The interference effects of multi-channel pion-pion scattering in final states of charmonia and bottomonia decays

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Introduction

- We considered practically all available data on the two-pion transitions of Υ mesons from the ARGUS, CLEO, CUSB, Crystal Ball, Belle, and *BaBar* Collaborations – $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2, 3, 4, 5,\ n = 1, 2, 3$) – to analyze contributions of multi-channel $\pi\pi$ scattering in the final-state interactions.
- The analysis was aimed at studying the scalar mesons and it was performed jointly considering the above bottomonia decays, the isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and the charmonium decay processes – $J/\psi \to \phi\pi\pi$, $\psi(2S) \to J/\psi\pi\pi$ – with data from the Crystal Ball, DM2, Mark II, Mark III, and BES II Collaborations.
- The multi-channel $\pi\pi$ scattering was described in our model-independent approach based on analyticity and unitarity and using an uniformization procedure.
- Possibility of using two-pion transitions of heavy quarkonia for studying the f_0 mesons is related to the expected fact that the dipion is produced in S wave whereas the final quarkonium is a spectator [D.Morgan, M.R.Pennington, PR D**48** (1993) 1185].

- Studying properties of scalar mesons is important but it is still far away to be solved completely [K.A.Olive et al. (PDG), Chin.Phys. C**38** (2014) 090001]. E.g., using our model-independent method in the 3-channel analyses of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ [Yu.S. Surovtsev et al., PR D81 (2010) 016001; PR D85 (2012) 036002] we obtained parameters of the $f_0(500)$ and $f_0(1500)$ which considerably differ from results of analyses based on other methods (mainly those based on dispersion relations and Breit–Wigner approaches).
- In the heavy-meson decay, explanation of the dipion mass distributions for the $\Upsilon(mS)$ $(m > 2)$ contains a number of surprises.
- E.g.,a distinction of the $\Upsilon(3S)$ decays from the $\Upsilon(2S)$ ones – in the former a phase space cuts off, as if, possible contributions which can interfere destructively with the $\pi\pi$-scattering contribution giving a characteristic two-humped shape of the dipion mass distribution in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$.
- In a number of works (see, e.g., Yu.A. Simonov and A.I. Veselov, PR D**79** (2009) 034024 and the references therein, and our discussion in Yu.S.Surovtsev et al., PR D**91** (2015) 037901) various (sometimes rather doubtful) assumptions were made to obtain the needed result.

- We have explained this effect on the basis of our previous conclusions without any additional assumptions.
- In (Yu.S.Surovtsev et al.,PR D**89** (2014) 036010; J.Phys.G: Nucl.Part.Phys.**41** (2014) 025006; PR D**86** (2012) 116002) we shown: If a wide resonance cannot decay into a channel which opens above its mass, but the resonance is strongly coupled to this channel (e.g. $f_0(500)$ and $K\overline{K}$ channel), then one should consider this resonance as a multi-channel state.
- In one's turn, the $\Upsilon(4S)$ and $\Upsilon(5S)$ are distinguished from the lower Υ-states by the fact that their masses are above the $B\overline{B}$ threshold. The dipion mass distributions of these decays have the additional mysteries, e.g. the sharp dips about 1 GeV in the two-pion transitions of these states to the basic ones.
- We show that the two-pion transitions both of bottomonia and of charmonia are explained by the unified mechanism which is based on our previous conclusions on the wide resonances [Yu.S.Surovtsev et al., J.Phys. G: Nucl.Part.Phys. **41** (2014) 025006; PR D**89** (2014) 036010] and is related with interference of the contributions of multi-channel $\pi\pi$ scattering in the final-state interactions.

The multi-channel $\pi\pi$ amplitude

- In the model-independent description of the multi-channel $\pi\pi$ scattering, we considered the 3-channel case, $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, because, as we have shown, this is a minimal number of channels needed for obtaining correct values of scalar-isoscalar resonance parameters. [Yu.S. Surovtsev et al., PR D**86** (2012) 116002; J.Phys. G: Nucl.Part.Phys. **41** (2014) 025006]

- Resonance representations on the 8-sheeted Riemann surface
  - The 3-channel S-matrix is determined on the 8-sheeted Riemann surface.
  - The matrix elements S_{ij}, where $i, j = 1, 2, 3$ denote channels, have the right-hand cuts along the real axis of the s complex plane (s is Mandelstam variable), starting with the channel thresholds s_i ($i = 1, 2, 3$), and the left-hand cuts related to the crossed channels.

- The Riemann-surface sheets are numbered according to the signs of analytic continuations of $\sqrt{s-s_i}$ $(i=1,2,3)$:

	I	II	III	IV	V	VI	VII	VIII
$\mathrm{Im}\sqrt{s-s_1}$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
$\mathrm{Im}\sqrt{s-s_2}$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
$\mathrm{Im}\sqrt{s-s_3}$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$

- Uniformizing variable mapping the Riemann surface to a plane is [Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D**85** (2012) 036002)]

$$w=\frac{\sqrt{(s-s_2)s_3}+\sqrt{(s-s_3)s_2}}{\sqrt{s(s_3-s_2)}} \qquad (s_2=4m_K^2 \text{ and } s_3=4m_\eta^2).$$

where we neglected the $\pi\pi$-threshold branch-point and took into account the $K\overline{K}$- and $\eta\eta$-threshold branch-points and the left-hand branch-point at $s=0$ related to the crossed channels.

- Resonance representations on the Riemann surface are obtained using formulas from [D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, NC A**109** (1996) 281], expressing analytic continuations of the S-matrix elements to all sheets in terms of those on the physical (I) sheet that have only the resonances zeros (beyond the real axis), at least, around the physical region.
- Then multi-channel resonances are classified. In the 3-channel case, there are *7 types* of resonances corresponding to 7 possible situations when there are resonance zeros on sheet I only in S_{11} – (**a**); S_{22} – (**b**); S_{33} – (**c**); S_{11} and S_{22} – (**d**); S_{22} and S_{33} – (**e**); S_{11} and S_{33} – (**f**); S_{11}, S_{22} and S_{33} – (**g**).
  The resonance of every type which is related to its nature is represented by the pair of complex-conjugate clusters (of poles and zeros on the Riemann surface).
  Let us explain in the 2-channel example how pole cluster describing resonance arises. In the 1-channel consideration of the scattering $1 \to 1$ the main model-independent contribution of resonance is given by a pair of conjugate poles on sheet II and by a pair of conjugate zeros on sheet I at the same points of complex energy in S_{11}.

In the 2-channel consideration of the processes $1 \to 1$, $1 \to 2$ and $2 \to 2$, we have

$$S_{11}^{\mathrm{II}} = \frac{1}{S_{11}^{\mathrm{I}}}, \qquad S_{11}^{\mathrm{III}} = \frac{S_{22}^{\mathrm{I}}}{S_{11}^{\mathrm{I}} S_{22}^{I} - (S_{12}^{\mathrm{I}})^2}, \qquad S_{11}^{\mathrm{IV}} = \frac{S_{11}^{\mathrm{I}} S_{22}^{\mathrm{I}} - (S_{12}^{\mathrm{I}})^2}{S_{22}^{I}},$$

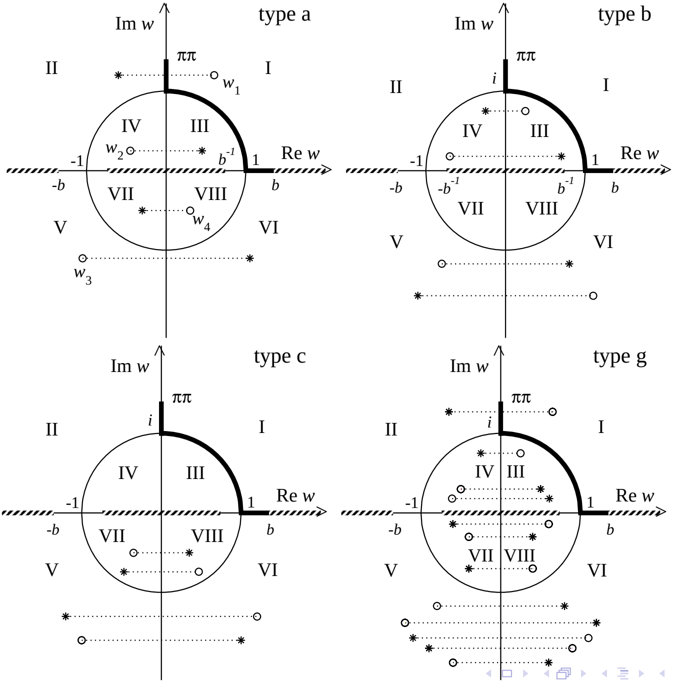
$$S_{22}^{\mathrm{II}} = \frac{S_{11}^{\mathrm{I}} S_{22}^{\mathrm{I}} - (S_{12}^{\mathrm{I}})^2}{S_{11}^{\mathrm{I}}}, \qquad S_{22}^{\mathrm{III}} = \frac{S_{11}^{\mathrm{I}}}{S_{11}^{\mathrm{I}} S_{22}^{\mathrm{I}} - (S_{12}^{\mathrm{I}})^2}, \qquad S_{22}^{\mathrm{IV}} = \frac{1}{S_{22}^{\mathrm{I}}},$$

$$S_{12}^{\mathrm{II}} = \frac{i S_{12}^{\mathrm{I}}}{S_{11}^{\mathrm{I}}}, \qquad S_{12}^{\mathrm{III}} = \frac{-S_{12}^{\mathrm{I}}}{S_{11}^{\mathrm{I}} S_{22}^{\mathrm{I}} - (S_{12}^{\mathrm{I}})^2}, \qquad S_{12}^{\mathrm{IV}} = \frac{i S_{12}^{\mathrm{I}}}{S_{22}^{\mathrm{I}}}.$$

In S_{11} a resonance is represented by a pair of conjugate poles on sheet II and by a pair of conjugate zeros on sheet I and also by a pair of conjugate poles on sheet III and by a pair of conjugate zeros on sheet IV at the same points of complex energy if the coupling of channels is absent ($S_{12} = 0$). If the resonance decays into both channels and/or takes part in exchanges in the crossing channels, the coupling of channels arises ($S_{12} \neq 0$). Then positions of the poles on sheet III (and of corresponding zeros on sheet IV) turn out to be shifted with respect to the positions of zeros on sheet I. Thus we obtain the cluster (of type (**a**)) of poles and zeros in the 2-channel case.

Process	I	II	III	IV	V	VI	VII	VIII
$1 \to 1$	S_{11}	$\frac{1}{S_{11}}$	$\frac{S_{22}}{D_{33}}$	$\frac{D_{33}}{S_{22}}$	$\frac{\det S}{D_{11}}$	$\frac{D_{11}}{\det S}$	$\frac{S_{33}}{D_{22}}$	$\frac{D_{22}}{S_{33}}$
$1 \to 2$	S_{12}	$\frac{iS_{12}}{S_{11}}$	$\frac{-S_{12}}{D_{33}}$	$\frac{iS_{12}}{S_{22}}$	$\frac{iD_{12}}{D_{11}}$	$\frac{-D_{12}}{\det S}$	$\frac{iD_{12}}{D_{22}}$	$\frac{D_{12}}{S_{33}}$
$2 \to 2$	S_{22}	$\frac{D_{33}}{S_{11}}$	$\frac{S_{11}}{D_{33}}$	$\frac{1}{S_{22}}$	$\frac{S_{33}}{D_{11}}$	$\frac{D_{22}}{\det S}$	$\frac{\det S}{D_{22}}$	$\frac{D_{11}}{S_{33}}$
$1 \to 3$	S_{13}	$\frac{iS_{13}}{S_{11}}$	$\frac{-iD_{13}}{D_{33}}$	$\frac{-D_{13}}{S_{22}}$	$\frac{-iD_{13}}{D_{11}}$	$\frac{D_{13}}{\det S}$	$\frac{-S_{13}}{D_{22}}$	$\frac{iS_{13}}{S_{33}}$
$2 \to 3$	S_{23}	$\frac{D_{23}}{S_{11}}$	$\frac{iD_{23}}{D_{33}}$	$\frac{iS_{23}}{S_{22}}$	$\frac{-S_{23}}{D_{11}}$	$\frac{-D_{23}}{\det S}$	$\frac{iD_{23}}{D_{22}}$	$\frac{iS_{23}}{S_{33}}$
$3 \to 3$	S_{33}	$\frac{D_{22}}{S_{11}}$	$\frac{\det S}{D_{33}}$	$\frac{D_{11}}{S_{22}}$	$\frac{S_{22}}{D_{11}}$	$\frac{D_{33}}{\det S}$	$\frac{S_{11}}{D_{22}}$	$\frac{1}{S_{33}}$

In Table, the superscript I is omitted to simplify the notation, $\det S$ is the determinant of the 3×3 S-matrix on sheet I, $D_{\alpha\beta}$ is the minor of the element $S_{\alpha\beta}$, that is, $D_{11} = S_{22}S_{33} - S_{23}^2$, $D_{22} = S_{11}S_{33} - S_{13}^2$, $D_{33} = S_{11}S_{22} - S_{12}^2$, $D_{12} = S_{12}S_{33} - S_{13}S_{23}$, $D_{23} = S_{11}S_{23} - S_{12}S_{13}$, etc. These formulas show how singularities and resonance poles and zeros are transferred from the matrix element S_{11} to matrix elements of coupled processes.


type a
Im w
Re w
ππ
I
II
III
IV
V
VI
VII
VIII
w1
w2
w3
w4
-1
1
-b
b
b-1
type b
i
-b-1
type c
type g

The S-matrix parametrization

- The S-matrix elements S_{ij} are parameterized via the Jost matrix determinant $d(w)$ using the Le Couteur-Newton relations [K.J.Le Couteur, Proc.R.London, Ser. A**256** (1960) 115; R.G.Newton, J.Math.Phys. **2** (1961) 188; M.Kato, Ann.Phys. **31** (1965) 130].

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \qquad S_{22} = \frac{d(-w^{-1})}{d(w)}, \qquad S_{33} = \frac{d(w^{-1})}{d(w)},$$

$$S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{*-1})}{d(w)}, \qquad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{*-1})}{d(w)}.$$

- The S-matrix elements are taken as the products $S = S_B S_{res}$
  - the main (model-independent) contribution of resonances, given by the pole clusters, is included in the resonance part S_{res};
  - possible remaining small (model-dependent) contributions of resonances and influence of channels not taken explicitly into account in the uniformizing variable are included in the background part S_B.

- The d-function: for the resonance part

$$d_{res}(w) = w^{-\frac{M}{2}} \prod_{r=1}^{M} (w + w_r^*) \quad (M \text{ is number of resonance zeros})$$

for the background part $\quad d_B = \exp[-i \sum_{n=1}^{3} \frac{\sqrt{s-s_n}}{2m_n}(\alpha_n + i\beta_n)]$,

$$\alpha_n = a_{n1} + a_{n\sigma}\frac{s-s_\sigma}{s_\sigma}\theta(s-s_\sigma) + a_{nv}\frac{s-s_v}{s_v}\theta(s-s_v),$$

$$\beta_n = b_{n1} + b_{n\sigma}\frac{s-s_\sigma}{s_\sigma}\theta(s-s_\sigma) + b_{nv}\frac{s-s_v}{s_v}\theta(s-s_v)$$

where s_σ is the $\sigma\sigma$ threshold; s_v is the combined threshold of the $\eta\eta',\ \rho\rho,\ \omega\omega$ channels.

- The resonance zeros w_r and the background parameters were fixed by fitting to data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$.

Results of the analysis of data on $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$

- For the data on multi-channel $\pi\pi$ scattering we used the results of phase analyses: phase shifts $\delta_{\alpha\beta}$ and modules of the S-matrix elements $\eta_{\alpha\beta} = |S_{\alpha\beta}|$ $(\alpha, \beta = 1,\ 2,\ 3)$:

$$S_{\alpha\alpha} = \eta_{\alpha\alpha} e^{2i\delta_{\alpha\alpha}}, \qquad S_{\alpha\beta} = i\eta_{\alpha\beta} e^{i\phi_{\alpha\beta}}.$$

- For the $\pi\pi$ scattering, the data are taken from the threshold to 1.89 GeV from [J.R.Batley et al, EPJ C**54** (2008) 411; B.Hyams et al., NP B**64** (1973) 134; **100** (1975) 205; A.Zylbersztejn et al., PL B**38** (1972) 457; P.Sonderegger, P.Bonamy, in: Proc. 5th Intern. Conf. on Elem. Part., Lund, 1969, paper 372; J.R.Bensinger et al., PL B**36** (1971) 134; J.P.Baton et al., PL B**33** (1970) 525, 528; P.Baillon et al., PL B**38** (1972) 555; L.Rosselet et al., PR D**15** (1977) 574; A.A.Kartamyshev et al., Pis'ma v ZhETF **25** (1977) 68; A.A.Bel'kov et al., Pis'ma v ZhETF **29** (1979) 652].

- For $\pi\pi \to K\overline{K}$, practically all the accessible data are used [W.Wetzel et al., NP B**115** (1976) 208; V.A.Polychronakos et al., PR D**19** (1979) 1317; P.Estabrooks, PR D**19** (1979) 2678 ; D.Cohen et al., PR D**22** (1980) 2595; G.Costa et al., NP B**175** (1980) 402; A.Etkin et al., PR D**25** (1982) 1786].
- For $\pi\pi \to \eta\eta$, we used data for $|S_{13}|^2$ from the threshold to 1.72 GeV [F.Binon et al., NC A**78** (1983) 313].
- More preferable scenario: the $f_0(500)$ is described by the cluster of type (**a**); the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, type (**c**) and $f_0'(1500)$, type (**g**); the $f_0(980)$ is represented only by the pole on sheet II and shifted pole on sheet III — this result is important for interpretation of the $f_0(980)$ as neither a $q\bar{q}$ state nor the $K\overline{K}$ molecule [Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D**85** (2012) 036002)].
- Analyzing these data, we have obtained two solutions which differ mainly in the width of $f_0(500)$.
  Further we show the solution preferable by the analysis of the data on decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ from the Mark III, DM2 and BES II Collaborations.

Table : The pole clusters for resonances on the $\sqrt{s}$-plane. $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$.

Sheet		$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
II	E_r	521.6±12.4	1008.4±3.1			1512.4±4.9	
	$\Gamma_r/2$	467.3±5.9	33.5±1.5			287.2±12.9	
III	E_r	552.5±17.7	976.7±5.8	1387.2±24.4		1506.1±9.0	
	$\Gamma_r/2$	467.3±5.9	53.2±2.6	167.2±41.8		127.8±10.6	
IV	E_r			1387.2±24.4		1512.4±4.9	
	$\Gamma_r/2$			178.2±37.2		215.0±17.6	
V	E_r			1387.2±24.4	1493.9±3.1	1498.8±7.2	1732.8±43.2
	$\Gamma_r/2$			261.0±73.7	72.8±3.9	142.3±6.0	114.8±61.5
VI	E_r	573.4±29.1		1387.2±24.4	1493.9±5.6	1511.5±4.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9		250.0±83.1	58.4±2.8	179.3±4.0	111.2±8.8
VII	E_r	542.5±25.5			1493.9±5.0	1500.4±9.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9			47.8±9.3	99.9±18.0	55.2±38.0
VIII	E_r				1493.9±3.2	1512.4±4.9	1732.8±43.2
	$\Gamma_r/2$				62.2±9.2	298.4±14.5	58.8±16.4

- Generally, *wide multi-channel states are most adequately represented by pole clusters*, because the pole clusters give the main model-independent effect of resonances. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model-dependent for wide resonances.
- The mass values are needed in some cases, e.g., in mass relations for multiplets. We stress that such parameters of the wide multi-channel states, as *masses, total widths and coupling constants with channels, should be calculated using the poles on sheets II, IV and VIII*, because only on these sheets the analytic continuations have the forms:

$$\propto 1/S_{11}^{\mathrm{I}}, \quad \propto 1/S_{22}^{\mathrm{I}} \ \text{ and } \quad \propto 1/S_{33}^{\mathrm{I}},$$

respectively, i.e., the pole positions of resonances are at the same points of the complex-energy plane, as the resonance zeros on the physical sheet, and are not shifted due to the coupling of channels.

E.g., if the resonance part of amplitude is taken as

$$T^{res} = \sqrt{s}\,\Gamma_{el}/(m_{res}^2 - s - i\sqrt{s}\,\Gamma_{tot}),$$

for the mass and total width, one obtains

$$m_{res} = \sqrt{\mathrm{E}_r^2 + (\Gamma_r/2)^2} \quad \text{and} \quad \Gamma_{tot} = \Gamma_r,$$

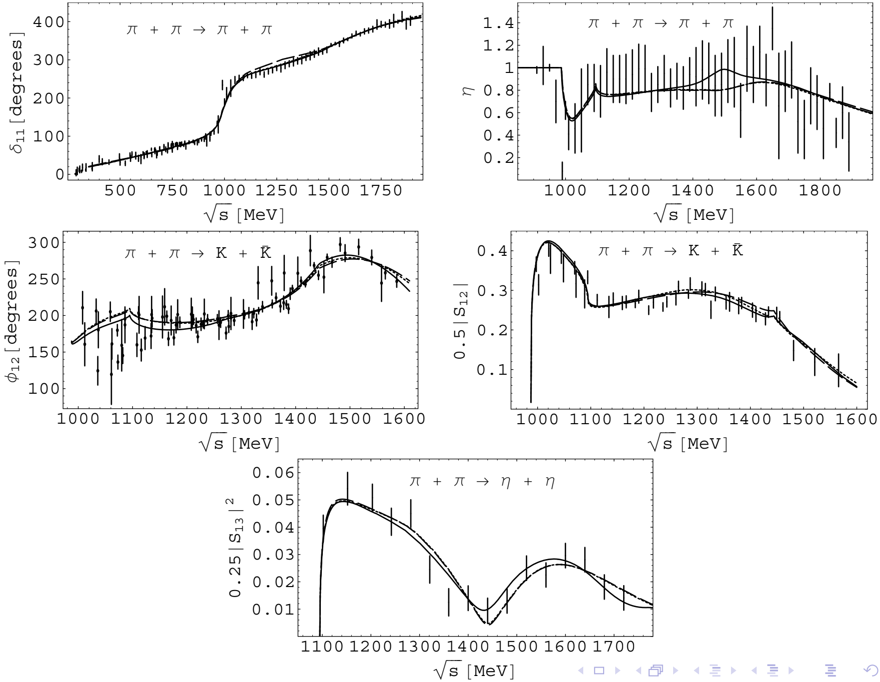
where the pole position $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$ must be taken on sheets II, IV, VIII, depending on the resonance classification.

Table : The masses and total widths of the f_0 resonances.

	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
m_{res}[MeV]	693.9±10.0	1008.1±3.1	1399.0±24.7	1495.2±3.2	1539.5±5.4	1733.8±43.2
Γ_{tot}[MeV]	931.2±11.8	64.0±3.0	357.0±74.4	124.4±18.4	571.6±25.8	117.6±32.8

- The obtained background parameters are very small which confirms our assumption $S = S_B S_{res}$ and also that representation of multi-channel resonances by the pole clusters on the uniformization plane is good and quite sufficient.
  It is also a criterion for the correctness of the approach
- Furthermore, this shows that the consideration of the left-hand branch-point at $s = 0$ in the uniformizing variable solves partly a problem of some approaches (see, e.g., N.N. Achasov, G.N. Shestakov, PR D**49** (1994) 5779) that the wide-resonance parameters are strongly controlled by the non-resonant background.
- Studying further the decays of charmonia and bottomonia, we investigated the role of the individual f_0 resonances in contributing to the shape of the dipion mass distributions. In this case we switched off only those resonances [$f_0(500)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$], removal of which can be somehow compensated by correcting the background (maybe, with elements of the pseudobackground) to have the more-or-less acceptable description of the multichannel $\pi\pi$ scattering. Therefore, below we show also the description of the multichannel $\pi\pi$ scattering more for two cases.

- First, when leaving out a minimal set of the f_0 mesons consisting of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$, which is sufficient to achieve a description of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ with a total $\chi^2/\text{ndf} \approx 1.20$.
- Second, from above-indicated three mesons only the $f_0(500)$ can be switched off while still obtaining a reasonable description of multichannel $\pi\pi$ scattering (though with an appearance of the pseudobackground) with a total $\chi^2/\text{ndf} \approx 1.43$.
- In the following figures we show the obtained description of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.


π + π → π + π
δ11 [degrees]
0
100
200
300
400
500
750
1000
1250
1500
1750
√s [MeV]
π + π → π + π
η
0.2
0.4
0.6
0.8
1
1.2
1.4
1000
1200
1400
1600
1800
√s [MeV]
π + π → K + K̄
φ12 [degrees]
100
150
200
250
300
1000
1100
1200
1300
1400
1500
1600
√s [MeV]
π + π → K + K̄
0.5|S12|
0.1
0.2
0.3
0.4
1000
1100
1200
1300
1400
1500
1600
√s [MeV]
π + π → η + η
0.25|S13|^2
0.01
0.02
0.03
0.04
0.05
0.06
1100
1200
1300
1400
1500
1600
1700
√s [MeV]

The contribution of multi-channel $\pi\pi$ scattering in the final states of decays of Ψ- and Υ-meson families

- The amplitudes of decays are related with the scattering amplitudes T_{ij} $(i,j = 1-\pi\pi, 2-K\overline{K}, 3-\eta\eta)$ as follows [D.Morgan, M.R.Pennington, PR D**48** (1993) 1185].

$$F\big(J/\psi \to \phi\pi\pi\big) = c_1(s)T_{11} + \Big(\frac{\alpha_2}{s-\beta_2} + c_2(s)\Big)T_{21} + c_3(s)T_{31},$$

$$F\big(\psi(2S) \to \psi(1S)\pi\pi\big) = d_1(s)T_{11} + d_2(s)T_{21} + d_3(s)T_{31},$$

$$F\big(\Upsilon(mS) \to \Upsilon(nS)\pi\pi\big) = e_1^{(mn)}T_{11} + e_2^{(mn)}T_{21} + e_3^{(mn)}T_{31},$$

$$m > n,\ \ m = 2,3,4,5,\ \ n = 1,2,3$$

where $c_i = \gamma_{i0} + \gamma_{i1}s$, $d_i = \delta_{i0} + \delta_{i1}s$ and $e_i^{(mn)} = \rho_{i0}^{(mn)} + \rho_{i1}^{(mn)}s$; indices m and n correspond to $\Upsilon(mS)$ and $\Upsilon(nS)$, respectively.

- The free parameters α_2, β_2, γ_{i0}, γ_{i1}, δ_{i0}, δ_{i1}, $\rho_{i0}^{(mn)}$ and $\rho_{i1}^{(mn)}$ depend on the couplings of J/ψ, $\psi(2S)$ and the $\Upsilon(mS)$ to the channels $\pi\pi$, $K\overline{K}$ and $\eta\eta$.

- It is assumed that pairs of pseudo-scalar mesons of final states have $I = J = 0$ and only they undergo strong interactions, whereas a final vector meson (ϕ, ψ, Υ) acts as a spectator.
- For $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ we have taken data from [W.Lockman, Proc.Hadron'89 Conf., ed. F.Binon et al.(Mark III), (Editions Frontières, Gif-sur-Yvette,1989) p.109; A.Falvard et al.(DM2), PR D**38** (1988) 2706; M.Ablikim et al.(BES II), PL B**607** (2005) 243]; for $\psi(2S) \to J/\psi(\pi^+\pi^-)$ from [G.Gidal et al.(Mark II), PL B**107** (1981) 153]; for $\psi(2S) \to J/\psi(\pi^0\pi^0)$ from [M.Oreglia et al.(Crystal Ball(80)), PRL **45** (1980) 959]; for $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ from [H.Albrecht et al.(Argus), PL B**134** (1984) 137; D.Besson et al.(CLEO), PR D**30** (1984) 1433; V.Fonseca et al.(CUSB), NP B**242** (1984) 31; D.Gelphman et al.(Crystal Ball(85)), PR D**32** (1985) 2893 (1985)]; for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ and $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$ from [D.Cronin-Hennessy et al.(CLEO(07)), PR D**76** (2007) 072001; F.Butler et al.(CLEO(94)), PR D**49** (1994) 40]; finally, for $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ from [B.Aubert et al.(BaBar(06)), PRL **96** (2006) 232001; A.Sokolov et al.(Belle(07)), PR D**75** (2007) 071103; A.Bondar et al.(Belle(12)), PRL **108** (2012) 122001].

The pole term in the middle equation in front of T_{21} is an approximation of possible ϕK states, not forbidden by OZI rules.
The expression for decay $J/\psi \to \phi\pi\pi$

$$N|F|^2\sqrt{(s-s_i)[m_\psi^2-(\sqrt{s}-m_\phi)^2][m_\psi^2-(\sqrt{s}+m_\phi)^2]}$$

and the analogues relations for $\psi(2S) \to \psi(1S)\pi\pi$ and $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ give the di-meson mass distributions.
N (normalization to experiment) is: for $J/\psi \to \phi\pi\pi$ 0.5172 (Mark III), 0.1746 (DM 2) and 3.8 (BES II); for $\psi(2S) \to J/\psi\pi^+\pi^-$ 1.746 (Mark II); for $\psi(2S) \to J/\psi\pi^0\pi^0$ 1.6891 (Crystal Ball(80)); for $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$ 4.1758 (ARGUS), 2.0445 (CLEO(94)) and 1.0782 (CUSB); for $\Upsilon(2S) \to \Upsilon(1S)\pi^0\pi^0$ 0.0761 (Crystal Ball(85)); for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^- \text{ and } \pi^0\pi^0)$ 19.8825 and 4.622 (CLEO(07)); for $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^- \text{ and } \pi^0\pi^0)$ 1.6987 and 1.1803 (CLEO(94)); for $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ 4.6827 (BaBar(06)) and 0.3636 (Belle(07)); for $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$, 37.9877 (BaBar(06)); for $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(3S)\pi^+\pi^-$ respectively 0.2047, 2.8376 and 6.9251 (Belle(12)).

Satisfactory combined description of all considered processes is obtained with the total $\chi^2/\text{ndf} = 736.457/(710 - 118) \approx 1.24$;
for the $\pi\pi$ scattering, $\chi^2/\text{ndf} \approx 1.15$;
for $\pi\pi \to K\overline{K}$, $\chi^2/\text{ndf} \approx 1.65$;
for $\pi\pi \to \eta\eta$, $\chi^2/\text{ndp} \approx 0.87$;
for decays $J/\psi \to \phi(\pi^+\pi^-)$, $\chi^2/\text{ndf} \approx 1.05$
for $\psi(2S) \to J/\psi(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndp} \approx 2.29$;
for $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 1.11$;
for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 1.01$,
for $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$, $\chi^2/\text{ndf} \approx 0.70$,
for $\Upsilon(4S) \to \Upsilon(1S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 0.25$,
for $\Upsilon(4S) \to \Upsilon(2S)(\pi^+\pi^-)$, $\chi^2/\text{ndp} \approx 0.25$,
for $\Upsilon(5S) \to \Upsilon(1S)(\pi^+\pi^-)$, $\chi^2/\text{ndf} \approx 1.78$,
for $\Upsilon(5S) \to \Upsilon(2S)(\pi^+\pi^-)$, $\chi^2/\text{ndf} \approx 1.10$,
for $\Upsilon(5S) \to \Upsilon(3S)(\pi^+\pi^-)$, $\chi^2/\text{ndf} \approx 1.30$.

The total χ^2/ndf for the considered decays of bottomonia and charmonia are about 1.14 and 1.65, respectively.

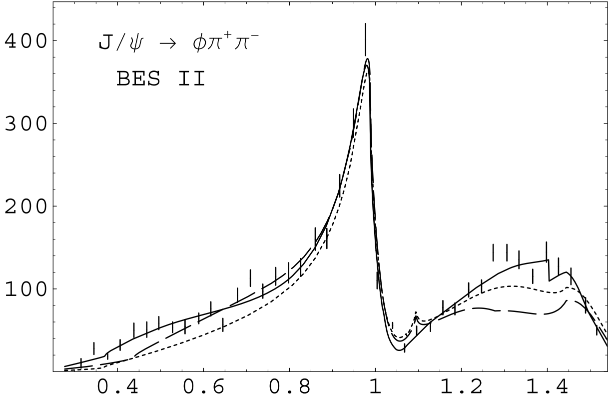


Figure : The $J/\psi \to \phi\pi\pi$ decay; the data of BES II Collaboration. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.

Important role of the BES II data: Namely this di-pion mass distribution rejects the solution with the narrower $f_0(500)$. The corresponding curve lies considerably below the data from the threshold to about 850 MeV.

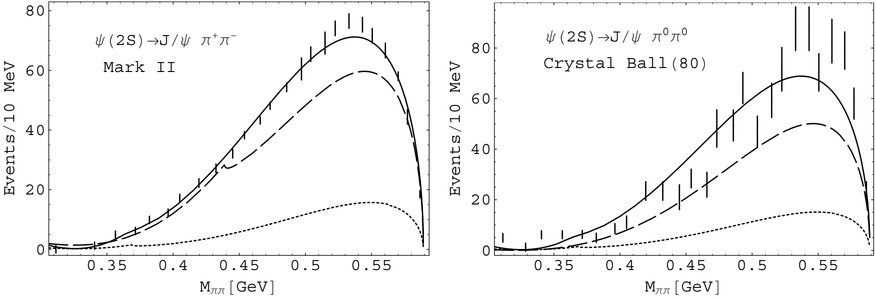


Figure : The $\psi(2S) \to J/\psi\pi\pi$ decay. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.

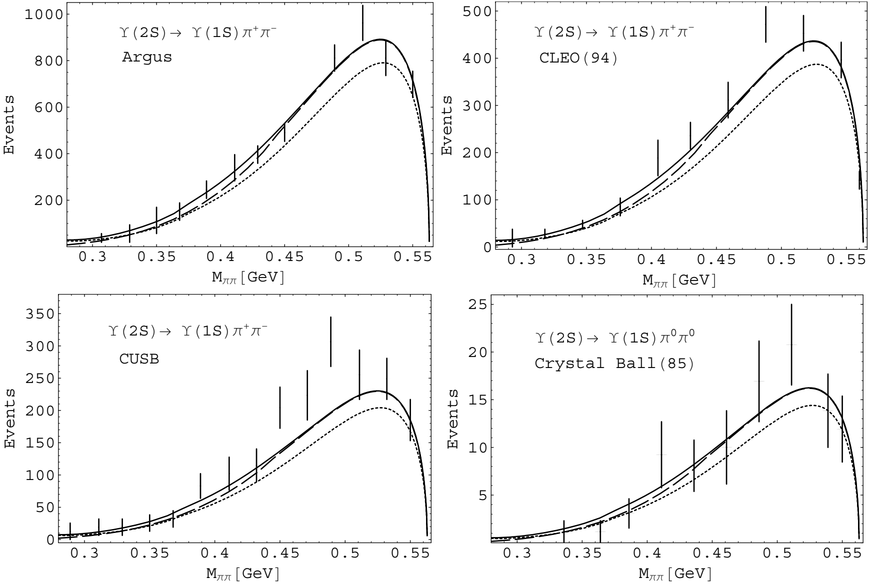


Figure : The $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ decay.

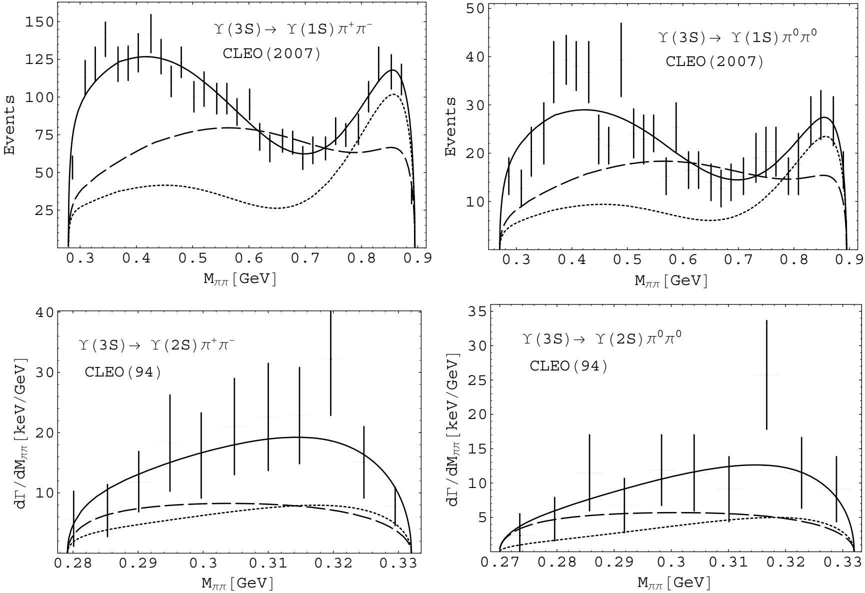


Figure : The decays $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$.

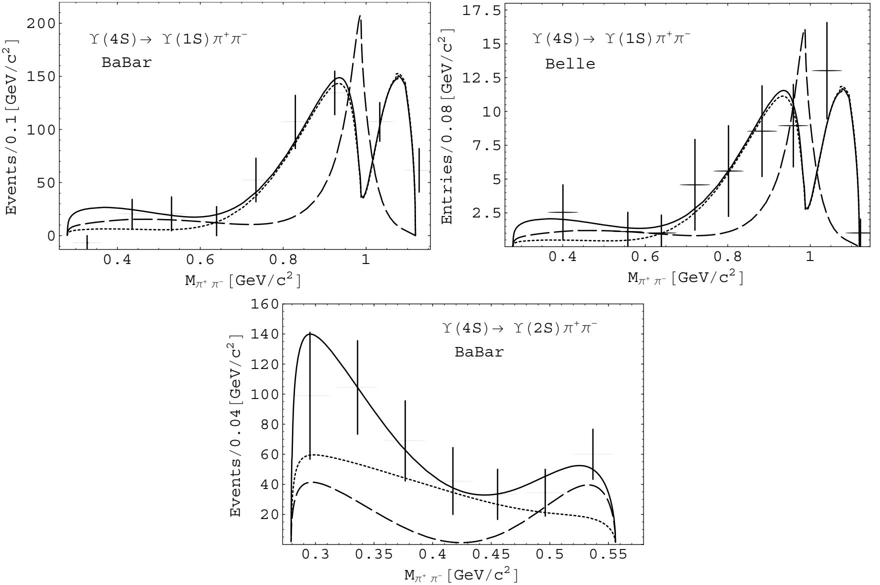


Figure : The decays $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.

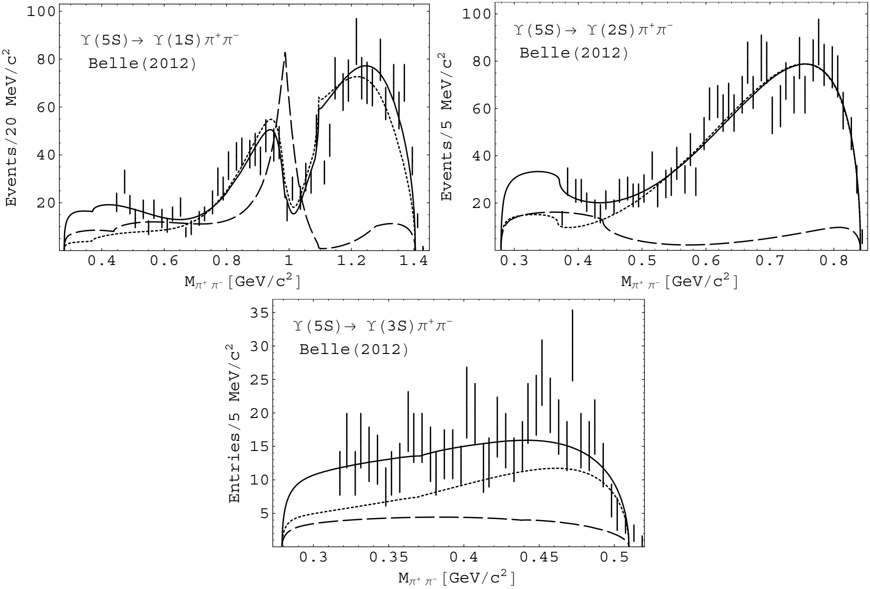


Figure : The decays $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ $(n = 1, 2, 3)$. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.

- The curves of the Υ decays demonstrate interesting behavior, beginning from the second radial excitation and higher, — a bell-shaped form in the near-$\pi\pi$-threshold region, smooth dips about 0.6 GeV in the $\Upsilon(4S,5S) \to \Upsilon(1S)\pi^+\pi^-$, about 0.45 GeV in the $\Upsilon(4S,5S) \to \Upsilon(2S)\pi^+\pi^-$, and about 0.7 GeV in the $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-,\pi^0\pi^0)$, and also sharp dips about 1 GeV in the $\Upsilon(4S,5S) \to \Upsilon(1S)\pi^+\pi^-$. Obviously, this shape of dipion mass distributions is explained by the interference between the $\pi\pi$ scattering, $K\overline{K} \to \pi\pi$ and $\eta\eta \to \pi\pi$ contributions to the final states of these decays — by the constructive one in the near-$\pi\pi$-threshold region and by the destructive one in the dip regions.
- Whereas the data on $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$ confirm the sharp dips about 1 GeV, the scarce data on $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ do not allow for such a unique conclusion yet. Here there helps us the consideration of a role of the individual f_0 resonances in making up a shape of the dipion mass distributions. Switching off the $f_0(500)$, we see that the sharp dips about 1 GeV in decays $\Upsilon(4S,5S) \to \Upsilon(1S)\pi^+\pi^-$ are related with the $f_0(500)$ contribution to the interfering amplitudes of $\pi\pi$ scattering, $K\overline{K} \to \pi\pi$ and $\eta\eta \to \pi\pi$ processes.

Conclusions

- The combined analysis was performed for data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on the decays of the charmonia — $J/\psi \to \phi\pi\pi$, $\psi(2S) \to J/\psi\,\pi\pi$ — and of the bottomonia — $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2, 3, 4, 5$, $n = 1, 2, 3$) from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, BES II, *BaBar*, and Belle Collaborations.
- It is shown that the dipion mass spectra in the above-indicated decays of charmonia and bottomonia are explained by the unified mechanism which is based on our previous conclusions on wide resonances [Yu.S.Surovtsev et al., J.Phys. G: Nucl.Part.Phys. **41** (2014) 025006; PR D**89** (2014) 036010] and is related to contributions of the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ coupled channels including their interference.
- It is shown that in the final states of these decays (except $\pi\pi$ scattering) the contribution of coupled processes, e.g., $K\overline{K}, \eta\eta \to \pi\pi$, is important even if these processes are energetically forbidden. This is in accordance with our previous conclusions on the wide resonances [Yu.S.Surovtsev et al., J.Phys. G: Nucl.Part.Phys. **41** (2014) 025006; PR D**89** (2014) 036010]: If a wide resonance cannot decay into some channels which open above its mass but the resonance is strongly connected with these channels (e.g. the $f_0(500)$ and the $K\overline{K}$ and $\eta\eta$ channels), one should consider this resonance as a multi-channel state.

- The role of the individual f_0 resonances in making up the shape of the dipion mass distributions in the charmonia and bottomonia decays is considered. Note the unexpected result — a considerable contribution of the $f_0(1370)$ to the bell-shaped form of the dipion mass spectra of bottomonia decays in the near-$\pi\pi$-threshold region.
- Since describing the bottomonia decays, we did not change resonance parameters in comparison with the ones obtained in the combined analysis of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and charmonia decays, the results of this analysis confirm all of our earlier conclusions on the scalar mesons, main of which are:
- Confirmation of the $f_0(500)$ with a mass of about 700 MeV and a width of 930 MeV (the pole on sheet II is $521.6 \pm 12.4 - i(467.3 \pm 5.9)$ MeV). This mass value is in line with prediction ($m_\sigma \approx m_\rho$) on the basis of mended symmetry by S.Weinberg [PRL **65** (1990) 1177] and with an analysis using the large-N_c consistency conditions between the unitarization and resonance saturation suggesting $m_\rho - m_\sigma = O(N_c^{-1})$ [J.Nieves, E.Ruiz Arriola, PR D**80** (2009) 045023]. Also the prediction of a soft-wall AdS/QCD approach [T.Gutsche et al., PR D**87** (2013) 056001] for the mass of the lowest f_0 meson – 721 MeV – practically coincides with the value obtained in our work.

- Indication for the $f_0(980)$ (the pole on sheet II is 1008.1 ± 3.1 $-i(32.0 \pm 1.5)$ MeV) to be neither a $q\bar{q}$ state nor the $K\overline{K}$ molecule, but possibly the bound $\eta\eta$ state, basing on the earlier-proposed test [D.Morgan, M.R.Pennington, PR D**48** (1993) 1185; KMS, NC A**109** (1996) 281].
- Indication for the $f_0(1370)$ and $f_0(1710)$ to have a dominant $s\bar{s}$ component. This is in agreement with a number of experiments: Conclusion on the $f_0(1370)$ agrees with the work of Crystal Barrel Collaboration [C.Amsler et al., PL B**355** (1995) 425] where the $f_0(1370)$ is identified as $\eta\eta$ resonance in $\bar{p}p \to \pi^0\eta\eta$. This explains also why one did not find this state considering only the $\pi\pi$ scattering [W.Ochs, arXiv:1001.4486[hep-ph]; P.Minkowski, W.Ochs, EPJ C**9** (1999) 283; arXiv: hep-ph/0209225]. Conclusion on the $f_0(1710)$ is consistent with the facts that this state is observed in $\gamma\gamma \to K_S K_S$ [S.Braccini, Frascati Phys. Series **XV** (1999) 53] and not observed in $\gamma\gamma \to \pi^+\pi^-$ [R.Barate et al., PL B**472** (2000) 189].
- Indication for 2 states in the 1500-MeV region: the $f_0(1500)$ ($m_{res} \approx 1495$ MeV, $\Gamma_{tot} \approx 124$ MeV) and $f_0'(1500)$ ($m_{res} \approx 1539$ MeV, $\Gamma_{tot} \approx 574$ MeV). The latter is interpreted as a glueball allowing for its biggest width among the enclosing states [V.V.Anisovich et al., NP Proc.Suppl.A**56** (1997) 270].

APPENDICES

The obtained background parameters are:
$a_{11}=0.0,\ a_{1\sigma}=0.0199,\ a_{1v}=0.0,\ b_{11}=b_{1\sigma}=0.0,\ b_{1v}=0.0338,$
$a_{21}=-2.4649,\ a_{2\sigma}=-2.3222,\ a_{2v}=-6.611,\ b_{21}=b_{2\sigma}=0.0,$
$b_{2v}=7.073,\ b_{31}=0.6421,\ b_{3\sigma}=0.4851,\ b_{3v}=0;\ s_\sigma=1.6338\ \mathrm{GeV}^2,$
$s_v=2.0857\ \mathrm{GeV}^2.$

First, when leaving out a minimal set of the f_0 mesons consisting of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$, which is sufficient to achieve a description of the processes $\pi\pi\to\pi\pi, K\overline{K}, \eta\eta$ with a total $\chi^2/\mathrm{ndf}\approx 1.20$.
The obtained, adjusted background parameters are:
$a_{11}=0.0,\ a_{1\sigma}=0.0321,\ a_{1v}=0.0,\ b_{11}=-0.0051,\ b_{1\sigma}=0.0,$
$b_{1v}=0.04;\ a_{21}=-1.6425,\ a_{2\sigma}=-0.3907,\ a_{2v}=-7.274,\ b_{21}=0.1189,$
$b_{2\sigma}=0.2741,\ b_{2v}=5.823;\ b_{31}=0.7711,\ b_{3\sigma}=0.505,\ b_{3v}=0.0.$

Second, from above-indicated three mesons only the $f_0(500)$ can be switched off while still obtaining a reasonable description of multichannel $\pi\pi$ scattering (though with an appearance of the pseudobackground) with a total $\chi^2/\mathrm{ndf}\approx 1.43$ and with the corrected background parameters:
$a_{11}=0.3513,\ a_{1\sigma}=-0.2055,\ a_{1v}=0.207,\ b_{11}=-0.0077,\ b_{1\sigma}=0.0,$
$b_{1v}=0.0378;\ a_{21}=-1.8597,\ a_{2\sigma}=0.1688,\ a_{2v}=-7.519,\ b_{21}=0.161,$
$b_{2\sigma}=0.0,\ b_{2v}=6.94;\ b_{31}=0.7758,\ b_{3\sigma}=0.4985,\ b_{3v}=0.0.$

These mesons predominantly decays into pairs of the B-meson family because these modes are not suppressed by the OZI rule: the $\Upsilon(4S)$ decays into the $B\overline{B}$ pairs form $> 96\%$ in the total width, the $\Upsilon(5S)$ decays into the pairs of the B-meson family in sum compose about 90%. In contrast, strongly reduced decay modes are $\Upsilon(4S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(4S) \to \Upsilon(2S)\pi\pi$ of about $(8.1 \pm 0.6) * 10^{-5}\%$ and $(8.6 \pm 1.3) * 10^{-5}\%$, and $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi\pi$ with $(5 \div 8) * 10^{-3}\%$. The total widths of $\Upsilon(5S)$ and $\Upsilon(4S)$ are 110 MeV and 20.5 MeV, respectively, and the one of the $\Upsilon(3S)$ is 20.32 keV. The partial decay widths of $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi\pi$ are almost of the same order as the ones of the decays $\Upsilon(3S) \to \Upsilon(1S, 2S)\pi\pi$. The decay widths of $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi\pi$ are even smaller than the latter ones by about two orders of magnitude. [K.A.Olive et al.(PDG), Chin.Phys. C**38** (2014) 090001]. Above comparison of decay widths implies that in the two-pion transitions of $\Upsilon(4S)$ and $\Upsilon(5S)$ the basic mechanism, which explains the dipion mass distributions, cannot be related to the $B\bar{B}$ transition dynamics.

One can formulate *a model-independent test as a necessary condition* to distinguish a bound state of colorless particles (*e.g.*, a $K\overline{K}$ molecule) and a $q\bar{q}$ bound state [D.Morgan, M.R.Pennington, PR D**48** (1993) 1185; KMS, NC A**109** (1996) 281]. In the 1-channel case, the existence of the particle bound-state means the presence of a pole on the real axis under the threshold on the physical sheet.

In the 2-channel case, existence of the bound-state in channel 2 ($K\overline{K}$ molecule) that, however, can decay into channel 1 ($\pi\pi$ decay), would imply the presence of the pair of complex conjugate poles on sheet II under the second-channel threshold without the corresponding shifted pair of poles on sheet III.

In the 3-channel case, the bound state in channel 3 ($\eta\eta$) that, however, can decay into channels 1 ($\pi\pi$ decay) and 2 ($K\overline{K}$ decay), is represented by the pair of complex conjugate poles on sheet II and by the pair of shifted poles on sheet III under the $\eta\eta$ threshold without the corresponding poles on sheets VI and VII.

According to this test, earlier we rejected interpretation of the $f_0(980)$ as the $K\overline{K}$ molecule because this state is represented by the cluster of type (**a**) in the 2-channel analysis of processes $\pi\pi \to \pi\pi, K\overline{K}$ and, therefore, does not satisfy the necessary condition to be the $K\overline{K}$ molecule [D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, NC A**109** (1996) 281].