Rare B-Decays in the SM and Hints of BSM Physics from Data

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Interest in Rare B Decays

- Rare B Decays $(b \to (s,d)\gamma, b \to (s,d)\ell^+\ell^-, ...)$ are Flavour-Changing-Neutral-Current (FCNC) processes $(|\Delta B| = 1, |\Delta Q| = 0)$; not allowed at the Tree level in the SM
- They are governed by the GIM mechanism, which imparts them sensitivity to higher scales in the SM (m_t, m_W)
- In the SM, they determine the weak mixing CKM matrix elements V_{td}, V_{ts} and V_{tb}
- In principle sensitive to physics beyond the SM (BSM), such as supersymmetry. Precise experiments and theory are needed to establish or definitively rule out BSM effects in Flavor physics
- Rare B-decays have enjoyed great attention in the current & past experimental programme in flavour physics, with the present frontier being LHC

Rare B-decays in the Standard Model

- SM Lagrangian and the CKM Matrix
- QCD Effects in Weak Decays
- Operator product Expansion
- The Standard Candle in Rare B-Decays: $\mathbf{B} \to X_s\gamma$
- Exclusive Radiative Decays $\mathbf{B} \to K^*\gamma$ & $\mathbf{B}_s \to \phi\gamma$
- Electroweak Penguins: $\mathbf{B} \to X_s\ell^+\ell^-$
- Exclusive Decays $\mathbf{B} \to (K, K^*, \pi)\ell^+\ell^-$
- Current Frontier of Rare B Decays: $\mathbf{B}_s \to \mu^+\mu^-$ & $\mathbf{B}_d \to \mu^+\mu^-$
- Summary and Outlook

Standard Model Lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm GSW} + \mathcal{L}_{\rm QCD}$$

QCD [SU(3)]

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + i \sum_q \bar{\psi}^\alpha_q \gamma^\mu (D_\mu)_{\alpha\beta} \psi^\beta_q$$

with $F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu - g_s f_{abc} A^{(b)}_\mu A^{(c)}_\nu; \;\; a, b, c = 1, ..., 8$

and $(D_\mu)_{\alpha\beta} = \delta_{\alpha\beta} \partial_\mu + i g_s \sum_a \frac{1}{2} \lambda^{(a)}_{\alpha\beta} A^{(a)}_\mu$

Electroweak $[SU(2)_I \times U(1)_Y]$

$$\mathcal{L}_{\rm GSW} = \mathcal{L}_{\rm gauge}(W_i, B, \psi_j) + \mathcal{L}_{\rm Higgs}(\phi_k, W_i, B, \psi_j)$$

$$\mathcal{L}_{\rm gauge}(W_i, B, \psi_j) = -\frac{1}{4} F^i_{\mu\nu} F^{\mu\nu}_i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_{\psi_L} \overline{\psi_L} i D_\mu \gamma^\mu \psi_L + \sum_{\psi_R} \overline{\psi_R} i D_\mu \gamma^\mu \psi_R$$

Standard Model Lagrangian-Contd.

$$\mathcal{L}_{\text{Higgs}}(\phi_k, W_i, B, \psi_j) = \mathcal{L}_{\text{Higgs}}(\text{gauge}) + \mathcal{L}_{\text{Higgs}}(\text{fermions})$$

$$\mathcal{L}_{\text{Higgs}}(\text{gauge}) = (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi)$$

$$D_\mu \Phi = (\mathbf{I}(\partial_\mu + i\tfrac{g_1}{2} B_\mu) + i g_2 \tfrac{\tau}{2} \cdot \text{W-})\Phi; V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\mathcal{L}_{\text{Higgs}}(\text{fermions}) = Y_u^{ij} \bar{Q}_{L,i} \tilde{\Phi} u_{R,j} + Y_d^{ij} \bar{Q}_{L,i} \Phi d_{R,j} + \text{h.c.} + ...$$

- 3 Quark families: $Q_{L_j} = (u_L,\ d_L); (c_L,\ s_L); (t_L;\ b_L); \bar{u}_R,\ \bar{d}_R; ...$
- Flavour mixing reside in the Higgs-Yukawa sector of the theory
- Flavour symmetry broken by Yukawa interactions

$$Q_i Y_d^{ij} d_j \phi \longrightarrow Q_i M_d^{ij} d_j$$

$$Q_i Y_u^{ij} u_j \phi^c \longrightarrow Q_i M_u^{ij} u_j$$

$$M_d = \textbf{diag}(m_d,\ m_s,\ m_b);\ M_u^\dagger = \textbf{diag}(m_u,\ m_c,\ m_t) \times V_{\text{CKM}}$$

- V_{CKM} a (3×3) unitary matrix is the only source of flavour violation

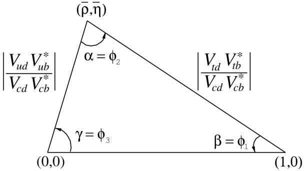
The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Customary to use the handy Wolfenstein parametrization

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3\left(1-\rho-i\eta\right) & -A\lambda^2\left(1+i\lambda^2\eta\right) & 1 \end{pmatrix}$$

- Four parameters: $A,\ \lambda,\ \rho,\ \eta;\ \bar{\rho} = \rho(1-\lambda^2/2),\ \bar{\eta} = \eta(1-\lambda^2/2)$
- The CKM-Unitarity triangle $[\phi_1 = \beta;\ \phi_2 = \alpha;\ \phi_3 = \gamma]$



Phases and sides of the UT

$$\alpha \equiv \arg\left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right), \qquad \beta \equiv \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right), \qquad \gamma \equiv \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right)$$

- β and γ have simple interpretation

$$V_{td} = |V_{td}| e^{-i\beta}, \qquad V_{ub} = |V_{ub}| e^{-i\gamma}$$

- α defined by the relation: $\alpha = \pi - \beta - \gamma$

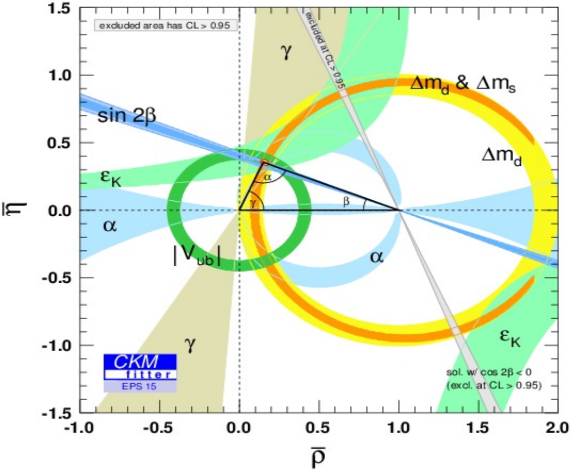
- The Unitarity Triangle (UT) is defined by:

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1$$

$$R_b \equiv \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left|\frac{V_{ub}}{V_{cb}}\right|$$

$$R_t \equiv \frac{|V_{tb}^* V_{td}|}{|V_{cb}^* V_{cd}|} = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left|\frac{V_{td}}{V_{cb}}\right|$$

Current Status of the CKM-Unitarity Triangle [CKMfitter: 2015]



- Direct and indirect measurements of angles agree well; largest Pull is on $\mathbf{\sin 2\beta}$ $(= \mathbf{1.6}\ \sigma)$

QCD Effects in Weak decays: Basic Formalism

- Renormalization procedure in QCD

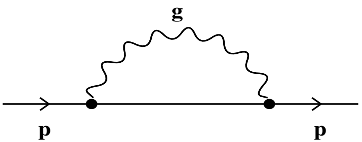
$$A^a_{0\mu} = Z_3^{1/2} A^a_\mu \qquad q_0 = Z_q^{1/2} q$$
$$g_{0,s} = Z_g g_s \mu^\varepsilon \qquad m_0 = Z_m m$$

- The index "0" indicates unrenormalized quantities. A^a_μ and q are renormalized fields, g_s is the renormalized QCD coupling and m the renormalized quark mass
- Dimensional Regularization is used in which Feynman diagrams are evaluated in $D = 4 - 2\varepsilon$ space-time dimensions and the singularities are extracted as $1/\varepsilon$ poles
- The simplest renormalization scheme is the *Minimal Subtraction Scheme* MS in which only divergences ($1/\varepsilon$ poles) are subtracted

$$Z_i = \frac{\alpha_s}{4\pi}\frac{a_{1i}}{\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{a_{2i}}{\varepsilon^2} + \frac{b_{2i}}{\varepsilon}\right) + \mathcal{O}(\alpha_s^3)$$

a_{ji} and b_{ji} are μ-independent constants.

Example: Quark Self-Energy Correction in the MS scheme:



$$i\Sigma_{\alpha\beta} = i \not{p}\, C_F\, \delta_{\alpha\beta} \frac{\alpha_s}{4\pi}\left[\frac{1}{\varepsilon} + \ln 4\pi - \gamma_E + \ln \frac{\mu^2}{-p^2} + 1\right]$$

$C_F = 4/3$; γ_E is the Euler constant $\gamma_E = 0.5772...$

- The $\overline{\text{MS}}$-scheme is defined by: $\mu_{\overline{MS}} = \mu e^{\gamma_E/2}(4\pi)^{-1/2}$

$$(i\Sigma_{\alpha\beta})_{div} = iC_F\delta_{\alpha\beta}\frac{\alpha_s}{4\pi}(\not{p} - 4m)\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

- Adding the counter-term $i\delta_{\alpha\beta}[(Z_q - 1)\not{p} - (Z_q Z_m - 1)m]$

and requiring the final result to be zero yields the Renormalization constants

$$Z_q = 1 - \frac{\alpha_s}{4\pi} C_F \frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

$$Z_m = 1 - \frac{\alpha_s}{4\pi} 3C_F \frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

Renormalization contd.

- Z_3 and Z_g calculated from the gluon propagator and the $g\bar{q}q$ vertex

$$Z_3 = 1 - \frac{\alpha_s}{4\pi}\left[\frac{2}{3}f - \frac{5}{3}N\right]\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

$$Z_g = 1 - \frac{\alpha_s}{4\pi}\left[\frac{11}{6}N - \frac{2}{6}f\right]\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

Basic RG Equations in QCD & their Solutions

- Scale-dependence of coupling $g_s(\mu)$ ($g \equiv g_s$) and quark mass $m(\mu)$:

$$\frac{dg(\mu)}{d\ln\mu} = \beta(g(\mu),\varepsilon)$$

$$\frac{dm(\mu)}{d\ln\mu} = -\gamma_m(g(\mu))m(\mu)$$

where

$$\beta(g(\mu),\varepsilon) = -\varepsilon g + \beta(g),$$

$$\beta(g) = -g\frac{1}{Z_g}\frac{dZ_g}{d\ln\mu}, \qquad \gamma_m(g) = \frac{1}{Z_m}\frac{dZ_m}{d\ln\mu}$$

Compendium of Useful Results

- $\beta(g)$, $\gamma(\alpha_s)$ and $Z_{q,1}(\alpha_s)$ up to two-loops are

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

$$\gamma_m(\alpha_s) = \gamma_m^{(0)} \frac{\alpha_s}{4\pi} + \gamma_m^{(1)} \left(\frac{\alpha_s}{4\pi}\right)^2$$

$$Z_{q,1}(\alpha_s) = a_1 \frac{\alpha_s}{4\pi} + a_2 \left(\frac{\alpha_s}{4\pi}\right)^2$$

where

$$\beta_0 = \frac{11N - 2f}{3} \qquad \beta_1 = \frac{34}{3}N^2 - \frac{10}{3}Nf - 2C_F f$$

$$\gamma_m^{(0)} = 6C_F \qquad \gamma_m^{(1)} = C_F\left(3C_F + \frac{97}{3}N - \frac{10}{3}f\right)$$

$$a_1 = -C_F \qquad a_2 = C_F\left(\frac{3}{4}C_F - \frac{17}{4}N + \frac{1}{2}f\right)$$

$$C_F = \frac{N^2 - 1}{2N}$$

Running Coupling Constant

- The RG equation for $g(\mu)$ can be written as:

$$\frac{d\alpha_s}{d\ln\mu} = -2\beta_0 \frac{\alpha_s^2}{4\pi} - 2\beta_1 \frac{\alpha_s^3}{(4\pi)^2}$$

- The solution is:

$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda_{\overline{MS}}^2)} - \frac{\beta_1}{\beta_0^3} \frac{\ln\ln(\mu^2/\Lambda_{\overline{MS}}^2)}{\ln^2(\mu^2/\Lambda_{\overline{MS}}^2)}$$

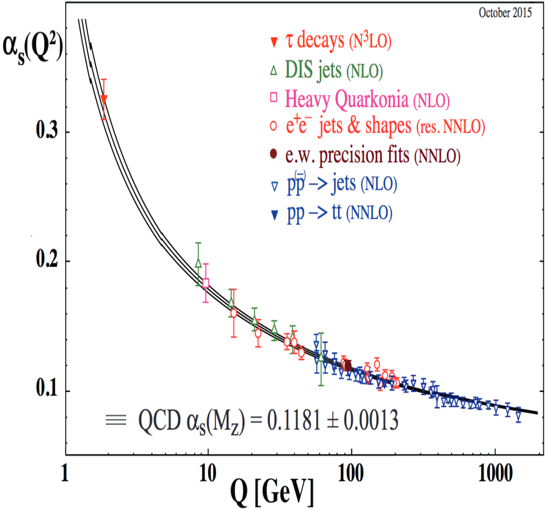
- $\Lambda_{\overline{MS}}$ is a QCD scale characteristic for the $\overline{\text{MS}}$ scheme.

- $\Lambda_{\overline{MS}}$ and $\alpha_s(\mu)$ depend on f, the number of "effective" flavours present, which depends on the scale μ. As a working procedure $f = 6$ for $\mu > m_t$, $f = 5$ for $m_b \leq \mu \leq m_t$ etc.

- Denoting by $\alpha_s^{(f)}(\mu)$ the effective coupling constant for a theory with f effective flavours, the current world average is

$$\alpha_s^{(5)}(M_Z) = 0.1181 \pm 0.0013$$

QCD Coupling constant $\alpha_s(\mu)$ [PDG: 2016]



Running Quark Masses

- The RG equation for $m(\mu)$ can be written as:

$$\frac{dm(\mu)}{d\ln\mu} = -\gamma_m(g)m(\mu)$$

- With $dg/d\ln\mu = \beta(g)$ the solution is:

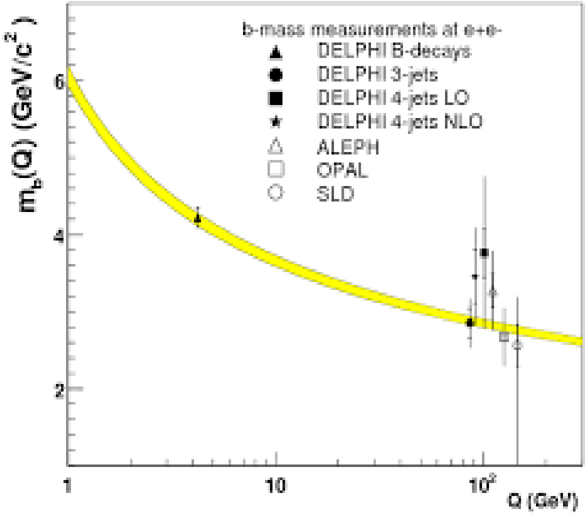
$$m(\mu) = m(\mu_0)\exp\left[-\int_{g(\mu_0)}^{g(\mu)} dg' \frac{\gamma_m(g')}{\beta(g')}\right]$$

- $m(\mu_0)$ is the value of the running quark mass at the scale μ_0. Inserting the expansions for $\gamma_m(g)$ and $\beta(g)$ and expanding in α_s gives:

$$m(\mu) = m(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_m^{(0)}}{2\beta_0}}\left[1 + \left(\frac{\gamma_m^{(1)}}{2\beta_0} - \frac{\beta_1\gamma_m^{(0)}}{2\beta_0^2}\right)\frac{\alpha_s(\mu)-\alpha_s(\mu_0)}{4\pi}\right]$$

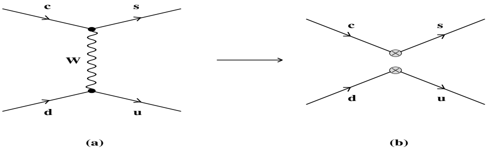
- Since $\frac{\gamma_m^{(0)}}{2\beta_0}$ is a positive number, quark masses $m(\mu)$ decrease as μ increases, and they require a scheme and a scale to be quantified much like $\alpha_s(\mu)$

Bottom-quark-mass running $m_b(\mu)$ [HERA & LEP]



Operator Product Expansion in Weak decays

- Consider the quark level transition $c \to su\bar{d}$



- The tree-level W-exchange amplitude is:

$$A = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{M_W^2}{k^2 - M_W^2} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A}$$
$$= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \mathcal{O}(\frac{k^2}{M_W^2})$$

where $(\bar{s}c)_{V-A} \equiv \bar{s}\gamma_\mu (1-\gamma_5) c$

- Ignoring $\mathcal{O}(k^2/M_W^2)$ terms, the amplitude A may also be obtained from

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \textbf{High D Operators}$$

Basic idea of OPE

- Product of two current operators is expanded into a series of local operators, weighted by the eff. coupling constants, Wilson Coefficients
  OPE & Short-distance QCD Effects
- Rewriting the $c \to s u \bar{d}$ transition to make the quark color-indices explicit

$$\mathcal{H}_{eff}^{(0)} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

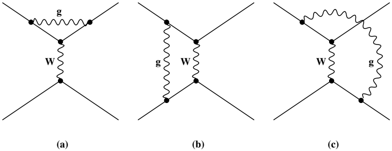
- With QCD effects $\mathcal{H}_{eff}^{(0)}$ is generalized to

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1(\mu) Q_1 + C_2(\mu) Q_2)$$

  where

$$Q_1 = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$
$$Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

- In addition to the original operator Q_2 , a new operator Q_1 with the *same flavour* form but *different colour structure* is generated, as is evident from the colour structure



- The Wilson coefficients C_1 and C_2, become calculable nontrivial functions of α_s, M_W and the renormalization scale μ.

Calculation of Wilson Coefficients

- They are determined by the requirement that the amplitude A_{full} in the SM is reproduced by the amplitude in the effective theory A_{eff}

$$A_{full} = A_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1 \langle Q_1 \rangle + C_2 \langle Q_2 \rangle)$$

- There are three steps involved in this procedure, outlined below

Step 1: Calculation of A_{full}

- In the SM, A_{full} to $\mathcal{O}(\alpha_s)$ ($m_i = 0, p^2 < 0$):

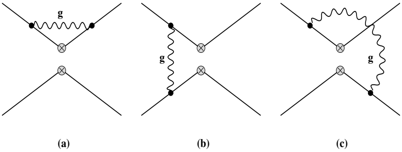
$$A_{full} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Big[\Big(1 + 2C_F \frac{\alpha_s}{4\pi} (\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2}) \Big) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} S_2 \\ -3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} S_1 \Big]$$

Here S_1 and S_2 are the tree level matrix elements of Q_1 and Q_2

$$S_1 \equiv \langle Q_1 \rangle_{tree} = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$

$$S_2 \equiv \langle Q_2 \rangle_{tree} = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

- The singularity $1/\varepsilon$ can be removed by quark field renormalization



Step 2: Calculation of Matrix Elements $\langle Q_i \rangle$

- The unrenormalized matrix elements of Q_1 and Q_2 are found at $\mathcal{O}(\alpha_s)$ by calculating the diagrams in the effective theory

$$\begin{aligned}\langle Q_1\rangle^{(0)} &= \left(1+2C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)\right)S_1+\frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_1\\ &\quad -3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_2\end{aligned}$$

$$\begin{aligned}\langle Q_2\rangle^{(0)} &= \left(1+2C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)\right)S_2+\frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_2\\ &\quad -3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_1\end{aligned}$$

- Divergence in the first terms can again be removed by quark field renormalization. However, one needs *Operator renormalization* to remove the residual divergence

$$Q_i^{(0)} = Z_{ij}Q_j$$

Relation between unrenormalized & renormalized Green functions

- The relation between the unrenormalized ($\langle Q_i \rangle^{(0)}$) and the renormalized amputated Green functions ($\langle Q_i \rangle$) is:

$$\langle Q_i \rangle^{(0)} = Z_q^{-2} \hat{Z}_{ij} \langle Q_j \rangle$$

- Z_q^{-2} removes the $1/\varepsilon$ divergences in the first terms discussed above. $\hat{Z}_{ij}$ removes the remaining divergences. In the $\overline{\mathrm{MS}}$-scheme:

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} \begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}$$

- The renormalized matrix elements $\langle Q_i \rangle$ are given by

$$\langle Q_1 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right) S_1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_1 - 3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_2$$

$$\langle Q_2 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_2 - 3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_1$$

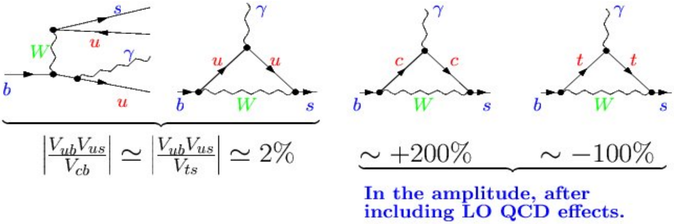
Step 3: Extraction of C_i

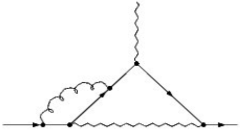
- Inserting $\langle Q_i \rangle$ in A_{eff} and comparing with A_{full} yields the Wilson coefficients C_1 and C_2

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}\,, \qquad C_2(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}$$

- The Wilson coefficients in the meanwhile have been calculated to next-next-leading-order (NNLO) precision. Their expressions are too long to give here. Their numerical values will be quoted.

Examples of leading electroweak diagrams for $B \to X_s \gamma$





- QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR($B \to X_s \gamma$) more than twice
- Effective field theory (obtained by integrating out heavy fields) used for resummation of such large logarithms

The effective Lagrangian for $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$

$$\mathcal{L} = \mathcal{L}_{QCD \times QED}(q, l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$
$$(q = u, d, s, c, b,\ l = e, \mu, \tau)$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu(\gamma_5) l), & i = 9, (10) \quad |C_i(m_b)| \sim 4 \end{cases}$$

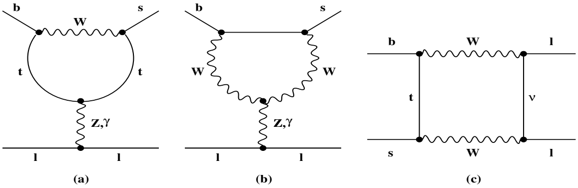
Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

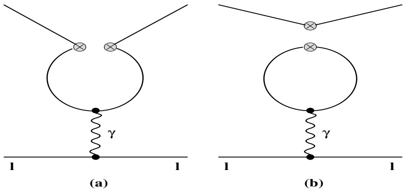
Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

The decay $b \to s\ell^+\ell^-$: Leading Feynman diagram



Diagrams in the full theory



Diagrams in the effective theory

Structure of the SM calculations for $\bar{B} \to X_s \gamma$ & $\bar{B} \to X_s \ell^+ \ell^-$

$$\mathcal{H}_{\text{eff}} \sim \sum_{i=1}^{10} C_i(\mu) O_i$$

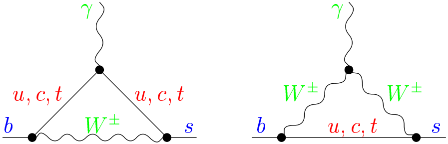
- $\mathcal{H}_{\text{eff}}$ independent of the scale μ, while $C_i(\mu)$ and $O_i(\mu)$ depend on μ $\Longrightarrow$ Renormalization Group Equation (RGE) for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^{\mathrm{T}} C_j(\mu)$$

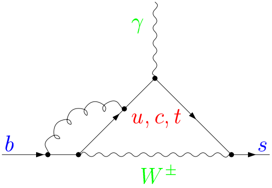
- γ_{ij}: anomalous dimension matrix
- Matching usually done at high scale $(\mu_0 \sim M_W, m_t)$
- SM and the matrix elements of the operators have the same large logs
  $\mu_0 \sim O(M_W)$
  $\downarrow$ RGE
  $\mu_b \sim O(m_b)$: matrix elements of the operators at this scale don't have large logs; they are contained in the $C_i(\mu_b)$
- Evaluation of the on-shell amplitudes at $\mu_b \sim m_b$

Examples of SM diagrams for the matching of $C_7(\mu_0)$

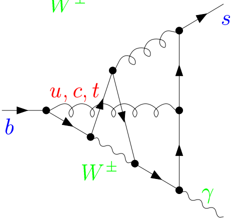
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, Misiak, 2004]



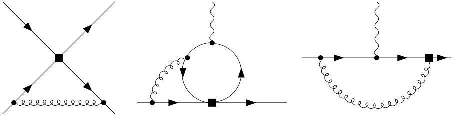
Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in $b \to s\gamma$ amplitude

RGE for the Wilson coefficients $\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$

- Renormalization constants $\Longrightarrow$ γ_{ij}: $C_j(\mu)$ known to NLL accuracy

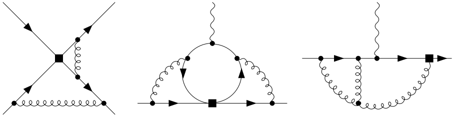
Z_{22} Z_{27} Z_{87}

LO



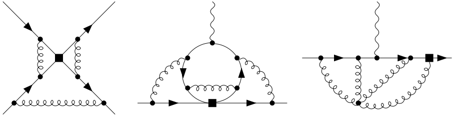
[Gaillard, Lee, 1974] [Altarelli, Maiani, 1974] — [Grinstein *et al.*, 1990] — [Shifman *et al.*, 1978] [Grigjanis *et al.*, 1988]

NLO



[Altarelli *et al.*, 1981] [Buras, Weisz, 1990] — [Chetyrkin *et al.*, 1997] — [Misiak, Münz, 1995]

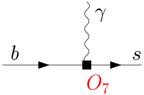
NNLO



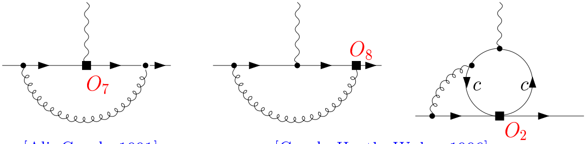
[Gambino *et al.*, 2004] — in progress: Czakon, — in progress: Gambino,

Perturbative on-shell amplitudes

LO



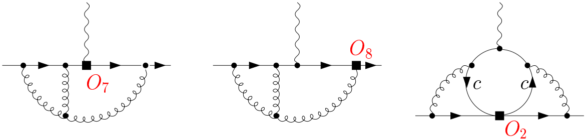
NLO



[Ali, Greub, 1991]

[Greub, Hurth, Wyler, 1996]

NNLO



in progress: Asatrian, Greub, Hurth

[Bieri *et al*, 2003] ($\mathcal{O}(\alpha_s^2 n_f)$)
in progress: Steinhauser, Misiak
(extrapolation in m_c)

Wilson Coefficients in the SM

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of the dipole and semileptonicr Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

- Obtained for the following input:

$$\mu_b = 4.6 \text{ GeV} \qquad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV} \qquad \sin^2\theta_W = 0.23$$

$\mathcal{B}(B \to X_s \gamma)$: Experiment vs. SM & BSM Effects

[Misiak et al., PRL 114 (2015) 22, 221801]

- Expt.: CLEO, Belle, BaBar [HFAG 2014]: ($E_\gamma > \mathbf{1.6}$ GeV):

$$\mathcal{B}(B \to X_s \gamma) = (\mathbf{3.43} \pm \mathbf{0.21} \pm \mathbf{0.07}) \times \mathbf{10}^{-4}$$

- SM [NNLO]: $\mathcal{B}(B \to X_s \gamma) = (\mathbf{3.36} \pm \mathbf{0.23}) \times \mathbf{10}^{-4}$

- Expt./SM $= \mathbf{1.02} \pm \mathbf{0.08}$
- Excellent agreement; restricts most NP models

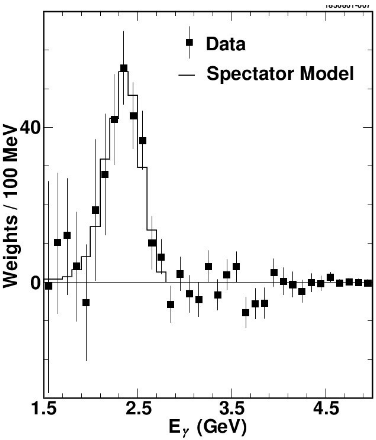
- BSM effects can be parametrized as additive contributions to the Wilson Coeffs. of the dipole operators C_7 and C_8

$$\mathcal{B}(B \to X_s \gamma) \times 10^4 = (3.36 \pm 0.23) - 8.22\Delta C_7 - 1.99\Delta C_8$$

- In 2HDM, $\mathcal{B}(B \to X_s \gamma)$ puts strict bounds on M_{H^+}

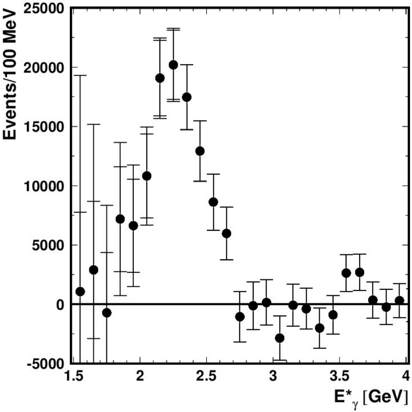
Photon Energy Spectrum in $B \to X_s \gamma$

Spectator Model: Greub, AA; PLB 259, 182 (1991)



CLEO

PRL 87 (2001) 251807



BELLE

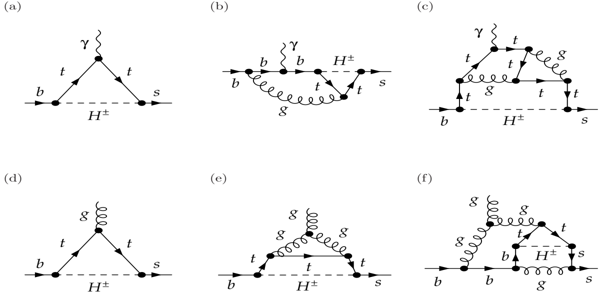
PRL 93 (2004) 061803

$B \to X_s\gamma$ in 2HDM

- NNLO in 2HDM [Hermann, Misiak, Steinhauser; JHEP 1211 (2012) 036] ;
  Updated [Misiak et al., Phys. Rev. Lett. 114 (2015) 22, 221801]

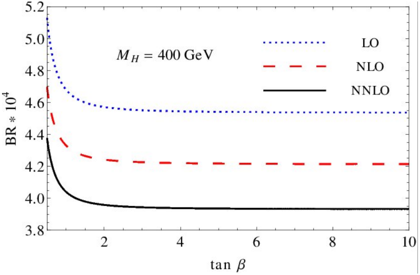
$$\mathcal{L}_{H^+} = (2\sqrt{2}G_F)^{1/2}\Sigma_{i,j=1}^{3}\bar{u}_i(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R) d_j H^* + h.c.$$

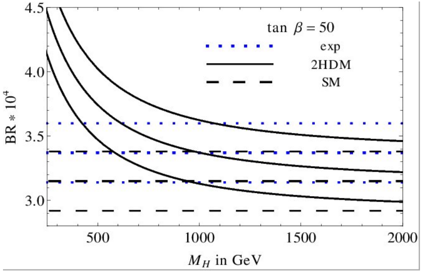
- 2HDM contributions to the Wilson coefficients are proportional to $A_i A_j^*$
  - 2HDM of type-I: $A_u = A_d = \frac{1}{\tan\beta}$
  - 2HDM of type-II: $A_u = -1/A_d = \frac{1}{\tan\beta}$



$B \to X_s \gamma$ in Type-II 2HDM

[Hermann, Misiak, Steinhauser JHEP 1211 (2012) 036]





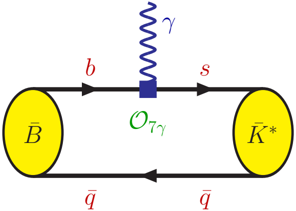
- Updated NNLO [Misiak et al., Phys. Rev. Lett. 114 (2015) 22, 221801]

- $M_{H^+} > \mathbf{480}$ GeV (at 95% C.L.)
- $M_{H^+} > \mathbf{358}$ GeV (at 99% C.L.)

- Limits on 2HDM competitive to direct $H^\pm$ searches at the LHC

The decay $B \to K^* \gamma$

- In LO, only the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes to the $B \to K^* \gamma$ amplitude; involves the form factor $T_1^{(K^*)}(0)$



$$\mathcal{M}^{\rm LO} \propto V_{tb} V_{ts}^* C_7^{(0)\rm eff} \frac{e \bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) \left[(Pq)(e^* \varepsilon^*) - (e^* P)(\varepsilon^* q) + i \, {\rm eps}(e^*, \varepsilon^*, P, q) \right]$$

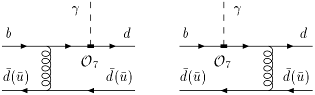
Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; e^μ is its polarization vector; ε^μ is the K^*-meson polarization vector

- Branching ratio:

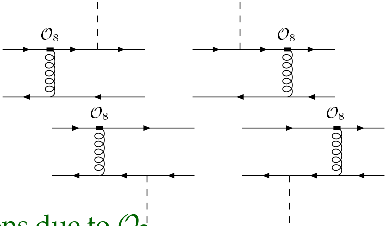
$$\mathcal{B}^{\rm LO}(B \to K^* \gamma) = \tau_B \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b) \, |C_7^{(0)\rm eff}(\mu_b)|^2 \, |T_1^{(K^*)}(0)|^2$$

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$

$B \to K^* \gamma$ decay rates in NLO

- Perturbative approaches: QCD-F; PQCD; SCET

Factorization Ansatz (QCDF):

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

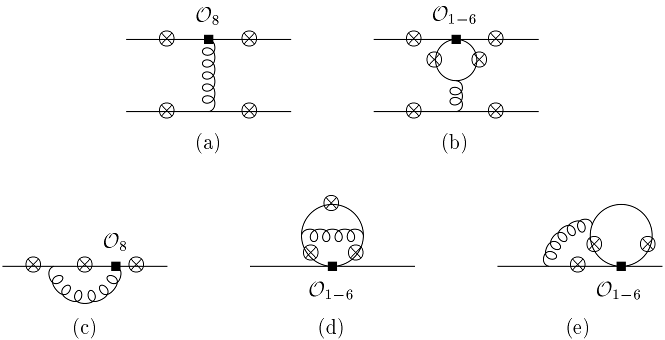
$$\langle V\gamma|Q_i|\bar{B}\rangle = t_i^I \zeta_{V_\perp} + t_i^{II} \otimes \phi_+^B \otimes \phi_\perp^V + \mathcal{O}(\frac{\Lambda_{\text{QCD}}}{m_b})$$

- $\zeta_{V_\perp}$ (form factor) and $\phi^{B,V}$ (LCDAs) are non-perturbative functions
- t^I and t^{II} are perturbative hard-scattering kernels

$$t^I = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + ..., \qquad t^{II} = \mathcal{O}(\alpha_s) + ...$$

- The kernels t^I and t^{II} are known at $\mathcal{O}(\alpha_s)$;
  include Hard-scattering and Vertex corrections
  [Parkhomenko, AA; Bosch, Buchalla; Beneke, Feldmann, Seidel 2001]

Nonfactorizable α_s Corrections



- First line: hard-spectator corrections
- Second line: $b \to s\gamma$ vertex corrections

SCET factorization formula for $B \to K^* \gamma$

[Chay, Kim '03; Grinstein, Grossman, Ligeti '04; Becher, Hill, Neubert '05]

$$\langle V\gamma|Q_i|\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + (\Delta_i C^{B1} \otimes j_\perp) \otimes \phi_\perp^V \otimes \phi_+^B$$

- $\zeta_{V_\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- Hard-scattering kernels t^I, t^{II} = SCET matching coefficients

$$t_i^I = \Delta_i C^A(m_b); \quad t_i^{II} = \Delta_i C^{B1}(m_b) \otimes j_\perp(\sqrt{m_b \Lambda}) \quad \textbf{(subfactorization)}$$

- Derivation of factorization in SCET
  1) QCD $\to$ SCET$_I$: Integrate out m_b; defines vertex corrections $\Delta_i C^A = t_i^I$

$$Q_i \to \Delta_i C^A(m_b) J^A + \Delta_i C^{B1}(m_b) \otimes J^{B1} + \ldots$$

  2) SCET$_I$ $\to$ SCET$_{II}$: Integrate out $\sqrt{m_b \Lambda_{\rm QCD}}$; defines spectator corr.

$$J^{B1} \to j_\perp(\sqrt{m_b \Lambda_{\rm QCD}}) \otimes O^{B1,{\rm SCET}_{II}}(\Lambda_{\rm QCD})$$

  3) Large logs in t_i^{II} resummed by solving RG equations

$B \to K^* \gamma$ in SCET at NNLO

[Pecjak, Greub, AA '07]

Vertex Corrections

$$\Delta_i C^A = \Delta_7 C^{A(0)} \left[\Delta_{i7} + \frac{\alpha_s(\mu)}{4\pi} \Delta_i C^{A(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2} \Delta_i C^{A(2)} \right]$$

- Contr. from O_7 and O_8 exact to NNLO $O(\alpha_s^2)$
- Contr. from O_2 exact at NLO $O(\alpha_s)$ but only large-β_0 limit at $O(\alpha_s^2)$

Spectator Corrections at $O(\alpha_s^2)$

$$t_i^{II(1)}(u, \omega) = \Delta_i C^{B1(1)} \otimes j_\perp^{(0)} + \Delta_i C^{B1(0)} \otimes j_\perp^{(1)}$$

- The one-loop jet-function $j_\perp^{(1)}$ known; [Becher and Hill '04; Beneke and Yang '05]
- The one-loop hard coefficient $\Delta_7 C^{B1(1)}$ known; [Beneke, Kiyo, Yang '04; Becher and Hill '04]
- The one-loop hard coefficient $\Delta_8 C^{B1(1)}$ known; [Pecjak, Greub, AA '07]
- $\Delta_i C^{B1(1)}$ $(i = 1, ..., 6)$ remain unknown (require two loops)

Estimates of BR($B \to K^* \gamma$) in SCET at NNLO

[Pecjak, Greub, AA; EPJ C55: 577 (2008)]

Estimates at NNLO in units of 10^{-5}

$$\mathcal{B}(B^+ \to K^{*+}\gamma) = 4.6 \pm 1.2[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt. 4.2 ± 0.18 (HFAG 2012)];

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = 4.3 \pm 1.1[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt.: 4.33 ± 0.15 (HFAG 2012)];

$$\mathcal{B}(B_s \to \phi\gamma) = 4.3 \pm 1.1[\zeta_{\phi}] \pm 0.3[m_c] \pm 0.3[\lambda_B] \pm 0.1[\mu]$$

[Expt.: $5.7^{+2.1}_{-1.8}$ (BELLE); 3.9 ± 0.5 (LHCb)]

Comparison with current experiments

- $\frac{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm exp}} = 1.10 \pm 0.35[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B^0 \to K^{*0}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*0}\gamma)_{\rm exp}} = 1.00 \pm 0.32[\text{theory}] \pm 0.04[\text{exp}]$

$B \to X_s l^+ l^-$

- There are two $b \to s$ semileptonic operators in SM:

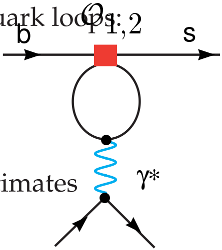
$$O_i = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu (\gamma_5) l), \qquad i = 9, (10)$$

- Their Wilson Coefficients have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

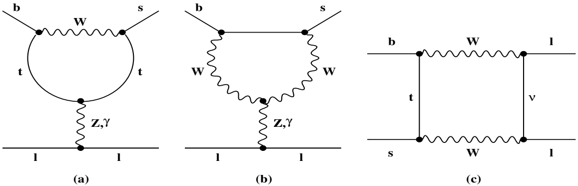
- The term $C_9^{(-1)}(\mu)$ reproduces the electroweak logarithm that originates from the photonic penguins with charm quark loops:



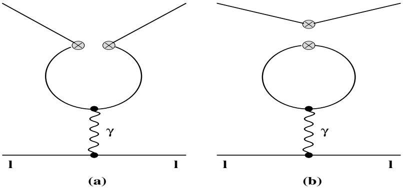
$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s) \simeq 2$$

- $C_9^{(0)}(m_b) \simeq 2.2$; need to calculate NNLO for reliable estimates

The decay $b \to s\ell^+\ell^-$: Leading Feynman diagram



Diagrams in the full theory



Diagrams in the effective theory

NNLO Calculations of $\mathcal{B}(B \to X_s \ell^+ \ell^-)$

- 2-loop matching, 3-loop mixing and 2-loop matrix elements are available
  - Matching: [Bobeth, Misiak, Urban]
  - Mixing: [Gambino, Gorbahn, Haisch]
  - Matrix elements: [Asatryan, Asatrian, Greub, Walker; Asatrian, Bieri, Greub, Hovhannissyan; Ghinculov, Hurth, Isidori, Yao; Bobeth, Gambino, Gorbahn, Haisch]
- Power corrections in $B \to X_s \ell^+ \ell^-$ decays
  - $1/m_b$ corrections [A. Falk et al.; AA, Handoko, Morozumi,Hiller; Buchalla, Isidori]
  - $1/m_c$ corrections [Buchalla, Isidori, Rey]
- NNLO Phenomenological analysis of $B \to X_s \ell^+ \ell^-$ decays

[AA, Greub, Hiller, Lunghi, Phys. Rev. D66, 034002 (2002)]

  - $\mathbf{BR}(B \to X_s \mu^+ \mu^-);\quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$
  - $\mathbf{BR}(B \to X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

Dilepton Invariant Mass in $B \to X_s \ell^+ \ell^-$

$$\frac{d\Gamma(B \to X_s \ell^+ \ell^-)}{d\hat{s}} = \left(\frac{\alpha_{em}}{4\pi}\right)^2 \frac{G_F^2 m_{b,pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\pi^3} (1-\hat{s})^2$$

$$\times \left((1+2\hat{s}) \left(\left|\tilde{C}_9^{\text{eff}}\right|^2 + \left|\tilde{C}_{10}^{\text{eff}}\right|^2 \right) + 4\,(1+2/\hat{s}) \left|\tilde{C}_7^{\text{eff}}\right|^2 + 12\text{Re}\left(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}\right) \right)$$

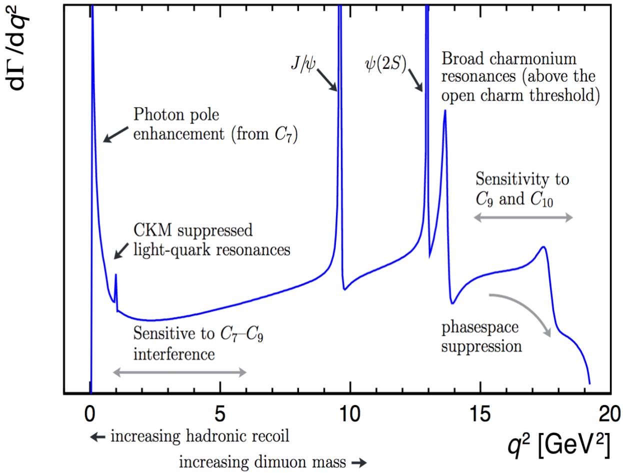
$$\tilde{C}_7^{\text{eff}} = \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_7(\hat{s})\right) A_7 - \frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)}F_1^{(7)}(\hat{s}) + C_2^{(0)}F_2^{(7)}(\hat{s}) + A_8^{(0)}F_8^{(7)}(\hat{s})\right)$$

$$\tilde{C}_9^{\text{eff}} = \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) \left(A_9 + T_9\, h(\hat{m}_c^2, \hat{s}) + U_9\, h(1,\hat{s}) + W_9\, h(0,\hat{s})\right) - \frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)}F_1^{(9)}(\hat{s}) + C_2^{(0)}F_2^{(9)}(\hat{s}) + A_8^{(0)}F_8^{(9)}(\hat{s})\right)$$

$$\tilde{C}_{10}^{\text{eff}} = \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) A_{10}$$

- $A_7, A_8, A_9, A_{10}, T_9, U_9, W_9$ are functions of the Wilson coefficients

Sensitivity of the different q^2 regions to SD- & LD-pieces



Forward-Backward Asymmetry in $B \to X_s \ell^+ \ell^-$

[Proposed in AA, Mannel, Morozumi, PLB 273, 505 (1991)]

[NNLL: Asatrian, Bieri, Greub, Hovhannisyan; Ghinculov, Hurth, Isidori, Yao]

Normalized FB Asymmetry

$$\overline{A}_{\rm FB}(\hat{s}) = \frac{\int_{-1}^{1} \frac{d^2\Gamma(B\to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz}\,{\rm sgn}(z)\,dz}{\int_{-1}^{1} \frac{d^2\Gamma(B\to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz}\,dz}$$

$$\int_{-1}^{1} \frac{d^2\Gamma(b\to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz}\,{\rm sgn}(z)\,dz = \left(\frac{\alpha_{\rm em}}{4\,\pi}\right)^2 \frac{G_F^2\, m_{b,\rm pole}^5\,\left|V_{ts}^* V_{tb}\right|^2}{48\,\pi^3}(1-\hat{s})^2$$

$$\times\left[-3\,\hat{s}\,{\rm Re}(\widetilde{C}_9^{\rm eff}\widetilde{C}_{10}^{\rm eff*})\left(1+\frac{2\alpha_s}{\pi}f_{910}(\hat{s})\right) - 6\,{\rm Re}(\widetilde{C}_7^{\rm eff}\widetilde{C}_{10}^{\rm eff*})\left(1+\frac{2\alpha_s}{\pi}f_{710}\right.\right.$$

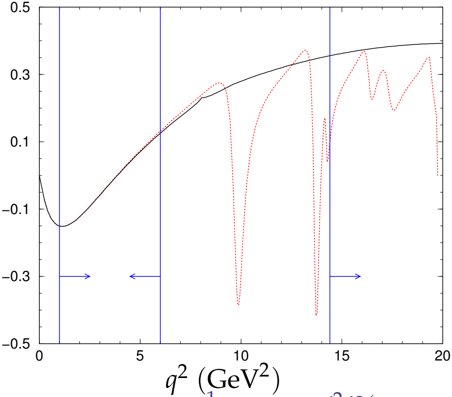
- NNLL stabilize the scale ($=\mu$) dependence of the FB Asymmetry

  $A_{\rm FB}^{\rm NLL}(0) = -(2.51 \pm 0.28)\times 10^{-6}$;
  $A_{\rm FB}^{\rm NNLL}(0) = -(2.30 \pm 0.10)\times 10^{-6}$

- Zero of the FB Asymmetry is a precise test of the SM, correlating $\widetilde{C}_7^{\rm eff}$ and $\widetilde{C}_9^{\rm eff}$

Normalized FB-Asymmetry in $\bar{B} \to X_s \ell^+ \ell^-$

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \; \frac{d^2\mathcal{B}(B \to X_s \ell^+ \ell^-)}{dq^2 \; d\cos\theta_\ell} \mathrm{sgn}(\cos\theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$q_0^2 \;=\; (3.90 \pm 0.25)\ \mathrm{GeV}^2$ [Ghinculov, Hurth, Isidori, Yao 200

Comparison of $B \to X_s \ell^+ \ell^-$ with Data

[AA,Greub, Hiller, Lunghi 2001 (AGHL); Ghinculov, Hurth, Isidori, Yao 2004 (GHIY); Huber, Lunghi, Misiak, Wyler 2005 (HLMW); Bobeth, Gambino, Gorbahn, Haisch 2003]

- Inclusive $B \to X_s \ell^+ \ell^-$ BRs

$$\mathcal{B}(B \to X_s \ell^+ \ell^-)(M_{\ell\ell} > 0.2 \text{ GeV}) = (3.66^{+0.76}_{-0.77}) \times 10^{-6} \quad [\text{HFAG}'12]$$

$$SM: (4.2 \pm 0.7) \times 10^{-6} \quad [\text{AGHL}]; \quad (4.6 \pm 0.8) \times 10^{-6} \quad [\text{GHIY}]$$

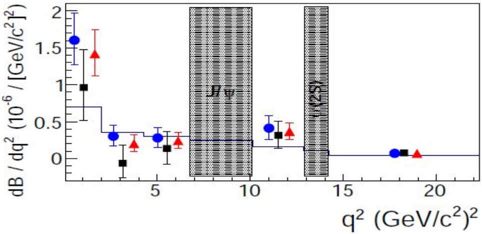
- Partial BRs (integrated over lower range of q^2)

$\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)$; $q^2 \in [1,6]$ GeV^2 $= (1.63 \pm 0.20) \times 10^{-6}$ [GHIY]
$\mathcal{B}(\bar{B} \to X_s \mu^+ \mu^-)$; $q^2 \in [1,6]$ GeV^2 $= (1.59 \pm 0.11) \times 10^{-6}$ [HLMW]
$\mathcal{B}(\bar{B} \to X_s e^+ e^-)$; $q^2 \in [1,6]$ GeV^2 $= (1.63 \pm 0.11) \times 10^{-6}$ [HLMW]
Expt:: $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)$ $q^2 \in [1,6]$ GeV^2 $= (1.60 \pm 0.51) \times 10^{-6}$

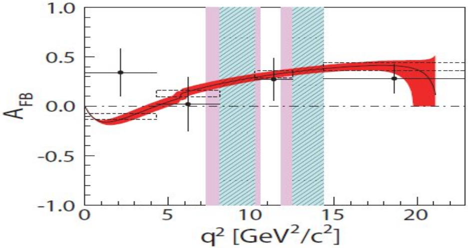
- Partial BRs (integrated over higher range of q^2)

$\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)$; $q^2 > 14$ GeV^2 $= (4.04 \pm 0.78) \times 10^{-7}$ [GHIY]
• $\mathcal{B}(\bar{B} \to X_s \mu^+ \mu^-)$; $q^2 > 14.4$ GeV^2 $= 2.40(1^{+0.29}_{-0.26}) \times 10^{-7}$ [HLMW]
$\mathcal{B}(\bar{B} \to X_s e^+ e^-)$; $q^2 > 14.4$ GeV^2 $= 2.09(1^{+0.32}_{-0.30}) \times 10^{-7}$ [HLMW]
Expt: $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)$ $q^2 > 14.4$ GeV^2 $= (4.4 \pm 1.2) \times 10^{-7}$

Dilepton invariant mass spectrum in $B \to X_s \ell^+ \ell^-$ [BaBar 2013]



Forward-Backward Asymmetry in $B \to X_s \ell^+ \ell^-$ [Belle 2014]



Exclusive Decays $B \to (K, K^*)\ell^+\ell^-$

- $B \to K$ & $B \to K^*$ transitions involve the currents:

$$\Gamma^1_\mu = \bar{s}\gamma_\mu(1-\gamma_5)b, \;\; \Gamma^2_\mu = \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$$

- $\Longrightarrow$ 10 non-perturbative q^2-dependent functions (Form factors)

$$\langle K|\Gamma^1_\mu|B\rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle K|\Gamma^2_\mu|B\rangle \supset f_T(q^2)$$

$$\langle K^*|\Gamma^1_\mu|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle K^*|\Gamma^2_\mu|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- Data on $B \to K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/SCET-approach allows to reduce the number of independent form factors from 10 to 3 in low-q^2 domain $(q^2/m_b^2 \ll 1)$

Experimental data vs. SM in $B \to (X_s, K, K^*)\ell^+\ell^-$ Decays

Branching ratios (in units of 10^{-6})   [HFAG: 2012]

SM: [A.A., Greub, Hiller, Lunghi PR D66 (2002) 034002]

Decay Mode	Expt. (BELLE & BABAR)	Theory (SM)
$B \to K\ell^+\ell^-$	0.45 ± 0.04	0.35 ± 0.12
$B \to K^* e^+e^-$	$1.19^{+0.17}_{-0.16}$	1.58 ± 0.49
$B \to K^* \mu^+\mu^-$	$1.15^{+0.16}_{-0.15}$	1.19 ± 0.39
$B \to X_s \mu^+\mu^-$	4.2 ± 1.3	4.2 ± 0.7
$B \to X_s e^+e^-$	4.7 ± 1.3	4.2 ± 0.7
$B \to X_s \ell^+\ell^-$	4.5 ± 1.3	4.2 ± 0.7

Test of Lepton Universality using $B^{\pm} \to K^{\pm} \ell^+ \ell^-$ decays

[R.Aaij *et al.* (LHCb) PRL 113, 151601 (2014)]

- Precise measurements of the differential branching ratios in $B^{\pm} \to K^{\pm} e^+ e^-$ & $B^{\pm} \to K^{\pm} \mu^+ \mu^-$

$$R_K \equiv \frac{\int_{1\ GeV^2}^{6\ GeV^2} d\Gamma/dq^2 [B^{\pm} \to K^{\pm} \mu^+ \mu^-] dq^2}{\int_{1\ GeV^2}^{6\ GeV^2} d\Gamma/dq^2 [B^{\pm} \to K^{\pm} e^+ e^-] dq^2} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

- SM Predictions [Bobeth, Hiller, Piranishvili, JHEP 12 (2007) 040]

$$R_K = 1.0003 \pm 0.0001 \implies 2.6\sigma \text{ deviation}$$

- Radiative corrections for the experimental setup is an issue
- BRs(expt.) smaller than the SM for both $K^{\pm} \mu^+ \mu^-$ and $K^{\pm} e^+ e^-$

$$\mathcal{B}(B \to K e^+ e^-) = \left(1.56^{+0.19-0.06}_{-0.15-0.04}\right) \times 10^{-7}$$

$$\mathcal{B}(B \to K \mu^+ \mu^-) = (1.20 \pm 0.09 \pm 0.07) \times 10^{-7}$$

$$\mathcal{B}^{\rm SM}(B \to K \mu^+ \mu^-) = \mathcal{B}^{\rm SM}(B \to K e^+ e^-) = \left(1.75^{+0.60}_{-0.29}\right) \times 10^{-7}$$

Test of Lepton Universality from the ratio $B \to D^{(*)}\tau\nu_\tau / B \to D^{(*)}\ell\nu_\ell$

[J.P. Lees *et al.* (BaBar), Phys. Rev. D88, 072012 (2013); M. Husche *et al.* (Belle) Phys. Rev. D92, 072014 (2015); R. Aaij *et al.* (LHCb) PRL 115, 159901 (2015)]

- A $\mathbf{3.9}\sigma$ deviation from $\tau/\ell;\ (\ell = e, \mu)$ universality in charged current semileptonic $B \to D^{(*)}$ decays is reported by BaBar, Belle and LHCb

$$R^{\tau/\ell}_{D^{(*)}} \equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)/\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)_{\rm SM}}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)/\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)_{\rm SM}}$$

$$R^{\tau\ell}_{D} = 1.37 \pm 0.17; \quad R^{\tau\ell}_{D^*} = 1.28 \pm 0.08$$

- A 30% deviation from the SM in a tree-level charged current interaction calls for a drastic contribution to an effective 4-fermi interaction proportional to the *LL* operator $(\bar{c}\gamma_\mu b_L)(\tau_L\gamma_\mu\nu_L)$
- Lepton non-universality in loop-induced R_K can be due to an *LL* operator $(\bar{s}_L\gamma_\mu b_L)(\bar{\mu}_L\gamma_\mu\mu_L)$, or an *RL* operator $(\bar{s}_L\gamma_\mu b_R)(\bar{\mu}_L\gamma_\mu\mu_L)$

Leptoquark models for R_K and $B \to D^{(*)}\tau\nu_\tau / B \to D^{(*)}\ell\nu_\ell$ anomalies

- Several suggestions along these lines have been made involving a leptoquark mediator

- A leptoquark model, with the leptoquark ϕ transforming as $(3, 3, -1/3)$ under the SM gauge groups, yielding an LL operator for muons:

$$\mathcal{L} = -\lambda_{b\mu}\phi^* q_3 \ell_2 - \lambda_{s\mu}\phi^* q_2 \ell_2$$

- A leptoquark model with an RL operator for electrons, with ϕ transforming as $(3, 2, 1/6)$

$$\mathcal{L} = -\lambda_{be}\phi(\bar{b}P_L\ell_e) - \lambda_{se}\phi(\bar{s}P_L\ell_e)$$

[G. Hiller, M. Schmaltz, Phys.Rev. D90, 054014 (2014)]

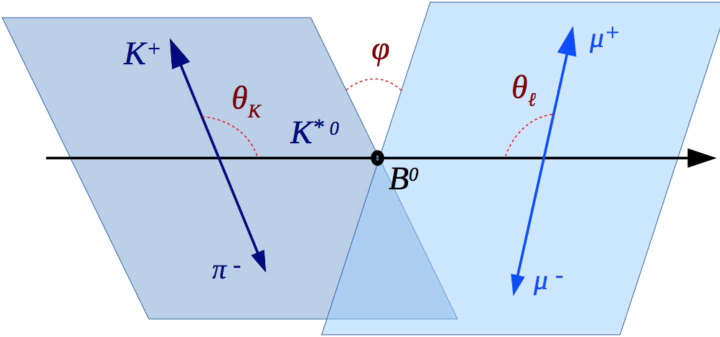
- A scalar leptoquark ϕ transforming as $(3, 1, -1/3)$ under the SM gauge groups, with $m_\phi = O(1)$ TeV and $O(1)$ couplings
[M. Bauer, M. Neubert, arxiv 1511.01900 (2015)]

- Anomalies in B decays and $U(2)$ flavor symmetry
[R. Barbieri *et al.*, Eur.Phys. J. C (2016) 76]

Angular analysis in $B \to K^* \mu^+ \mu^-$

$$\boldsymbol{B^0 \to K^{*0}(\to K^+ \pi^-)\mu^+\mu^-}$$

- Decay is $P \to VV'$ (since $K^*(892)^0$ is $J^P = 1^-$).
- System fully described by q^2 and three angles $\vec{\Omega} = (\cos\theta_l, \cos\theta_K, \phi)$



Observables in $B \to K^* \mu^+ \mu^-$

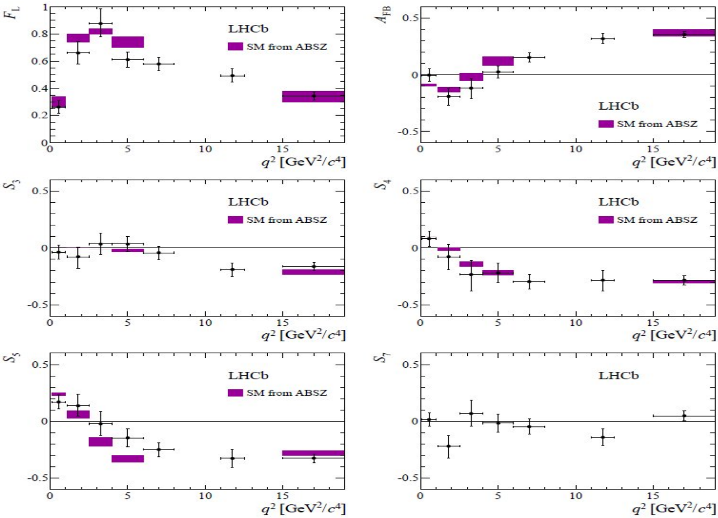
$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2} \frac{\mathrm{d}^4(\Gamma+\bar{\Gamma})}{\mathrm{d}q^2\,\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K$$
$$+\tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_l$$
$$-F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$
$$+S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+\tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_l + S_7\sin 2\theta_K\sin\theta_l\sin\phi$$
$$+S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]\,.$$

Optimized variables with reduced FF uncertainties

$$P_1 = 2S_3/(1-F_L);\;\; P_2 = 2A_{FB}/3(1-F_L);\;\; P_3 = -S_9/(1-F_L)$$
$$P_{4,5,6,8} = S_{4,5,6,8}/\sqrt{F_L(1-F_L)}$$

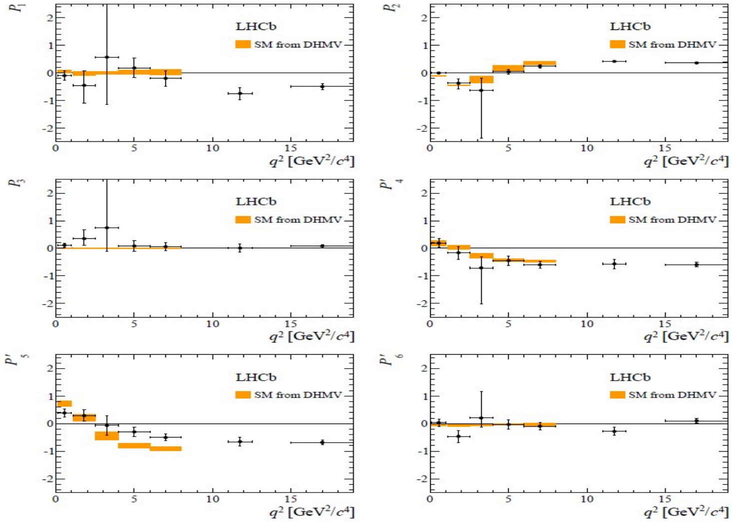
Latest Update from the LHCb: LHCb-Paper-2015-051

SM Estimates: Altmannshofer & Straub, EPJC 75 (2015) 382



Analysis of the optimised angular variables: LHCb-Paper-2015-051

SM Estimates: Descotes-Genon, Hofer, Matias, Virto; JHEP 12 (2014) 125



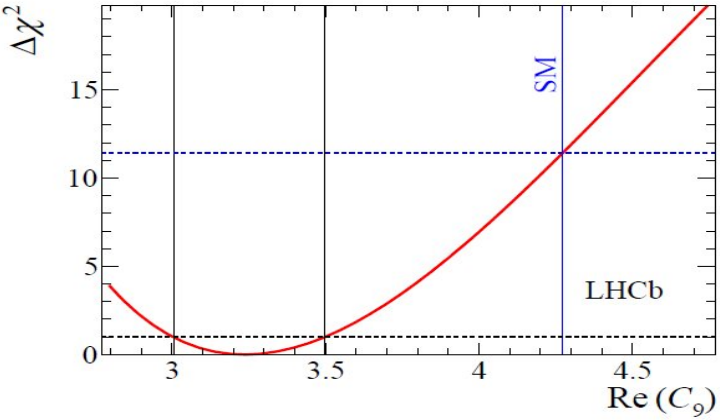
Recent Updates: Pull on the SM [Altmannshofer, Straub (2015)]

W. Altmannshofer & D.M. Straub, EPJ C75 (2015) 8, 382

Decay	obs.	q^2 bin	SM pred.	measurement		pull
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	[2, 4.3]	0.44 ± 0.07	0.29 ± 0.05	LHCb	+1.8
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	[16, 19.25]	0.47 ± 0.06	0.31 ± 0.07	CDF	+1.8
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	F_L	[2, 4.3]	0.81 ± 0.02	0.26 ± 0.19	ATLAS	+2.9
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	F_L	[4, 6]	0.74 ± 0.04	0.61 ± 0.06	LHCb	+1.9
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	S_5	[4, 6]	-0.33 ± 0.03	-0.15 ± 0.08	LHCb	−2.2
$B^- \to K^{*-}\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	[4, 6]	0.54 ± 0.08	0.26 ± 0.10	LHCb	+2.1
$\bar{B}^0 \to \bar{K}^0\mu^+\mu^-$	$10^8 \frac{dBR}{dq^2}$	[0.1, 2]	2.71 ± 0.50	1.26 ± 0.56	LHCb	+1.9
$\bar{B}^0 \to \bar{K}^0\mu^+\mu^-$	$10^8 \frac{dBR}{dq^2}$	[16, 23]	0.93 ± 0.12	0.37 ± 0.22	CDF	+2.2
$B_s \to \phi\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	[1, 6]	0.48 ± 0.06	0.23 ± 0.05	LHCb	+3.1
$B \to X_s e^+e^-$	10^6 BR	[14.2, 25]	0.21 ± 0.07	0.57 ± 0.19	BaBar	−1.8

Tension on the SM from $B \to K^* \mu^+ \mu^-$ measurements

- Perform χ^2 fit of the measured observables $F_L, A_{FB}, S_3, ..., S_9$
- Float the generic vector coupling, i.e., $\mathrm{Re}(C_9)$
- Best fit: $\Delta\mathrm{Re}(C_9) = \mathrm{Re}(C_9)^{\mathrm{LHCb}} - \mathrm{Re}(C_9)^{\mathrm{SM}} = -1.04 \pm 0.25$



Effective Weak $b \to d$ Hamiltonian

$$H_{\rm eff}^{(b\to d)} = -\frac{4G_F}{\sqrt{2}}\Big[V_{tb}^* V_{td} \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$
$$+V_{ub}^* V_{ud} \sum_{i=1}^{2} C_i(\mu)\Big(\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu)\Big)\Big] + \text{h.c.}$$

- G_F (Fermi constant), $C_i(\mu)$ (Wilson coefficients), and $\mathcal{O}_i(\mu)$ (dimension-six operators) are the same (modulo $s \to d$) as in $H_{\rm eff}^{(b\to s)}$
- CKM structure of the matrix elements more interesting in $H_{\rm eff}^{(b\to d)}$, as $V_{tb}^* V_{td} \sim V_{ub}^* V_{ud} \sim \lambda^3$are of the same order in $\lambda = \sin\theta_{12}$
- Anticipate sizable CP-violating asymmetries in $b \to d$transitions compared to $b \to s$

Operator Basis

- Tree operators

$$\mathcal{O}_1 = \left(\bar{d}_L \gamma_\mu T^A c_L\right)\left(\bar{c}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2 = \left(\bar{d}_L \gamma_\mu c_L\right)\left(\bar{c}_L \gamma^\mu b_L\right)$$

$$\mathcal{O}_1^{(u)} = \left(\bar{d}_L \gamma_\mu T^A u_L\right)\left(\bar{u}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2^{(u)} = \left(\bar{d}_L \gamma_\mu u_L\right)\left(\bar{u}_L \gamma^\mu b_L\right)$$

- Dipole operators

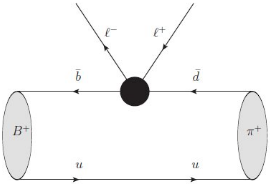
$$\mathcal{O}_7 = \frac{e\, m_b}{g_{\text{st}}^2}\left(\bar{d}_L \sigma^{\mu\nu} b_R\right) F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{m_b}{g_{\text{st}}}\left(\bar{d}_L \sigma^{\mu\nu} T^A b_R\right) G^A_{\mu\nu}$$

- Semileptonic operators

$$\mathcal{O}_9 = \frac{e^2}{g_{\text{st}}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \ell\right), \quad \mathcal{O}_{10} = \frac{e^2}{g_{\text{st}}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \gamma_5 \ell\right)$$

$B \to \pi$ transition matrix elements

Momentum transfer:
$q = p_B - p_\pi = p_{\ell^+} + p_{\ell^-}$



The Feynman diagram for the $B^+ \to \pi^+\ell^+\ell^-$ decay.

$$\langle \pi(p_\pi)|\bar{b}\gamma^\mu d|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu\right) + \left[f_0(q^2) - f_+(q^2)\right]\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

$$\langle \pi(p_\pi)|\bar{b}\sigma^{\mu\nu}q_\nu d|B(p_B)\rangle = \frac{if_T(q^2)}{m_B + m_\pi}\left[\left(p_B^\mu + p_\pi^\mu\right)q^2 - q^\mu\left(m_B^2 - m_\pi^2\right)\right]$$

- Dominant theoretical uncertainty is in the form factors $f_+(q^2), f_0(q^2)$, $f_T(q^2)$; require non-perturbative techniques, such as Lattice QCD
- Their determination is the main focus of the theory

$B \to \pi \ell^+ \nu_\ell$ decay

$$\langle \pi | \bar{u} \gamma^\mu b | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

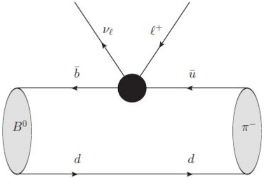
- $f_0(q^2)$contribution is suppressed by m_ℓ^2 / m_B^2 for $\ell = e, \mu$
- Differential decay width

$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^- \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

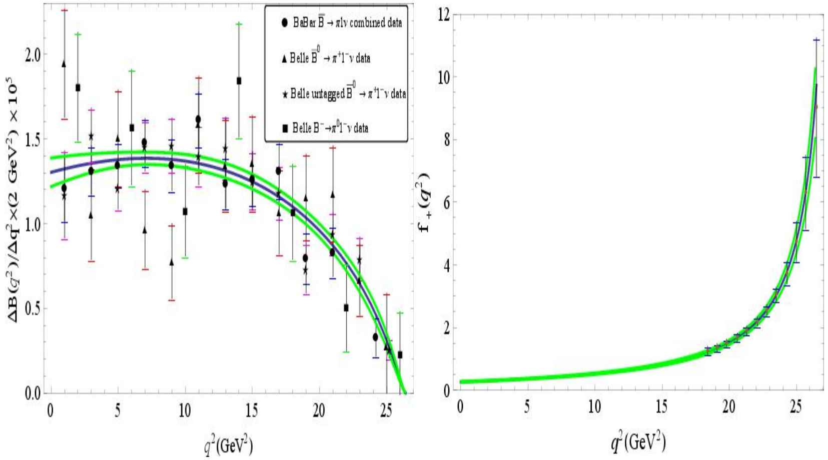
with $\lambda(q^2) = \left(m_B^2 + m_\pi^2 - q^2\right)^2 - 4 m_B^2 m_\pi^2$

- Assuming Isospin symmetry: $f_+(q^2)$ and $f_0(q^2)$in charged current $B \to \pi \ell \nu_\ell$ and neutral current $B \to \pi \ell^+ \ell^-$decays are equal
- Global fit of the CKM matrix element [PDG, 2012]

$$|V_{ub}| = (3.51^{+0.15}_{-0.14}) \times 10^{-3}$$



Fits of the data on $B \to \pi^+ \ell^- \nu_\ell$ yield $f_+(q^2)$



Heavy-Quark Symmetry (HQS) relations

- Including symmetry-breaking corrections, Heavy Quark Symmetry relates $f_+(q^2), f_0(q^2)$ and $f_T(q^2)$ (for $q^2/m_b^2 \ll 1$) [Beneke, Feldmann (2000)]

$$f_0(q^2) = \left(\frac{m_B^2 + m_\pi^2 - q^2}{m_B^2}\right)\left[\left(1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left(2 - 2L(q^2)\right)\right) f_+(q^2) + \frac{\alpha_s(\mu)C_F}{4\pi}\frac{m_B^2(q^2 - m_\pi^2)}{(m_B^2 + m_\pi^2 - q^2)^2}\Delta F_\pi\right],$$

$$f_T(q^2) = \left(\frac{m_B + m_\pi}{m_B}\right)\left[\left(1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left(\ln\frac{m_b^2}{\mu^2} + 2L(q^2)\right)\right) f_+(q^2) - \frac{\alpha_s(\mu)C_F}{4\pi}\frac{m_B^2}{m_B^2 + m_\pi^2 - q^2}\Delta F_\pi\right],$$

$$L(q^2) = \left(1 + \frac{m_B^2}{m_\pi^2 - q^2}\right)\ln\left(1 + \frac{m_\pi^2 - q^2}{m_B^2}\right), \quad \Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{N_c m_B}\left\langle l_+^{-1}\right\rangle_+ \left\langle \bar{u}^{-1}\right\rangle_\pi$$

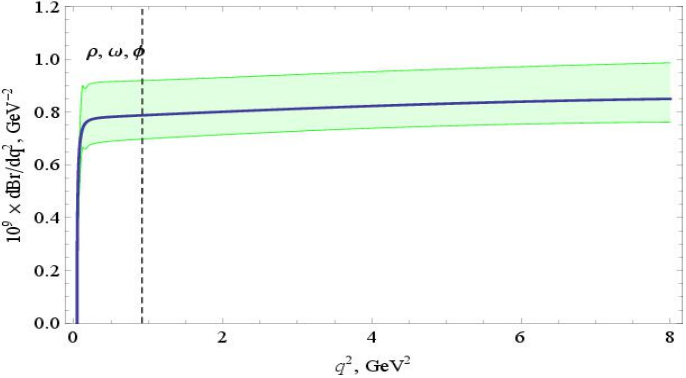
$B^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$ at large hadronic recoil ($q^2/m_b^2 \ll 1$)

[AA, A. Parkhomenko, A. Rusov; Phys. Rev. D89 (2014) 094021]

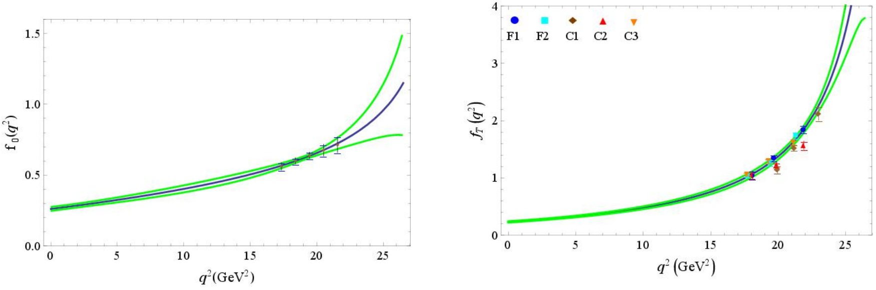
- Partially integrated branching fractions for $B^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$

  $$\mathbf{BR_{SM}}(B^{+} \to \pi^{+}\mu^{+}\mu^{-}; 1\,\mathrm{GeV}^2 \leq q^2 \leq 8\,\mathrm{GeV}^2) = \left(0.57^{+0.07}_{-0.05}\right) \times 10^{-8}$$

- Dimuon invariant mass spectrum at large hadronic recoil



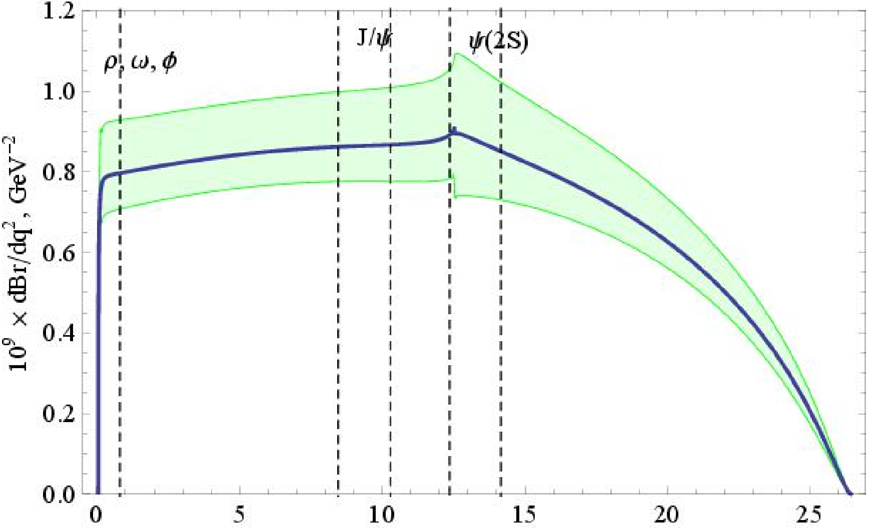
Determination of $f_0^{B\pi}(q^2)$ and $f_T^{B\pi}(q^2)$ and comparison with Lattice QCD



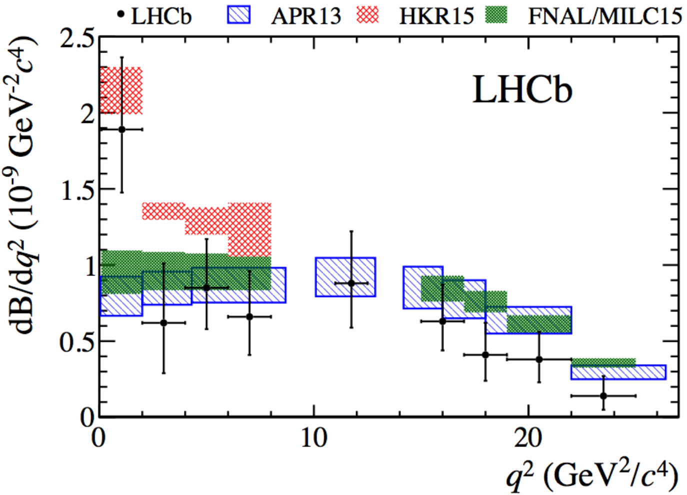
- FFs are obtained by the z-expansion [Boyd, Grinstein, Lebed] and constraints from data in low-q^2
- Lattice data (in high-q^2 are obtained by the HPQCD Collab.
  for $f_0^{B\pi}(q^2)$ from [arXiv:hep-lat/0601021]
  for $f_T^{B\pi}(q^2)$ from [arXiv:1310.3207]
- In almost the entire q^2-domain, the form factors are now determined accurately. Recent Fermilab/MILC lattice results are in agreement

$B^+ \to \pi^+\mu^+\mu^-$ in the entire range of q^2

[AA, A. Parkhomenko, A. Rusov; Phys. Rev. D89 (2014) 094021]



Dimuon invariant mass spectrum in $B \to \pi \ell^+ \ell^-$



- In excellent agreement with the APR2013 predictions, as well as with the Lattice results

SM vs. experimental data

- SM theoretical estimate of the total branching fraction [AA, A. Parkhomeno, A. Rusov; Phys. Rev. D89 (2014) 094021] :

$$\mathrm{BR_{SM}}(B^\pm \to \pi^\pm \mu^+ \mu^-) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}$$

- Uncertainty from the form factors is now reduced greatly.
  Residual theoretical uncertainty is mainly from the scale dependence and the CKM matrix elements

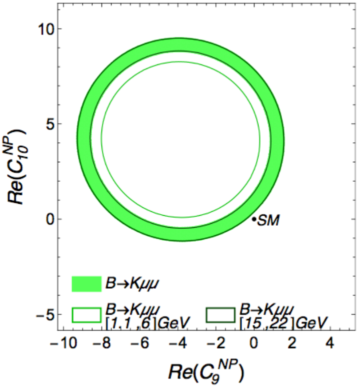
- LHCb has measured the BR and dimuon invariant mass distribution in $B^\pm \to \pi^\pm \mu^+ \mu^-$) based on 3 fb^{-1} integrated luminosity [LHCb-PAPER-2015-035; arXiv:1509.00414] :

$$\mathrm{BR_{exp}}(B^\pm \to \pi^\pm \mu^+ \mu^-) = (1.83 \pm 0.24(\mathrm{stat}) \pm 0.05(\mathrm{syst})) \times 10^{-8}$$

- Excellent agreement with SM-based APR2013-theory within errors, but significant improvement expected from the future analysis

Determination of Wilson Coeffs. from $B \to (\pi/K)\mu^+\mu^-$

[Fermilab/MILC, arxiv:1510.02349]



$B_s \to \mu^+\mu^-$ in the SM & BSM

- Effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}{}^* V_{tb} \sum_i \left[C_i(\mu) \mathcal{O}_i(\mu) + C_i'(\mu) \mathcal{O}_i'(\mu) \right]$$

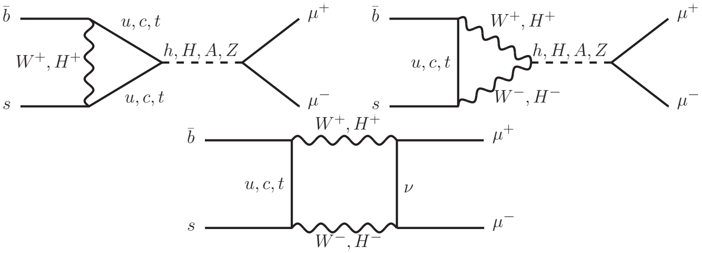
- Operators: O_i (SM) & O_i' (BSM)

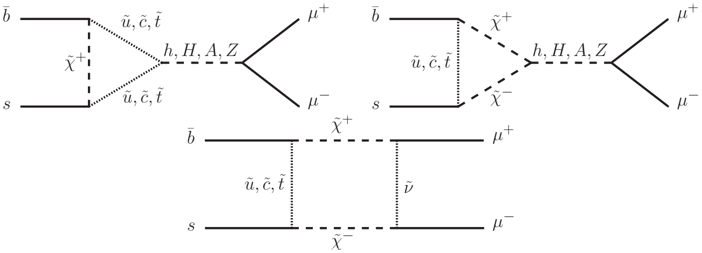
$$\mathcal{O}_{10} = (\bar{s}_\alpha \gamma^\mu P_L b_\alpha)\left(\bar{l}\gamma_\mu \gamma_5 l\right), \qquad \mathcal{O}_{10}' = (\bar{s}_\alpha \gamma^\mu P_R b_\alpha)\left(\bar{l}\gamma_\mu \gamma_5 l\right)$$
$$\mathcal{O}_S = m_b\,(\bar{s}_\alpha P_R b_\alpha)\left(\bar{l} l\right), \qquad \mathcal{O}_S' = m_s\,(\bar{s}_\alpha P_L b_\alpha)\left(\bar{l} l\right)$$
$$\mathcal{O}_P = m_b\,(\bar{s}_\alpha P_R b_\alpha)\left(\bar{l}\gamma_5 l\right), \qquad \mathcal{O}_P' = m_s\,(\bar{s}_\alpha P_L b_\alpha)\left(\bar{l}\gamma_5 l\right)$$

$$\mathbf{BR}\left(\bar{B}_s \to \mu^+\mu^-\right) = \frac{G_F^2 \alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3} |V_{ts}{}^* V_{tb}|^2 \sqrt{1 - 4\hat{m}_\mu^2} \times \left[\left(1 - 4\hat{m}_\mu^2\right) |F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2 \right]$$

$$F_{S,P} = m_{B_s}\left[\frac{C_{S,P} m_b - C_{S,P}' m_s}{m_b + m_s}\right], \quad F_{10} = C_{10} - C_{10}', \; \hat{m}_\mu = m_\mu / m_{B_s}$$

Leading diagrams for $B_s \to \mu^+\mu^-$ in SM, 2HDM & MSSM





$B_s \to \mu^+\mu^-$ in the SM

- SM predictions depend somewhat on the input parameters [Blanke & Buras, arxiv: 1602.04021; Bobeth et al., Phys. Rev. Lett. 112, 101801 (2014)]

$$\mathcal{B}\left(\bar{B}_s \to \mu^+\mu^-\right) = (3.65 \pm 0.06) \times 10^{-9} \left(\frac{m_t(m_t)}{163.5\ \text{GeV}}\right)^{3.02} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{0.032}$$

$$R_s = \left(\frac{F_{B_s}}{227.7\ \text{MeV}}\right)^2 \left(\frac{\tau_{B_s}}{1.516\ \text{ps}}\right) \left(\frac{0.938}{r(y_s)}\right) \left(\frac{|V_{ts}|}{41.5 \times 10^{-3}}\right)^2$$

- $\Delta\Gamma_s$ effects are taken into account through $r(y_s) = 1 - y_s$, with $y_s = \tau_{B_s}\Delta\Gamma_s/2 = 0.062 \pm 0.005$

$$\mathcal{B}\left(\bar{B}_s \to \mu^+\mu^-\right) = (3.78 \pm 0.23)[3.65 \pm 0.23] \times 10^{-9}$$

$$\mathcal{B}\left(B_d \to \mu^+\mu^-\right) = (1.06 \pm 0.02) \times 10^{-10} \left(\frac{m_t(m_t)}{163.5\ \text{GeV}}\right)^{3.02} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{0.032}$$

$$R_d = \left(\frac{F_{B_d}}{190.5\ \text{MeV}}\right)^2 \left(\frac{\tau_{B_d}}{1.519\ \text{ps}}\right) \left(\frac{|V_{td}|}{8.8 \times 10^{-3}}\right)^2$$

$$\mathcal{B}\left(B_d \to \mu^+\mu^-\right) = (1.02 \pm 0.08[1.06 \pm 0.09]) \times 10^{-10}$$

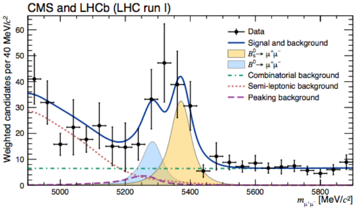
Compatibility of the SM with $B^0_{(s)} \to \mu^+\mu^-$ measurements

$B_s^0 \to \mu^+\mu^-$

Combined analysis with CMS

[Nature 522(2015)]

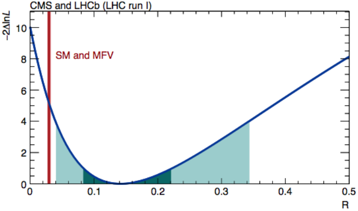
- **First observation** of $B_s^0 \to \mu^+\mu^-$ and **evidence** for $B^0 \to \mu^+\mu^-$.
  - 6.2σ and 3.2σ respectively.



- Measurement of **branching fractions** and **ratio** of branching fractions.

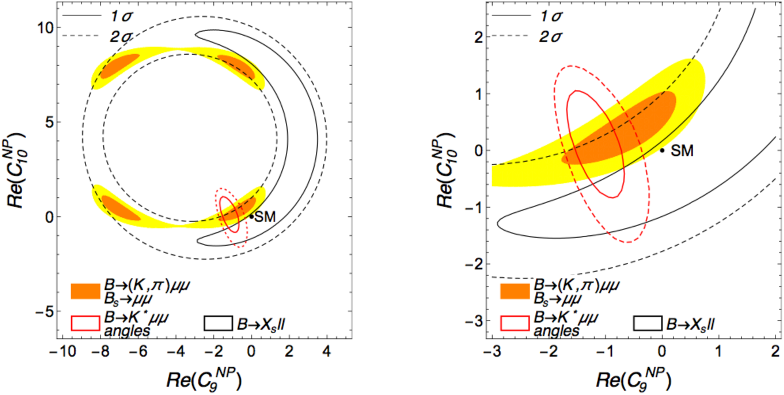
$$\mathcal{B}\left[B_s^0 \to \mu^+\mu^-\right] = 2.8^{+0.7}_{-0.6} \times 10^{-9}$$
$$\mathcal{B}\left[B^0 \to \mu^+\mu^-\right] = 3.9^{+1.6}_{-1.4} \times 10^{-10}$$

- Ratio found to be compatible with SM to 2.3σ.



Test of the SM in Semileptonic B-decays and $B_s \to \mu^+\mu^-$

[Fermilab/MILC, arxiv:1510.02349]



Summary and outlook

- Lattice QCD, QCD sum rules, and heavy quark symmetry provide a controlled theoretical framework for B-meson physics
- Despite this impressive progress, some non-perturbative power corrections remain to be calculated quantitatively, limiting the current theoretical precision
- B-decays have been measured over 9 orders of magnitude and are found to be compatible with the SM, in general
- There is some tension on the SM in a number of rare B decays, typically 2 -3 σ; whether this is due to New Physics or QCD remains to be seen
- FCNC processes remain potentially very promising to search for physics beyond the SM, and they complement direct searches of BSM physics
- We look forward to improved theory and even more precise measurements at the LHC and the Super-B factory at KEK