (2013). → *Needs compensation mechanism*

Don't compensate QCD vacuum contribution

Vacuum compensation problem on the QCD scale remains a debated issue in cosmology

QCD ground state and cosmology

Many approaches to describe QCD vacuum

1. Instanton ensemble. *(Edward V. SHURYAK, THEORY AND PHENOMENOLOGY OF THE QCD VACUUM, PHYSICS REPORTS (Review Section of Physics Letters) 115, Nos. 4 & 5 (1984) 151—314. North-Holland, Amsterdam)*

Predicts QCD vacuum contribution to the early Universe after deconfinement phase transition $\Lambda_{\rm inst} \equiv \varepsilon_{vac(top)} \simeq -(5 \pm 1) \times 10^9\,{\rm MeV}^4$

2. Lattice calculations. *(Shuryak, E.V. Rev.Mod.Phys. 65 (1993) 1-46)*
3. FRG based approach. *(Fischer, C. S. et al. Annals Phys. 324 (2009) 2408-2437 arXiv:0810.1987 [hep-ph])*

The same infrared behaviour of gluon propagator

Confirms Savvidy vacuum model

4. **Savvidy vacuum.** *(S. G. Matinyan and G. K. Savvidy, Nucl. Phys. B 134, 539 (1978), H. Pagels and E. Tomboulis, Nucl. Phys. B 143, 485 (1978))*
5. ...

Savvidy vacuum

Ovsyannlkov-Callan-Symanzik equation for effective YM action

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(g)\frac{\partial}{\partial g} + \gamma(g)\int d^4x\, \mathcal{A}_\mu \frac{\delta}{\delta \mathcal{A}_\mu}\right]\Gamma = 0$$

Vacuum polarisation effects define effective YM Lagrangian

$$L_{\rm YM} = -\frac{11}{128\pi^2}\frac{F^a_{\mu\nu}F^{\mu\nu}_a}{\sqrt{-g}}\ln\left(\frac{J}{\Lambda^4_{\rm QCD}}\right), \qquad J = \frac{1}{\xi^4}\frac{|F^a_{\alpha\beta}F^{\alpha\beta}_a|}{\sqrt{-g}}.$$

Non-trivial vacuum field

$$(F^a_{\mu\nu})^2 = \lambda^4 \Rightarrow T^{\nu,YM}_{\mu} = \delta^\nu_\mu C$$

Application to inflation λ≈10^16 GeV (GUT) and DE λ≈10^-3 eV (Mirror SM) *(Y. Zhang, Phys. Lett. B 340 (1994) 18. doi:10.1016/0370-2693(94)91291-2, P. Dona, A. Marciano, Y. Zhang and C. Antolini, arXiv:1509.05824 [gr-qc].)*

Mirror QCD (mQCD).

Mirror QCD

Class of the models with additional non-Abelian gauge group *(M.J. Strassler, K.M. Zurek, Phys. Lett. B 651, 374 (2007) doi:10.1016/j.physletb.2007.06.055 [hep-ph/0604261]; Z. Chacko, D. Curtin and C. B. Verhaaren, Phys. Rev. D 94, no. 1, 011504 (2016) doi:10.1103/PhysRevD.94.011504 [arXiv:1512.05782 [hep-ph]].)*

Supersymmetric models (folded supersymmetry FSUSY)

stops Have EW charge, but neutral over SM color

Quirky little Higgs models

quirks with "infracolor" compensate top loops

Twin Higgs models

Twin copy of SM

TeV extra dimensions, string theory and so on

Main features

1. Suppressed interaction with SM particles: through loops of heavy particles with botn charges.
2. Hidden valley of light mQCD particles.
3. Different gauge groups, often mirror QCD gauge group.
4. Usually high scale - 1 GeV < Λ_V < 1 TeV
5. In confinment: several v-hadrons and v-glueballs.

Mirror QCD (mQCD).

Particular case

(Z. Chacko, D. Curtin and C. B. Verhaaren, Phys. Rev. D 94, no. 1, 011504 (2016) doi:10.1103/PhysRevD.94.011504 [arXiv:1512.05782 [hep-ph]].)

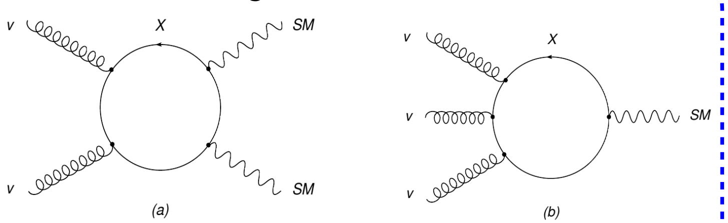
No light quarks:

1. Stable glueballs.
2. Top partners mass below few TeV.
3. Glueball mass 10-60 GeV.
4. Glueball mixes with Higgs and decay to pair of SM heavy quarks.

Experimental signatures

1. Can be looked for on the colliders.
2. *Displaced vertices*
3. Stable v-hadrons: missing energy and Dark Matter
4. Can have different life-time.
5. Different final states: two SM fermions + v-hadron, ggg, WW, ZZ

Possible diagrams:



From J.E. Juknevich, D. Melnikov and M.J. Strassler, JHEP 0907 (2009) 055 doi:10.1088/1126-6708/2009/07/055

Vacuum compensation from positive beta-function.

Gluon anomaly induced vacuum contribution

$$\epsilon^{\rm QCD} \equiv \frac{1}{4}\langle 0|T^{\mu,{\rm QCD}}_{\mu}|0\rangle\,, \qquad T^{\mu,{\rm QCD}}_{\mu} = \frac{\beta(\bar{g}_s^2)}{2} F^a_{\mu\nu}F^{\mu\nu}_a$$

One loop beta-function:

$$\beta(\bar{g}_s^2) = -\frac{b\bar{g}_s^2}{16\pi^2} < 0\,, \qquad \bar{g}_s^2 = \frac{16\pi^2}{b\ln(Q^2/\Lambda^2_{\rm QCD})}\,, \qquad \langle F^2{}_{\rm QCD}\rangle > 0$$

Big negative QCD vacuum contribution:

$$\epsilon^{\rm QCD} = -\frac{b}{32}\langle 0|\frac{\alpha_s}{\pi}F^a_{\mu\nu}F^{\mu\nu}_a|0\rangle \simeq -(5\pm 1)\times 10^9\ {\rm MeV}^4$$

Compensation needed $\epsilon^{\rm QCD} \simeq -\epsilon^{\rm mQCD}$

As mQCD gluons are singlet over SM color – mQCD vacuum doesn't change SM hadron properties

Vacuum compensation from positive beta-function.

Compensation can be reached if:

$$\epsilon^{\mathrm{mQCD}} > 0\,, \qquad \langle F^2_{\mathrm{mQCD}} \rangle > 0\,, \qquad \beta(\bar{g}^2) > 0$$

Whether mQCD beta-function can become positive – non-monotonic coupling behaviour ?

(M. Baldicchi, A. V. Nesterenko, G. M. Prosperi, D. V. Shirkov and C. Simolo, Phys. Rev. Lett. 99, 242001 (2007) doi:10.1103/PhysRevLett.99.242001 [arXiv:0705.0329 [hep-ph]]; D. Shirkov, arXiv:0807.1404 [hep-ph].)

mQCD in deeply nonperturbative regime on the SM hadron scale (corresponding to QCD phase transition temperature)

Can become possible due to → $\Lambda_{\mathrm{mQCD}} \gg \mu_g \simeq 1.2\ \mathrm{GeV}$

Effective Yang-Mills theory in the expanding Universe: exact quasiclassical solution with positive beta-function.

Concrete model of mQCD vacuum, leading to the supposed compensation scenario:

Conformal coordinates are used

$$S_{\rm eff}[\mathcal{A}] = \int \mathcal{L}_{\rm eff}\sqrt{-g}\,d^4x\,, \qquad \mathcal{L}_{\rm eff} = \frac{J}{4\bar{g}^2(J)}\,, \qquad J = -\frac{\mathcal{F}^2}{\sqrt{-g}}$$

1. Coupling depends on field gauge invariant J.
2. Like in Savvidy vacuum model, but don't use perturbative beta-function.

RG-equation

$$2J\frac{d\bar{g}^2}{d(J)} = \bar{g}^2\,\beta(\bar{g}^2)$$

Defines coupling running

Reproduces gluon anomaly

$$T^{\nu}_{\mu} = \frac{1}{\bar{g}^2}\left[1-\frac{1}{2}\beta(\bar{g}^2)\right]\left(-\frac{\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a}{\sqrt{-g}} - \frac{1}{4}\delta^{\nu}_{\mu}J\right) - \frac{\delta^{\nu}_{\mu}\beta(\bar{g}^2)}{8\bar{g}^2}J$$

Traceless part

Effective Yang-Mills theory in the expanding Universe: exact quasiclassical solution with positive beta-function.

YM equation of motion from general effective Lagrangian. *(H. Pagels and E. Tomboulis, Nucl. Phys. B 143, 485 (1978))*

$$\left(\frac{\delta^{ab}}{\sqrt{-g}}\partial_\nu\sqrt{-g} - f^{abc}\mathcal{A}^c_\nu\right)\left[\frac{\mathcal{F}^{\mu\nu}_b}{\bar{g}^2\sqrt{-g}}\left(1-\frac{1}{2}\beta(\bar{g}^2)\right)\right] = 0$$

In quasiclassical approximation

Non-fixed form of beta-function in non-perturbative case

If beta-function becomes positive and big in non-perturbative region, solution can exist

$$\beta\left(\bar{g}^2(J)\right) = 2$$

The necessary positivness corresponds to exact analytic solution

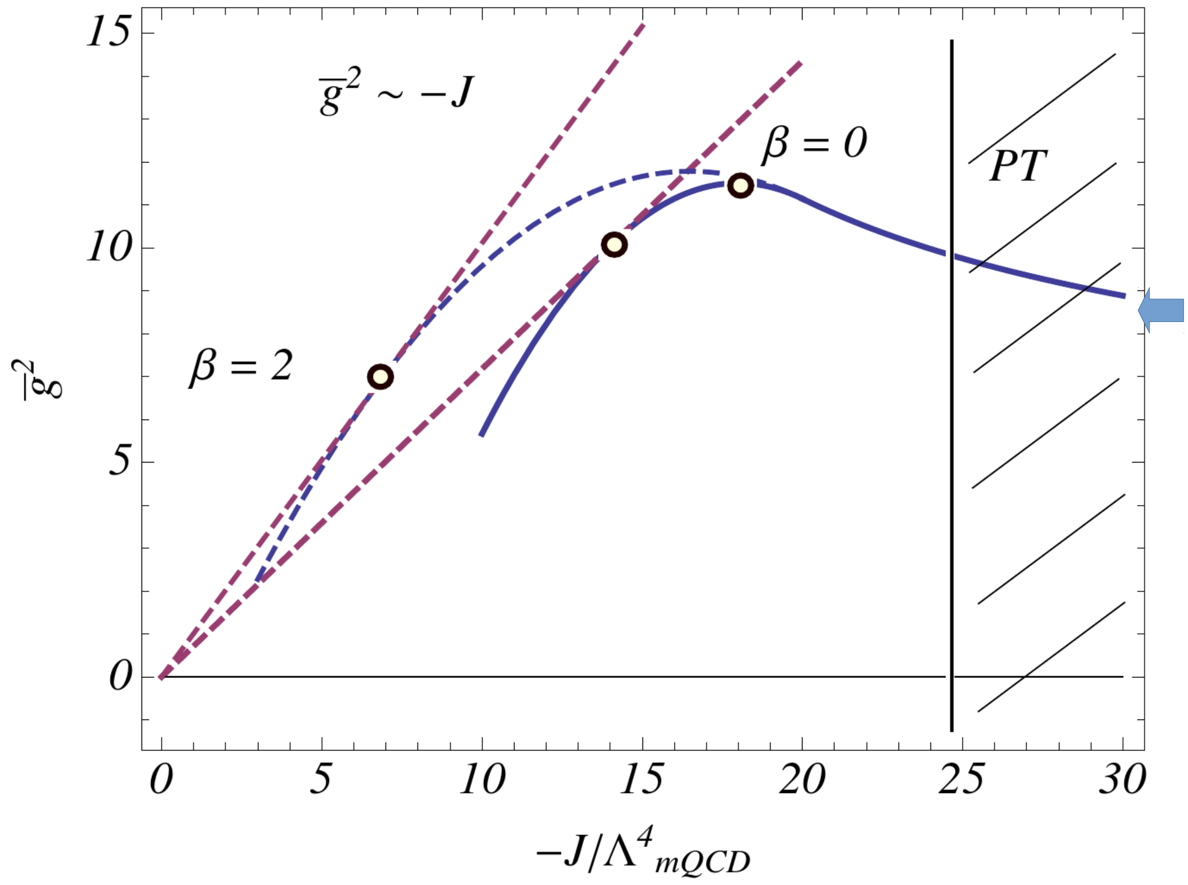
$$J(t) = J(t=0) = J_0$$

Constant field solution *(H. Pagels, E. Tomboulis, Nucl. Phys. B 143, 485 (1978), S. N. Nedelko, V. E. Voronin, Phys. Rev. D 93 (2016) no.9, 094010 doi:10.1103/PhysRevD.93.094010 [arXiv:1603.01447 [hep-ph]].)*

Vacuum compensation can be reached in nonperturbative case.

Effective Yang-Mills theory in the expanding Universe: exact quasiclassical solution with positive beta-function.

Prediction for the running mQCD coupling behaviour.



Leads to the existence of exact analytic solution.

Effective Yang-Mills theory in the expanding Universe: exact quasiclassical solution with positive beta-function.

Compensation condition

Fine-tuning is still necessary

$$\epsilon^{\rm QCD} \simeq \frac{J_0^{\rm mQCD}}{4\bar{g}_0^2} < 0\,, \qquad J_0^{\rm mQCD} < 0$$

Universe expansion after the compensation

Contribution of other vacuum subsystems (can be produced from graviton-mediated interactions)

$$\frac{3}{\varkappa}\frac{(a')^2}{a^4} = \epsilon_{\rm mat} + \epsilon_{\rm CC}\,, \qquad \epsilon_{\rm CC} \equiv \epsilon^{\rm QCD} + \epsilon^{\rm mQCD} + \epsilon_{\rm vac}$$

Friedmann equation

compensate each other (exactly or partly)

Non-constant fields, additional solution

$$\frac{d\ln \bar{g}^2}{d\ln\big(-J/(\xi\Lambda_{\rm mQCD})^4\big)} = \frac{1}{2}\beta\big(\bar{g}^2\big) = 1$$

Saturated beta-function, frozen near constant value 2

$$\bar{g}^2(J) = \bar{g}_0^2\,\frac{J}{J_0}\,, \qquad \bar{g}_0^2 \equiv \bar{g}^2(J_0)\,, \qquad J_0 = J(t=0)\,.$$

The same cosmological predictions

Main results

- **The mechanism of QCD vacuum compensation on the basis of Savvidy vacuum fluctuations of mQCD fields was suggested.**
- **Compensation realizes at the expense of change of beta-function sign.**
- **The necessary qualitative form of the nonperturbative coupling**

  $\overline{g}^{\,2}(J)$ is constructed.
- **The solution with non-constant fields is suggested.**

Thank you for attention!

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$$2J\frac{dg^2_{\rm YM}}{dJ} = g^2_{\rm YM}\,\beta(g^2_{\rm YM})$$

$$\beta(g^2_{\rm YM}) = -\frac{b g^2_{\rm YM}}{16\pi^2}\,, \qquad g^2_{\rm YM} = \frac{32\pi^2}{b\ln(|J|/\lambda^4)}$$

$$e^a_i \mathcal{A}^a_k \equiv \mathcal{A}_{ik}\,, \qquad e^a_i e^a_k = \delta_{ik}\,, \qquad e^a_i e^b_i = \delta_{ab}\,.$$

$$\frac{6}{\varkappa}\frac{a''}{a^3} = (\epsilon - 3p) + 4\bar{\Lambda} + T^{\mu,{\rm YM}}_{\mu}\,, \qquad T^{\mu,{\rm YM}}_{\mu} = \frac{3b}{16\pi^2 a^4}\left[(U')^2 - \frac{1}{4}U^4\right]$$

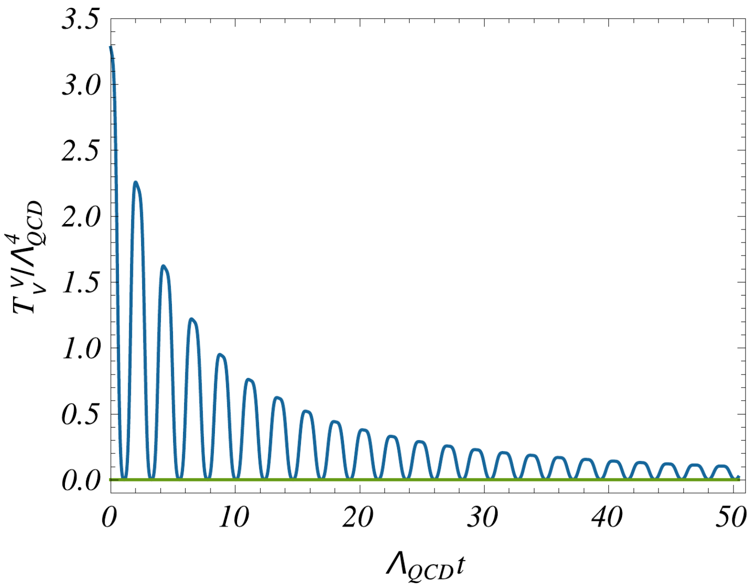
$$\frac{\partial}{\partial\eta}\left(U'\,\ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4(\xi\lambda)^4}\right) + \frac{1}{2}U^3\,\ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4(\xi\lambda)^4} = 0\,,$$

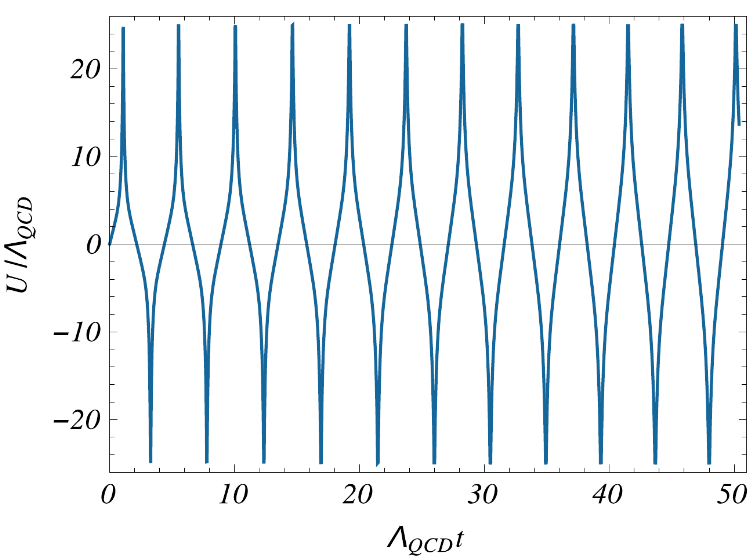
$$\frac{3}{\varkappa}\frac{(a')^2}{a^4} = \epsilon + \bar{\Lambda} + T^{0,{\rm YM}}_{0}\,,$$

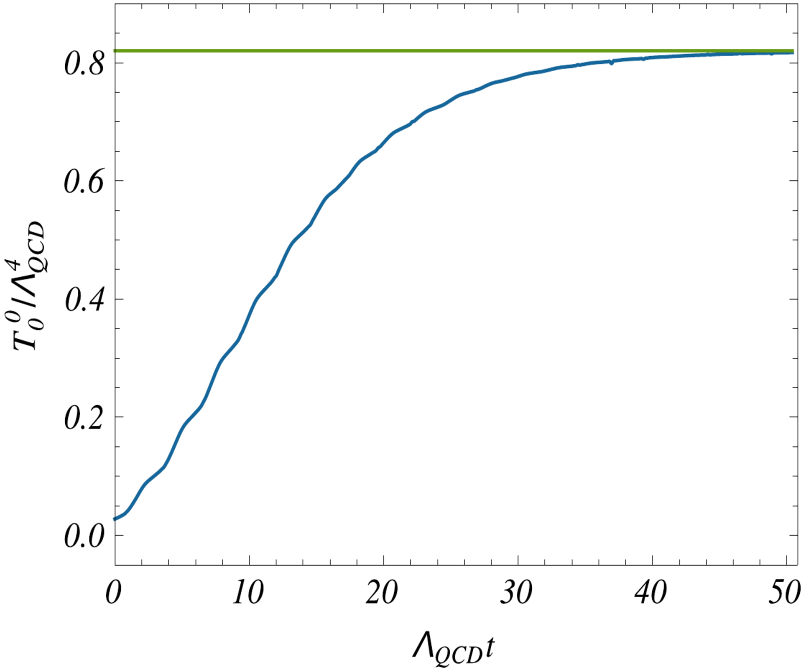
$$T^{0,{\rm YM}}_{0} = \frac{3b}{64\pi^2 a^4}\left(\left[(U')^2 + \frac{1}{4}U^4\right]\,\ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4(\xi\lambda)^4} + (U')^2 - \frac{1}{4}U^4\right)$$

$$|Q| \equiv \frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4(\xi\lambda)^4} = 1$$

$$\int_{\widetilde{U}_0}^{\widetilde{U}} \frac{du}{\sqrt{\frac{1}{4}u^4 - 1}}$$







0.0
0.2
0.4
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$T_0^{\,0}/\Lambda_{QCD}^4$
0
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20
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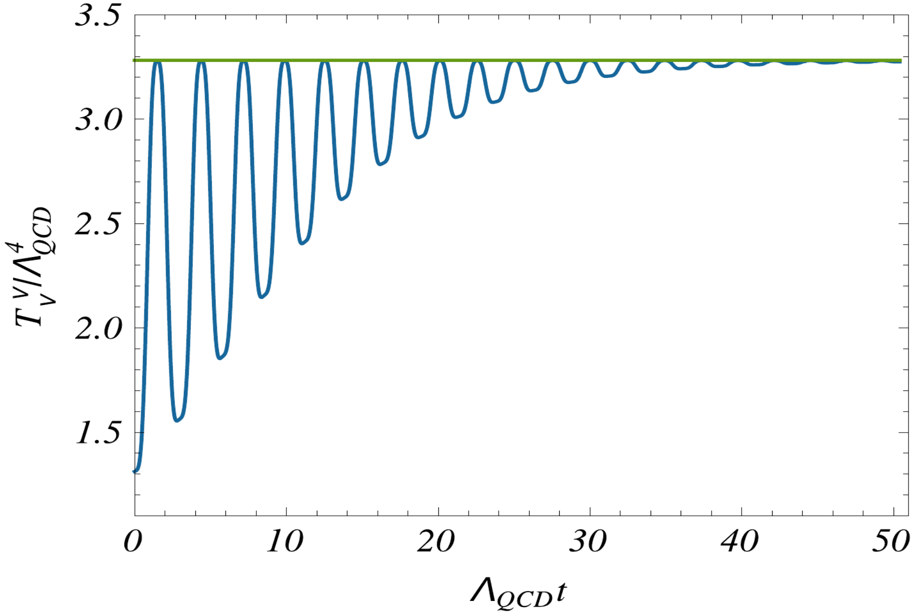
$\Lambda_{QCD}t$



1.5
2.0
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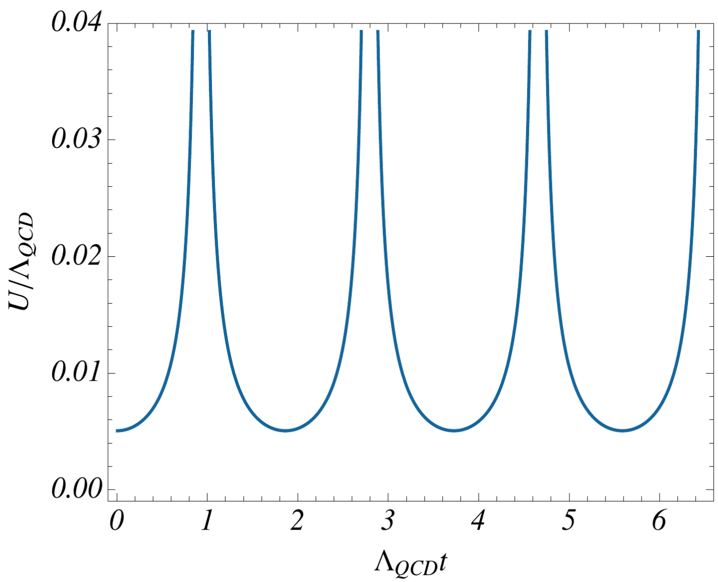

0.00
0.01
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0.03
0.04
U/Λ_{QCD}
0
1
2
3
4
5
6
$\Lambda_{QCD}t$

$$\delta\epsilon(w) = \frac{-4\rho_{DE} w(1+w) e^{-3t_0\sqrt{\frac{-\rho_{DE}\varkappa w}{3}}}}{\left(\sqrt{-w} + \sqrt{-w} e^{-3t_0\sqrt{\frac{-\rho_{DE}\varkappa w}{3}}} + e^{-3t_0\sqrt{\frac{-\rho_{DE}\varkappa w}{3}}} - 1\right)^2}$$

$$Kx = \coth(x)\,,\; K = \frac{2}{t_0\sqrt{3\rho\varkappa}} \approx 0.84\,,\; x \approx 1.36\,,\; w_{min} = -\frac{4x^2}{3\rho t_0^2 \varkappa} \approx -1.30$$

$$Kx = \tanh(x)\,,\; x \approx 0.77\,, w_{max} = -\frac{4x^2}{3\rho t_0^2 \varkappa} \approx -0.42\,.$$

$$a(t) \simeq \frac{a^*}{3}\left(4 - \sqrt{\frac{C}{4\varepsilon_0}}\right) e^{t\sqrt{\frac{\varkappa C}{12}}} + \frac{a^*}{3}\left(\sqrt{\frac{C}{4\varepsilon_0}} - 1\right) e^{-2t\sqrt{\frac{\varkappa C}{12}}}$$

$$T_0^0(t) \simeq \frac{C}{4}\left(1 - \left(1 - \frac{4\varepsilon_0}{C}\right) e^{-3\sqrt{\frac{\varkappa C}{12}}t}\right),$$

$$T_\mu^\mu(t) \simeq C - \frac{C}{4}(g(t)+1)\left(1 - \frac{4\varepsilon_0}{C}\right) e^{-3\sqrt{\frac{\varkappa C}{12}}t}\,.$$