Domain wall nework as QCD vacuum: confinement, chiral symmetry, hadronization

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July 29, 2016

An overall task pursued by most of approaches to QCD vacuum structure is an identification of the properties of nonperturbative gauge field configurations able to provide a coherent resolution of the confinement, chiral symmetry breaking, the $U_{\mathrm{A}}(1)$ symmetry realization and the strong CP problems, both in terms of color-charged fields and colorless hadrons.

Present approach: QCD Vacuum is considered as a medium characterized by condensates describable in terms of the nonperturbative gauge field configurations.

The other side of this task is identification of the conditions for deconfinement and chiral symmetry restoration.

45 years ago - Harmonic confinement – 4-dim. oscillator
R. P. Feynman, M, Kislinger, and F. Ravndal, Phys. Rev. D **3** (1971) 2706.

- **Confinement of both static and dynamical quarks** $\longrightarrow$

$$W(C) = \langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu} \rangle$$
$$S(x,y) = \langle \psi(y)\bar{\psi}(x) \rangle$$

- **Dynamical Breaking of chiral** $SU_L(N_f) \times SU_R(N_f)$ **symmetry** $\longrightarrow \langle \bar{\psi}(x)\psi(x) \rangle$

- $U_A(1)$ **Problem** $\longrightarrow \eta'$ (χ, Axial Anomaly)

- **Strong CP Problem** $\longrightarrow Z(\theta)$

- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes:
  hadron masses and "wave functions",
  form factors, scattering

Light mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**

What would be a formalism for coherent simultaneous description of all these nonperturbative features of QCD?

QCD vacuum as a medium characterized by certain condensates,
quarks and gluons - elementary coloured excitations (confined),
mesons and baryons - collective colorless excitations (masses, form factors, etc)

Quantum effective action of QCD!

- QCD effective action and vacuum gluon configurations
- Gluon condensates and domain wall network as QCD vacuum
- Testing the domain model - static characteristics of QCD vacuum
- Bosonization – Effective meson action
- Meson mass spectrum, decay constants
- Form factors
- Comparison with other approaches: FRG, DSE+BS, AdS/QCD and Light-Front Holography
- Summary

QCD effective action and vacuum gluon configurations

In Euclidean functional integral for YM theory one has to allow the gluon condensates to be nonzero:

$$Z = N \int\limits_{\mathcal{F}_B} DA \int\limits_{\Psi} D\psi D\bar{\psi} \exp\{-S[A,\psi,\bar{\psi}]\}$$

$$\mathcal{F}_B = \left\{ A : \lim_{V\to\infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2 \right\}.$$

B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017

L. D. Faddeev, [arXiv:0911.1013 [math-ph]]

H. Leutwyler, Nucl. Phys. B 179 (1981) 129

$A^a_\mu = B^a_\mu + Q^a_\mu$, background gauge fixing condition $D(B)Q = 0$:

$$1 = \int\limits_{\mathcal{B}} DB\Phi[A,B] \int\limits_{\mathcal{Q}} DQ \int\limits_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega]\delta[D(B^\omega)Q^\omega]$$

Q^a_μ – local (perturbative) fluctuations of gluon field with zero gluon condensate: $Q \in \mathcal{Q}$;
B^a_μ are long range field configurations with nonzero condensate: $B \in \mathcal{B}$.

$$Z = N' \int\limits_{\mathcal{B}} DB \int\limits_{\mathcal{Q}} DQ \int\limits_{\Psi} D\psi D\bar{\psi} \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q,\psi,\bar{\psi}]\}$$

The character of long range fields has yet to be identified by the dynamics of fluctuations:

$$Z = N' \int_{\mathcal{B}} DB \int_{\Psi} D\psi D\bar{\psi} \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S_{\rm QCD}[B+Q,\psi,\bar{\psi}]\}$$

$$= \int_{\mathcal{B}} DB \exp\{-S_{\rm eff}[B]\}$$

Global minima of $S_{\rm eff}[B]$ – field configurations that are dominant in the thermodynamic limit $V \to \infty$. Homogeneous Abelian (anti-)self-dual fields are of particular interest.

$$\langle F^2 \rangle : \quad A_\mu = -\frac{1}{2} n F_{\mu\nu} x_\nu, \ \tilde{F}_{\mu\nu} = \pm F_{\mu\nu}$$

$$n = T^3 \ \cos\xi + T^8 \ \sin\xi.$$

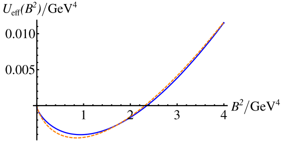
P. Minkowski, Nucl. Phys. B177 (1981) 203
H. Leutwyler, Nucl. Phys. B 179 (1981) 129

$$G\left(z^2\right) \sim \frac{e^{-Bz^2}}{z^2}, \quad \tilde{G}\left(p^2\right) \sim \frac{1}{p^2}\left(1 - e^{-p^2/B}\right)$$

H. Leutwyler, Phys. Lett. B 96 (1980) 154

Gluon propagator ⇒ Regge trajectories

G.V. Efimov, and S.N. Nedelko, Phys. Rev. D 51 (1995)



A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D 83, 045014 (2011)

Gluon condensates and domain wall network

$$\mathcal{L}_{\rm eff} = -\frac{1}{4B^2}\left(D^{ab}_{\nu}F^{b}_{\rho\mu}D^{ac}_{\nu}F^{c}_{\rho\mu} + D^{ab}_{\mu}F^{b}_{\mu\nu}D^{ac}_{\rho}F^{c}_{\rho\nu}\right) - U_{\rm eff}$$

$$U_{\rm eff} = \frac{B^4}{12}\mathrm{Tr}\left(C_1F^2 + \frac{4}{3}C_2F^4 - \frac{16}{9}C_3F^6\right),$$

B.V. Galilo, S.N. Nedelko, Phys. Part. Nucl. Lett., 8 (2011) 67

D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]

where

$$D^{ab}_{\mu} = \delta^{ab}\partial_{\mu} - iA^{ab}_{\mu} = \partial_{\mu} - iA^{c}_{\mu}(T^{c})^{ab},$$

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - if^{abc}A^{b}_{\mu}A^{c}_{\nu},$$

$$F_{\mu\nu} = F^{a}_{\mu\nu}T^{a}, \quad T^{a}_{bc} = -if^{abc}$$

$$C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

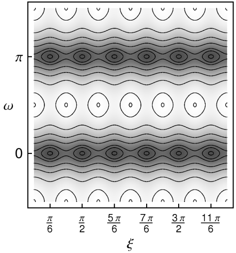
U_{eff} possesses 12 degenerate discrete minima:

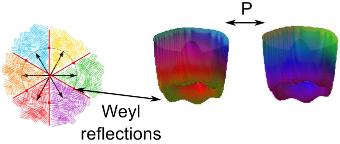
$$A_\mu = -\frac{1}{2} n_k F_{\mu\nu} x_\nu, \ \tilde{F}_{\mu\nu} = \pm F_{\mu\nu},$$

where the matrix n_k belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3 \ \cos\left(\xi_k\right) + T^8 \ \sin\left(\xi_k\right),$$

$$\xi_k = \frac{2k+1}{6}\pi, \ k = 0, 1, \ldots, 5.$$





Domain wall network

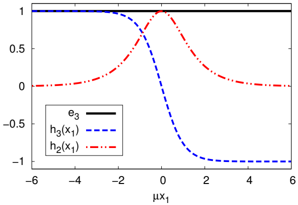
$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\Lambda^2 b_{\text{vac}}^2 \partial_\mu\omega\partial_\mu\omega - b_{\text{vac}}^4\Lambda^4 \left(C_2 + 3C_3 b_{\text{vac}}^2\right)\sin^2\omega,$$

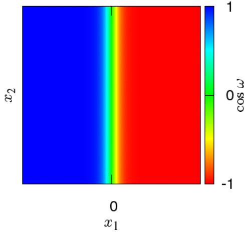
leads to sine-Gordon equation

$$\partial^2\omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\text{vac}}^2\Lambda^2 \left(C_2 + 3C_3 b_{\text{vac}}^2\right),$$

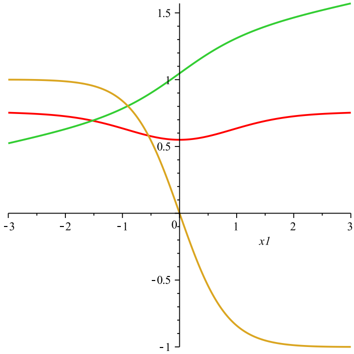
and the standard kink solution

$$\omega(x_\nu) = 2\ \text{arctg}\left(\exp(\mu x_\nu)\right)$$

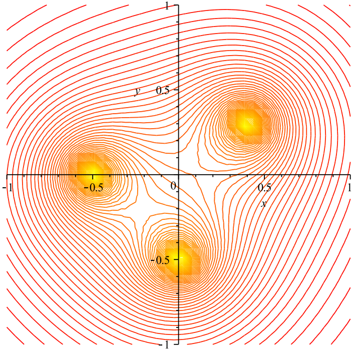




"Domain wall involving the topological charge density, $su(3)$ angle ξ and the field strength simultaneously"



"Non-topological solitonic excitations - extended objects in QCD vacuum under extreme conditions"



The general kink configuration can be parametrized as

$$\zeta(\mu_i, \eta^i_\nu x_\nu - q^i) = \frac{2}{\pi} \arctan\exp(\mu_i(\eta^i_\nu x_\nu - q^i)).$$

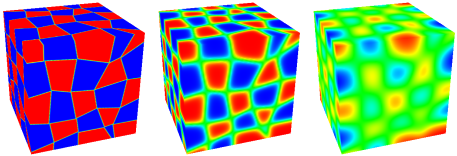
A single lump in two, three and four dimensions is given by

$$\omega(x) = \pi \prod_{i=1}^{k} \zeta(\mu_i, \eta^i_\nu x_\nu - q^i).$$

for $k = 4, 6, 8$, respectively. The general kink network is then given by the additive superposition of lumps

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{ij}, \eta^{ij}_\nu x_\nu - q^{ij})$$

S.N. Nedelko, V.E. Voronin, Eur.Phys.J. A51 (2015) 4



$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle = B^2$

$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle \ll B^2$

H. Pagels, and E. Tomboulis, Nucl. Phys. B 143 (1978) 485

P. Minkowski, Phys. Lett. B 76 (1978) 439

H. Leutwyler, Nucl. Phys. B 179 (1981);

G.V. Efimov, and S.N. Nedelko, Phys. Rev. D 51 (1995) 176

Domain bulk - harmonic confinement

Elementary color charged excitations - fluctuations decaying in all four directions.

Eigenvalue problem for scalar field in $\mathbb{R}^4$: *H. Leutwyler, Nucl. Phys. B 179 (1981);*

$$B_\mu = B_{\mu\nu}x_\nu, \widetilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu}.$$

$$-\left(\partial_\mu - iB_\mu\right)^2 G = \delta \quad \longrightarrow \quad G\left(x-y\right) \sim \frac{e^{-B(x-y)^2/4}}{(x-y)^2}$$

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = \lambda\Phi \quad \longrightarrow \quad \left[\beta_\pm^+\beta_\pm + \gamma_+^+\gamma_+ + 1\right]\Phi = \frac{\lambda}{4B}\Phi,$$

$$\beta_\pm = \frac{1}{2}(\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2}(\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}}x_\mu + \partial_\mu,$$

$$\beta_\pm^+ = \frac{1}{2}(\alpha_1^+ \pm i\alpha_2^+), \quad \gamma_\pm^+ = \frac{1}{2}(\alpha_3^+ \pm i\alpha_4^+), \quad \alpha_\mu^+ = \frac{1}{\sqrt{B}}x_\mu - \partial_\mu.$$

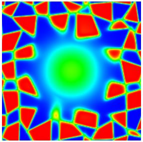
The eigenfunctions and eigenvalues - 4-dim. harmonic oscillator

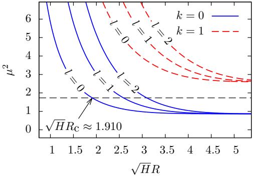
$$\Phi_{nmkl}(x) = \frac{1}{\pi^2\sqrt{n!m!k!l!}}\left(\beta_+^+\right)^k\left(\beta_-^+\right)^l\left(\gamma_+^+\right)^n\left(\gamma_-^+\right)^m\Phi_{0000}, \ \Phi_{0000} = e^{-\frac{1}{2}Bx^2}$$

$$\lambda_r = 4B(r+1), \ r = k+n \text{ self-dual field}, \ r = l+n \text{ anti-self-dual field}$$

Domain wall junctions - deconfinement

S.N. Nedelko, V.E. Voronin, Eur.Phys.J. A51 (2015) 4





The color charged scalar field inside junction:

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = 0, \quad \Phi(x) = 0, \quad x \in \mathcal{T} = \left\{x_1^2 + x_2^2 < R^2, \ (x_3, x_4) \in \mathrm{R}^2\right\}$$

The solutions are quasi-particle excitations

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a^+_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3}\right] e^{il\vartheta} \phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b^+_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3}\right] e^{-il\vartheta} \phi_{alk}(r),$$

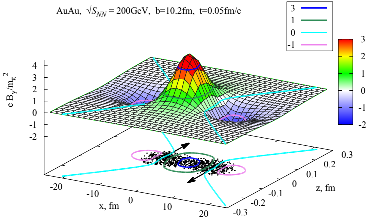
$$p_0^2 = p_3^2 + \mu^2_{akl}, \quad p_0 = \pm\omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu^2_{akl}},$$

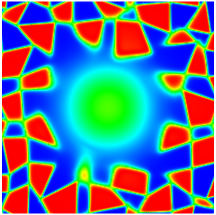
$$k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z},$$

"Polarization of QCD vacuum by the strong electromagnetic fields"

- **Relativistic heavy ion collisions - strong electromagnetic fields**

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)





Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!

B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017.

M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)

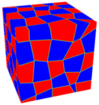
G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

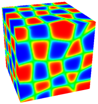
An ensemble of almost everywhere (in R^4) homogeneous Abelian (anti-)self-dual gluon fields

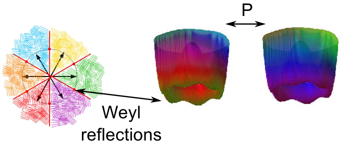
P. Minkowski, Phys. Lett. **B 76** (1978) 439.
H. Pagels, and E. Tomboulis, Nucl. Phys. B **143** (1978) 485.
P. Minkowski, Nucl. Phys. B**177** (1981) 203.
H. Leutwyler, Nucl. Phys. B **179** (1981) 129.

$$\langle : g^2 F^2 : \rangle \neq 0, \qquad \chi = \int d^4x \langle Q(x) Q(0) \rangle \neq 0, \qquad \langle Q(x) \rangle = 0$$

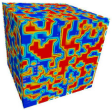
Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x) \tilde{F}^a_{\mu\nu}(x)$







Domain wall network (S.N., V.E. Voronin, EPJA (2015); A. Kalloniatis, S.N., PRD (2001))



Lattice confining configuration (P.J. Moran, D.B. Leinweber, arXiv:0805.4246v1 [hep-lat])

Testing the model: characteristics of the domain wall network ensemble

Spherical domains

A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006), Eur.Phys.J. A51 (2015), arXiv:1603.01447 [hep-ph] (2016)

Area law
Spontaneous chiral symmetry breaking
$U_A(1)$ is broken by anomaly
There is no strong CP violation

Hadronization

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995); Phys. Rev. D 54 (1996)
A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006)

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi\mathcal{D}\bar{\psi} \int_{\mathcal{Q}} \mathcal{D}Q\delta[D(B)Q]\Delta_{\mathrm{FP}}[B,Q]e^{-S^{\mathrm{QCD}}[Q+B,\psi,\bar{\psi}]} =$$

$$\int dB \int_{\Psi} \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left\{\int dx\bar{\psi}\left(i\not{\partial} + g\not{B} - m\right)\psi\right\} W[j]$$

$$W[j] = \int_{\mathcal{Q}} \mathcal{D}Q\delta[D(B)Q]\Delta_{\mathrm{FP}}[B,Q]\exp\left\{-\frac{1}{2}\int dx\ \mathrm{Tr}G^2\left[B+Q\right] + g\int dx\ Q^a_\mu j^a_\mu\right\},$$

$$j^a_\mu = \bar{\psi}\gamma_\mu t^a\psi$$

Recalling the definition of Green's functions

$$G^{a_1\dots a_n}_{\mu_1\dots\mu_n}(x_1,\dots,x_n|B) = \frac{1}{g^n}\frac{\delta^n \ln W[j]}{\delta j^{a_1}_{\mu_1}(x_1)\dots\delta j^{a_n}_{\mu_n}(x_n)},$$

we obtain

$$W[j] = \exp\left\{\sum_n \frac{g^n}{n!}\int dx_1\dots\int dx_n j^{a_1}_{\mu_1}(x_1)\dots j^{a_n}_{\mu_n}(x_n)G^{a_1\dots a_n}_{\mu_1\dots\mu_n}(x_1,\dots,x_n|B)\right\}$$

$W[j]$ is truncated up to the term including two-point gluon correlation function

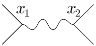
$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar\psi \exp\left\{ \int dx \bar\psi \left(i\partial\!\!\!/ + gB\!\!\!/ - m \right) \psi + \frac{g^2}{2} \int dx_1 dx_2 G^{a_1 a_2}_{\mu_1 \mu_2}(x_1, x_2|B) j^{a_1}_{\mu_1}(x_1) j^{a_2}_{\mu_2}(x_2) \right\}$$

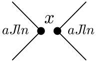
Fierz transform, center of mass coordinates $\longrightarrow \int dz dx G(z|B) J^{aJ}(x,z) J^{aJ}(x,z)$

α_s  $= \alpha_s(0)$  $\left[1 + \Pi^{\mathrm{R}}(p^2) \right]; \ \Pi^{\mathrm{R}}(0) = 0$

0  $z \ \rightarrow \dfrac{e^{-\frac{1}{4}Bz^2}}{4\pi^2 z^2}$

$\rightarrow \alpha_s(p^2) \frac{1 - \exp(-p^2/B)}{p^2}$

$\int dx_1 \, dx_2$ 

$= \int dx \sum_{aJln}$ 

$$J^{aJ}(x,z) = \sum_{nl} \left(z^2 \right)^{l/2} f^{nl}_{\mu_1 \ldots \mu_l}(z) J^{aJln}_{\mu_1 \ldots \mu_l}(x), \quad J^{aJln}_{\mu_1 \ldots \mu_l}(x) = \bar q(x) V^{aJln}_{\mu_1 \ldots \mu_l} \left(\frac{\overleftrightarrow{D}(x)}{B} \right) q(x),$$

$$f^{nl}_{\mu_1 \ldots \mu_l} = L_{nl}\left(z^2 \right) T^{(l)}_{\mu_1 \ldots \mu_l}(n_z), \quad n_z = \frac{z}{\sqrt{z}}.$$

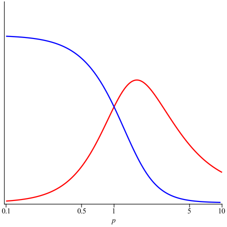
$T^{(l)}_{\mu_1 \ldots \mu_l}$ are irreducible tensors of four-dimensional rotational group

$$\int_0^\infty du \rho_l(u) L_{nl}(u) L_{n'l}(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u} \leftrightarrow \frac{e^{-Bz^2}}{z^2} \quad \text{gluon propagator}$$

Running coupling constant included:

Propagator: $\alpha_s(p^2)[1-\exp(-p^2/B)]/p^2$

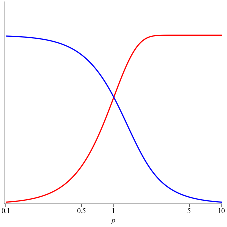
Dressing function: $\alpha_s(p^2)[1-\exp(-p^2/B)]$

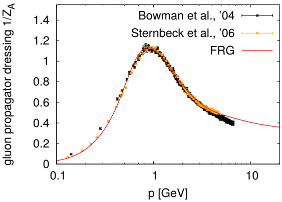


Tree level approximation:

$\alpha_s(0)[1-\exp(-p^2/B)]/p^2$

$\alpha_s(0)[1-\exp(-p^2/B)]$





M. Mitter, J. M. Pawlowski and N. Strodthoff, Phys. Rev. D **91**, 054035 (2015) doi:10.1103/PhysRevD.91.054035 [arXiv:1411.7978 [hep-ph]].
P. O. Bowman, U. M. Heller, D. B. Leinweber, M. B. Parappilly and A. G. Williams, Phys. Rev. D **70**, 034509 (2004) doi:10.1103/PhysRevD.70.034509 [hep-lat/0402032].
A. Sternbeck, E.-M. Ilgenfritz, M. Muller-Preussker, A. Schiller and I. L. Bogolubsky, PoS LAT **2006**, 076 (2006) [hep-lat/0610053].

Effective meson action for composite colorless fields:

$$Z = \mathcal{N} \lim_{V\to\infty} \int D\Phi_{\mathcal{Q}} \exp\left\{ -\frac{B}{2}\frac{h_{\mathcal{Q}}^2}{g^2 C_{\mathcal{Q}}} \int dx \Phi_{\mathcal{Q}}^2(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\}, \quad \mathcal{Q} = (aJln)$$

$$1 = \frac{g^2 C_{\mathcal{Q}}}{B} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(-M_{\mathcal{Q}}^2|B), \quad h_{\mathcal{Q}}^{-2} = \frac{d}{dp^2} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(p^2)|_{p^2=-M_{\mathcal{Q}}^2}.$$

$$W_k[\Phi] = \sum_{\mathcal{Q}_1\dots\mathcal{Q}_k} h_{\mathcal{Q}_1}\dots h_{\mathcal{Q}_k} \int dx_1 \dots \int dx_k \Phi_{\mathcal{Q}_1}(x_1)\dots\Phi_{\mathcal{Q}_k}(x_k) \Gamma^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k|B)$$

$$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} = \overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)} - \Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1} G^{(1)}_{\mathcal{Q}_2}},$$

$$\Gamma^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3} = \overline{G^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3}(x_1,x_2,x_3)} - \frac{3}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2) G^{(1)}_{\mathcal{Q}_3}(x_3)}$$
$$+\frac{1}{2}\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(1)}_{\mathcal{Q}_2}(x_2) G^{(1)}_{\mathcal{Q}_3}(x_3)},$$

$$\Gamma^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4} = \overline{G^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_1,x_2,x_3,x_4)} - \frac{4}{3}\Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(3)}_{\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_2,x_3,x_4)}$$
$$-\frac{1}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2) G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$
$$+\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(1)}_{\mathcal{Q}_2}(x_2) G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$
$$-\frac{1}{6}\Xi_4(x_1,x_2,x_3,x_4)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1) G^{(1)}_{\mathcal{Q}_2}(x_2) G^{(1)}_{\mathcal{Q}_3}(x_3) G^{(1)}_{\mathcal{Q}_4}(x_4)}.$$

$$\overline{G^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k)} = \int dB_j \mathrm{Tr} V_{\mathcal{Q}_1}\left(x_1|B^{(j)}\right) S\left(x_1,x_2|B^{(j)}\right)\dots$$
$$\dots V_{\mathcal{Q}_k}\left(x_k|B^{(j)}\right) S\left(x_k,x_1|B^{(j)}\right)$$

$$\overline{G^{(l)}_{\mathcal{Q}_1\dots\mathcal{Q}_l}(x_1,\dots,x_l) G^{(k)}_{\mathcal{Q}_{l+1}\dots\mathcal{Q}_k}(x_{l+1},\dots,x_k)} =$$
$$\int dB_j \mathrm{Tr}\left\{V_{\mathcal{Q}_1}\left(x_1|B^{(j)}\right) S\left(x_1,x_2|B^{(j)}\right)\dots V_{\mathcal{Q}_k}\left(x_l|B^{(j)}\right) S\left(x_l,x_1|B^{(j)}\right)\right\}$$
$$\times\ \mathrm{Tr}\left\{V_{\mathcal{Q}_{l+1}}\left(x_{l+1}|B^{(j)}\right) S\left(x_{l+1},x_{l+2}|B^{(j)}\right)\dots V_{\mathcal{Q}_k}\left(x_k|B^{(j)}\right) S\left(x_k,x_{l+1}|B^{(j)}\right)\right\},$$

Bar denotes integration over all configurations of the background field with measure dB_j.

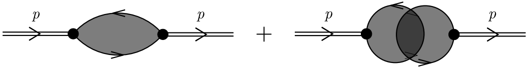
$$\langle \exp(iB_{\mu\nu}J_{\mu\nu})\rangle = \frac{\sin W}{W}$$

$$W = \sqrt{2B^2\left(J_{\mu\nu}J_{\mu\nu} \pm J_{\mu\nu}\widetilde{J}_{\mu\nu}\right)}$$

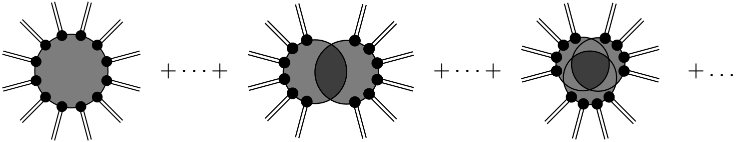
$J_{\mu\nu}$ is a tensor, composed of the momenta $p_{1\mu_1}\dots p_{n\mu_n}$ - arguments of the meson vertex

$$\tilde{\Gamma}^{(n)}(p_{1\mu_1}\dots p_{n\mu_n})$$

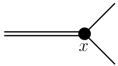
$$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} =$$



$$\Gamma^{(n)}_{\mathcal{Q}_1\mathcal{Q}_2\ldots\mathcal{Q}_n} =$$



Meson-quark vertex operators $\Leftarrow J^{aJln}_{\mu_1\ldots\mu_l} = \bar{q}(x)V^{aJln}_{\mu_1\ldots\mu_l}q(x)$



$$V^{aJln}_{\mu_1\ldots\mu_l}(x) = M^a\Gamma^J \left\{\left\{F_{nl}\left(\frac{\overset{\leftrightarrow}{D}{}^2(x)}{B^2}\right)T^{(l)}_{\mu_1\ldots\mu_l}\left(\frac{1}{i}\frac{\overset{\leftrightarrow}{D}(x)}{B}\right)\right\}\right\},$$

$$F_{nl}(s) = s^n\int_0^1 dt t^{n+l}\exp(st) = \int_0^1 dt t^{n+l}\frac{\partial^n}{\partial t^n}\exp(st),$$

$$\overset{\leftrightarrow}{D} = \overset{\leftarrow}{D}\xi_{f'} - \vec{D}\xi_f, \xi_f = \frac{m_f}{m_f + m_{f'}}$$

Quark propagator in homogeneous Abelian (anti-)self-dual field

$$\longrightarrow = \frac{\longrightarrow}{m(0)}\left[1 + \Sigma^{\mathrm{R}}(p^2)\right];\ \Sigma^{\mathrm{R}}(0) = 0 \qquad S(x,y) = \exp\left(-\frac{i}{2}x_\mu B_{\mu\nu}y_\nu\right)H(x-y),$$

$$\widetilde{H}_f(p|B) = \frac{1}{\upsilon B^2}\int_0^1 ds e^{(-p^2/\upsilon B^2)s}\left(\frac{1-s}{1+s}\right)^{m_f^2/2\upsilon B^2}\left[p_\alpha\gamma_\alpha \pm is\gamma_5\gamma_\alpha\frac{B_{\alpha\beta}}{\upsilon B^2}p_\beta + \right.$$

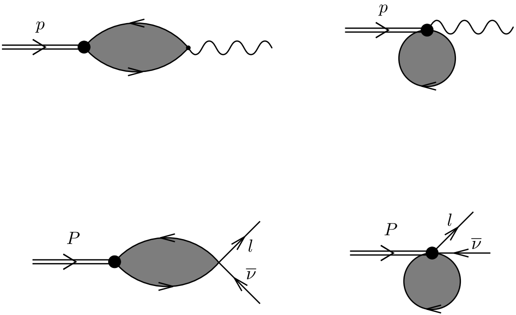
$$\left. + m_f\left(P_\pm + P_\mp\frac{1+s^2}{1-s^2} - \frac{i}{2}\gamma_\alpha\frac{B_{\alpha\beta}}{\upsilon B^2}\gamma_\beta\frac{s}{1-s^2}\right)\right]$$

The parameters of the model are

$$\alpha_s(0) \quad m_{u/d}(0) \quad m_s(0) \quad m_c(0) \quad m_b(0) \quad B \quad R$$

$$\langle\alpha_s F^2\rangle = \frac{B^2}{\pi} \qquad \chi_{\mathrm{YM}} = \frac{B^4R^4}{128\pi^2}$$

Weak and electromagnetic interactions



Masses of radially excited mesons

η and η'!

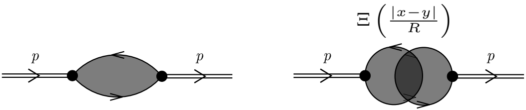


Table : Model parameters fitted to the masses of $\pi, \rho, K, K^*, \eta', J/\psi, \Upsilon$ and used in calculation of all other quantities.

$m_{u/d}$, MeV	m_s, MeV	m_c, MeV	m_b, MeV	Λ, MeV	α_s	R, fm
145	376	1566	4879	416	3.45	1.12

Polarization operator

Polarization operation for $l = 0$:

$$\Pi_J^{nn'}\left(-M^2; m_f, m_{f'}; B\right) =$$

$$\frac{B}{4\pi^2}\mathrm{Tr}_v \int\limits_0^1 dt_1 \int\limits_0^1 dt_2 \int\limits_0^1 ds_1 \int\limits_0^1 ds_2 \left(\frac{1-s_1}{1+s_1}\right)^{m_f^2/4vB} \left(\frac{1-s_2}{1+s_2}\right)^{m_{f'}^2/4vB} \times$$

$$\times\, t_1^n t_2^{n'} \frac{\partial^n}{\partial t_1^n} \frac{\partial^{n'}}{\partial t_2^{n'}} \frac{1}{\Phi_2^2} \left[\frac{M^2}{B}\frac{F_1^{(J)}}{\Phi_2^2} + \frac{m_f m_{f'}}{B}\frac{F_2^{(J)}}{(1-s_1^2)(1-s_2^2)} + \frac{F_3^{(J)}}{\Phi_2}\right] \exp\left\{\frac{M^2}{2vB}\frac{\Phi_1}{\Phi_2}\right\}.$$

$$\Phi_1 = s_1 s_2 + 2\left(\xi_1^2 s_1 + \xi_2^2 s_2\right)(t_1+t_2)v,$$

$$\Phi_2 = s_1 + s_2 + 2(1+s_1 s_2)(t_1+t_2)v + 16(\xi_1^2 s_1 + \xi_2^2 s_2)t_1 t_2 v^2,$$

$$F_1^{(P)} = (1+s_1 s_2)\left[2(\xi_1 s_1 + \xi_2 s_2)(t_1+t_2)v + 4\xi_1\xi_2(1+s_1 s_2)(t_1+t_2)^2 v^2 + s_1 s_2(1-16\xi_1\xi_2 t_1 t_2 v^2)\right],$$

$$F_1^{(V)} = \left(1 - \frac{1}{3}s_1 s_2\right)\left[s_1 s_2 + 16\xi_1\xi_2 t_1 t_2 v^2 + 2(\xi_1 s_1 + \xi_2 s_2)(t_1+t_2)v\right] + 4\xi_1\xi_2(1-s_1^2 s_2^2)(t_1-t_2)^2 v^2,$$

$$F_2^{(P)} = (1+s_1 s_2)^2, \quad F_2^{(V)} = (1-s_1^2 s_2^2),$$

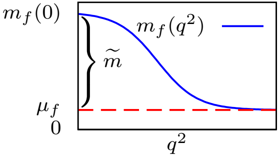
$$F_3^{(P)} = 4v(1+s_1 s_2)(1-16\xi_1\xi_2 t_1 t_2 v^2),\; F_3^{(V)} = 2v(1-s_1 s_2)(1-16\xi_1\xi_2 t_1 t_2 v^2).$$

Masses of radially excited mesons: light mesons

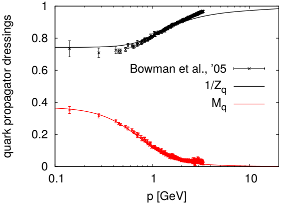
Asymptotic relation for spectrum (Regge trajectories):

$$M_n^2 \sim Bn,\ n \gg 1$$
$$M_l^2 \sim Bl,\ l \gg 1$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995)



$$\widetilde{m} = 136 \text{ MeV}$$
$$\mu_{u/d} = m_{u/d} - \widetilde{m}$$
$$\mu_s = m_s - \widetilde{m}$$
$$\frac{\mu_s}{\mu_{u/d}} = 26.7$$



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P. O. Bowman, U. M. Heller, D. B. Leinweber, M. B. Parappilly and P. O. Bowman, U. M. Heller, D. B. Leinweber, M. B. Parappilly, A. G. Williams and J. b. Zhang, Phys. Rev. D **71**, 054507 (2005) doi:10.1103/PhysRevD.71.054507 [hep-lat/0501019].

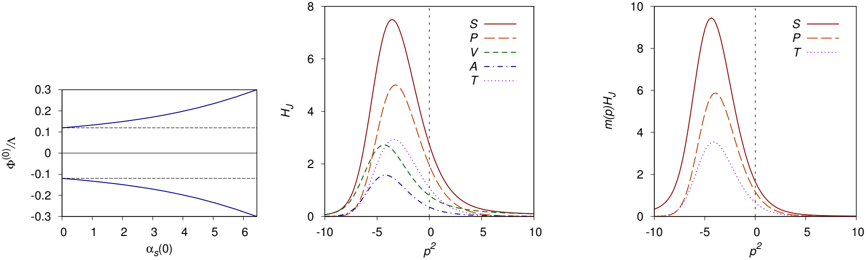


Figure : Scalar quark condensate (LHS). Momentum dependence of the scalar (solid line), pseudoscalar (long dash), vector (dash), axial (dash dot) and tensor (dot) form factors (central plot) in the quark propagator (1), and scalar, pseudoscalar and tensor form factors (RHS plot) multiplied by the quark mass.

$$\tilde{H}(p) = \frac{m}{2v\Lambda^2}\mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2v\Lambda^2}\mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2v\Lambda^2}\mathcal{H}_V(p^2) \pm i\gamma_5\gamma_\alpha \frac{f_{\alpha\beta}p_\beta}{2v\Lambda^2}\mathcal{H}_A(p^2) + \sigma_{\alpha\beta}\frac{mf_{\alpha\beta}}{4v\Lambda^2}\mathcal{H}_T(p^2). \quad (1)$$

Table : Masses of light mesons. $\tilde{M}$ denotes the value in the chiral limit.

Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	$\widetilde{M}$ (MeV)	Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	$\widetilde{M}$ (MeV)
π	0	140	140	0	ρ	0	775	775	769
$\pi(1300)$	1	1300	1310	1301	$\rho(1450)$	1	1450	1571	1576
$\pi(1800)$	1	1812	1503	1466	ρ	2	1720	1946	2098
K	0	494	494	0	K^*	0	892	892	769
$K(1460)$	1	1460	1302	1301	$K^*(1410)$	1	1410	1443	1576
K	2		1655	1466	$K^*(1717)$	1	1717	1781	2098
η	0	548	621	0	ω	0	775	775	769
η'	0	958	958	872	ϕ	0	1019	1039	769
$\eta(1295)$	1	1294	1138	1361	$\phi(1680)$	1	1680	1686	1576
$\eta(1475)$	1	1476	1297	1516	ϕ	2	2175	1897	2098

Table : Masses of heavy-light mesons and their lowest radial excitations .

Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)
D	0	1864	1715	D^*	0	2010	1944
D	1		2274	D^*	1		2341
D	2		2508	D^*	2		2564
D_s	0	1968	1827	D_s^*	0	2112	2092
D_s	1		2521	D_s^*	1		2578
D_s	2		2808	D_s^*	2		2859
B	0	5279	5041	B^*	0	5325	5215
B	1		5535	B^*	1		5578
B	2		5746	B^*	2		5781
B_s	0	5366	5135	B_s^*	0	5415	5355
B_s	1		5746	B_s^*	1		5783
B_s	2		5988	B_s^*	2		6021
B_c	0	6277	5952	B_c^*	0		6310
B_c	1		6904	B_c^*	1		6938
B_c	2		7233	B_c^*	2		7260

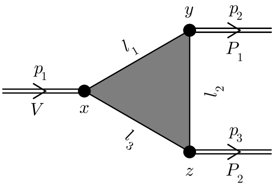
Table : Masses of heavy quarkonia.

Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)
$\eta_c(1S)$	0	2981	2751
$\eta_c(2S)$	1	3639	3620
η_c	2		3882
$J/\psi(1S)$	0	3097	3097
$\psi(2S)$	1	3686	3665
$\psi(3770)$	2	3773	3810
$\Upsilon(1S)$	0	9460	9460
$\Upsilon(2S)$	1	10023	10102
$\Upsilon(3S)$	2	10355	10249

Table : Decay and transition constants of various mesons

Meson	n	$f_P^{\exp}$ (MeV)	f_P (MeV)	Meson	n	$g_{V\gamma}^{\exp}$	$g_{V\gamma}$
π	0	130	140	ρ	0	0.2	0.2
$\pi(1300)$	1	—	29	ρ	1		0.034
K	0	156	175	ω	0	0.059	0.067
$K(1460)$	1	—	27	ω	1		0.011
D	0	205	212	ϕ	0	0.074	0.069
D	1	—	51	ϕ	1		0.025
D_s	0	258	274	J/ψ	0	0.09	0.057
D_s	1	—	57	J/ψ	1		0.024
B	0	191	187	Υ	0	0.025	0.011
B	1	—	55	Υ	1		0.0039
B_s	0	253	248				
B_s	1	—	68				
B_c	0	489	434				
B_c	1		135				

Stong decays: g_{VPP}



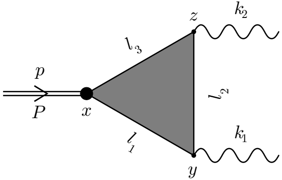
Decay	g_{VPP} [*]	g_{VPP}
$\rho^0 \to \pi^+\pi^-$	5.95	7.58
$\omega \to \pi^+\pi^-$	0.17	0
$K^{*\pm} \to K^\pm\pi^0$	3.23	3.54
$K^{*\pm} \to K^0\pi^\pm$	4.57	5.01
$\varphi \to K^+K^-$	4.47	5.02
$D^{*\pm} \to D^0\pi^\pm$	8.41	7.9
$D^{*\pm} \to D^\pm\pi^0$	5.66	5.59

local color gauge invariance

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Pion transition form factor – "BaBar puzzle"

$$T_a^{\mu\nu}(x,y,z) = h_P \sum_n u_n^a \int d\sigma_B \mathrm{Tr}\ t_a e_f^2 V^n(x)\gamma_5 S(x,y|B)\gamma_\mu S(y,z|B)\gamma_\nu S(z,x|B),$$



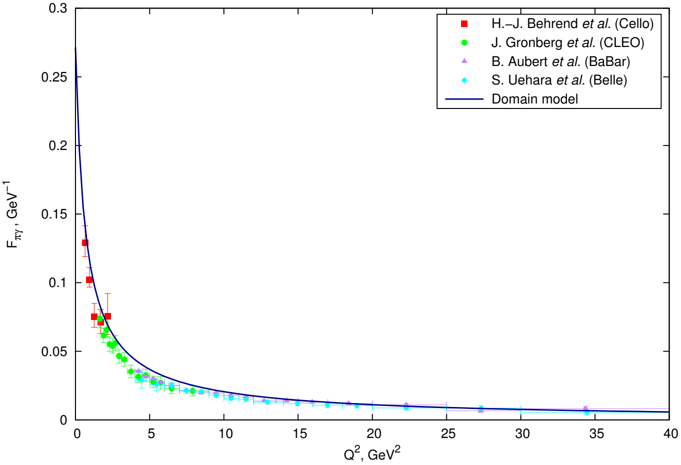
In momentum representation, the diagram has the following structure:

$$T_a^{\mu\nu}(p^2,k_1^2,k_2^2) = ie^2\delta^{(4)}(p-k_1-k_2)\varepsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}T_a(p^2,k_1^2,k_2^2).$$

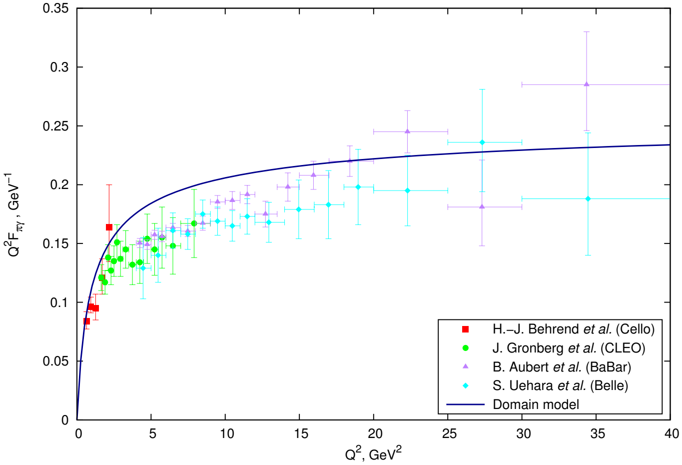
$$F_{P\gamma}\left(Q^2\right) = T\left(-M_P^2, Q^2, 0\right).$$

$$\Gamma(P\to\gamma\gamma) = \frac{\pi}{4}\alpha^2 M_P^3 g_{P\gamma\gamma}^2$$

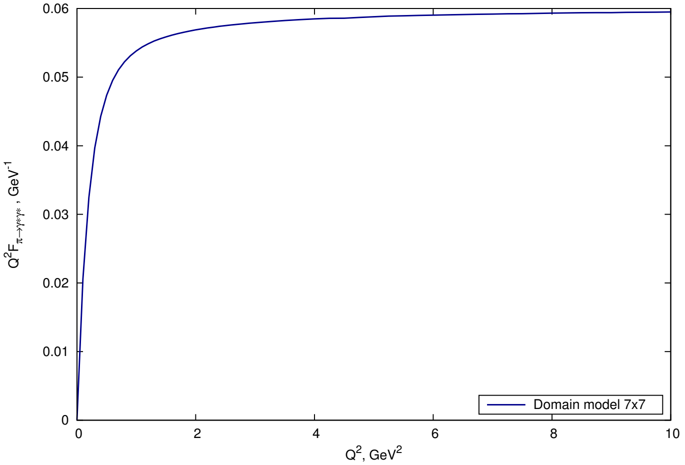
$$g_{P\gamma\gamma} = T(-M_P^2,0,0) = F_{P\gamma}(0).$$



$$g_{\pi\gamma\gamma} = 0.272\text{GeV}^{-1} \quad (g^{\text{exp}}_{\pi\gamma\gamma} = 0.274\text{GeV}^{-1}).$$



$$g_{\pi\gamma\gamma} = 0.272\mathrm{GeV}^{-1} \qquad (g^{\mathrm{exp}}_{\pi\gamma\gamma} = 0.274\mathrm{GeV}^{-1}).$$



$$F_{\pi\gamma^*\gamma^*}\left(Q^2\right) = T\left(-M_P^2, Q^2, Q^2\right).$$

Bethe-Salpeter approach

S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]
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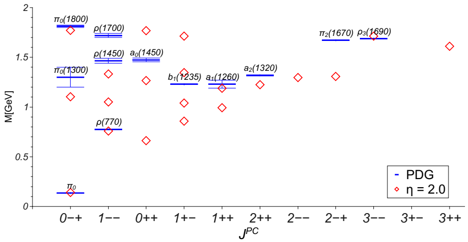
S. M. Dorkin, L. P. Kaptari, T. Hilger and B. Kampfer, Phys. Rev. C **89**, no. 3, 034005 (2014) [arXiv:1312.2721 [hep-ph]]

$$S^{-1}(p) = Z_2 S_0^{-1}(p) + 4\pi Z_2^2 C_F \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k+p)\gamma^\nu (\delta_{\mu\nu} - k_\mu k_\nu/k^2)\ \frac{\alpha_{\rm eff}(k^2)}{k^2},$$

$$\alpha_{\rm eff}(q^2) = \pi\eta^7 x^2 e^{-\eta^2 x} + \frac{2\pi\gamma_m\left(1-e^{-y}\right)}{\ln\left[e^2 - 1 + (1+z)^2\right]},\ x = q^2/\Lambda^2,\ y = q^2/\Lambda_t^2,\ z = q^2/\Lambda_{\rm QCD}^2$$



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Harmonic confinement – Light-Front Holography – soft-wall AdS/QCD

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G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. **94** (2005) 201601; Phys. Rev. Lett. **96** (2006) 201601; Phys. Rev. Lett. **102** (2009) 081601;

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$$\mathrm{S}_\Phi = \frac{(-1)^J}{2}\int d^dx\, dz \left(\frac{R}{z}\right)^{d+1} e^{-\kappa^2 z^2}\left(\partial_N\Phi_J\partial^N\Phi_J - \mu_J^2(z)\Phi_J\Phi^J\right)$$

$$M_{nJ} = 4\kappa^2\left(n + \tfrac{l+J}{2}\right)$$

$$\Phi_J(x,z) = \sum_n \phi_{nJ}(z)\Phi_{nJ}(x),$$

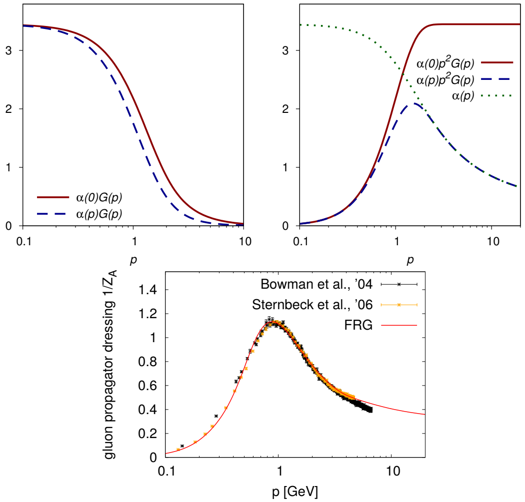
$$\phi_{nJ} = R^{J-(d-1)/2}\kappa^{1+l}z^{l-J+2}L_n^l(\kappa^2z^2)$$

Laguerre polynomials

$$\mathcal{S}_2 = -\frac{1}{2}\int d^4x\int d^4z D(z)\Phi^2_{Jc}(x,z)$$

$$-2g^2\int d^4xd^4x'd^4zd^4z'D(z)D(z')\Phi_{Jc}(x,z)\Pi_{Jc,J'c'}(x,x';z,z')\Phi_{J'c'}(x',z'),$$

$$\Phi^{aJ}(x,z) = \sum_{nl}\left(z^2\right)^{l/2}\varphi^{nl}(z)\Phi^{aJln}(x), \quad \Phi^{aJln}(x) = \bar q(x)V^{aJln}(x)\,q(x)$$



Functional RG, DSE, Lattice QCD

Summary

An ensemble of almost everywhere Abelian homogeneous (anti-)self-dual gluon fields represented by a domain wall networks – **highly promising framework** for studying mechanisms of confinement, chiral symmetry realisation and hadronization as well as new nonperturbative phenomena related to the hadronic matter under extreme conditions.

$$\langle g^2 F^2 \rangle \neq 0$$

Nonlocal form of quark and gluon propagators.
Nonlocal colorless hadron effective action.

Rich and quantitatively correct phenomenology.