Gravitational memory and Black Holes

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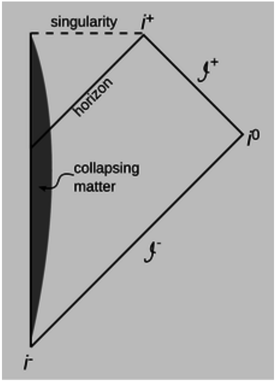
Outline

Motivation

- We want to describe gravitational radiation emitted by source and received by a distant observer
- We need to develop the idiom of asymptotic flatness
- We expect solution of Einstein vacuum equations will become algebraically special at infinity

Definition



Define the asymptotic flatness as the certain view of Penrose-Carter diagram

We want such view at infinity point J^+

The flat metric with corrections

Ansatz

$$ds^2 = -Ae^{2\beta}du^2 - 2e^{2\beta}dudr + r^2e^{2\gamma}(d\theta - Udu)^2 + r^2e^{-2\gamma}sin^2\theta d\phi^2$$

Can always choose such a coordinate system

Assume axisymmetry($\frac{\partial}{\partial\phi}$ is a killing vector)

Choose the gauge in which $r^4 sin^2\theta = g_{\theta\theta}g_{\phi\phi}$

The flat metric with corrections

When $r \to \infty$ we approach a region in which there are no gravity sources
$R_{\mu\nu} = 0$, $T_{\mu\nu} = 0$

$r \to \infty$

$$ds^2 = -du^2 - dudr + r^2 d\Omega^2$$

Hence
$A = 1 + O(\frac{1}{r})$
$\beta = O(\frac{1}{r})$
$U = O(\frac{1}{r})$
$\gamma = O(\frac{1}{r})$

The flat metric with corrections

$R_{\mu\nu} = 0$
$R_{\mu\phi} = 0$ from axisymmetry
Thus, we have 6 equations on functions(A, U, β, γ)

After evaluating, introducing $C = C(u, \theta)$

$$\gamma = \frac{C(u,\theta)}{r} + O(\frac{1}{r^2})$$
$$\beta = \frac{C^2(u,\theta)}{4r^2} + O(\frac{1}{r^3})$$
$$U = -\frac{\partial_\theta C + 2C cot\theta}{r^2} + O(\frac{1}{r^3})$$
$$A = 1 - \frac{2m(u,\theta)}{r} + O(\frac{1}{r^3})$$

The flat metric with corrections

Very close to answer

$$ds^2 = -(1 - \frac{2m}{r})du^2 - 2dudr + r^2 d\Omega^2 + 2U_0 dud\theta + 2cr(d\theta^2 - sin^2\theta d\phi^2) + O(\frac{1}{r^2})dudr + O(1)d\theta^2 + O(1)d\phi^2 + O(\frac{1}{r})dud\theta$$

$$U_0 = \partial_\theta C + 2C\cot\theta$$
$$\partial_u m = -(\partial_u C)^2 + \frac{1}{2}\partial_u(\partial_\theta^2 C + 3\partial\theta(C)\cot\theta - 2C)$$

Finally, make coordinate transformation

$$z = \cot\frac{\theta}{2}e^{i\phi}$$

The Bondi metric

The final form

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + D^z C_{zz}dudz + D^{\bar{z}} C_{\bar{z}\bar{z}}dud\bar{z} + \dots$$

$$\gamma_{z\bar{z}} = \frac{2}{(1+|z|^2)^2}$$

$$C_{zz} = \frac{4C(u,\theta)}{(1+|z|^2)^2}\frac{\bar{z}}{z}$$

$D^z C_{zz}$ is covariant derivative on sphere

The BMS group

The Bondi metric is invariant under such following transformations

Super translations

$$u \to u - f(z, \bar{z})$$
$$r \to r - D^z D_z f$$
$$z \to z + \frac{1}{r} D^2 f$$
$$\bar{z} \to \bar{z} + \frac{1}{r} \bar{D}^2 f$$

The S-T together with conformal $SL(2, \mathbb{C})$ transform acting on S^2 form the Bondi-Metzner-Sacks group

The Bondi mass and Bondi news

The Bondi News

$$N_{zz} = \partial_u C_{zz}$$

Bondi Mass aspect

$$\partial_u m = -\frac{1}{4} N_{zz} N^{zz} + \frac{1}{4}(D_z^2 N^{zz} + D_{\bar{z}}^2 N^{\bar{z}\bar{z}})$$

We define the Bondi mass to be the average value of the mass aspect over the sphere

Bondi Mass

$$M_B = \frac{1}{4\pi} \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} m(u, \theta, \phi) d\phi$$

The Bondi flux $\partial_u M_B = -\frac{1}{4\pi} \int d\Omega (\partial_u C)^2$

The scalar field

The K-G equation for massless scalar field

$$\Box\phi = \left[-\partial_t^2 + \frac{1}{r}\partial_r(r^2\partial_r) + \frac{\Delta_{\theta,\varphi}}{r^2}\right]\phi = 0$$

The compact solution of K-G equation

$$\phi = f(t+r)g(\theta,\phi) = f(u)g(z,\bar{z})$$
$$f(u) = e^{-\alpha u^2}$$
$$g(\theta,\phi) = e^{-\beta(1-\cos\theta)}$$

Einstein equations

At the beginning

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$-\Box h_{\mu\nu} + h_{\nu}{}^{\alpha}{}_{,\mu\alpha} + h_{\mu\alpha}{}^{,\alpha}{}_{\nu} - h_{,\mu\nu} - h^{\alpha\beta}{}_{,\alpha\beta}\eta_{\mu\nu} + \eta_{\mu\nu}\Box h = 16\pi T_{\mu\nu}$$

Gauge conditions

$$h^{\mu\alpha}_{,\alpha} = 0$$
$$h = 0$$

Finally

$$\Box h_{\mu\nu} = -16\pi T_{\mu\nu}$$

Einstein equations

Stress Energy tensor

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}\eta_{\mu\nu}(\partial\phi)^2$$

The Box operator

$$\Box = \partial_r^2 + \frac{2}{r}\partial_r - 2\partial_u\partial_r - \frac{2}{r}\partial_u + \frac{2}{r^2\gamma_{z\bar{z}}}\partial_z\partial_{\bar{z}}$$

The scalar field

$$\phi = f(u)g(z,\bar{z})$$

Corrections

Components

$$h_{uu} = -\frac{2 \cdot 4\pi g(z,\bar{z})^2 \int du \partial_u f(u)^2}{r} \equiv -\frac{2\widetilde{m}(u,z,\bar{z})}{r}$$

$$h_{zz} = r(\cdot \frac{a(1-z\bar{z})}{1+z\bar{z}} - \frac{b[4+(1-z\bar{z})\log z\bar{z}]}{1+z\bar{z}}) \equiv r\xi_{zz}$$

$$h_{uz} = C_0 - 8\pi \int dz d\bar{z} \gamma_{z\bar{z}} \partial_u f(u) f(u) g(z,\bar{z}) \partial_z g(z,\bar{z}) \equiv \Omega_z(u,z,\bar{z})$$

$$h_{z\bar{z}} = r \cdot 8\pi \int dz d\bar{z} \gamma_{z\bar{z}}^2 f_\infty \partial_u f_\infty g_\infty^2 \equiv r\Delta_{z\bar{z}}$$

$$h_{rr} = \frac{1}{r}\frac{h(z,\bar{z})}{2\pi} e^{-u}$$

The Final

The scalar corrections with pure gravity

$$ds^2 = -\left(1 - \frac{2m_B}{r} + \frac{2\widetilde{m}}{r}\right) du^2 - \left(2 - \frac{1}{r}\frac{f_u(z,\bar{z})}{\pi} e^{-u}\right) dudr + \left(2r^2\gamma_{z\bar{z}} + 2r\Delta_{z\bar{z}}\right) dzd\bar{z} + r\left(C_{zz} + \xi_{zz}\right) dz^2 + r\left(C_{\bar{z}\bar{z}} + \xi_{\bar{z}\bar{z}}\right) d\bar{z}^2 + \left(D^z C_{zz} + 2\Omega_z(u,z,\bar{z})\right) dudz + \left(D^{\bar{z}} C_{\bar{z}\bar{z}} + 2\Omega_{\bar{z}}(u,z,\bar{z})\right) dud\bar{z} + \frac{1}{r}\frac{f_r(z,\bar{z})}{2\pi} e^{-u} dr^2 + \ldots$$

Summary

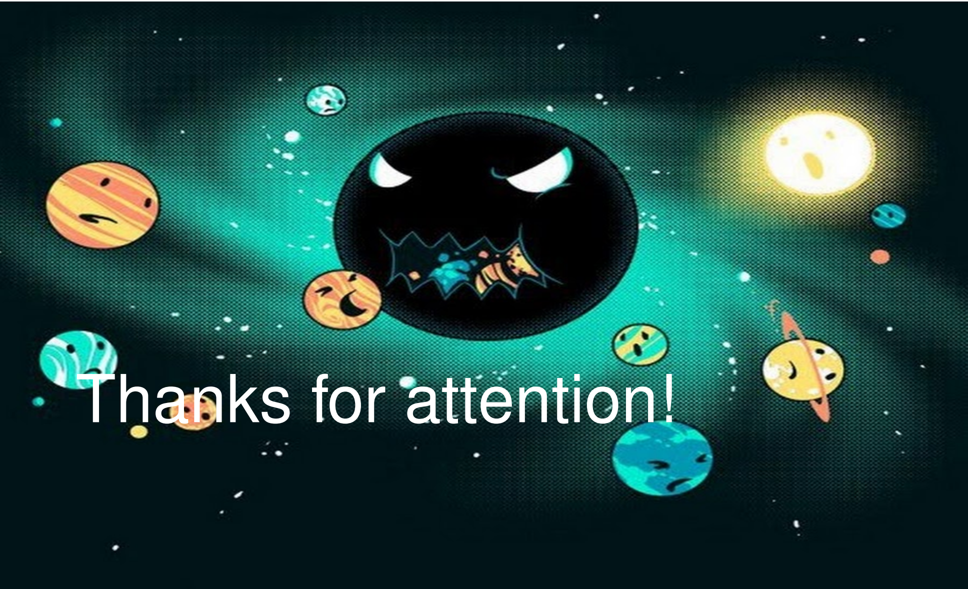
- The scalar field has the same corrections on metric as the pure gravity
- The scalar field has "gravitational memory"

- Outlook
  - The gauge field(the lorentz gauge condition)
  - The graviton field(the additional contribution from pseudo-tensor)
  - The behaviour of corrections on the horizon of Schwarzschild and Kerr Black holes(difficult, but not extremely)

For Further Reading I

Andrew Strominger, Alexander Zhiboedov
Gravitational Memory, BMS Supertranslations and Soft Theorems.
1411.5745, 2014

Sabrina Pasterski, Andrew Strominger, Alexander Zhiboedov
New Gravitational Memories.
1502.06120, 2015



Thanks for attention!