thermalization of a strongly interacting non-Abelian plasma

Pietro Colangelo
INFN – Bari, Italy

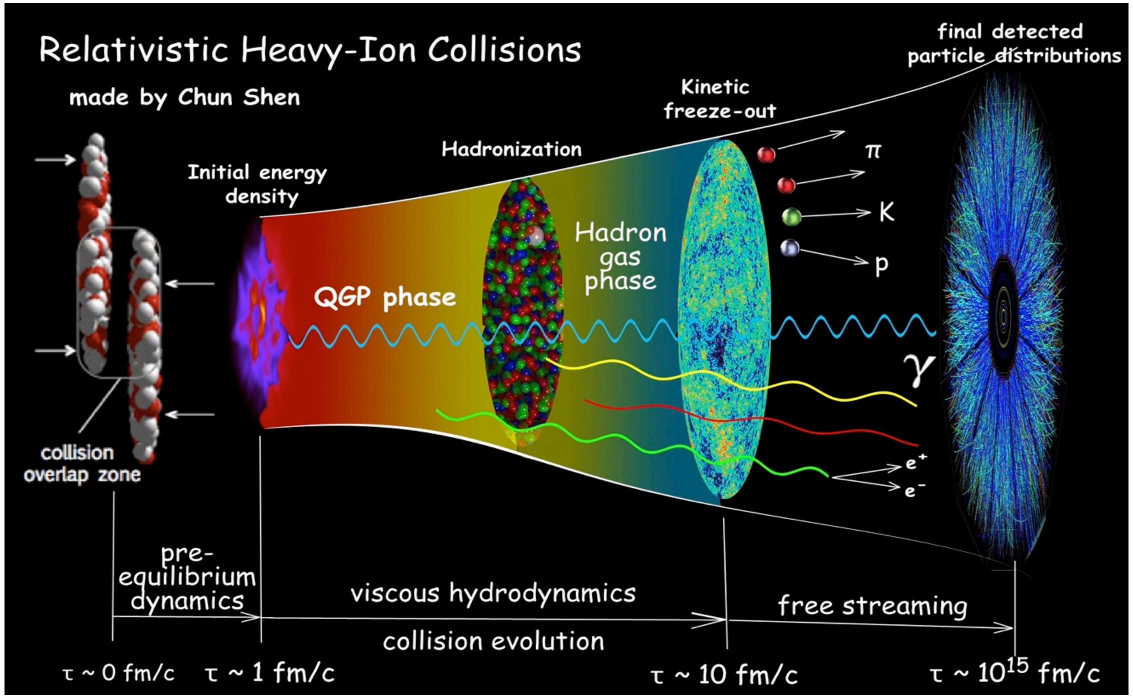
based on
L. Bellantuono, F. De Fazio, F. Giannuzzi, S. Nicotri, PC,
JHEP 07 (2015) 053
PRD 94 (2016) 025005

Helmholtz International Summer School
Quantum Field Theories at the Limits:
from Strong Fields to Heavy Quarks
BLTP - JINR, Dubna, Russia
18-30 July 2016

Outline:

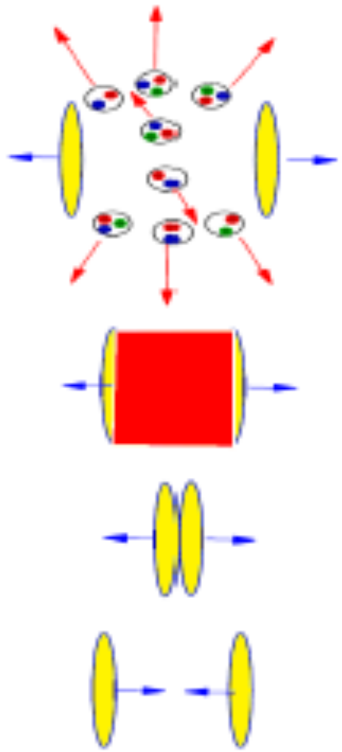
- Example of strongly coupled plasma
- Gauge/Gravity and Fluid/Gravity duality
- Quenches and out-of-equilibrium dynamics for a strongly interacting plasma
- Thermalization: local vs nonlocal observables
- Possible improvements and perspectives

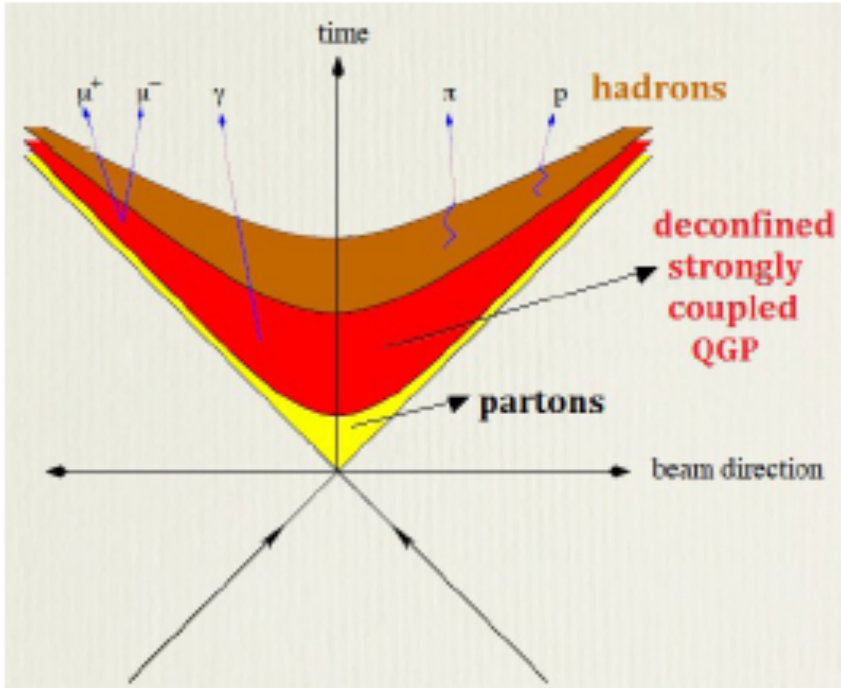
Example of strongly coupled non-Abelian system: plasma from ultrarelativistic heavy ion collisions

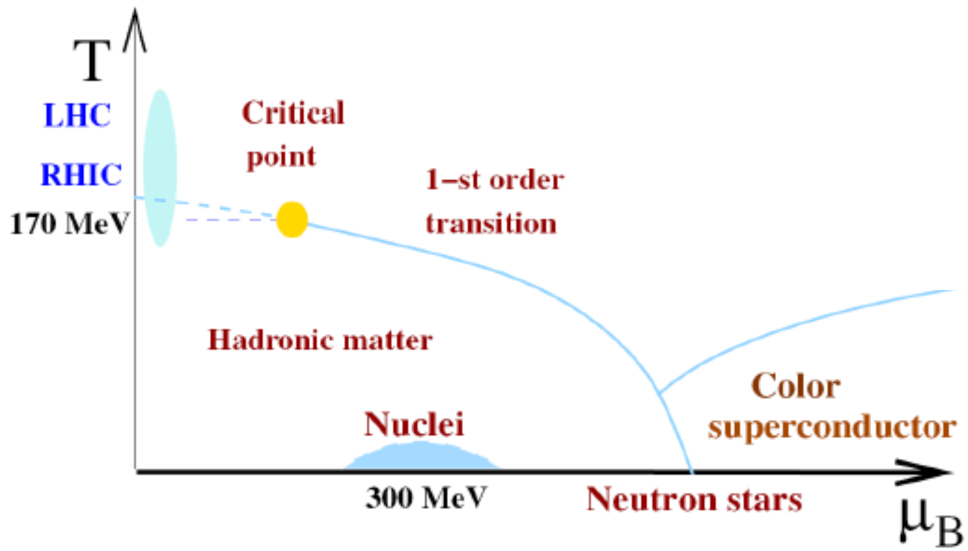


- state of matter (with deconfined quarks and gluons) produced in ultrarelativistic heavy ion collision experiments (RHIC, Pb-Pb at LHC)

- issue: description of the evolution from pre-equilibrium condition towards a thermalized final state




time
μ+
μ−
γ
π
p
hadrons
deconfined
strongly
coupled
QGP
partons
beam direction


T
LHC
RHIC
170 MeV
Critical
point
1–st order
transition
Hadronic matter
Nuclei
Color
superconductor
300 MeV
Neutron stars
μB

indications from heavy ion (HI) experiments (RHIC, LHC)

- models: after the pre-equilibrium phase, the hydrodynamic description is relevant
- simulations reproducing the elliptic flow, describing pressure anisotropy: almost perfect fluid behaviour
- small viscosity/entropy density ratio η/s
- fast thermalization time $\tau \cong O(1\ \mathrm{fm/c})$

issues

- role of the underlying gauge theory, QCD
- why QGP behaves like a perfect fluid?
- understanding the equilibration dynamics and the time scale

remarks

- perturbative QCD calculations give large values of viscosity/entropy density ratio
- strongly coupled QCD is the relevant regime at $T_c<T$
- lattice QCD able to describe static properties of QGP, not the real-time dynamics relevant for HI collisions

simple approach: models

models

- start from a particular classical gauge field configuration (color-electric flux tubes produced by color charges in the colliding nuclei)
- let color fields decay to particles
- particles propagate in medium colliding and interacting with the color field background
- particle creation and particle currents affect the color electric field
- implement abelian color field dynamics
- evolve numerically using some transport equation

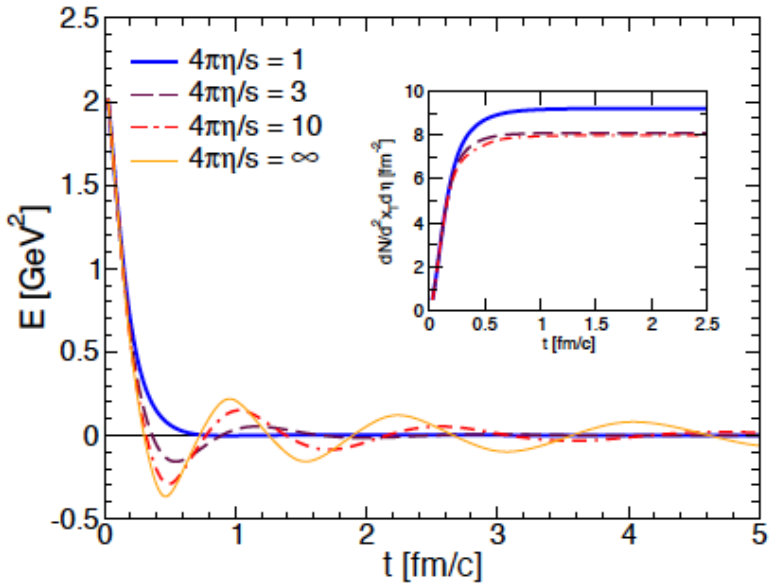
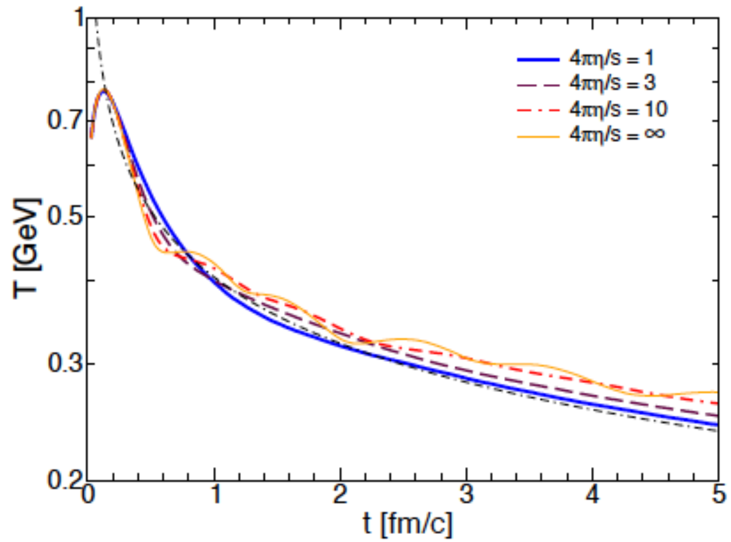


FIG. 3: Chromoelectric field strength (main panel) and particle number produced per unit of transverse area and rapidity (inset panel) as a function of time. The electric field is aver-



Ruggieri, Greco et al, arXiv:1505.08081

gauge/gravity duality methods: new tool to investigate the thermalization process

notice

- unknown gravity dual of QCD (if any)
- QGP a strongly coupled deconfined phase of QCD, duality arguments could be applicable (celebrated result: $\eta/s=1/4\pi$ Policastro, Son, Starinets, PRL 87 (2001) 081601)

gauge/gravity duality methods: new tool to investigate the thermalization process

caveat

- SU(3) gauge fields & fundamental quarks -> SU(N_c) gauge fields & adjoint matter
- strongly coupled plasma -> strongly coupled SYM ($\mathcal{N}$=4) at large N_c
- non-equilibrium evolution -> 5D gravitational problem

HI collisions and fluid/gravity duality

HI collisions produce a dense strongly interacting medium

- relevant degrees of freedom not the individual partons
- description as a fluid might be appropriate

to be determined

- stress-energy tensor (energy density, pressures)
- local temperature, entropy density
- scales of thermalization (UV vs IR)

AdS/CFT (Anti de Sitter/Conformal Field Theory) correspondence conjecture (or Maldacena conjecture) (late 90's)

a remarkable connection conjectured between certain string theory in certain curved space-times and certain gauge theories in flat (3+1) dimensional space-time

generalization of this idea useful for strong interaction physics

- gauge theories with large N_c
- strong coupling regime $\lambda = g^2_{YM} N_c >> 1$

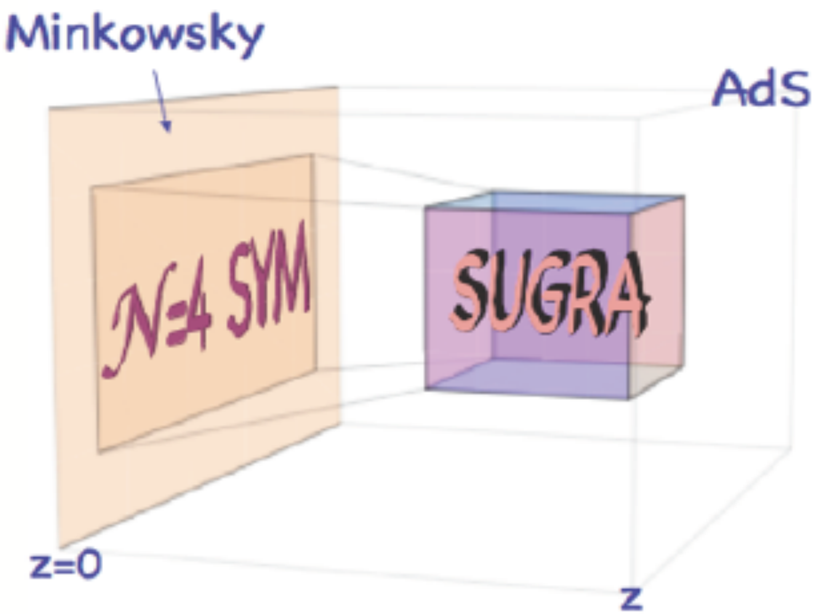
AdS/CFT correspondence

original proposal

correspondence between a low energy supergravity approximation to D=10 Type IIB string theory on $AdS_5 \times S^5$ and a N=4 SYM theory with gauge group $SU(N_C)$ at large N_C Maldacena '98

more general

equivalence (duality) between a gravity theory defined in $AdS_{d+1} \times C$ (C a compact manifold) and a conformal field theory (CFT) defined on the boundary of AdS_{d+1} (M_d)



realization of the holographic principle 't Hooft and Susskind peculiar role of the d+1 dimensional Anti de Sitter (AdS_{d+1}) space:

- solution of Einstein eqs. in the vacuum with negative cosmological constant

- negative curvature

- embedded in R^{d+2} (coordinates (X^0,…X^{d+1})) it is defined by $\left(X^0\right)^2 - \sum_{i=1}^{d}\left(X^i\right)^2 + \left(X^{d+1}\right)^2 = R^2$

- group of isometries SO(2,d) ((d+2)(d+1)/2 generators)

- it has a boundary M_d

- on the boundary M_d the coordinate transformation belonging to SO(2,d) are conformal transformations ((d+2)(d+1)/2 generators)

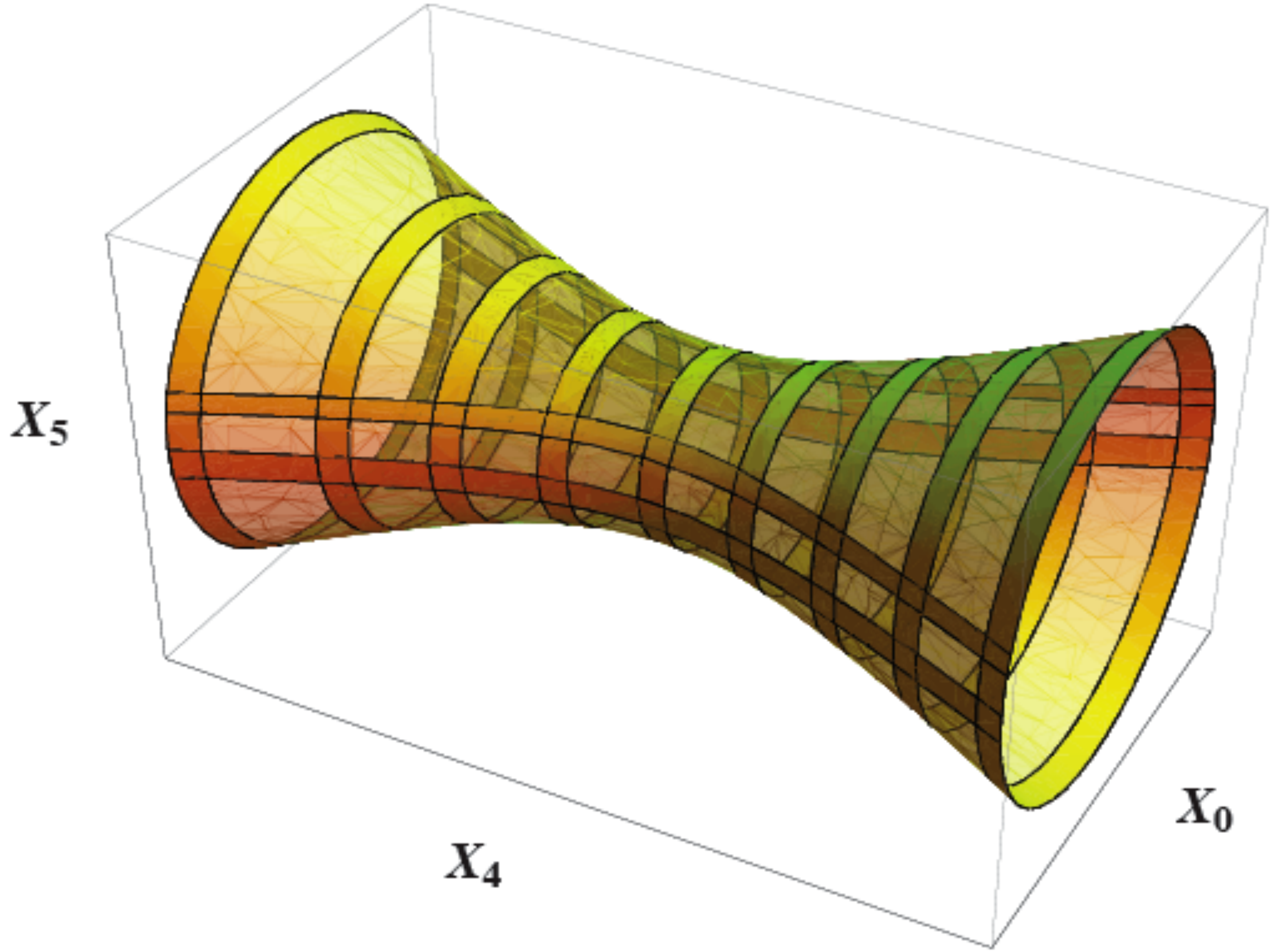
	Transformation	Generator
Translation	$x'^\mu = x^\mu + a^\mu$	$P_\mu = -i\,\partial_\mu$
Dilation	$x'^\mu = a\,x^\mu$	$D = -i\,x^\mu\,\partial_\mu$
Rotation	$x'^\mu = M^\mu{}_\nu\,x^\nu$	$L_{\mu\nu} = i\,(x_\mu\partial_\nu - x_\nu\partial_\mu)$
Special conf.	$x'^\mu = \dfrac{x^\mu - b^\mu x^2}{1 - 2\,b\cdot x + b^2\,x^2}$	$K_\mu = -i\,(2\,x_\mu x^\nu\partial_\nu - x^2\partial_\mu)$

d generators

1 generator

d (d-1)/2 generators

d generators


X_5
X_4
X_0

x ε AdS_{d+1} $\quad x=(x^0,x^1,\dots x^{d-1},z)$ $\quad ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$

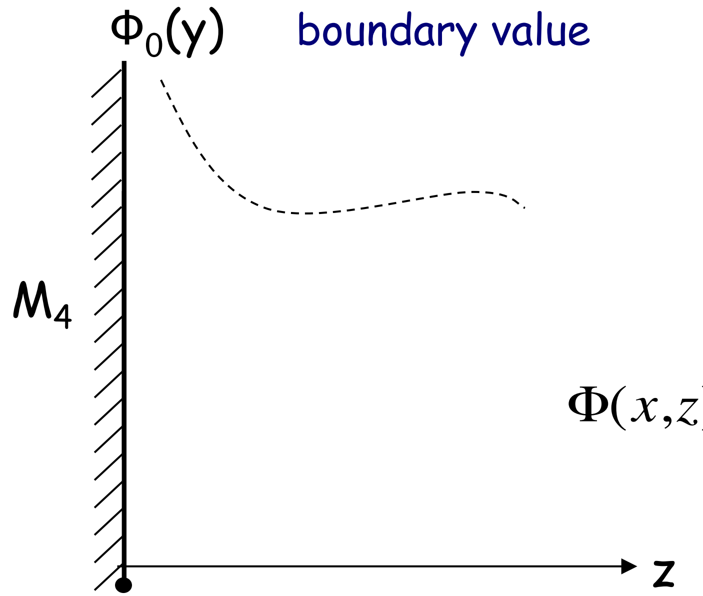
Fefferman-Graham coordinates

R Anti de Sitter radius

z>0 holographic coordinate

AdS/CFT correspondence

Maldacena,
Gubser, Klebanov and Polyakov
Witten



scalar massless bulk field in AdS_5

$\Phi\,(x,z)$

$$\Phi(x,z) = \int_{M_4} d^4y\, K(x,z;y)\, \Phi_0(y)$$

bulk-to-boundary propagator

At Φ_0 (y) an operator O(y) of CFT on M_4 is associated

Φ_0 (y) coupled to O via $\quad \int_{M_4} d^4y\, \Phi_0(\mathrm{y})\, O(y)$

$$\left\langle \exp \int_{M_4} \Phi_0 O \right\rangle_{CFT} \quad vs \quad \exp\left(-S_{AdS}(\Phi)\right)$$

Duality between gravity theory in AdS_{d+1} x C and the large N_c limit of a CFT is given by the generating functionals of the string and CFT correlation functions at the AdS boundary

Boundry theory generating functional
external source Φ_0

$$Z_{CFT}[\Phi_0] = \int dA \ \exp\left\{-S_{CFT} + \int d^d x \ \Phi_0 O\right\}$$

gravity partition function
in AdS_{d+1} x C with boundary value Φ_0

$$Z_{grav}\left[\Phi_0\right] = \int d[\Phi] \ \exp\left\{-S_{grav}\left[\Phi\right]\right\}$$

AdS/CFT correspondence conjecture

$$Z_{grav}[\Phi_0] = Z_{CFT}\left[\Phi_0\right]$$

Gubser, Klebanov and Polyakov
Witten

relation between the mass m_{d+1}^2 of the field Φ in the bulk and the dimension Δ of a p-form operator O on the boundary: $m_{d+1}^2 R^2 = (\Delta - p)(\Delta + p - d)$

$$\frac{R_{AdS_5}}{\ell_s} = \left(g_{YM}^2 N\right)^{\frac{1}{4}}$$

$R_{AdS_5} >> \ell_s$ $\left\{ \begin{array}{l} \text{supergravity approximation} \\ g_{YM}^2 N \text{ large} \end{array} \right.$

strongly coupled regime
dual to a semiclassical regime

stationary and finite temperature

(N=4 at T≠0 on $S^3 x S^1$ and $N_c \to \infty$)

E. Witten, Adv. Theor. Math. Phys. 2, 505

periodic Euclidean time τ extended to β'
T=1/ β'

Black Hole-AdS

$$ds^2 = \frac{R^2}{z^2}\left(f(z)dt^2 - d\bar{x}^2 - \frac{dz^2}{f(z)}\right)$$

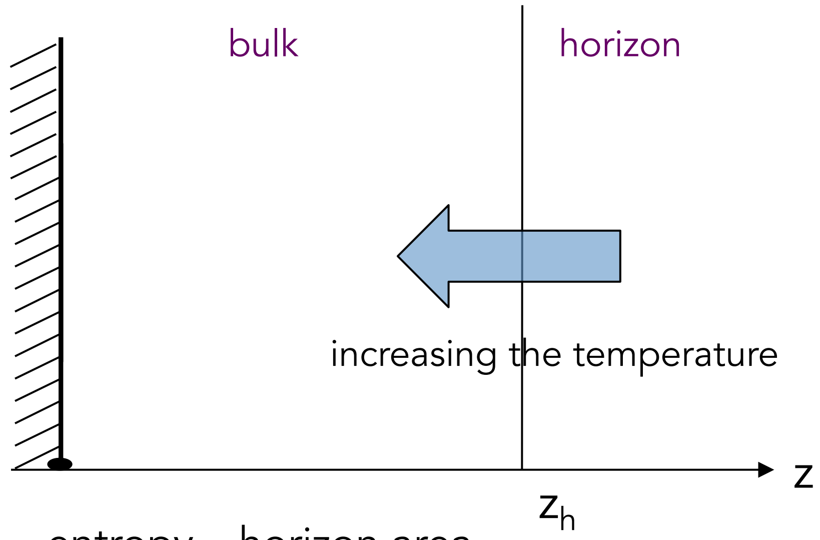
$$f(z) = 1 - \frac{z^4}{z_h^4} \qquad 0 < z < z_h$$

$$0 \leqslant \tau < \beta = \frac{1}{T}$$

black hole horizon

$$z_h = \frac{1}{\pi T}$$

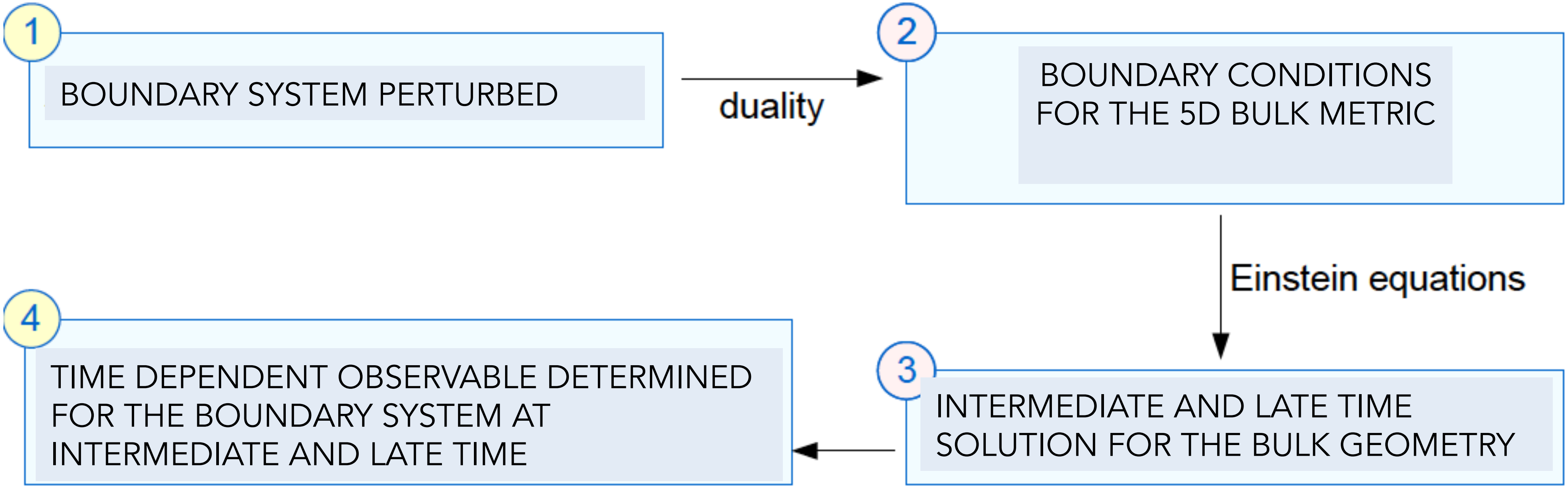
Hawking temperature



entropy – horizon area
Bekenstein

ANALYSIS OF AN OUT-OF-EQUILIBRIUM SYSTEM

SCHEME FOR CALCULATING AN EQUILIBRATION PROCESS USING GAUGE/GRAVITY DUALITY



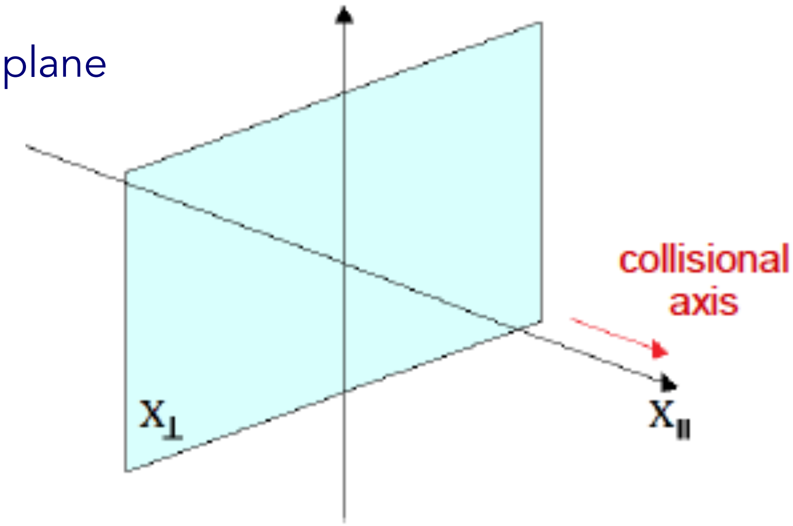
BOOST INVARIANT EXPANDING PLASMA

perfect fluid hydrodynamics: stress energy tensor under boost-invariance assumption

Bjorken, PRD 27 (1983) 140

assumptions
- boost invariance along the collision axis
- translational and rotational symmetries in the transverse plane

approximatively realized in the central region of QGP



local thermal equilibrium:
all portions of the fluid share
the same t-dependent temperature

coordinates & metrics

proper time τ, spatial rapidity y, transverse coordinates $x_\perp$

$$x^0 = \tau \cosh y \qquad x^1 = \tau \sinh y$$

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$$

diagonal stress-energy tensor $T_{\mu\nu}$
only depends on τ

$$T^\mu_\nu = \frac{N_c^2}{2\pi^2} diag(-\epsilon, p_\perp, p_\perp, p_\parallel)$$

diagonal stress-energy tensor only depends on τ

$$T_\mu^\nu = diag\left(-f(\tau),\, f(\tau)+\frac{1}{2}\tau f'(\tau),\, f(\tau)+\frac{1}{2}\tau f'(\tau),\, -f(\tau)-\tau f'(\tau)\right)$$

stress energy tensor in perfect fluid hydrodynamics

$$\epsilon = 3p$$

$$\boxed{\epsilon(\tau) = \frac{const}{\tau^{4/3}}}$$

$$\boxed{\begin{aligned} p_{\|}(\tau) &= -\epsilon(\tau) - \tau\epsilon'(\tau) \\ p_{\perp}(\tau) &= \epsilon(\tau) + \frac{\tau}{2}\epsilon'(\tau)\ . \end{aligned}}$$

subleading time corrections -> viscous hydrodynamics

$$\begin{aligned}
\epsilon(\tau) &= \frac{3\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1-\frac{2c_1}{(\Lambda\tau)^{2/3}}+\frac{c_2}{(\Lambda\tau)^{4/3}}+\mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right)\right] \\
p_{\|}(\tau) &= \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1-\frac{6c_1}{(\Lambda\tau)^{2/3}}+\frac{5c_2}{(\Lambda\tau)^{4/3}}+\mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right)\right] \\
p_{\perp}(\tau) &= \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1-\frac{c_2}{(\Lambda\tau)^{4/3}}+\mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right)\right]\ ,
\end{aligned}$$

$$c_1 = \frac{1}{3\pi} \text{ and } c_2 = \frac{1+2\log 2}{18\pi^2}$$

Heller, Janik, Witaszczyk, PRD 85 (12) 126002, PRL 110 (2013) 211602
Heller, Janik, PRD 76 (07) 025027
Baier et al, JHEP 0804 (08) 100
Lublinsky, Shuryak, PRD 80 (2009) 065026

pressure ratio & anisotropy

$$\frac{p_\parallel}{p_\perp} = 1 - \frac{6c_1}{(\Lambda\tau)^{2/3}} + \frac{6c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right),$$

$$\frac{\Delta p}{\epsilon} = \frac{p_\perp - p_\parallel}{\epsilon} = 2\left[\frac{c_1}{(\Lambda\tau)^{2/3}} + \frac{2c_1^2 - c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right)\right]$$

$$c_1 = \frac{1}{3\pi} \text{ and } c_2 = \frac{1 + 2\log 2}{18\pi^2}$$

an effective temperature can be defined $\epsilon(\tau) = \frac{3}{4}\pi^4 T_{eff}(\tau)^4$

$$T_{eff}(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}}\left[1 - \frac{1}{6\pi(\Lambda\tau)^{2/3}} + \frac{-1 + \log 2}{36\pi^2(\Lambda\tau)^{4/3}} + \frac{-21 + 2\pi^2 + 51\log 2 - 24(\log 2)^2}{1944\pi^3(\Lambda\tau)^2} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^{8/3}}\right)\right]$$

to drive the boundary system far from equilibrium: introduce a quench

duality

boundary gauge theory	bulk theory
$T_{\mu\nu}$	g_{MN}

compute changes of the boundary stress-energy tensor produced by modifications of the bulk geometry

connection provided by "holographic renormalization"

de Haro et al, Comm. Mat. Phys. 217 (01) 595
Kinoshita et al, Prog. Theor. Phys., 121 (09) 121

holographic renormalization

example: Fefferman-Graham coordinates:

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2}.$$

metric a solution of Einstein equations with negative cosmological constant

$$R_{\mu\nu} - \frac{1}{2} g^{(5D)}_{\mu\nu} R - 6\, g^{(5D)}_{\mu\nu} = 0$$

near boundary expansion (z->0)

$$g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x^\rho) + \ldots\ .$$

holographic renormalization

$$\langle T_{\mu\nu}(x^\rho)\rangle = \frac{N_c^2}{2\pi^2} \cdot g^{(4)}_{\mu\nu}(x^\rho)$$

two possibilities for using this approach

1. solution of the Einstein equations -> $T_{\mu\nu}$ in the boundary theory
2. given $T_{\mu\nu}$ -> dual bulk geometry
   criterion to select a solution: absence of singularities in various curvature invariants

Janik Peschanski, PRD 73 (2006) 045013

boundary sourcing

P. Chesler, L. Yaffe, PRL 102 (09) 211601
PRD 82 (10) 026006

L. Bellantuono et al., JHEP 07 (2015) 053
PRD 94 (2016) 025005

1. 4D gauge theory driven out-of-equilibrium
   -> 4D metric deformed by a quench $\gamma(\tau)$

$$ds_4^2 = -d\tau^2 + e^{\gamma(\tau)}dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)}dy^2$$

1

BOUNDARY SYSTEM PERTURBED $\xrightarrow{\text{duality}}$

boundary sourcing

P. Chesler, L. Yaffe, PRL 102 (09) 211601
PRD 82 (10) 026006

L. Bellantuono et al., JHEP 07 (2015) 053
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2. 5D metric of the dual theory in Eddington-Finkelstein coordinates

5D radial coordinate (boundary at r-> infinity)

$$ds_5^2 = 2 dr d\tau - A\ d\tau^2 + \Sigma^2 e^{B} dx_\perp^2 + \Sigma^2 e^{-2B} dy^2$$

1 BOUNDARY SYSTEM PERTURBED $\xrightarrow{\text{duality}}$ 2 BOUNDARY CONDITIONS FOR THE 5D BULK METRIC

boundary sourcing

P. Chesler, L. Yaffe, PRL 102 (09) 211601
PRD 82 (10) 026006

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1 BOUNDARY SYSTEM PERTURBED

duality

2 BOUNDARY CONDITIONS FOR THE 5D BULK METRIC

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - 6\, g_{\mu\nu} = 0}$$

Einstein equations

3 INTERMEDIATE AND LATE TIME SOLUTION FOR THE BULK GEOMETRY

boundary sourcing

P. Chesler, L. Yaffe, PRL 102 (09) 211601
PRD 82 (10) 026006

L. Bellantuono et al., JHEP 07 (2015) 053
PRD 94 (2016) 025005

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3. metric functions $A(r,\tau)$, $B(r,\tau)$, $\Sigma(r,\tau)$ obtained solving the Einstein eqs. with as boundary condition (r->∞)

$$\Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0$$

$$\Sigma(\dot{B})' + \frac{3}{2}\left(\Sigma'\dot{B} + B'\dot{\Sigma}\right) = 0$$

$$A'' + 3B'\dot{B} - 12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2} + 4 = 0$$

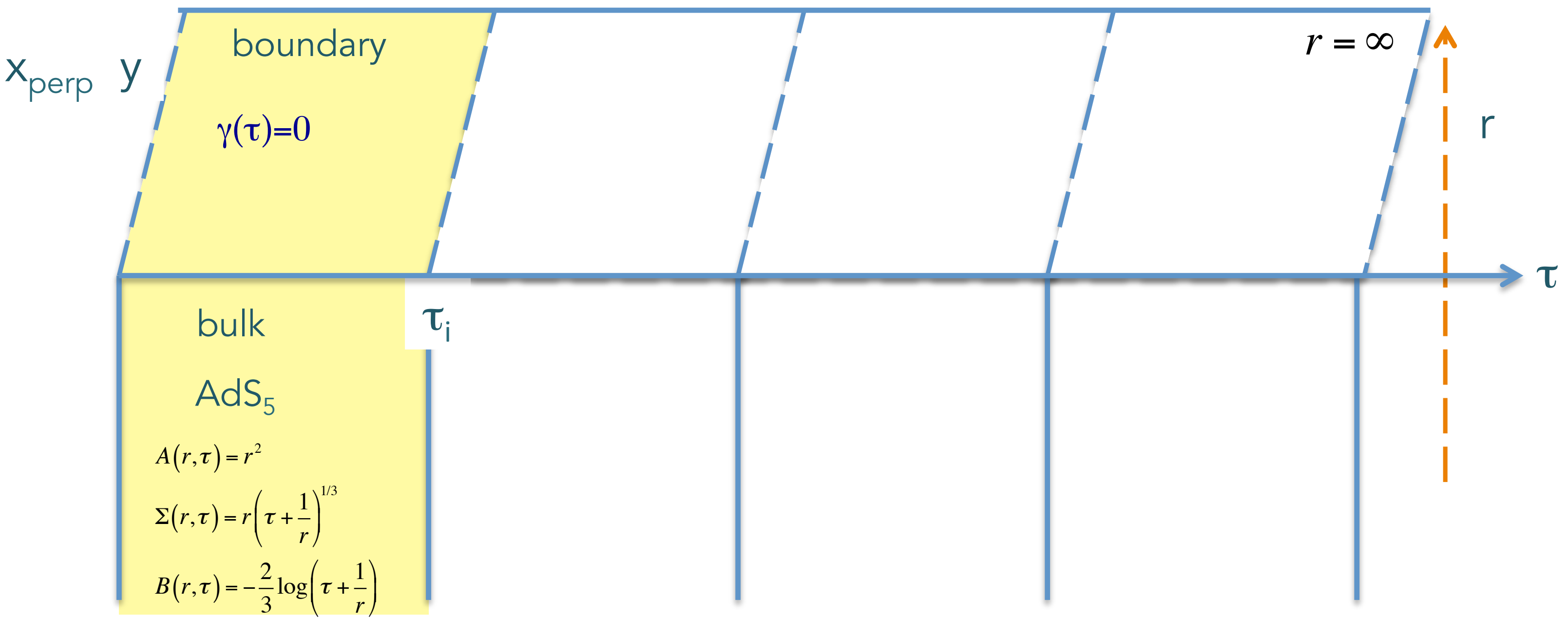
$$\ddot{\Sigma} + \frac{1}{2}\left(\dot{B}^2\Sigma - A'\dot{\Sigma}\right) = 0$$

$$\Sigma'' + \frac{1}{2}B'^2\Sigma = 0$$

$$f' = \partial_r f$$

$$\dot{f} = \partial_\tau f + \frac{1}{2} A \partial_r f$$

quench $\gamma(\tau)$ starts at $\tau=\tau_i$ -> metric AdS_5 at $\tau < \tau_i$



4D $$ds_4^2 = -d\tau^2 + e^{\gamma(\tau)}dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)}dy^2$$

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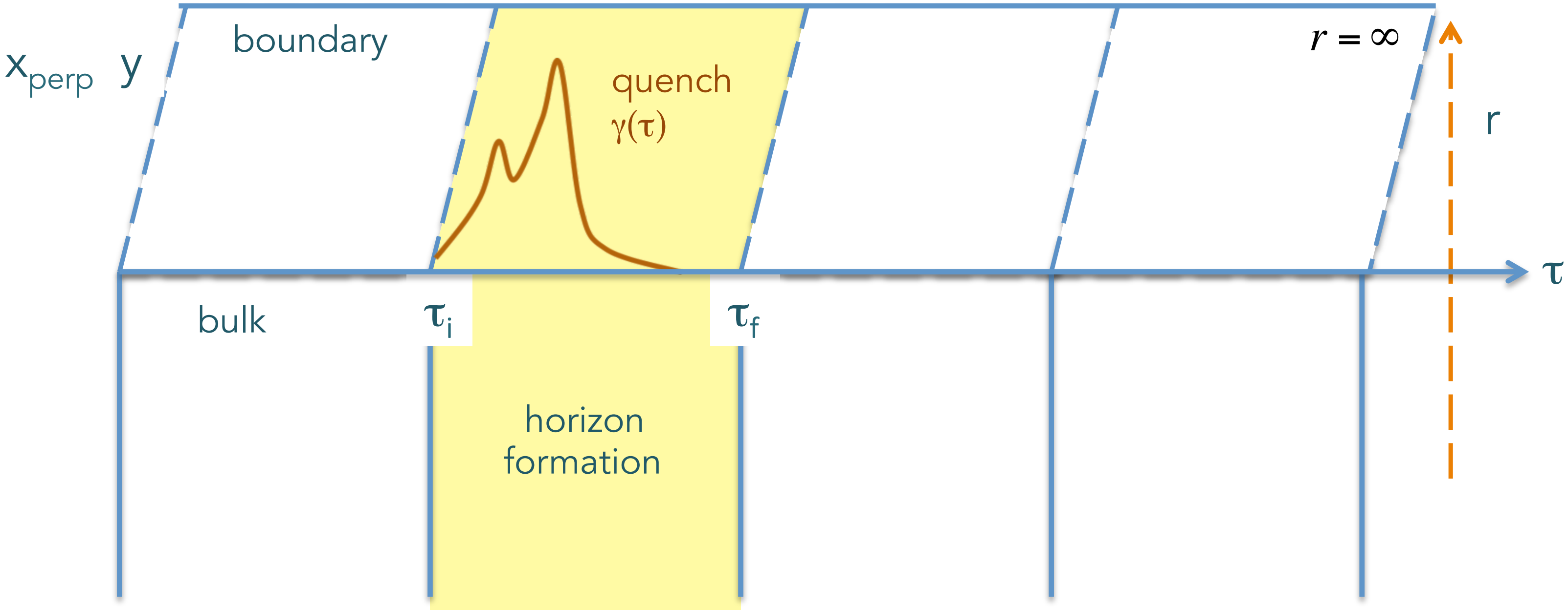
new algorithm for the PDE solution

boundary sourcing



4D $ds_4^2 = -d\tau^2 + e^{\gamma(\tau)}dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)}dy^2$

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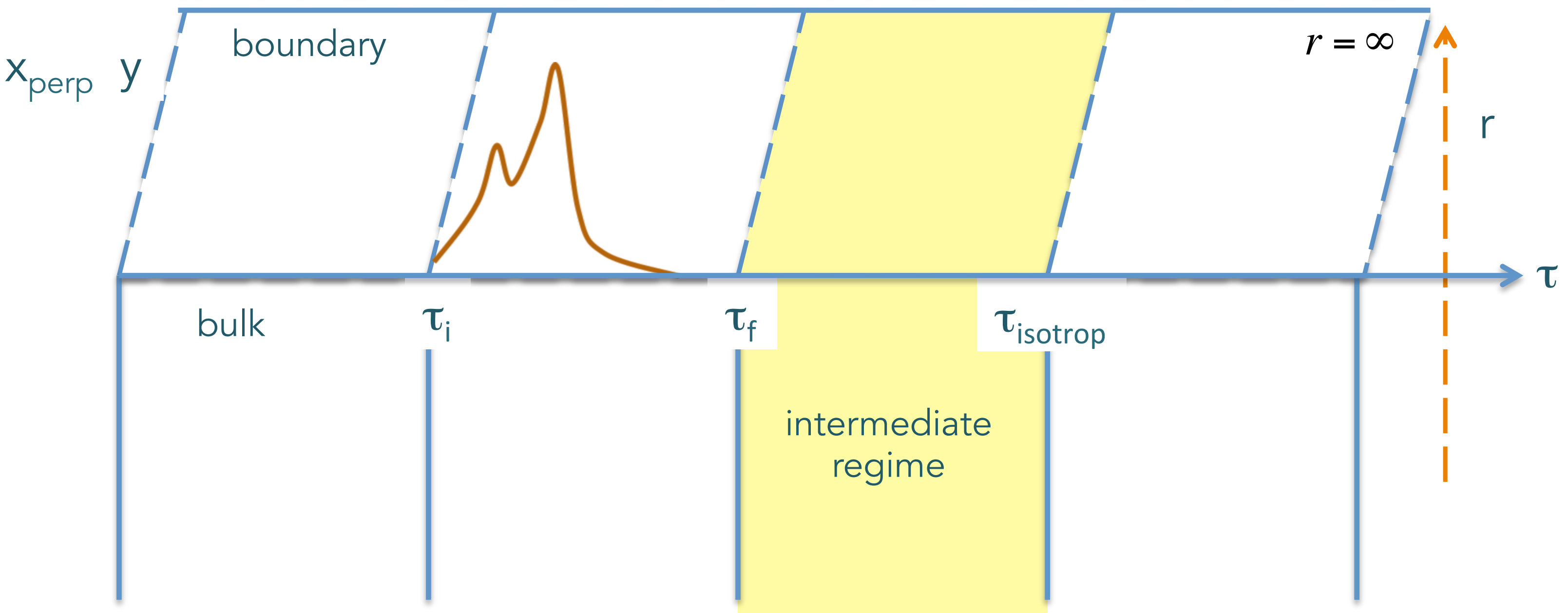

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new algorithm for the PDE solution

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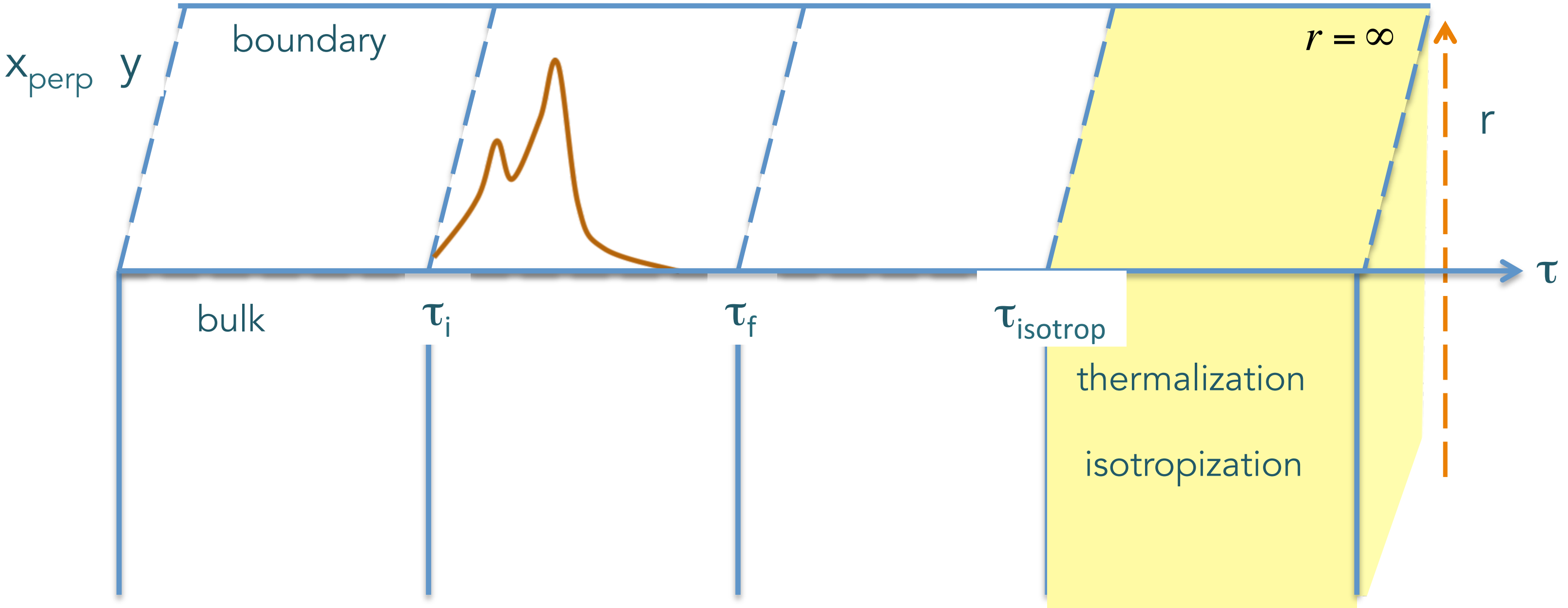

4D $\quad ds_4^2 = -d\tau^2 + e^{\gamma(\tau)} dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$

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new algorithm for the PDE solution

$$\Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0 \qquad (1)$$

$$\Sigma(\dot{B})' + \frac{3}{2}\left(\Sigma'\dot{B} + B'\dot{\Sigma}\right) = 0 \qquad (2)$$

$$A'' + 3B'\dot{B} - 12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2} + 4 = 0 \qquad (3)$$

$$\ddot{\Sigma} + \frac{1}{2}\left(\dot{B}^2\Sigma - A'\dot{\Sigma}\right) = 0 \qquad (4)$$

$$\Sigma'' + \frac{1}{2}B'^2\Sigma = 0 \qquad (5)$$

Einstein equations:

3 dynamical and 2 constraint PDE

efficient algorithm for the solution

$$f' = \partial_r f$$

$$\dot{f} = \partial_\tau f + \frac{1}{2}A\partial_r f$$

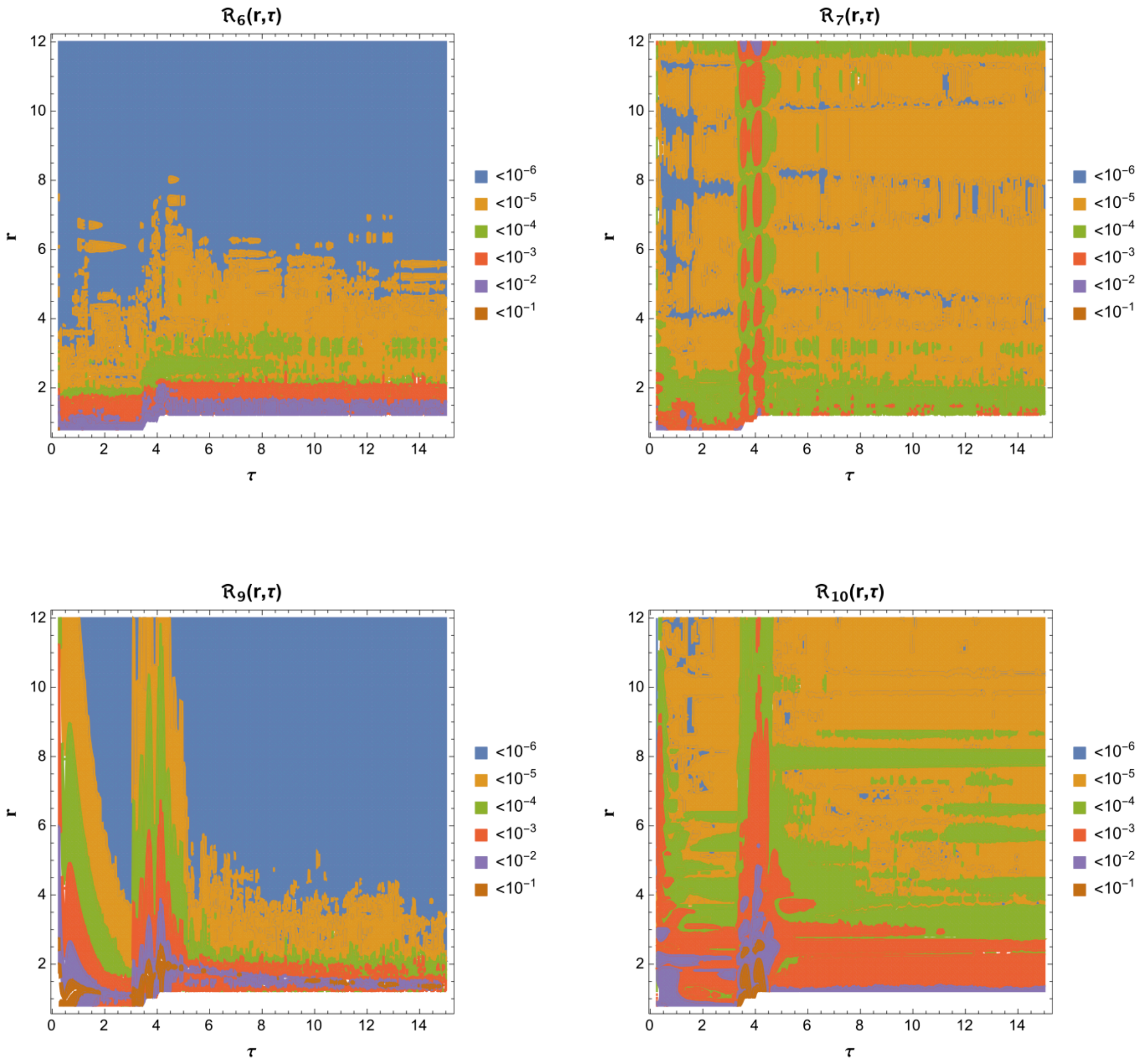
- at $\tau=\tau_i$ Σ, B, A known
- $\dot{\Sigma}(r,\tau_i)$ and $\dot{B}(r,\tau_i)$ from (1) and (2)
- Σ and B at a new time slice
- asymptotic large r expansion of the metric functions
- $\dot{\Sigma}(r,\tau_i+d\tau)$, $\dot{B}(r,\tau_i+d\tau)$ from (1) and (2)
- $A(r,\tau_i+d\tau)$ from (3)

stable evolution achieved

3 dynamical and 2 constraint PDE

efficient algorithm for the solution

example: one of the quench models



$$\mathcal{R}_6(r,\tau) = \frac{\Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2}{|\Sigma(\dot{\Sigma})'| + 2|\Sigma'\dot{\Sigma}| + 2|\Sigma^2|}$$

$$\mathcal{R}_7(r,\tau) = \frac{\Sigma(\dot{B})' + \frac{3}{2}\left(\Sigma'\dot{B} + B'\dot{\Sigma}\right)}{|\Sigma(\dot{B})'| + \frac{3}{2}\left(|\Sigma'\dot{B}| + |B'\dot{\Sigma}|\right)}$$

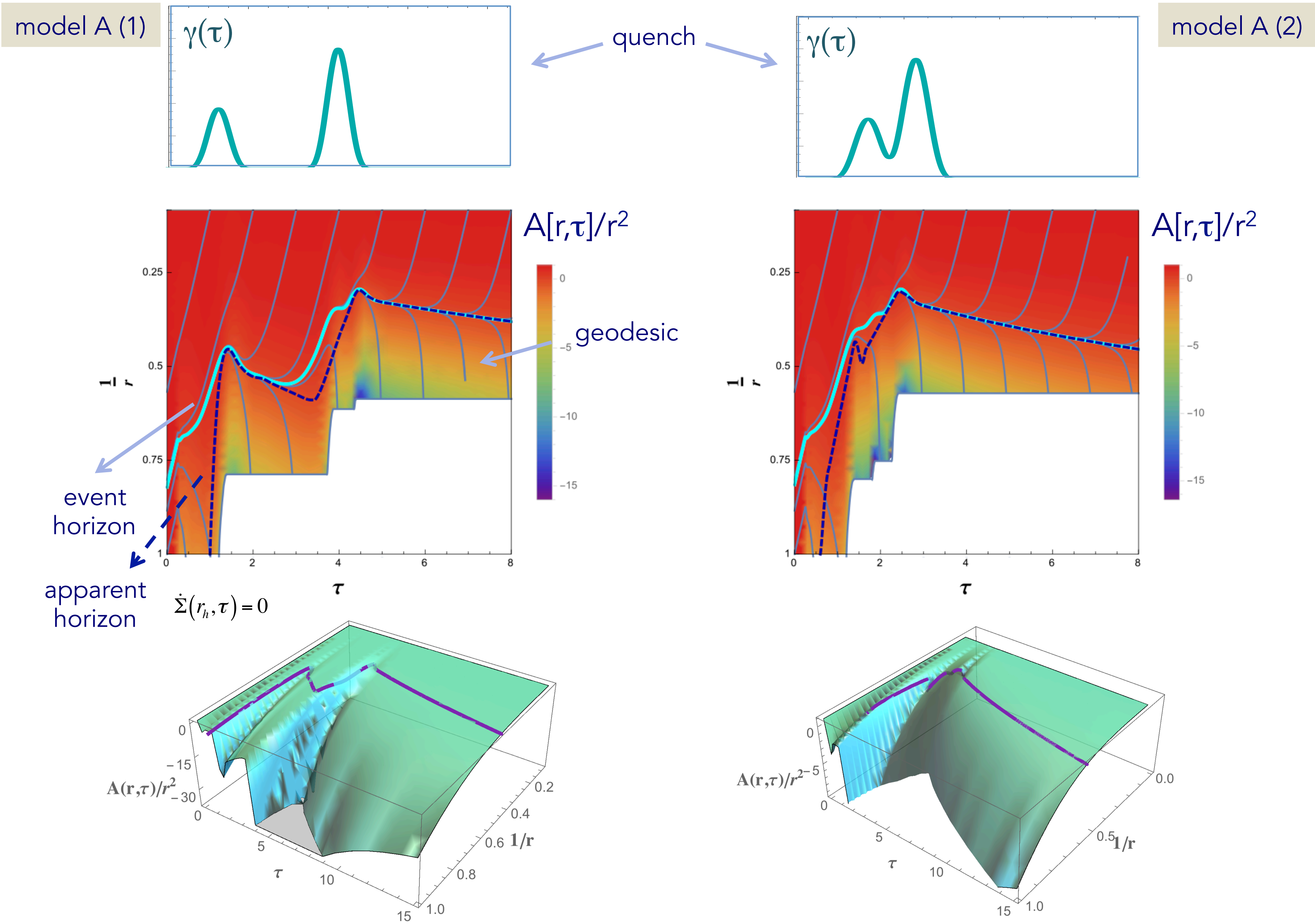
$$\mathcal{R}_8(r,\tau) = \frac{A'' + 3B'\dot{B} - 12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2} + 4}{|A''| + 3|B'\dot{B}| + |12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2}| + 4}$$

$$\mathcal{R}_9(r,\tau) = \frac{\ddot{\Sigma} + \frac{1}{2}\left(\dot{B}^2\Sigma - A'\dot{\Sigma}\right)}{|\ddot{\Sigma}| + \frac{1}{2}\left(|\dot{B}^2\Sigma| + |A'\dot{\Sigma}|\right)}$$

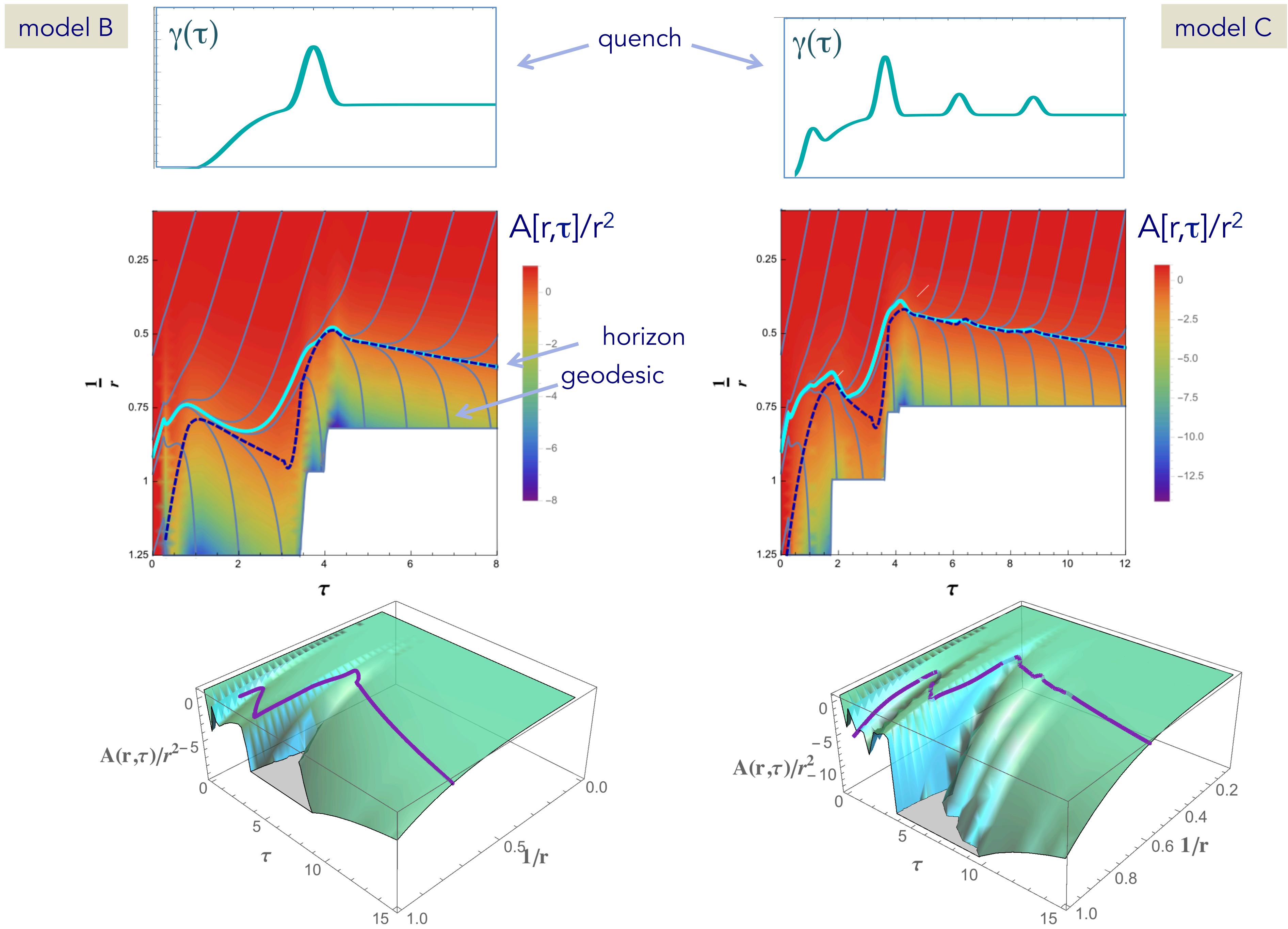
$$\mathcal{R}_{10}(r,\tau) = \frac{\Sigma'' + \frac{1}{2}B'^2\Sigma}{|\Sigma''| + \frac{1}{2}B'^2|\Sigma|}$$

self-adapting excision at small r

BULK GEOMETRY



model A (1)
model A (2)
γ(τ)
γ(τ)
quench
A[r,τ]/r²
A[r,τ]/r²
geodesic
event horizon
apparent horizon
$\dot{\Sigma}(r_h,\tau)=0$
1/r
τ
A(r,τ)/r²
1/r
τ
A(r,τ)/r²


model B
model C
quench
γ(τ)
γ(τ)
A[r,τ]/r²
A[r,τ]/r²
horizon
geodesic
1/r
τ
τ
A(r,τ)/r²
A(r,τ)/r²
1/r
1/r

boundary sourcing

P. Chesler, L. Yaffe, PRL 102 (09) 211601
PRD 82 (10) 026006

L. Bellantuono et al., JHEP 07 (2015) 053
PRD 94 (2016) 025005

1. 4D gauge theory driven out-of-equilibrium
   -> 4D metric deform ed by a quench $\gamma(\tau)$

$$ds_4^2 = -d\tau^2 + e^{\gamma(\tau)} dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$$

2. 5D metric of the dual theory in Eddington-Finkelstein coordinates

5D radial coordinate

$$ds_5^2 = 2dr d\tau - A\, d\tau^2 + \Sigma^2 e^B dx_\perp^2 + \Sigma^2 e^{-2B} dy^2$$

1 BOUNDARY SYSTEM PERTURBED

duality →

2 BOUNDARY CONDITIONS FOR THE 5D BULK METRIC

Einstein equations ↓

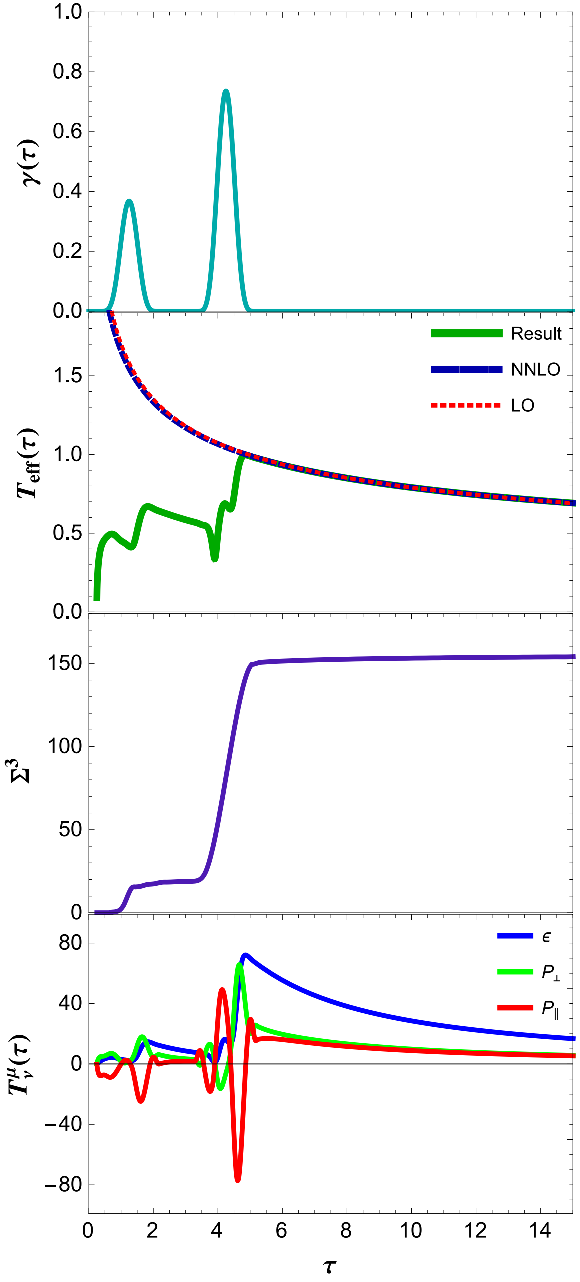
3 INTERMEDIATE AND LATE TIME SOLUTION FOR THE BULK GEOMETRY

←

4 TIME DEPENDENT OBSERVABLE DETERMINED FOR THE BOUNDARY SYSTEM

model A (1)

model A (2)

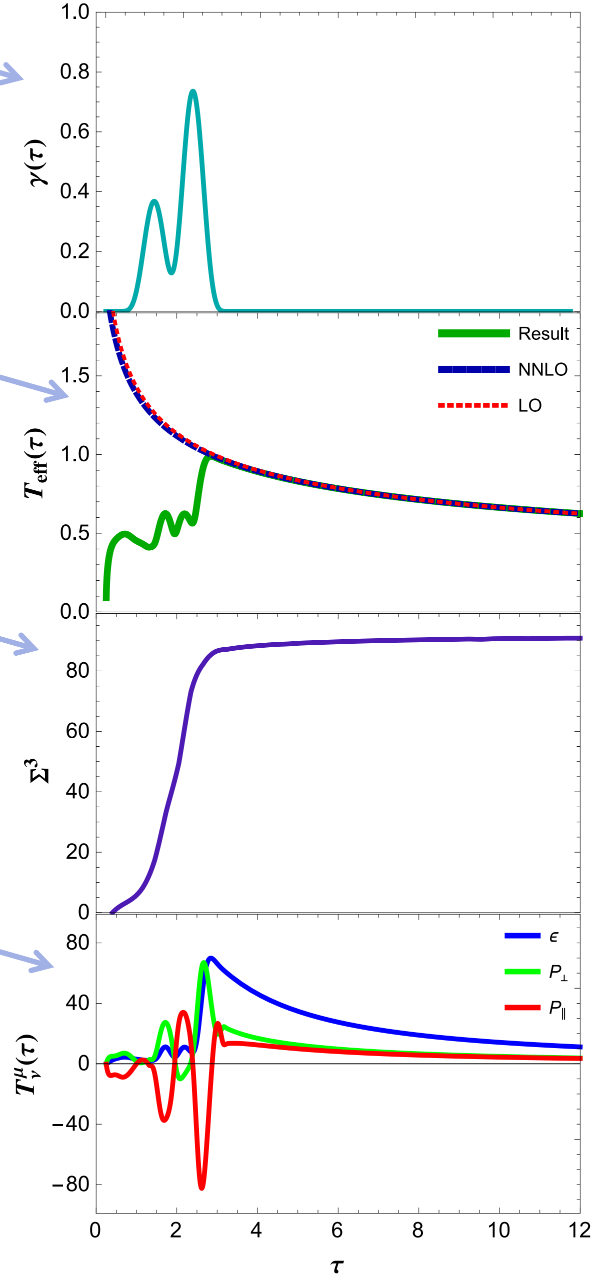


quench

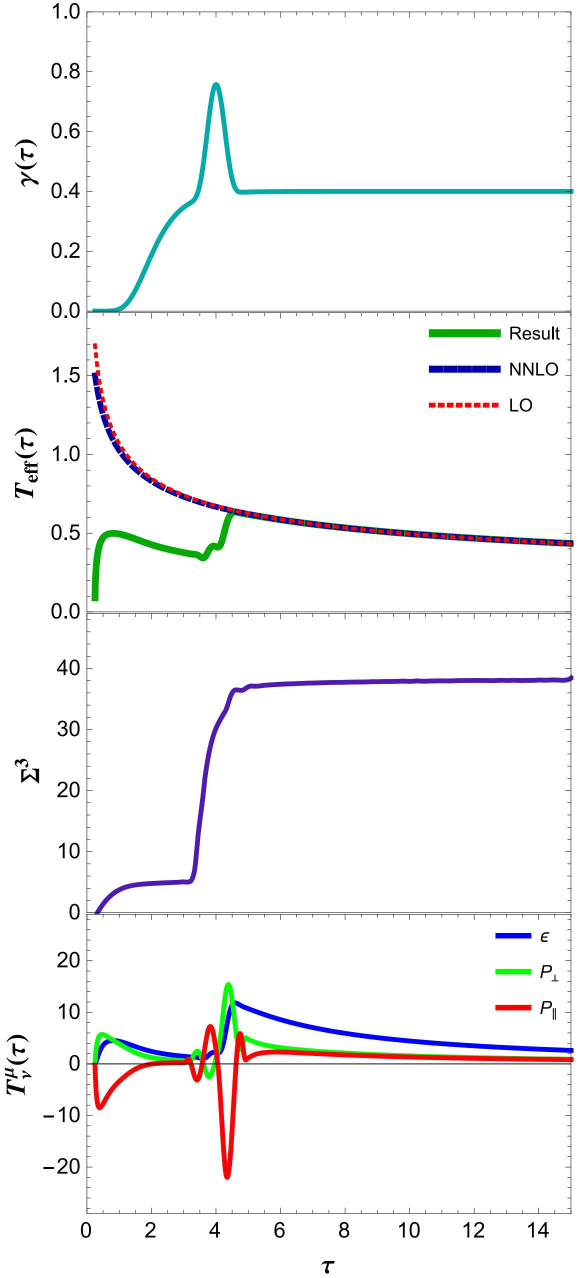
effective temperature

horizon area

energy density, pressures



model B



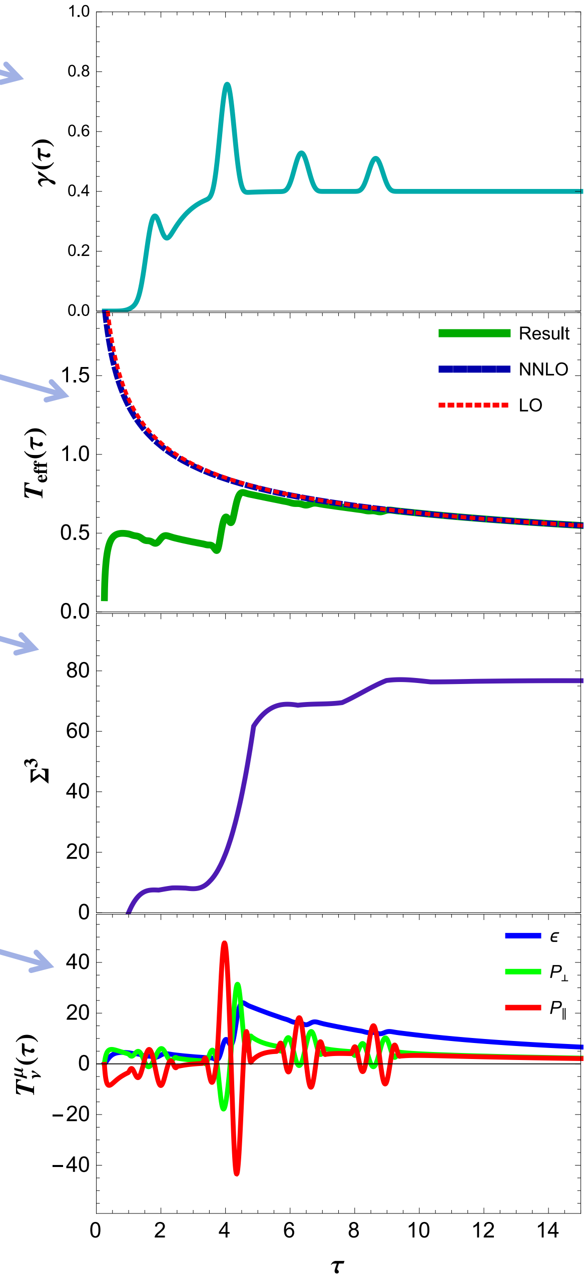
quench

effective temperature

horizon area

energy density, pressures

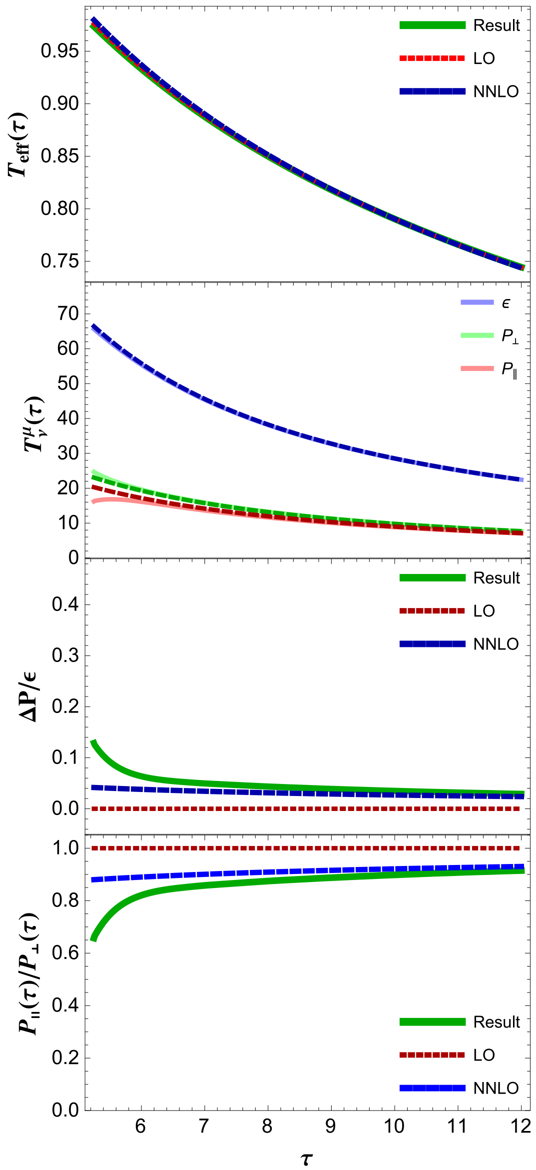
model C

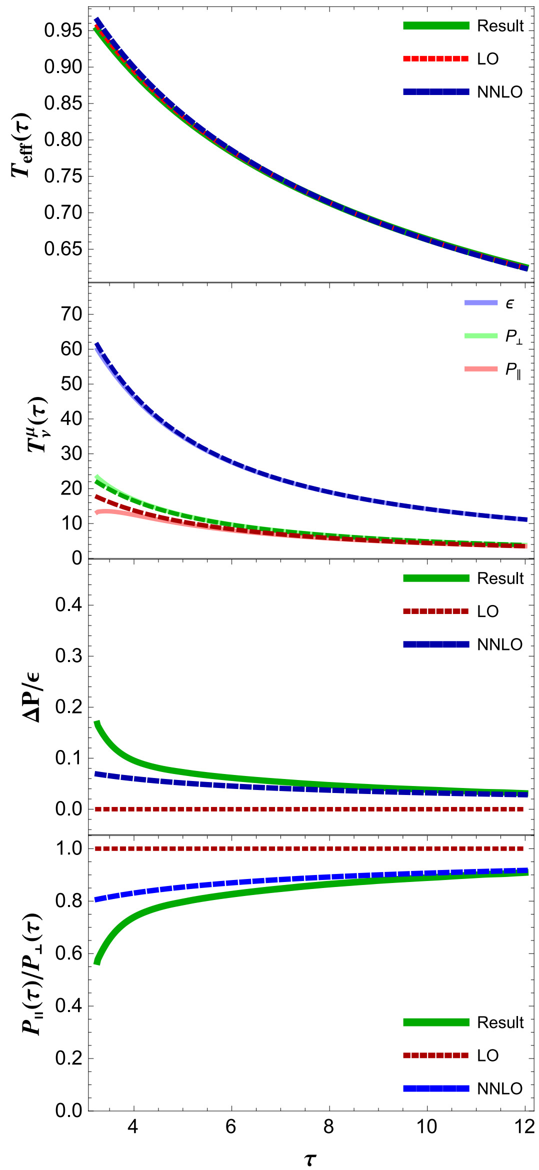


model A (1)

after the quench

model A (2)

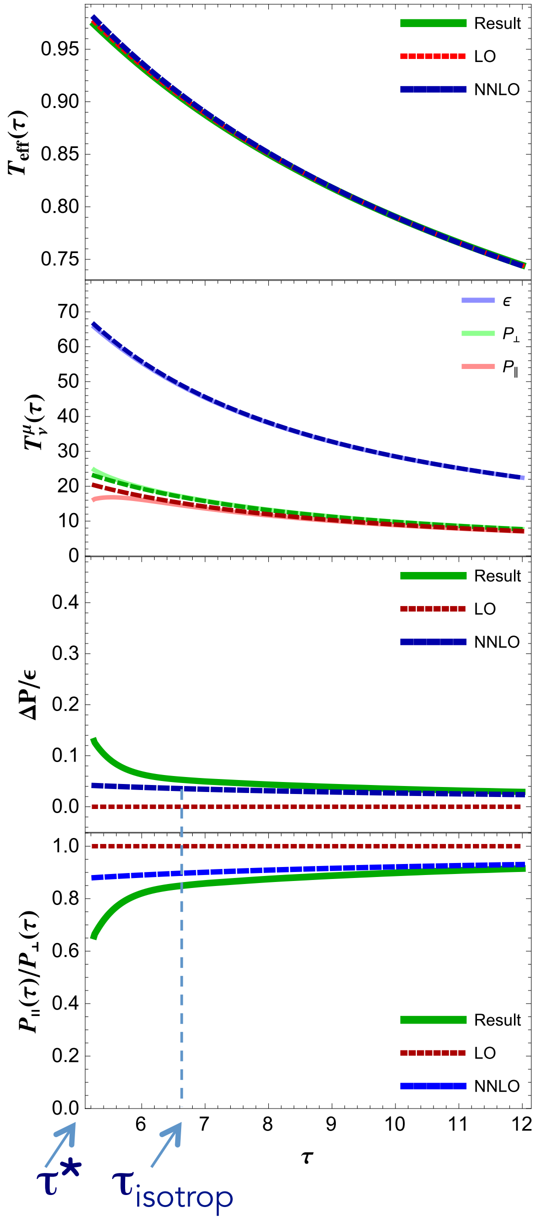




model A (1)

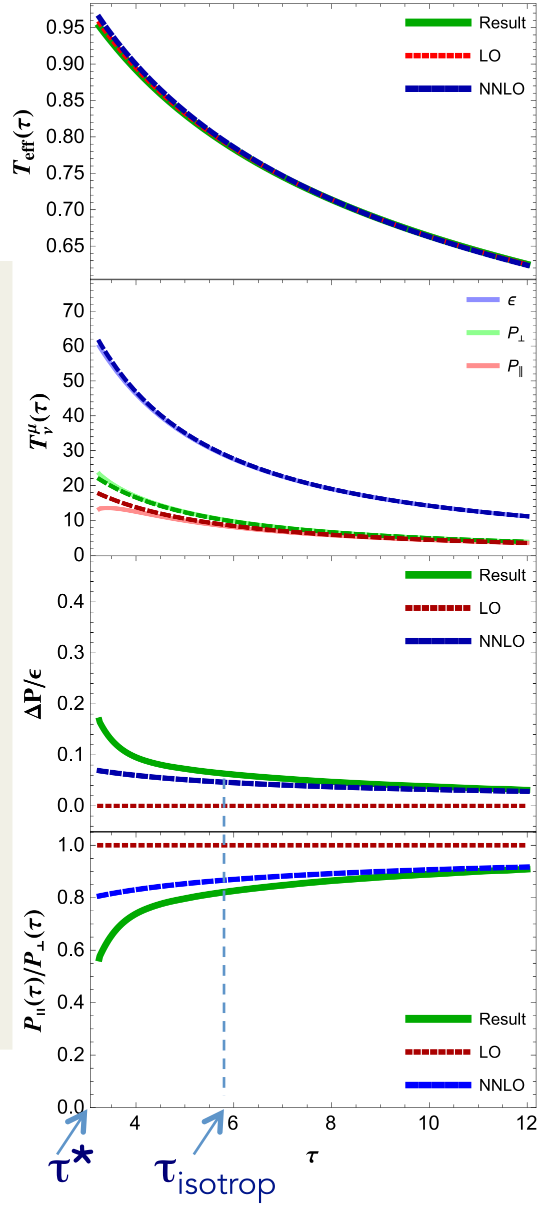
model A (2)

after the quench



in all cases
T_{eff} and ε reach the hydro τ-dependence right after the quench ($\tau=\tau^*$)

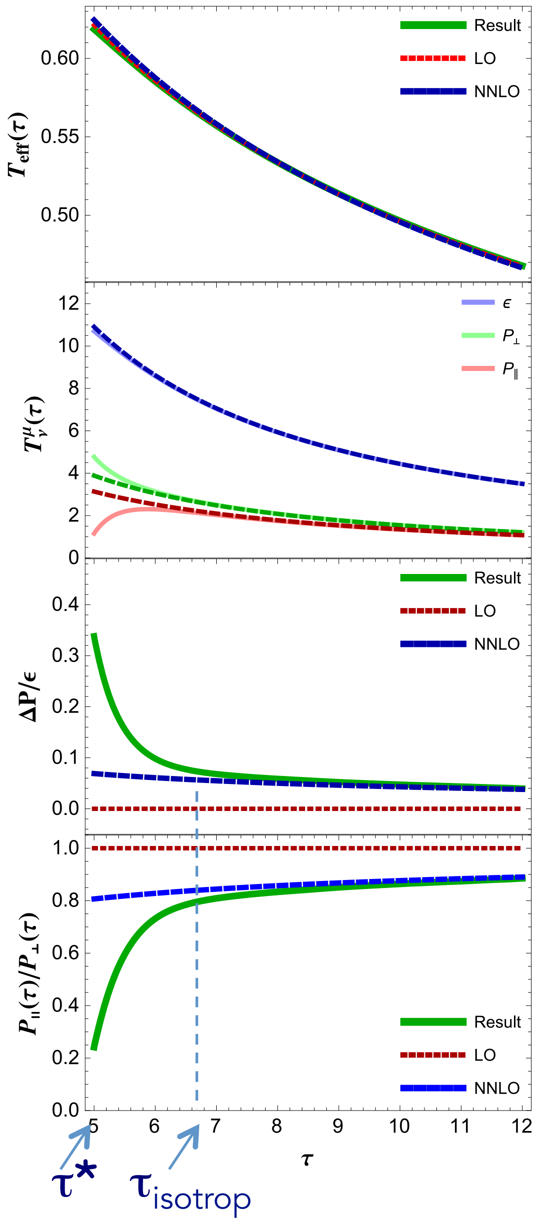
pressures take longer:
at $\tau=\tau_{\mathrm{isotrop}}$ they are within 5%
of the hydro NNLO value



model B

model C

after the quench



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T_{eff} and ε reach the
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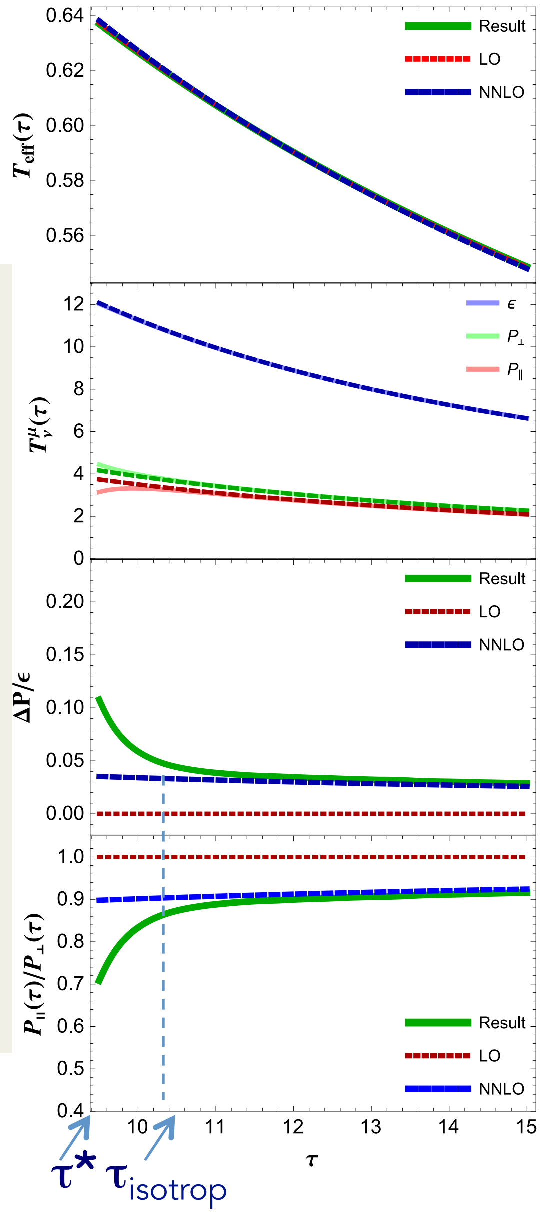


a scale can be introduced fixing T_{eff} at the end of the quench: T_{eff}=500 MeV

model	τ^*	$\tau_{isotrop}$	Λ	$\Delta\tau=\tau_{isotrop}-\tau^*$ (fm/c)
$\mathcal{A}$ (1)	5.25	6.8	2.25	0.60
$\mathcal{A}$ (2)	3.25	6.0	1.73	1.03
$\mathcal{B}$	5	6.74	1.12	0.42
$\mathcal{C}$	9.45	10.24	1.59	0.20

JHEP 07 (2015) 053

thermalization time (fraction of 1 fm)

main features independent of the details of the quench

local observables (energy density, pressures) from the dual metric close to the boundary

nonlocal probes of thermalization

Lin and Shuryak, PRD 78 (2008) 125018
Balasubramanian et al., PRL 106 (11) 191601
PRD 84 (11) 026010

computed in the dual space in terms of invariant geometric objects

related to minimal lengths, surfaces, volumes of various kinds in the bulk

probe deeper into the bulk spacetime, away from the boundary

sensitive to a wide range of energy scales in the boundary field theory

allow to investigate the thermalization mechanism, distinguishing between the possibilities:

- bottom-up (perturbative): hard quanta of gauge theory equilibrate radiating softer quanta
- top-down (strongly coupled gauge th.): energetic gauge field modes equilibrate first, soft modes last

nonlocal probes of thermalization

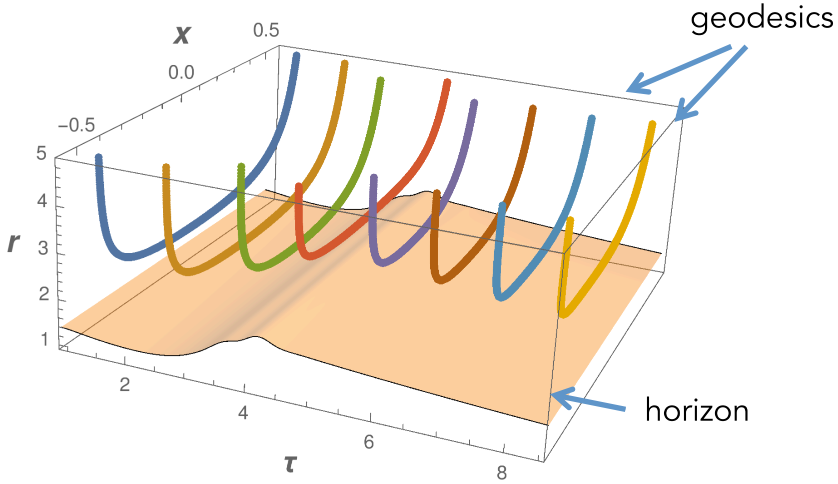
Lin and Shuryak, PRD 78 (2008) 125018
Balasubramanian et al., PRL 106 (11) 191601
PRD 84 (11) 026010

equal time two-point correlation function

geodesic approximation

$$\langle O(t,x)O(t,x')\rangle \approx e^{-\Delta L}$$

O operator of large Δ
L (regularized) geodesic length

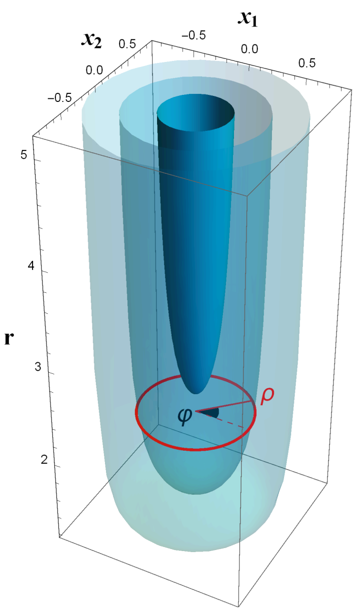


circular Wilson loop

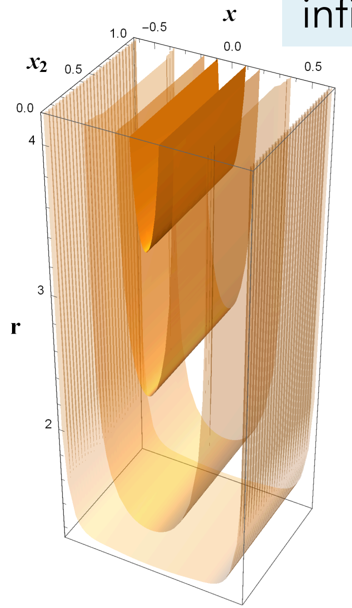
$$\langle W(C)\rangle \approx e^{-(1/\alpha')A(\Sigma_0)}$$

Σ_0 minimal area surface with boundary C

$A(\Sigma_0)$ (regularized) area



infinite rectangular Wilson loop



nonlocal probes of thermalization geodesics length

Spacelike geodesics connecting two boundary points:

P=(t_0, -ℓ/2, x_2, y) and Q=(t_0, ℓ/2, x_2, y) for fixed (x_2,y)
parametrized by r(x), τ(x) solutions of the geodesics equation
with BC in the middle point x=0 $r(0)=r_*$ $\tau(0)=\tau_*$ $r'(0)=\tau'(0)=0$
on the boundary $\tau\left(\pm\frac{\ell}{2}\right)=t_0$ $r\left(\pm\frac{\ell}{2}\right)=r_0$

$$\mathcal{L}=\int_P^Q d\lambda\sqrt{\pm g_{MN}\dot{x}^M\dot{x}^N},$$

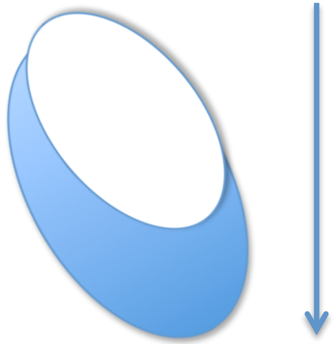
$$\boxed{\mathcal{L}=\int_{-\ell/2}^{\ell/2}dx\frac{\tilde{\Sigma}(r,\tau)}{\sqrt{\tilde{\Sigma}(r_*,\tau_*)}}}$$

$\tilde{\Sigma}(r,\tau)\equiv\Sigma(r,\tau)^2e^{B(r,\tau)}$ computed metric functions

nonlocal probes of thermalization
Wilson loops

$$W_C[A] = \frac{1}{N_c} Tr\left(P e^{-ig \oint_C dx^\mu A_\mu^a T^a}\right)$$

$$\langle W_C \rangle \sim e^{-S_{NG}}$$



bulk

$$S_{NG} = \frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{det\left[g_{MN}\partial_\alpha X^M \partial_\beta X^N\right]},$$

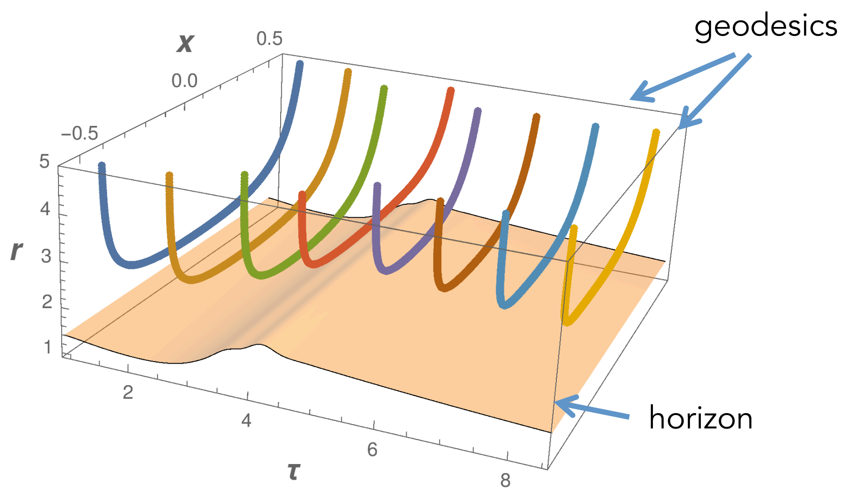
$$\mathcal{L} = \int_{\lambda_1}^{\lambda_2} d\lambda \left(-A(r,\tau)\dot{\tau}(\lambda)^2 + 2\dot{\tau}(\lambda)\dot{r}(\lambda) + \tilde{\Sigma}(r,\tau)\dot{x}(\lambda)^2\right)^{1/2}$$

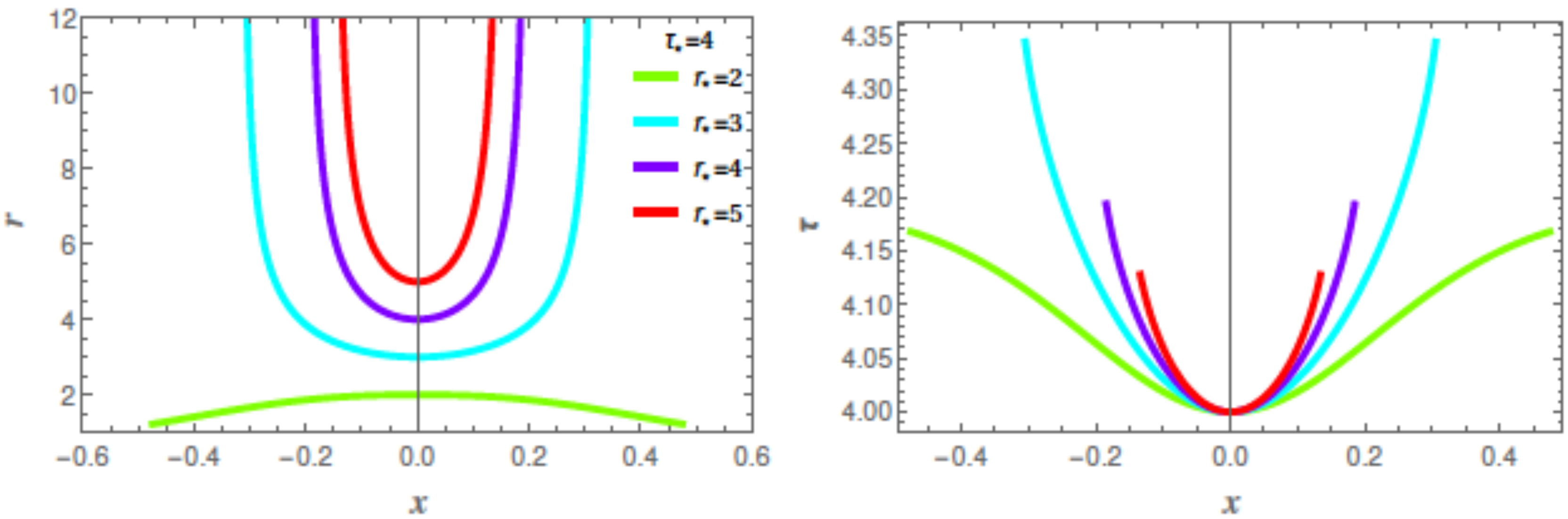
for a rectangular WL

$$\tilde{\Sigma}(r,\tau) \equiv \Sigma(r,\tau)^2 e^{B(r,\tau)}$$

computed metric functions

geodesics





(a) (b)

FIG. 3. Quench model $\mathcal{B}$. Geodesics $r(x)$ (a) and $\tau(x)$ (b), for $\tau_\star = 4$ and the values of $r_\star$ in the legenda.

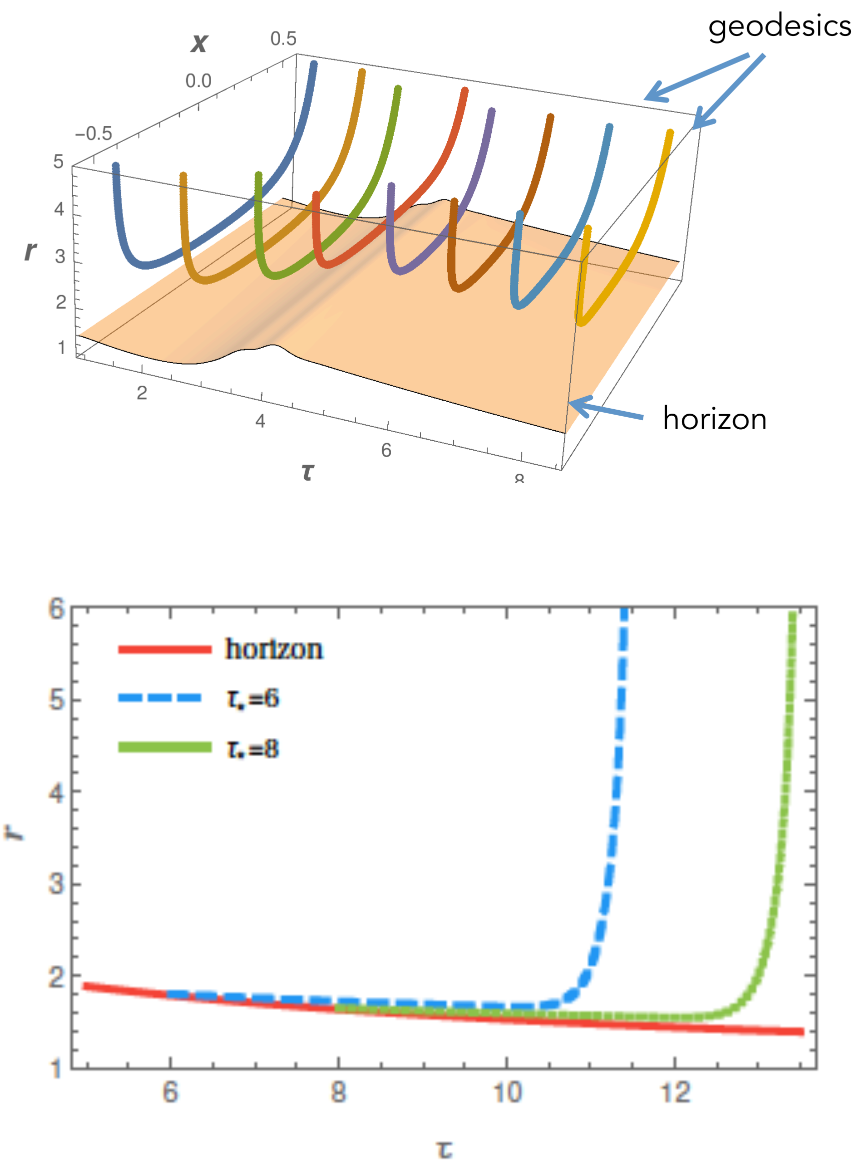


FIG. 4. Geodesics in the (τ, r) plane for ($\tau_* = 6$, $r_* \sim 1.80$) and ($\tau_* = 8$, $r_* \sim 1.65$) in quench model $\mathcal{B}$. Increasing τ_* (after the pulse in the quench) and for large ℓ, the radial coordinate closely follows the event horizon.

model B

model A (2)

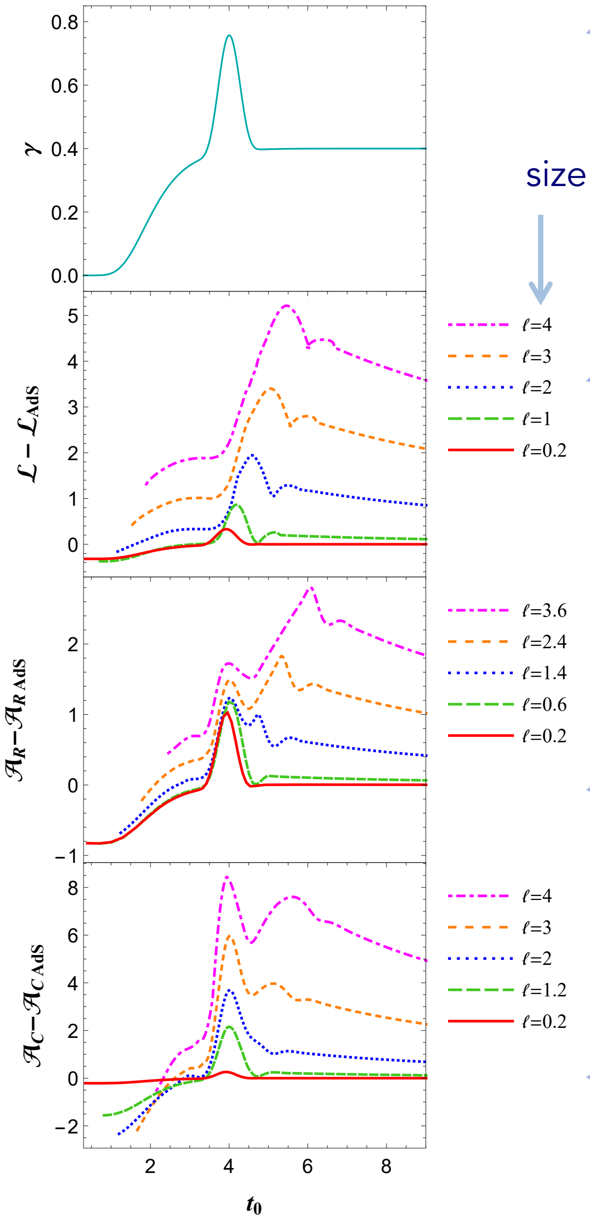
quench

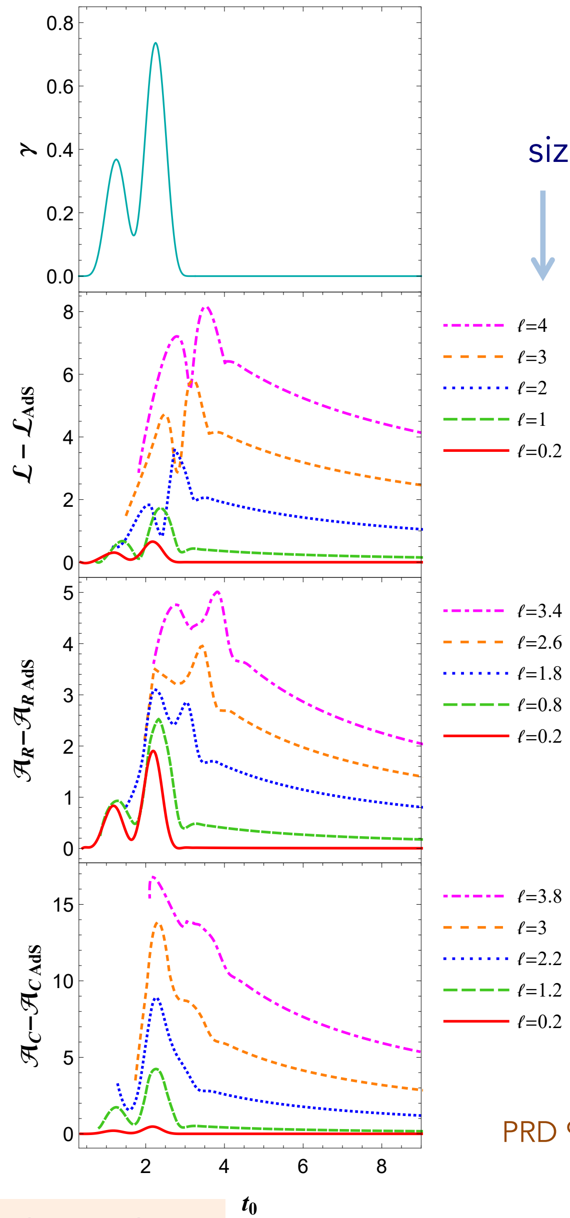
size of the probe

regularized length of the geodesic

regularized area rectangular WL

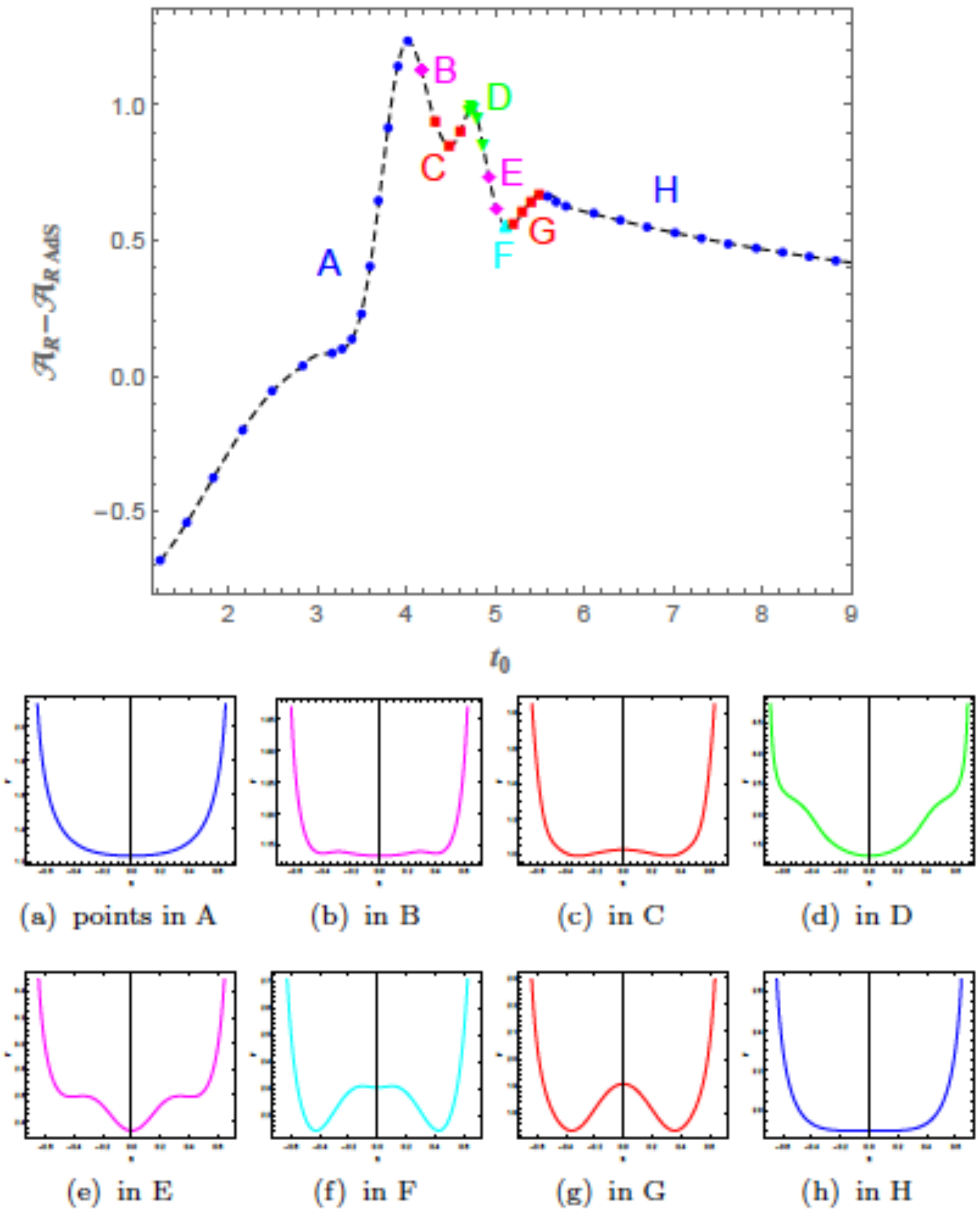
regularized area circular WL





PRD 94 (2016) 025005

quench profile followed in the three observables



WL profiles

in some cases
the profile exceeds
the event horizon

possible in a rapidly
time-changing setup

thermalization of the nonlocal probes:
comparison with viscous hydrodynamics

5D metric dual to the viscous hydrodynamics

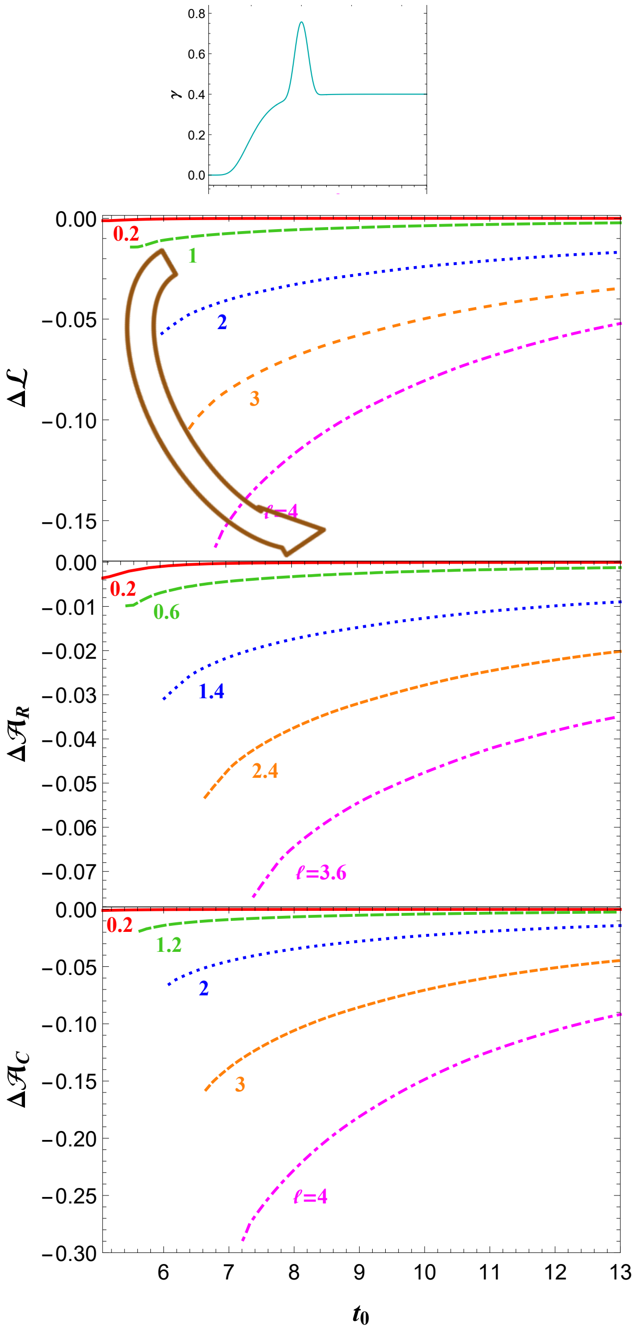
$$ds_5^2 = 2drd\tau - A^H d\tau^2 + \left[\Sigma^H\right]^2 e^{B^H} dx_\perp^2 + \left[\Sigma^H\right]^2 e^{-2B^H} dy^2$$

$$A^H(r,\tau) = r^2\left(1 - \frac{4}{3r^4}\varepsilon(\tau)\right)$$

$$\Sigma^H(r,\tau) = r\left(\tau + \frac{1}{r}\right)^{1/3}$$

$$B^H(r,\tau) = \frac{1}{r^4}\left(p_\perp(\tau) - p_\parallel(\tau)\right) - \frac{2}{3}\log\left(\tau + \frac{1}{r}\right)$$

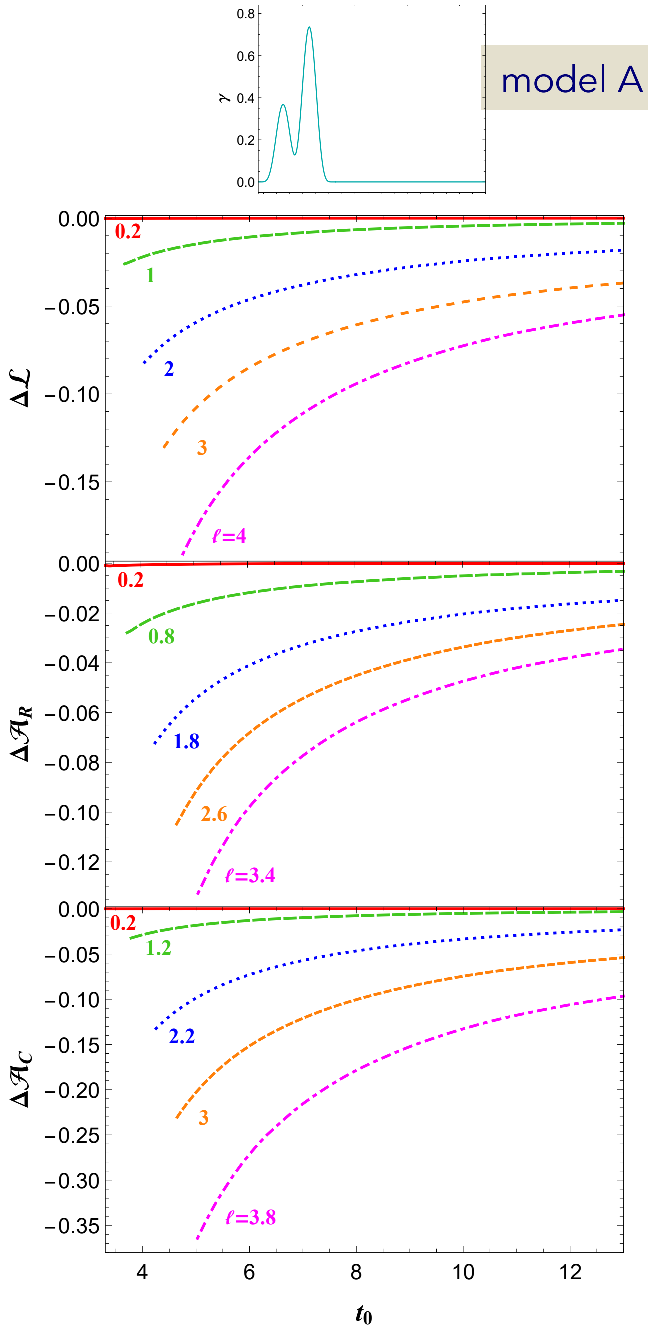
model B



$$\Delta L = L - L_{hydro}$$

$$\Delta A = A - A_{hydro}$$

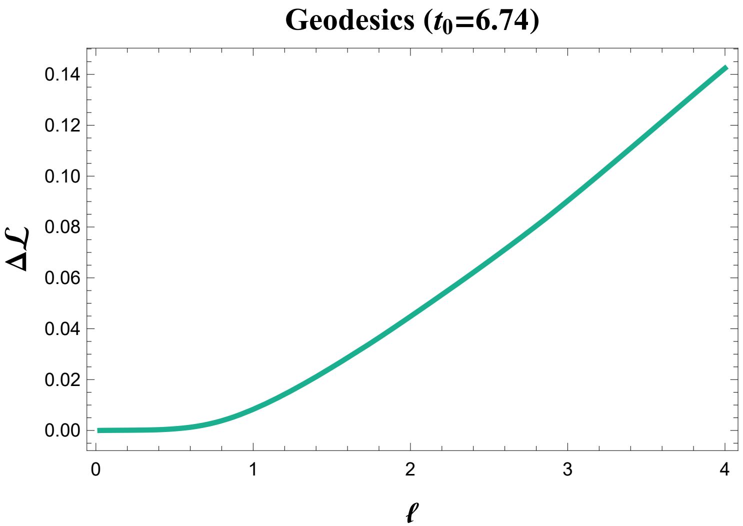
model A (2)

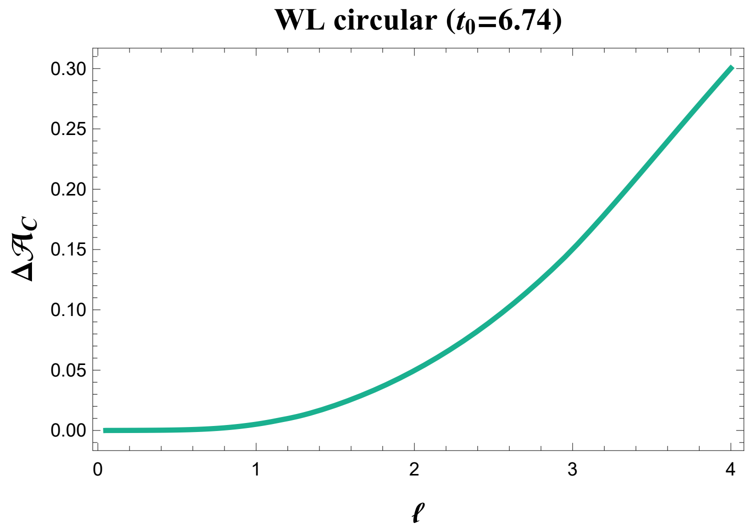


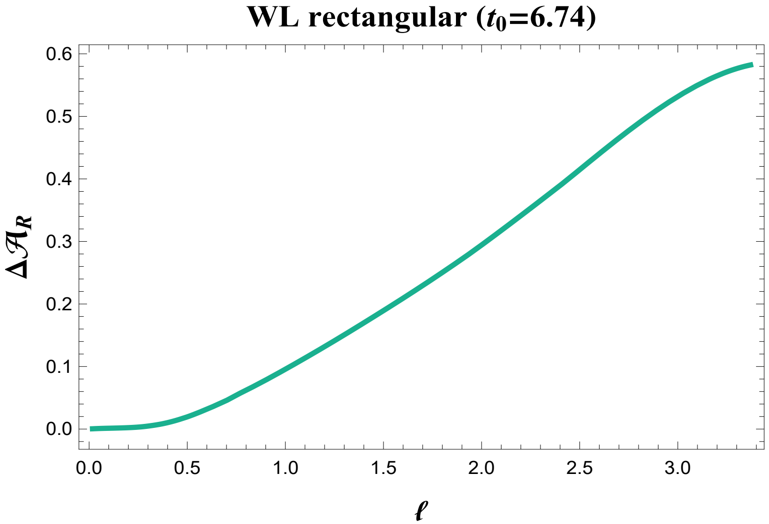
hydro regime reached at large time for large ℓ, almost immediately for small ℓ

local vs nonlocal probes
time scales

model B   at the time when local probes indicate the onset of thermalization



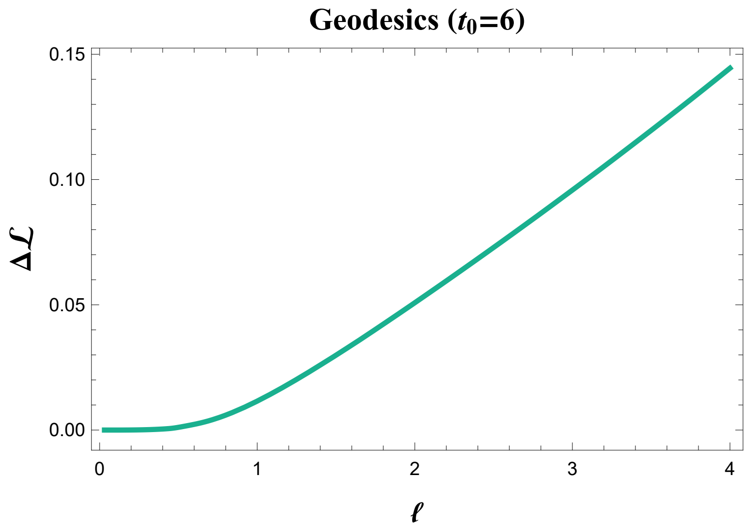


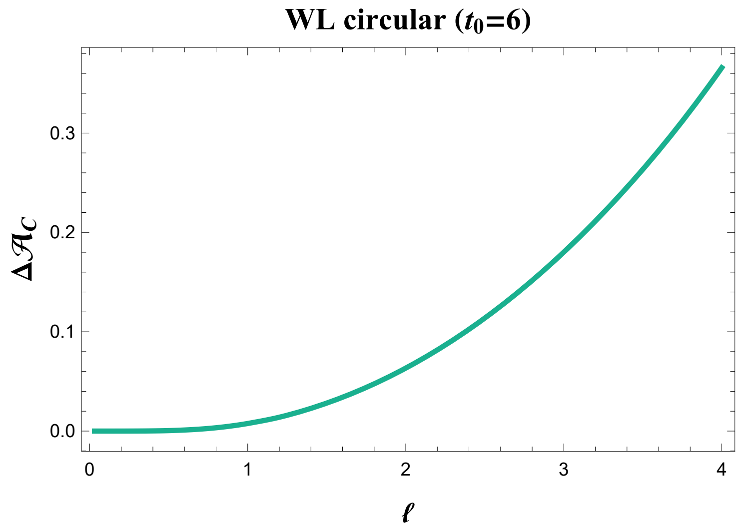


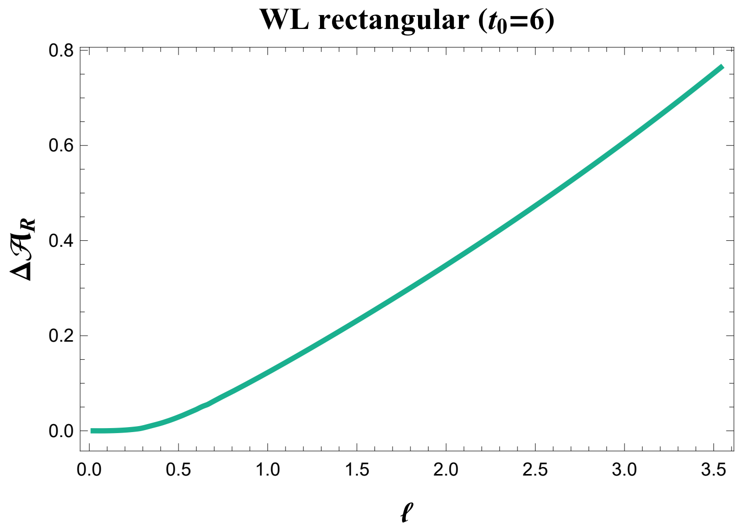
only for small ℓ thermalization occurred
geodesics lenght thermalizes for approx ℓ<0.5
circular WL area for ℓ<1
rectangular WL area for small ℓ

local vs nonlocal probes time scales

model A (2) at the time when local probes indicate the onset of thermalization

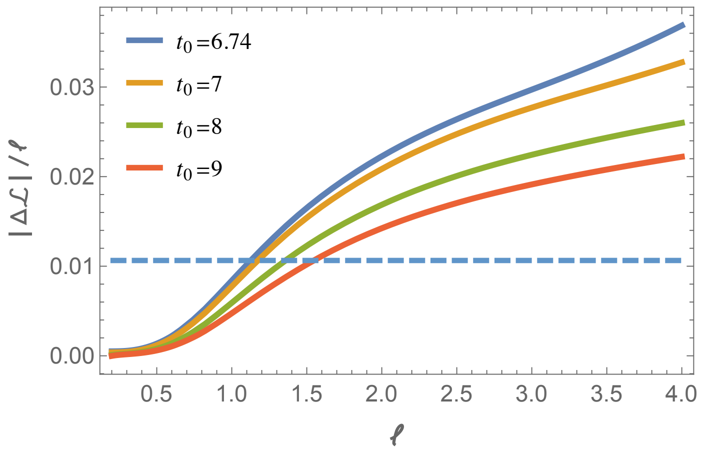






only for small ℓ thermalization occurred
geodesics lenght thermalizes for approx ℓ<1
circular WL area for ℓ<1
rectangular WL area for ℓ<0.5

model B



as time proceeds, larger probes thermalize

model B

thermalization criteria for extended probes

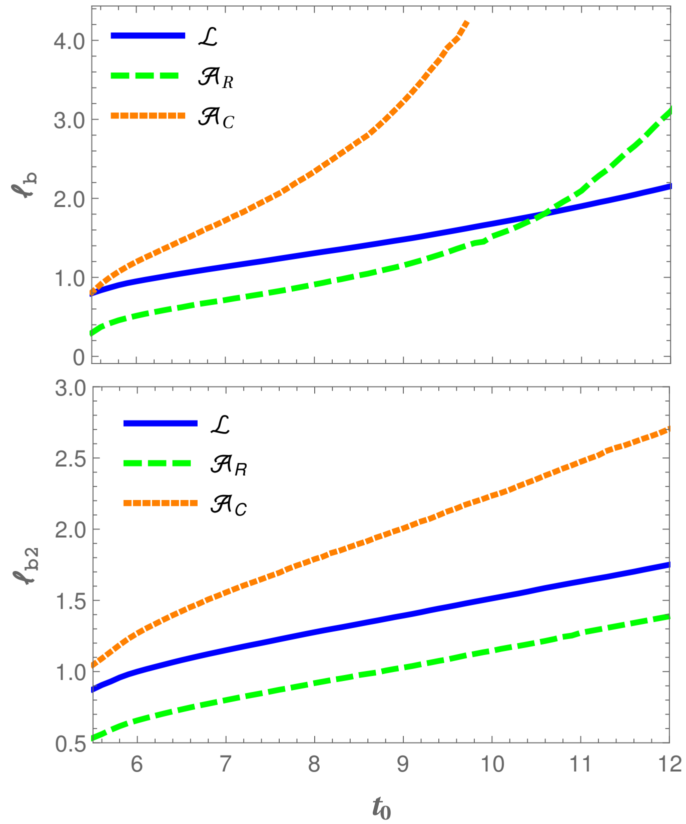
definition of critical sizes

$\ell_b:\quad \Delta L/\ell_b = 0.01$

......

$\ell_{b2}:\quad \Delta L/L = 0.001$

......



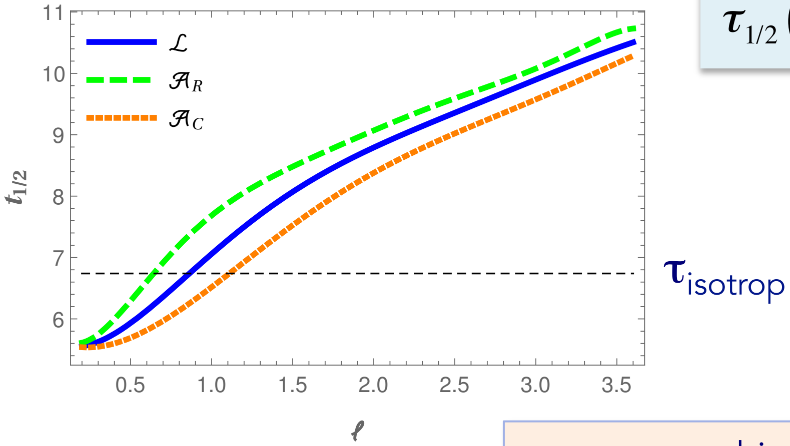
model B
$\tau_{isotrop}$=6.74

critical size	GL	RWL	CW	GL	RWL	CWL
$\hat{\ell}_1$	1	0.5	1	0.9	0.4	0.8
$\hat{\ell}_2$	0.7	0.3	0.7	0.6	0.3	0.6
$\hat{\ell}_b$	1.1	0.7	1.6	0.9	0.5	1.2
$\hat{\ell}_{b2}$	1.1	0.8	1.5	1.0	0.6	1.1

model A (2)
$\tau_{isotrop}$=6

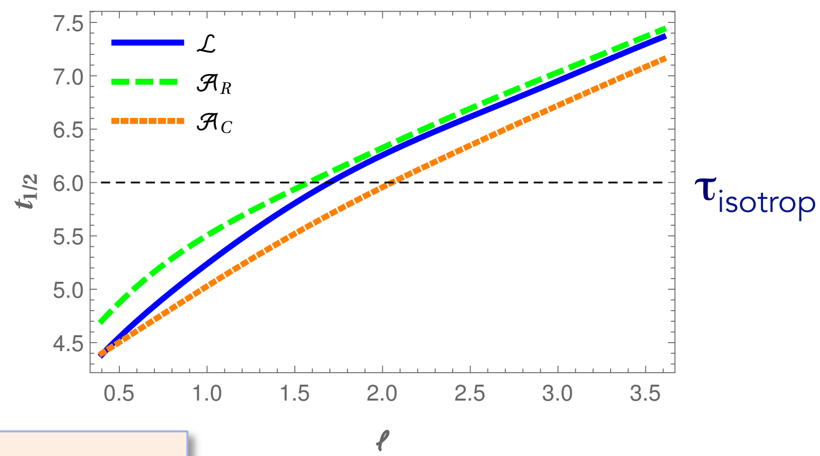
$\tau_{1/2}$: value of τ where ΔL , … is reduced by ½ with respect to the value after the quench

model B



$$\tau_{1/2}(\ell) \approx \ell$$

model A (2)



hierarchy:

$\tau^* < \tau_{1/2} < \tau_{\text{isotrop}}$ (small ℓ)

$\tau^* < \tau_{\text{isotrop}} < \tau_{1/2}$ (large ℓ)

thermal limit reached after a finite time that depends on the size of the probe
hierarchy among thermalization times: UV thermalizes first

equilibration proceeds top-down (typical for a strongly coupled system)

Conclusions

- Gauge/gravity duality methods for an out-of equilibrium system
- Short thermalization time obtained from local observables
- Nonlocal probes of thermalization computed in a time-dependent setup with boundary quenches
- Results for local and nonlocal observables indendent of the quench profile (after the end of the quench)
- Thermalization proceeds top-down

Ongoing & future

- Other different time regimes? Suggested from analyses using a particular time-dependent bulk geometry (Vaidya geometry), unclear if they are general Liu & Suh, PRD89 (2014) 066012

- Nonlocal probes for less symmetric systems

- Thermalization in presence of a confinement/deconfinement phase transition

- Many other possible analyses of out-of-equilibrium strongly coupled systems