

Heavy quark production at the LHC in the parton Reggeization approach

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in collaboration with

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Outline.

- ① Introduction
- ② Hard processes and Collinear Parton Model
 - Collinear factorization of the amplitudes, large log corrections
 - Single-scale processes, CPM-factorization formula
 - Multi-scale processes, TMD-factorization formula
- ③ Introduction to the parton Reggeization approach (PRA)
 - Reggeization of the amplitudes in QCD
 - Lipatov's Effective action
 - k_T -factorization and PRA
- ④ Selected results in LO PRA
- ⑤ Heavy quarkonium production in PRA (It has been presented in report by M. Nefedov)
- ⑥ D and B production in PRA
- ⑦ b -jet and $b\bar{b}$ -dijet production in PRA
- ⑧ $D\bar{D}$ and DD pair production at the LHCb
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Introduction

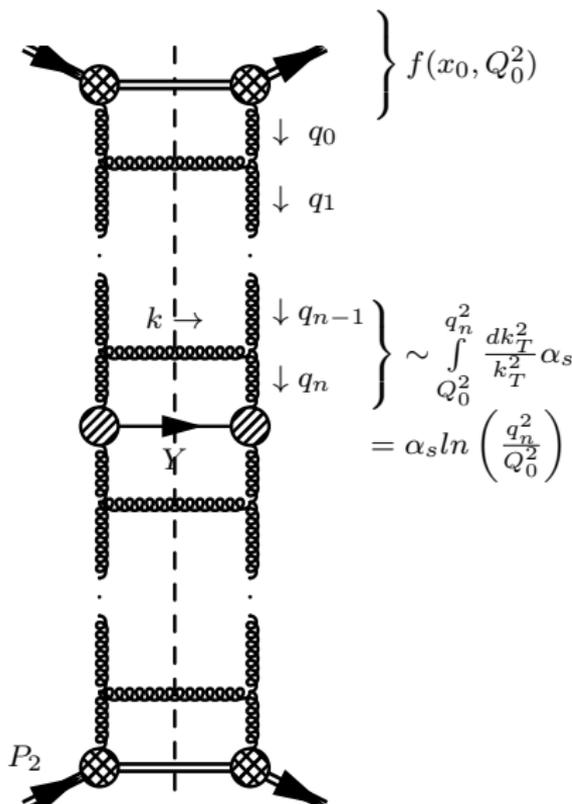
The hard processes (i. e. the inelastic processes involving high momentum transfer $Q^2 \gg 1 \text{ GeV}^2$) are the major tool to study the fundamental interactions, both QCD and EW, at hadron colliders, since the most interesting fundamental particles (W^\pm , Z^0 , H , t , b , \bar{t} , ...) are heavy.

Thanks to asymptotic freedom of QCD, and a special kinematics of the hard collision, it is possible to separate the perturbative and nonperturbative dynamics, systematically parameterize nonperturbative part, calculate hard subprocess in the perturbation theory, and therefore put the whole problem under quantitative control. Currently, the studies of the hard processes in pQCD are developing along the lines of four *complementary* approaches:

- Fixed-order calculations in the Collinear Parton Model (CPM)
- Soft gluon/logarithmic resummation techniques
- LO, NLO, (NNLO) + Parton Shower Monte-Carlo techniques
- Soft-Collinear Effective Theory (SCET)
- TMD factorization
- k_T -factorization

The talk will be devoted mostly to **the two of approaches**, which try to generalize the conventional Collinear Parton Model.

Collinear factorization, DGLAP evolution.



$$P_1^2 = P_2^2 = 0, \quad 2P_1 P_2 = S \gg \Lambda_{QCD}^2$$

$$q_0 = x_0 P_1 + q_{0T}, \quad q_0^2 = q_{0T}^2 = Q_0^2 \sim \Lambda_{QCD}^2$$

...

$$q_n = x P_1 + q_{nT}, \quad q_n^2 = q_{nT}^2 = Q^2 \gg \Lambda_{QCD}^2.$$

Collinear factorization for the amplitude ($k_T \rightarrow 0$):

$$|\overline{\mathcal{M}}_{n+1}|^2 = \frac{1}{k_T^2} \left(\frac{\alpha_s}{2\pi} P_{ij}(z) |\overline{\mathcal{M}}_n|^2 + O(k_T^2) \right),$$

\Rightarrow n collinear radiations will give:
 $\alpha_s^n \log^n(Q^2)$, which can be resummed
 into PDF by solving DGLAP equation:

$$\frac{\partial f_i(x, \mu^2)}{\partial \log(\mu^2)} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P_{ij}(z) f_j(x/z, \mu^2)$$

Approach of the Collinear Parton Model in the fixed order.

Factorization formula of the CPM:

$$d\sigma = \sum_{p_1, p_2} \int_0^1 dx_1 \int_0^1 dx_2 f_{p_1}(x_1, \mu_F^2) f_{p_2}(x_2, \mu_F^2) d\hat{\sigma}_{p_1 p_2}(q_1, q_2, \mu_F, \mu_R), + O\left(\frac{1}{(\mu_F^2)^\alpha}\right)$$

where $q_1 = x_1 P_1$, $q_2 = x_2 P_2$, $f_p(x, \mu_F)$ – (integrated) PDF of the parton p in proton, $d\hat{\sigma}$ – hard-scattering cross-section.

For the sufficiently inclusive **single-scale** observables (e. g. $d\sigma/dy dQ^2$ in Drell-Yan or $F_2(x, Q^2)$ in DIS), it is proven (see e. g. [Collins, 2011]), that the factorization-breaking terms are power-suppressed.

Now we can start to do perturbation theory. Possible problems:

- PT is complicated, LO – tree level, NLO – 1-loop+IR cancelations between real and virtual part, NNLO – 2-loops+ much more complicated IR cancelations, ...
- The PT expansion can be slow-convergent due to soft-gluon effects.
- In the case of multiscale processes, the large logarithms of the scale ratios come in – $\alpha_s \log(\mu_1/\mu_2)$.
- In the case of azimuthal correlations between final particles, we have start with $2 \rightarrow 3$ subprocess and include $2 \rightarrow 4, 5, ..$ to describe data for $\Delta\phi < \pi/2$.

Light-cone decomposition.

Let's introduce the Sudakov (or light-cone) notation. The protons are flying along the z -axis. For any 4-vector q :

$$q_\mu = \frac{1}{2}(q^+ n_\mu^- + q^- n_\mu^+) + q_{T\mu},$$

where $n^+ = \frac{2P_2}{\sqrt{S}}$, $n^- = \frac{2P_1}{\sqrt{S}}$, $n^+ n^- = 2$, $q^\pm = n^\pm q = q^0 \pm q^3$, $q_T n^\pm = 0$, and $\forall q, k$:

$$qk = \frac{1}{2}(q^+ k^- + q^- k^+) - \mathbf{q}_T \mathbf{k}_T, \quad q^2 = q^+ q^- - \mathbf{q}_T^2.$$

Rapidity – natural parameter for boosts along the z -axis:

$$y = \frac{1}{2} \log \left(\frac{q^+}{q^-} \right),$$

is closely related to pseudorapidity $\eta = -\log \tan(\theta/2)$. For massless particle:

$$\eta = y.$$

TMD factorization

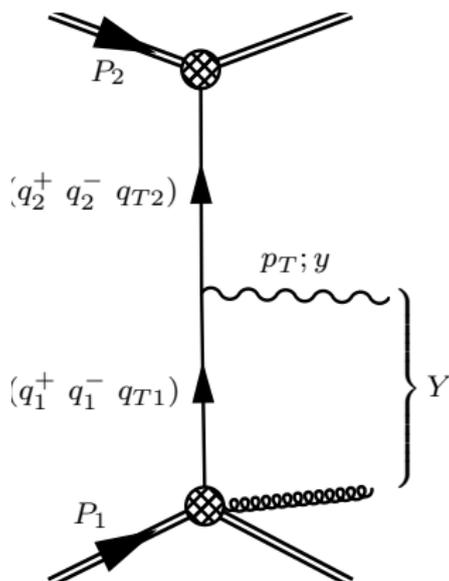
For the multiscale processes, like the Drell-Yan $d\sigma/dQ^2 dp_T$, the large log corrections of the form $\alpha_s \log(p_T^2/Q^2)$ are accumulated for $\Lambda_{QCD}^2 \ll p_T^2 \ll Q^2$. For this kind of processes, the TMD-factorization theorem is proven in all orders (see e. g. [Collins, 2011]):

$$d\sigma = \int dx_1 dx_2 \int d^2 \mathbf{q}_{T1} F(x_1, \mathbf{q}_{T1}^2, \mu_F^2, \mu_Y^2) F(x_2, \mathbf{q}_{T2}^2, \mu_F^2, \mu_Y^2) C_{low\ p_T}(x_1, x_2) + \\ + \int dx_1 dx_2 f(x_1, \mu_F^2) f(x_2, \mu_F^2) C_{high\ p_T}(x_1, x_2) + \text{power corrections},$$

where $\mathbf{q}_{T2} = \mathbf{p}_T - \mathbf{q}_{T1}$, F -TMD PDF where new log corrections are absorbed to. The hard-scattering coefficients $C_{low\ p_T}$ and $C_{high\ p_T}$ are free from large logarithms, **do not depend on \mathbf{q}_T** , and calculable in the PT. Regions of $p_T^2 < Q^2$ and $p_T^2 > Q^2$ are treated separately. The region of high p_T can be taken into account only order by order in PT. The factorization scale μ_F and rapidity evolution scale μ_Y can be considered separately.

TMD factorization and "classic" k_T -factorization.

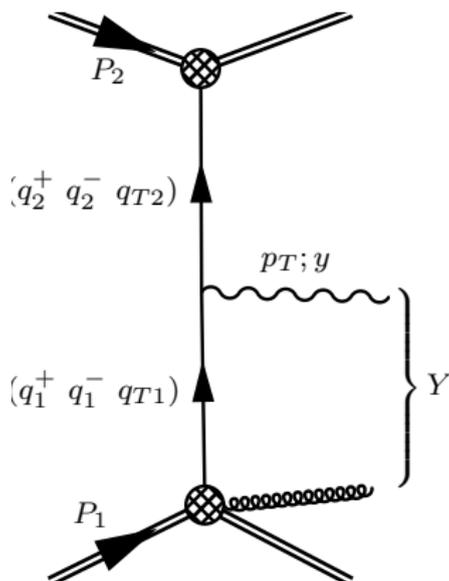
Momentum-flow diagram:



- Collinear factorization:
 - $q_1^+ \gg q_1^-$, $|\mathbf{q}_{T1}| \ll \mu_F$, Y - arbitrary,
 - q_1^- , \mathbf{q}_{T1} - integrated out $\Rightarrow f(x, \mu_F)$,
 - $\alpha_s \log(\mu_F)$ - resummed into PDF.
- TMD factorization:
 - $q_1^+ \gg q_1^-$, $|\mathbf{q}_{T1}| \ll \mu_F$, Y - arbitrary
 - q_1^- - integrated out $\Rightarrow F(x, \mathbf{q}_T, \mu_F, \mu_Y)$,
 - $\alpha_s \log(\mu_F)$, $\alpha_s \log^2(|\mathbf{q}_T|/\mu_F)$ - resummed into TMD PDF.
- k_T -factorization [Gribov *et. al.* 1983; Collins *et. al.* 1991; Catani *et. al.* 1991] ("Semihard processes" or "high-energy factorization"):
 - $|\mathbf{q}_{T1}| \sim \mu_F \ll \sqrt{S}$, $Y \gg 1 \Rightarrow q_1^+ \gg q_1^-$,
 - q_1^- - integrated out $\Rightarrow \Phi(x, \mathbf{q}_T)$,
 - $\alpha_s Y \sim \alpha_s \log(1/x)$ - resummed into unPDF.

Parton Reggeization Approach.

Momentum-flow diagram:



- Collinear factorization:
 - $q_1^+ \gg q_1^-$, $|\mathbf{q}_{T1}| \ll \mu_F$, Y - arbitrary,
 - q^- , \mathbf{q}_{T1} - integrated out $\Rightarrow f(x, \mu_F)$,
 - $\alpha_s \log(\mu_F)$ - resummed into PDF.
- PRA:
 - $|\mathbf{q}_{T1}|/\mu_F$ - arbitrary, Y - arbitrary, combines the TMD and k_T -factorization regions,
 - q_1^- - integrated out $\Rightarrow \Phi(x, \mathbf{q}_T, \mu_F)$,
 - $\alpha_s \log(\mu_F)$, $\alpha_s \log^2(|\mathbf{q}|/\mu_F)$ and $\alpha_s \log(1/x)$ - resummed into the unintegrated PDF.

Gauge-invariant amplitudes for the k_T -factorization.

In QCD, off-shell Green functions are not gauge-invariant, in general, so the separation of the contributions between hard subprocess and unPDF seems to be bad-defined.

The Reggeization of the amplitudes in QCD solves this problem. In present time three main approaches to generate the gauge-invariant amplitudes for k_T -factorization are proposed, which are related with Reggeization in one or another way:

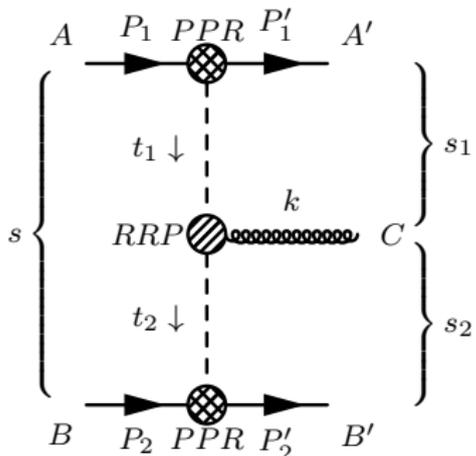
- The parton Reggeization approach (PRA).
- The method based on the extraction of the multi-Regge asymptotic of the amplitudes in the spinor-helicity representation [van Hameren *et. al.*, 2013]. This method is equivalent to the PRA at tree level.

The "classic- k_T -factorization which based on prescription for gluons ($\varepsilon^\mu(q) = \frac{q_T^\mu}{|q_T|}$) is no gauge-invariant. It is formulated only for off-shell gluon originally and conception of off-shell quarks is absent.

Reggeization of amplitudes in QCD.

PRA is based on the Reggeization of amplitudes in gauge theories (QED, QCD, Gravity). The *high energy asymptotic* of the $2 \rightarrow 2 + n$ amplitude is dominated by the diagram with t -channel exchange of the effective (Reggeized) particle and Multi-Regge (MRK) or Quasi-Multi-Regge Kinematics (QMRK) of final state.

In the limit $s \rightarrow \infty$, $s_{1,2} \rightarrow \infty$, $-t_1 \ll s_1$, $-t_2 \ll s_2$ (MRK limit), $2 \rightarrow 3$ amplitude reads:



$$\mathcal{A}_{AB}^{A'B'C} = \gamma_{A'A}^{R_1} \left(\frac{s_1}{s_0} \right)^{\omega(t_1)} \frac{1}{t_1} \times \\ \times \Gamma_{R_1 R_2}^C(q_1, q_2) \times \frac{1}{t_2} \left(\frac{s_2}{s_0} \right)^{\omega(t_2)} \gamma_{B'B}^{R_2}$$

$\Gamma_{R_1 R_2}^C(q_1, q_2)$ - RRP effective production vertex,

$\gamma_{A'A}^R$ - PPR effective scattering vertex,

$\omega(t)$ - Regge trajectory.

Three approaches to obtain this asymptotic:

- Direct study of the MRK limit of the amplitudes.
- BFKL-approach (Unitarity, renormalizability and gauge invariance), see e. g. [Ioffe, Fadin, Lipatov, 2010].
- Effective action approach [Lipatov, 1995].

Feynman rules. Quarks, gluons and photons.

Feynman Rules for Reggeized gluons [Antonov, Cherednikov, Kuraev, Lipatov, 2005]

Feynman Rules for Reggeized quarks [Lipatov, Vyazovsky, 2001]

"ReggeQCD" model for FeynArts and FormCalc calculations [Nefedov, 2016]

Initial state factors:

$$\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \rightarrow q \end{array} \begin{array}{c} \pm \\ \pm \end{array} = \frac{q^\pm}{2\sqrt{-q^2}},$$

$$\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \rightarrow q \end{array} \begin{array}{c} \pm \\ \pm \end{array} = u(q^\parallel).$$

Propagators ($\hat{P}_\pm = \frac{1}{4}\hat{n}^\mp \hat{n}^\pm$):

$$\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \rightarrow q \end{array} \begin{array}{c} \pm \\ \pm \end{array} = \hat{P}_\pm \frac{i\hat{q}}{q^2},$$

$$\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \rightarrow q \end{array} \begin{array}{c} \pm \\ \pm \end{array} = \frac{i\hat{q}}{q^2} \hat{P}_\pm.$$

$$\begin{array}{c} \downarrow \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \downarrow \end{array} \begin{array}{c} \pm \\ \mp \end{array} = -ig_s T^a \hat{n}^\pm,$$

$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ q_{21} \downarrow \mp \end{array} p = -ig_s T^a \left(\hat{n}^\pm + 2 \frac{\hat{q}_1}{q_2^\mp} \right),$$

$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ q_{21} \downarrow \mp \end{array} p = -2ie g_s T^a \frac{\hat{q}_1 n_\mu^\mp}{p^\mp q_2^\mp},$$

$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ q_2 \downarrow \mp \end{array} p = -ie \left(\gamma_\mu + \hat{q}_1 \frac{n_\mu^\mp}{p^\mp} + \hat{q}_2 \frac{n_\mu^\pm}{p^\pm} \right),$$

$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ \downarrow \end{array} p = -ie \left(\gamma_\mu + \hat{q}_1 \frac{n_\mu^\mp}{p^\mp} \right),$$

$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ \downarrow \end{array} \begin{array}{c} p_2 \\ p_1 \end{array} = -ie^2 \hat{q}_1 \frac{n_{\mu_1}^\mp n_{\mu_2}^\mp}{p_1^\mp p_2^\mp}.$$

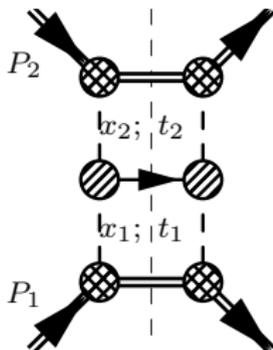
$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ q_2 \downarrow \mp \end{array} \begin{array}{c} p_2 \\ p_1 \end{array} = ie^2 \left(\hat{q}_2 \frac{n_{\mu_1}^\pm n_{\mu_2}^\pm}{p_1^\pm p_2^\pm} - \hat{q}_1 \frac{n_{\mu_1}^\mp n_{\mu_2}^\mp}{p_1^\mp p_2^\mp} \right),$$

$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ q_2 \downarrow \mp \end{array} \begin{array}{c} p_3 \\ p_2 \\ p_1 \end{array} = -ie^3 \left(\hat{q}_2 \frac{n_{\mu_1}^\pm n_{\mu_2}^\pm n_{\mu_3}^\pm}{p_1^\pm p_2^\pm p_3^\pm} + \hat{q}_1 \frac{n_{\mu_1}^\mp n_{\mu_2}^\mp n_{\mu_3}^\mp}{p_1^\mp p_2^\mp p_3^\mp} \right),$$

$$\begin{array}{c} q_1 \downarrow \pm \\ \bullet \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \bullet \\ q_2 \downarrow \mp \end{array} \begin{array}{c} p_2 \\ p_1 \end{array} = -2ie^2 g_s T^a \frac{\hat{q}_1 n_{\mu_1}^\mp n_{\mu_2}^\mp}{p_1^\mp p_2^\mp q_2^\mp}.$$

Factorization of the cross-section.

Factorization:



Collinear limit holds for the amplitude:

$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2} \rightarrow 0} |\mathcal{M}|^2_{PRA} = |\mathcal{M}|^2_{CPM}$$

 k_T -factorization formula:

$$d\sigma = \int \frac{d^2 \mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \times \\ \times \int \frac{d^2 \mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}$$

Where Φ - Unintegrated PDFs. The factorization is known to hold in the LLA ($\alpha_s \log(1/x)$) [BFKL, 1978], and NLLA ($\alpha_s^2 \log(1/x)$) [Fadin, Lipatov, 1998; Camici, Ciafaloni, 1998; Bartels, *et. al.*, 2006].

Normalization of the unPDF in KMR model:

$$\int^{\mu^2} dt \Phi(x, t, \mu^2) = xf(x, \mu^2),$$

where $f(x, \mu^2)$ - collinear PDF.

The Kimber-Martin-Ryskin unPDF.

In the present numerical computations we use the KMR unPDF from [M. Kimber, A. Martin, and M. Ryskin, 2000].

KMR prescription to obtain unintegrated PDF from collinear one is based on the mechanism of last step parton k_T -dependent radiation and the assumption of strong angular ordering:

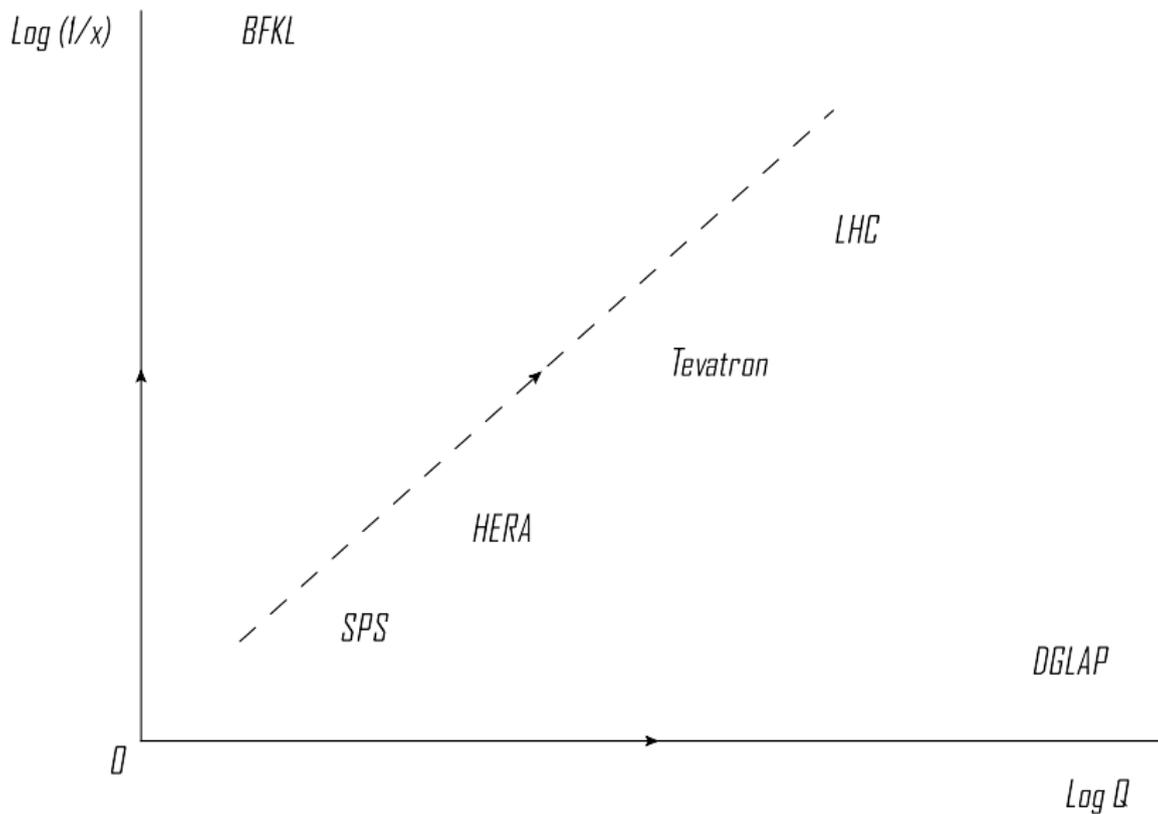
$$k_T^2 \Phi_q(x, k_T^2, \mu^2) = \frac{\alpha_s(k_T^2)}{(2\pi)} T_q(k_T^2, \mu^2) \int_x^{1-\Delta} dz \left[P_{qg}(z) f_g\left(\frac{x}{z}, q^2\right) + P_{qq}(z) f_q\left(\frac{x}{z}, q^2\right) \right],$$

where $P_{qg}(z)$, $P_{qq}(z)$ - LO DGLAP splitting functions, $T_q(k^2, \mu^2)$ - Sudakov formfactor:

$$T_q(k_T^2, \mu^2) = \exp \left\{ - \int_{k_T^2}^{\mu^2} \frac{dq_T^2}{q_T^2} \frac{\alpha_s(q_T^2)}{2\pi} \sum_{a'} \int_0^{1-\Delta} P_{qa'}(z') dz' \right\}$$

where $\Delta = \frac{k_T}{\mu + k_T}$ ensures the **rapidity ordering of the last emission and particles produced in the hard subprocess.**

BFKL versus DGLAP



Selected results in LO PRA

- Single jet and prompt photon production
 - V. A. Saleev, “Prompt photon photoproduction at HERA within the framework of the quark Reggeization hypothesis,” *Phys. Rev. D* **78**, 114031 (2008); “Deep inelastic scattering and prompt photon production within the framework of quark Reggeization hypothesis,” *Phys. Rev. D* **78**, 034033 (2008).
 - B. A. Kniehl, V. A. Saleev, A. V. Shipilova and E. V. Yatsenko, “Single jet and prompt-photon inclusive production with multi-Regge kinematics: From Tevatron to LHC,” *Phys. Rev. D* **84**, 074017 (2011).

Single jet production in PRA via Fadin-Kuraev-Lipatov effective vertex (1974 !!):

$$R + R \rightarrow g$$

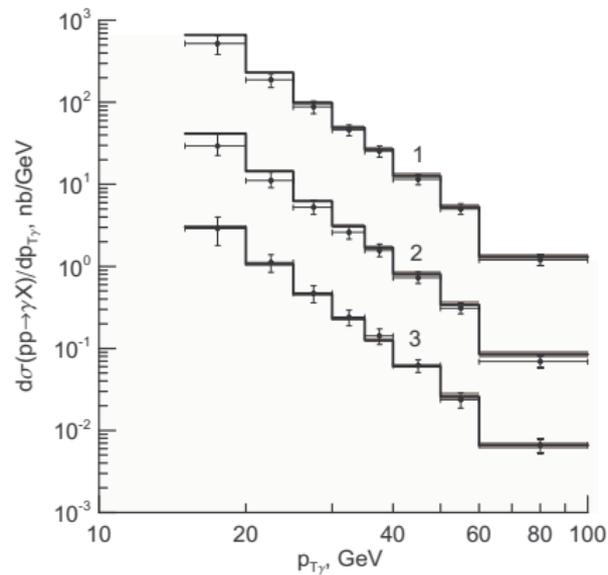
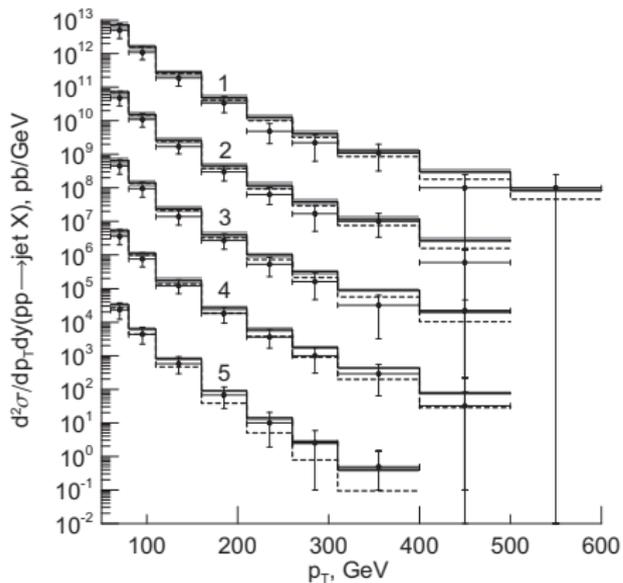
Single prompt photon production in PRA via Fadin-Sherman effective vertex (1976 !!):

$$Q + \bar{Q} \rightarrow g(\gamma)$$

Because $\vec{p}_T = \vec{q}_{1T} + \vec{q}_{2T}$ and $|M(RR \rightarrow g, Q\bar{Q} \rightarrow g)|^2 \sim \alpha_S p_T^2$, one has

$$\frac{d\sigma(pp \rightarrow jet + X)}{dp_T} \sim \Phi_g(x_1, q_{1T}^2, \mu^2) \otimes \Phi_g(x_2, q_{2T}^2, \mu^2)$$

Single jet and prompt photon at the LHC.

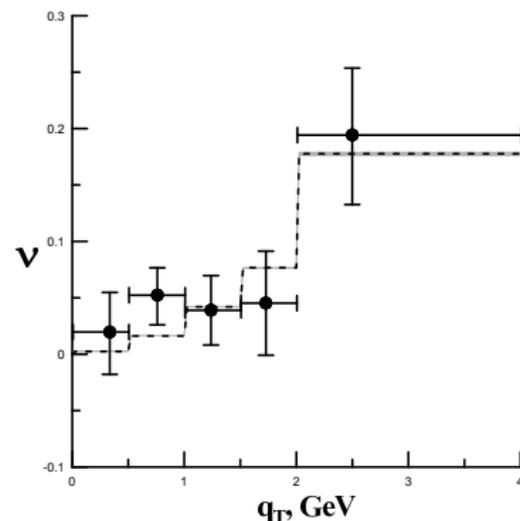
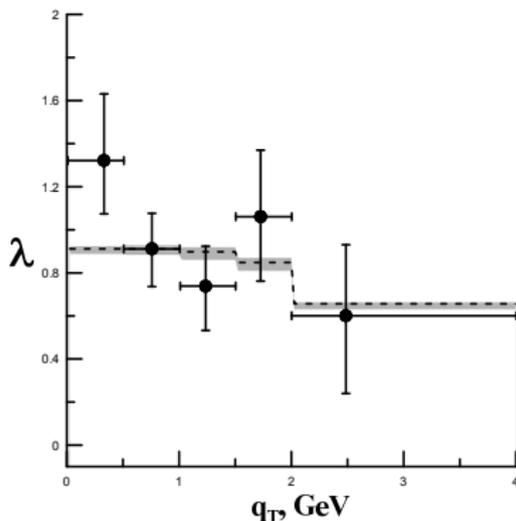


Selected results in LO PRA

- Dijet and jet+gamma associated production
 - M. A. Nefedov, V. A. Saleev and A. V. Shipilova, “Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach,” [Phys. Rev. D **87** \(2013\) no.9, 094030](#)
 - B. A. Kniehl, M. A. Nefedov and V. A. Saleev, “Prompt-photon plus jet associated photoproduction at HERA in the parton Reggeization approach,” [Phys. Rev. D **89**, no. 11, 114016 \(2014\)](#)

- DY pair production

- M. A. Nefedov, N. N. Nikolaev and V. A. Saleev, “Drell-Yan lepton pair production at high energies in the Parton Reggeization Approach,” *Phys. Rev. D* **87**, no. 1, 014022 (2013).



Angular coefficients λ and ν as function of q_T . The data are from NuSea

Collaboration: $4.5 < Q < 15$ GeV, $0 < q_T < 4$ GeV, $0 < x_F < 0.8$, $\sqrt{S} = 39$ GeV.

Heavy quarkonium production in PRA

It has been presented in report by M. Nefedov, 26.07.2016

Inclusive $D(B)$ -meson production at Tevatron and the LHC

- A. V. Karpishkov, M. A. Nefedov, V. A. Saleev and A. V. Shipilova, “B-meson production in the Parton Reggeization Approach at Tevatron and the LHC,” *Int. J. Mod. Phys. A* **30**, no. 04n05, 1550023 (2015) [arXiv:1411.7672 [hep-ph]].
- A. V. Karpishkov, M. A. Nefedov, V. A. Saleev and A. V. Shipilova, “Open charm production in the parton Reggeization approach: Tevatron and the LHC,” *Phys. Rev. D* **91**, no. 5, 054009 (2015)
- B. A. Kniehl, A. V. Shipilova and V. A. Saleev, “Open charm production at high energies and the quark Reggeization hypothesis,” *Phys. Rev. D* **79**, 034007 (2009) [arXiv:0812.3376 [hep-ph]].

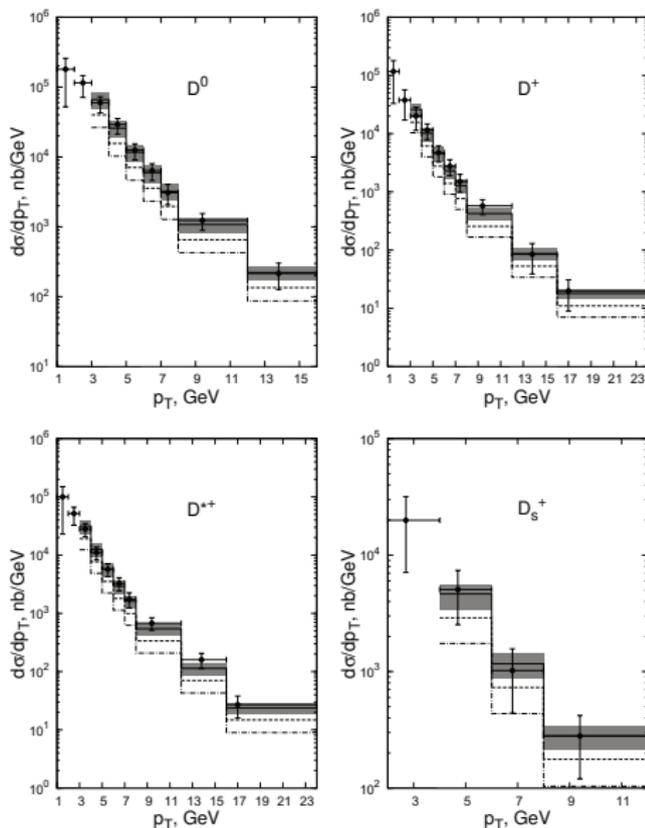
$D(B)$ -meson production: PRA + Fragmentation Function Model

- $R + R \rightarrow g$ with $g \rightarrow D(B)$ and $R + R \rightarrow c(b) + \bar{c}(\bar{b})$ with $c(b) \rightarrow D(B)$
- Hadronization is calculated with the help of the **scale-dependent fragmentation functions** of T. Kneesch, B. A. Kniehl, G. Kramer and I. Schienbein, Nucl. Phys. B **799**, 34 (2008); KKKS08-package:
<http://laph.cnrs.fr/ffgenerator/>

Direct Fragmentation	CPM	$\sim \alpha_S^2$	PRA	$\sim \alpha_S^2$
	$g + g \rightarrow c + \bar{c}$		$R + R \rightarrow c + \bar{c}$	
	$g + g \rightarrow g + g$	$\sim \alpha_S^3 \log(p_T/m)$	$R + R \rightarrow g$	$\sim \alpha_S^2 \log(p_T/m)$

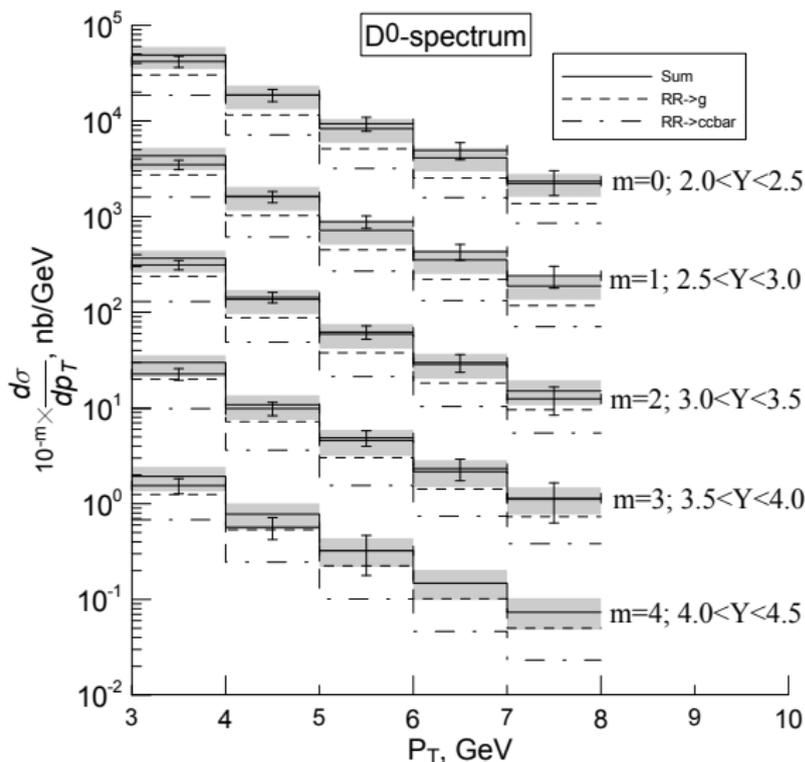
It was shown that at high p_T region the gluon into the final heavy meson fragmentation in $R + R \rightarrow g$ with $g \rightarrow D(B)$ is dominating production mechanism instead of heavy quark fragmentation in $R + R \rightarrow c(b) + \bar{c}(\bar{b})$ with $c(b) \rightarrow D(B)$

D meson production at the LHC (central)



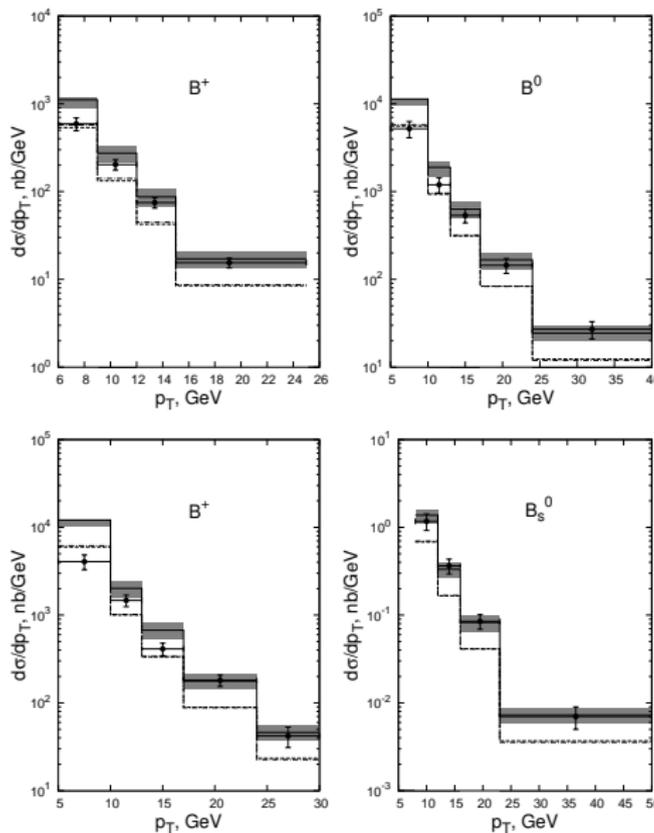
Transverse momentum distributions of D -meson production at the LHC (ALICE), $\sqrt{S} = 7.0$ TeV. Dashed line represents the contribution of gluon fragmentation, dash-dotted line – the c -quark fragmentation contribution, solid line is their sum.

D meson production at the LHC (forward)



Transverse momentum distributions of D -meson production at the LHC (LHCb), $\sqrt{S} = 7.0$ TeV. Dashed line represents the contribution of gluon fragmentation, dash-dotted line – the c -quark fragmentation contribution, solid line is their sum.

B meson production at the Tevatron and LHC.



Transverse momentum distributions of B^+ -meson production at Tevatron (CDF), $\sqrt{S} = 1.96$ TeV (left-top); B^0 (right-top), B^+ (left-bottom), and B_s^0 (right-bottom) mesons at LHC (CMS), $\sqrt{S} = 7$ TeV. Dashed line represents the contribution of gluon fragmentation, dash-dotted line – the b -quark fragmentation contribution, solid line is their sum.

b -jet production at Tevatron and the LHC

- V. A. Saleev and A. V. Shipilova, “Inclusive b -jet and $b\bar{b}$ -dijet production at the LHC via Reggeized gluons,” *Phys. Rev. D* **86**, 034032 (2012) [[arXiv:1201.4640 \[hep-ph\]](#)].
- B. A. Kniehl, V. A. Saleev and A. V. Shipilova, “Inclusive b and b anti- b production with quasi-multi-Regge kinematics at the Tevatron,” *Phys. Rev. D* **81**, 094010 (2010) [[arXiv:1003.0346 \[hep-ph\]](#)].

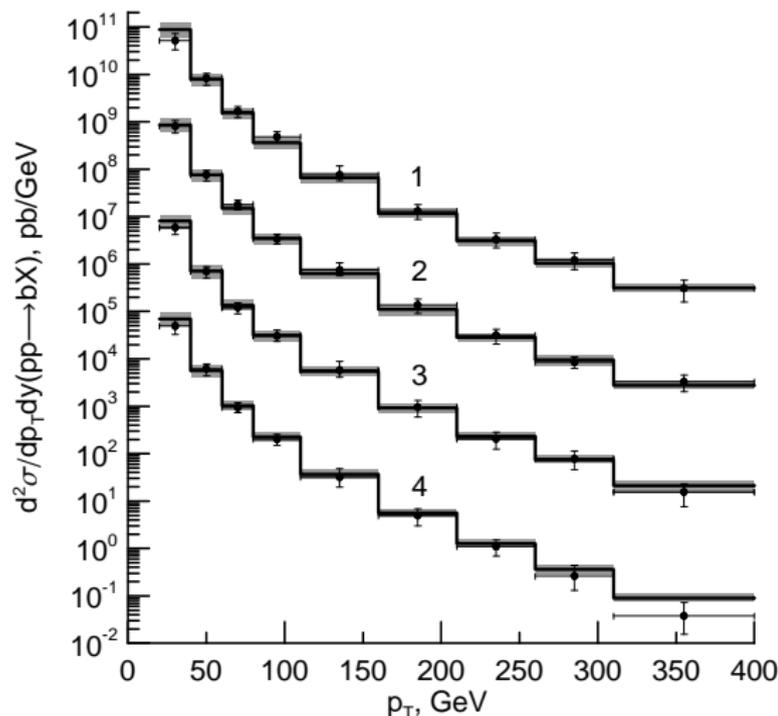
$$R + R \rightarrow b + \bar{b} \quad , \quad R + R \rightarrow g \rightarrow b\bar{b}$$

$$\frac{d\hat{\sigma}^{frag}}{dp_T} = \frac{d\hat{\sigma}^g}{dp_T} n_g(\mu), \quad n_g \sim \alpha_S(\mu) \ln \frac{\mu}{m_b}, \quad \mu \sim p_T$$

$$n_g(\mu) = \int_0^1 D_{g \rightarrow b}(z, \mu) dz$$

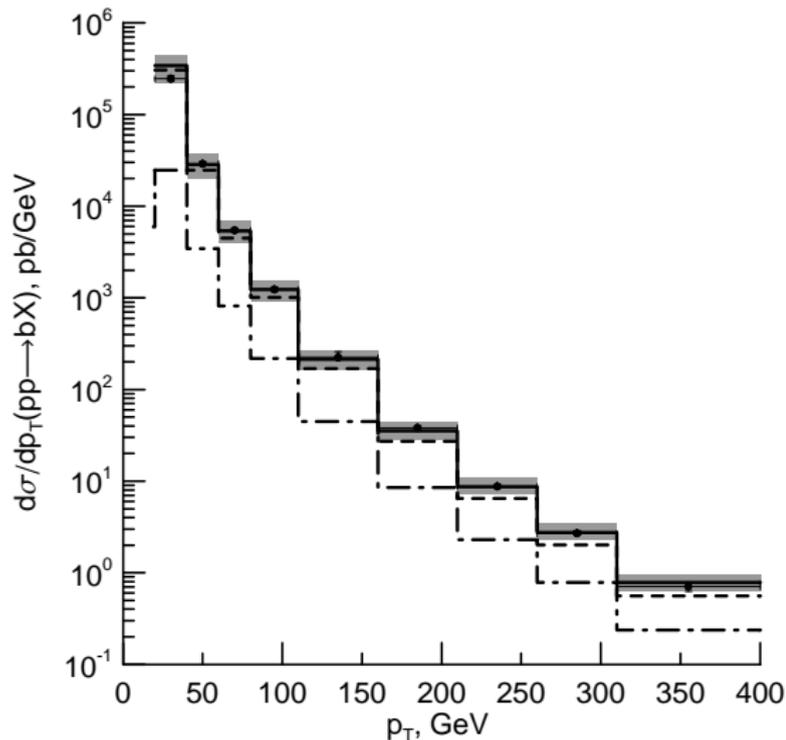
Inclusive b -jet production at the LHC

Inclusive double-differential b -jet cross-sections as a functions of p_T for the different rapidity ranges:
 (1) $|y| < 0.3$ ($\times 10^6$),
 (2) $0.3 < |y| < 0.8$
 ($\times 10^4$), (3)
 $0.8 < |y| < 1.2$ ($\times 10^2$)
 and (4) $1.2 < |y| < 2.1$.
 The data are from ATLAS Collaboration



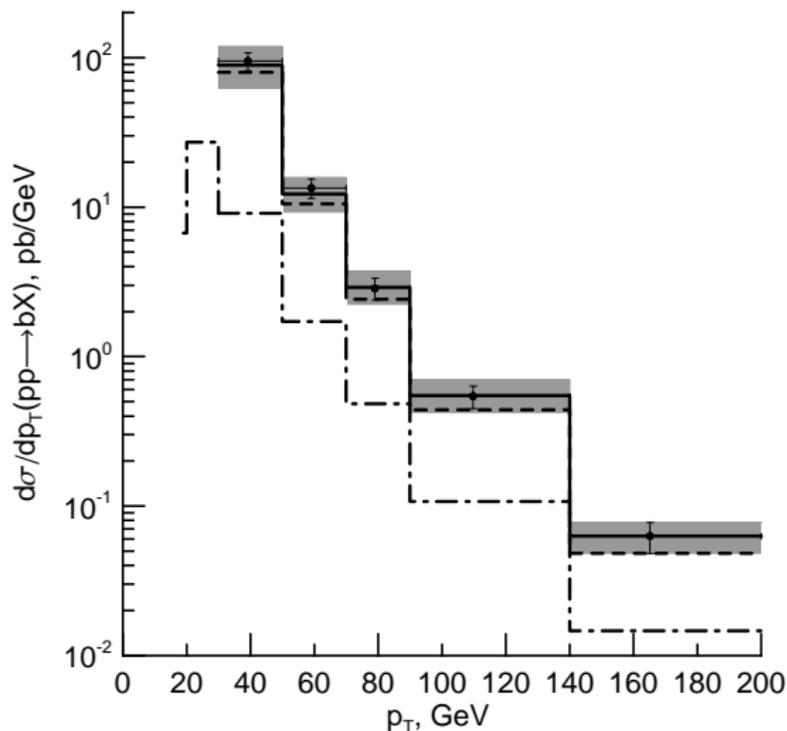
Inclusive b -jet production at the LHC

Inclusive differential b -jet cross-section as a function of p_T for b -jets with $|y| < 2.1$. The data are from ATLAS Collaboration. The dashed polyline corresponds to contribution of the open b -quark production, the dashed-dotted one – the gluon-to-bottom-pair fragmentation, the solid – sum of their all.



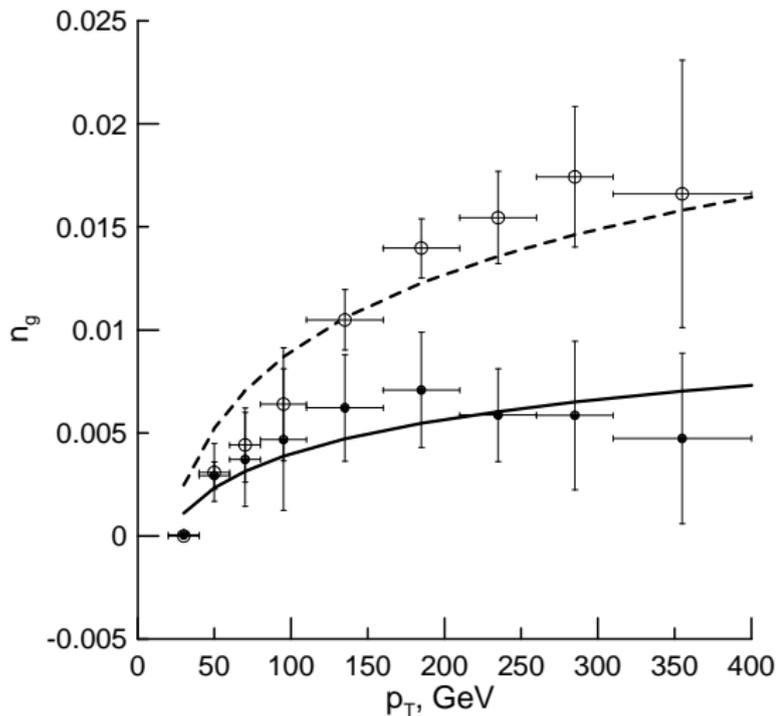
Inclusive b -jet production at Tevatron

Inclusive differential b -jet cross-section as a function of p_T for b -jets with $|y| < 2.0$. The data are from CMS Collaboration. The dashed polyline corresponds to contribution of the open b -quark production, the dashed-dotted one – the gluon-to-bottom-pair fragmentation, the solid – sum of their all.



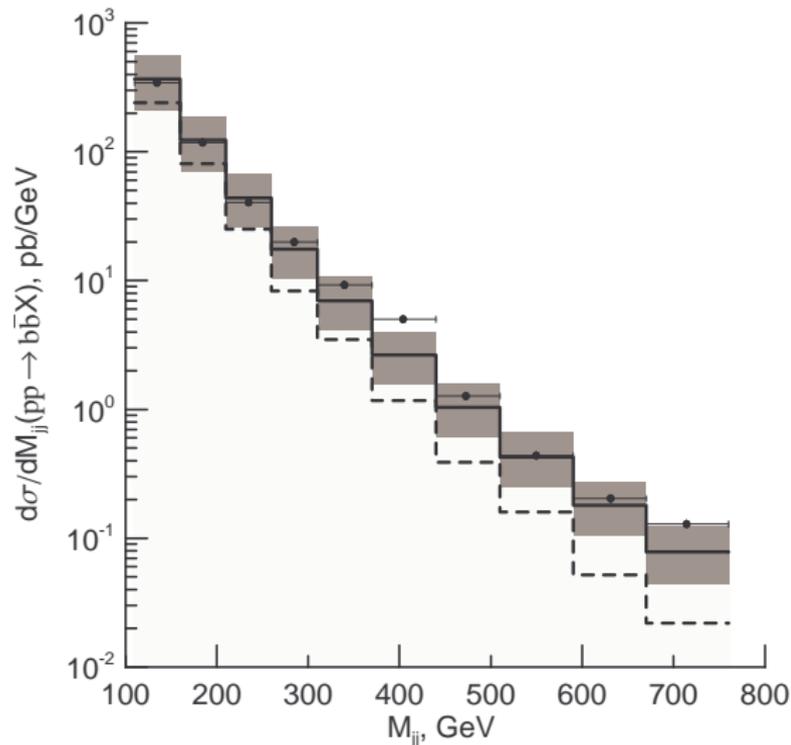
b -quark multiplicity in a gluon jet.

The $b\bar{b}$ -pair multiplicity n_g in a gluon jet as a function of p_T extracted from the ATLAS data for the inclusive b -jet production spectra. The open circles and dashed fitting line correspond to Blümlein unintegrated PDF, the black circles and solid fitting line correspond to KMR unintegrated PDF.



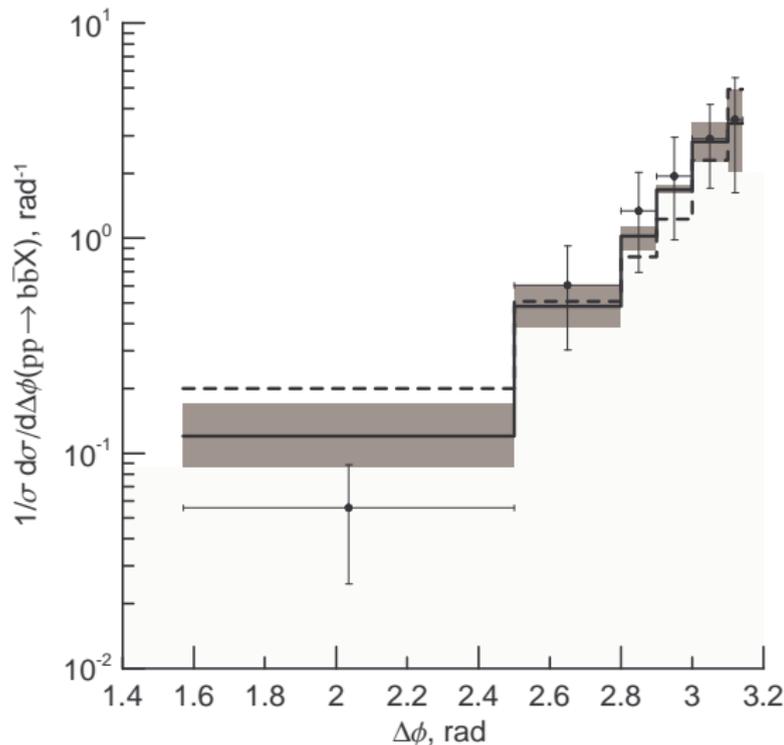
$b\bar{b}$ -dijet production at the LHC.

The $b\bar{b}$ -dijet cross-section as a function of dijet invariant mass M_{jj} for b -jets with $p_T > 40$ GeV, $|y| < 2.1$. The data are from ATLAS Collaboration. The solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF (BFKL based)



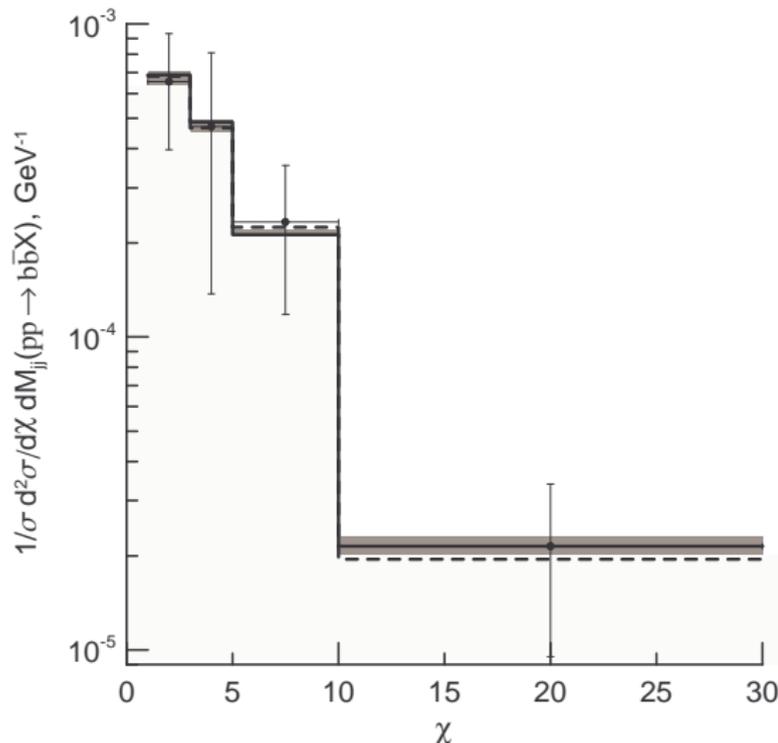
$b\bar{b}$ -dijet production at the LHC.

The $b\bar{b}$ -dijet cross-section as a function of the azimuthal angle difference between the two jets for b -jets with $p_T > 40$ GeV, $|y| < 2.1$ and a dijet invariant mass of $M_{jj} < 110$ GeV. The data are from ATLAS Collaboration, the solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF



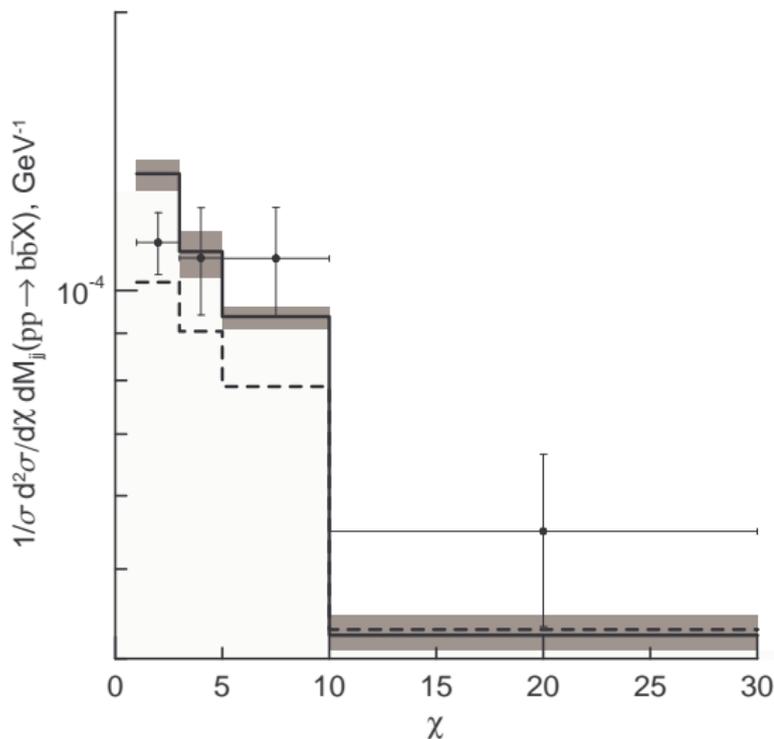
$b\bar{b}$ -dijet production at the LHC.

The $b\bar{b}$ -dijet cross-section as a function of $\chi = \exp |y_1 - y_2|$ for b -jets with $p_T > 40$ GeV, $|y| < 2.1$ and $|y_{boost}| = \frac{1}{2}|y_1 + y_2| < 1.1$, for dijet invariant mass range $110 < M_{jj} < 370$ GeV. The data are from ATLAS Collaboration, the solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF



$b\bar{b}$ -dijet production at the LHC.

The $b\bar{b}$ -dijet cross-section as a function of $\chi = \exp |y_1 - y_2|$ for b -jets with $p_T > 40$ GeV, $|y| < 2.1$ and $|y_{boost}| = \frac{1}{2}|y_1 + y_2| < 1.1$, for dijet invariant mass range $370 < M_{jj} < 850$ GeV. The data are from ATLAS Collaboration, the solid polyline corresponds to KMR unintegrated PDF, the dashed one — to Blümlein PDF.



$D\bar{D}$ pair production at the LHCb

In PRA, we include $2 \rightarrow 2$ subprocesses:

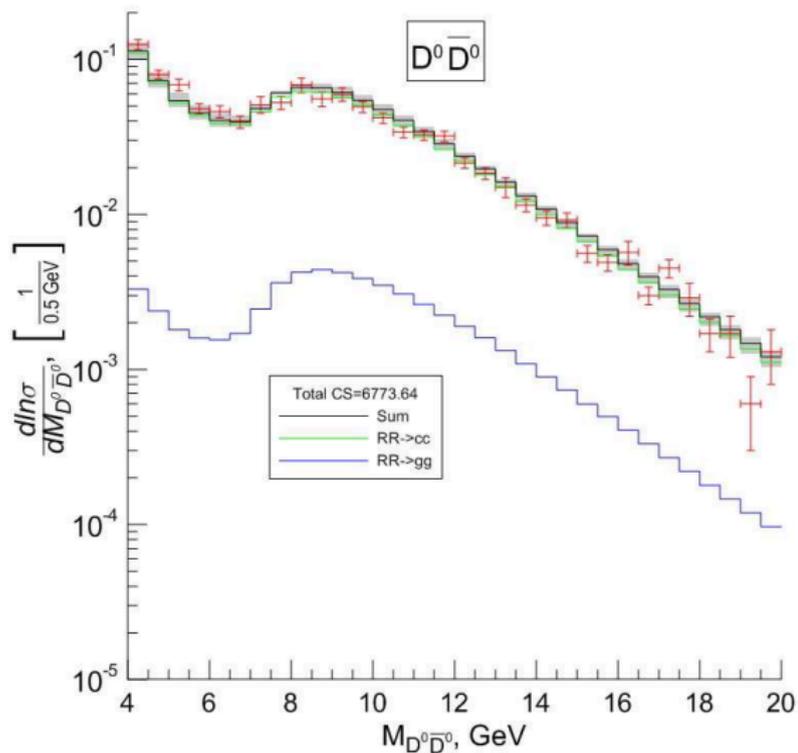
$$R + R \rightarrow c(D) + \bar{c}(\bar{D}), \quad \gg \quad R + R \rightarrow g(D) + g(\bar{D})$$

In CPM, we should start with $2 \rightarrow 3$ subprocesses and include $2 \rightarrow 4$ subprocesses:

$$g + g \rightarrow c + \bar{c} + g, \quad g + g \rightarrow b + \bar{b} + g + g, \quad \dots$$

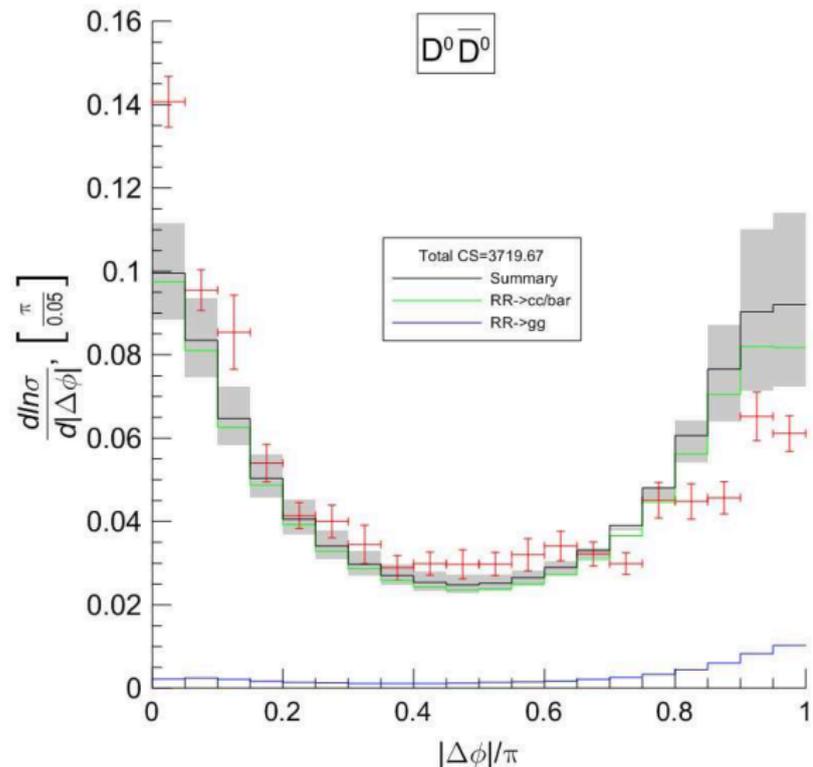
$D\bar{D}$ pair production at the LHCb

$D\bar{D}$ -pair invariant mass spectrum. The data are from LHCb Collaboration.



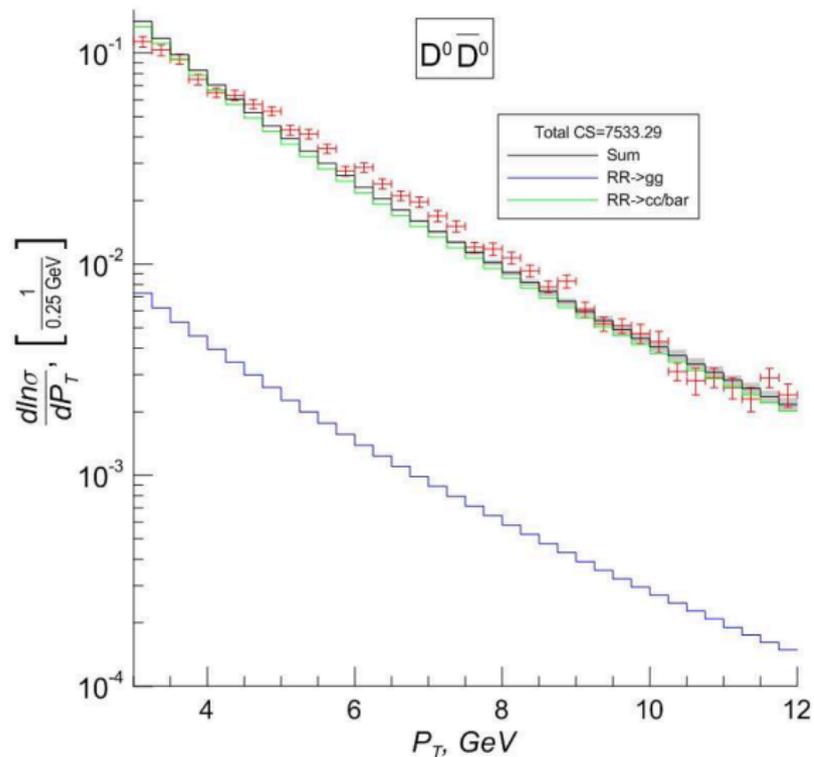
$D\bar{D}$ pair production at the LHCb

$D\bar{D}$ -pair azimuthal angle difference spectrum. The data are from LHCb Collaboration.



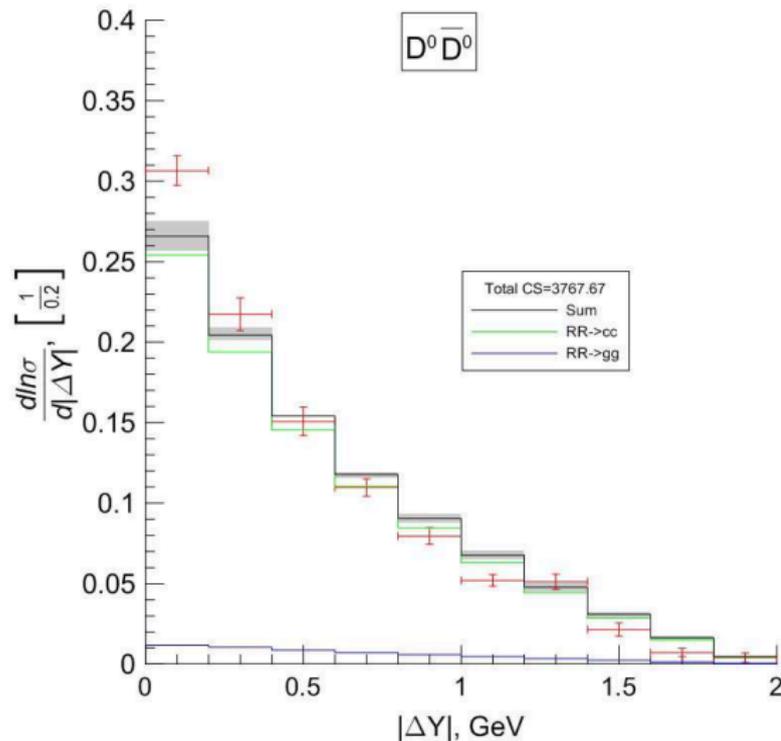
$D\bar{D}$ pair production at the LHCb

$D\bar{D}$ -pair transverse momentum difference spectrum. The data are from LHCb Collaboration.



$D\bar{D}$ pair production at the LHCb

$D\bar{D}$ -pair rapidity difference spectrum. The data are from LHCb Collaboration.



DD pair production at the LHCb

R. Maciula, V. A. Saleev, A. V. Shipilova and A. Szczurek, “**New mechanisms for double charmed meson production at the LHCb**,” *Phys. Lett. B* **758** (2016) 458

In CPM or k_T -factorization, the LO contribution is originated from $2 \rightarrow 4$ subprocess:

$$g + g \rightarrow c + \bar{c} + c + \bar{c}, \quad R + R \rightarrow c + \bar{c} + c + \bar{c}$$

Here, we suggest that $c \rightarrow D$ transition is described using Peterson fragmentation function $D_c(z)$.

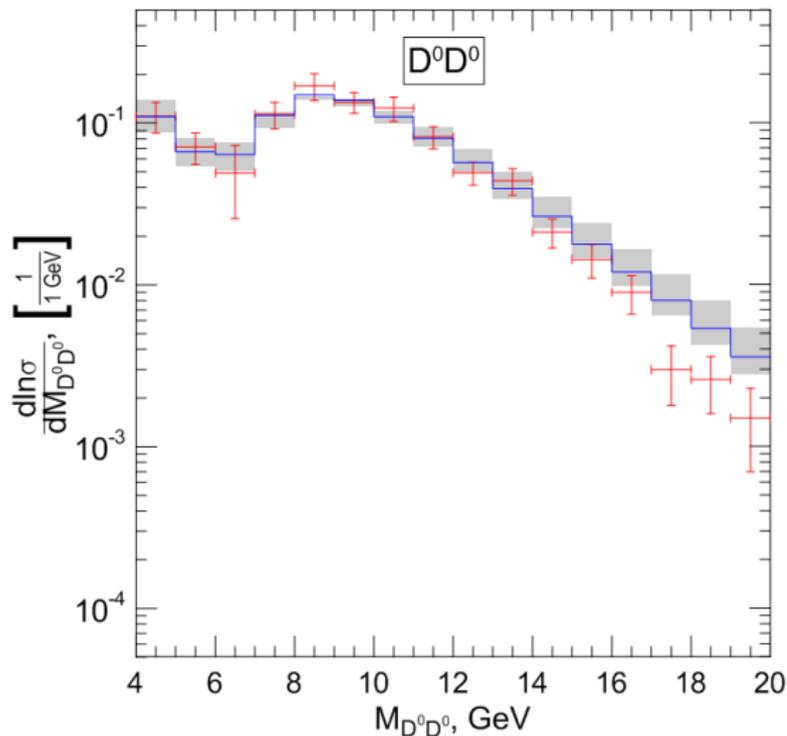
In model with scaled dependent fragmentation functions (KKKS08) we should include following subprocesses:

$$g + g \rightarrow c(D) + \bar{c} + g(D), g(D) + g(D) + g, \dots$$

$$R + R \rightarrow g(D) + g(D)$$

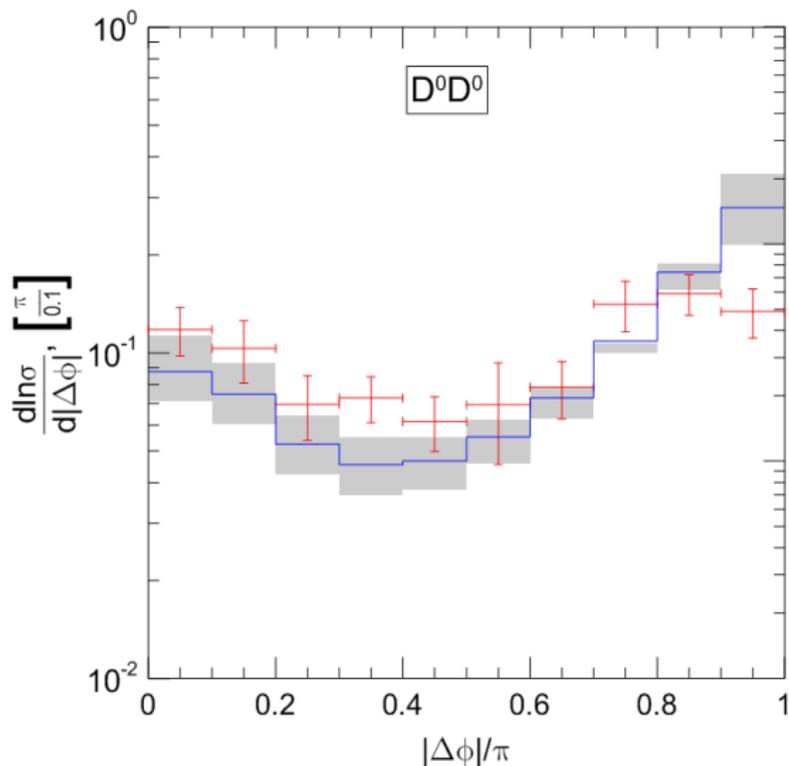
DD pair production at the LHCb

DD -pair invariant mass spectrum. The data are from LHCb Collaboration.



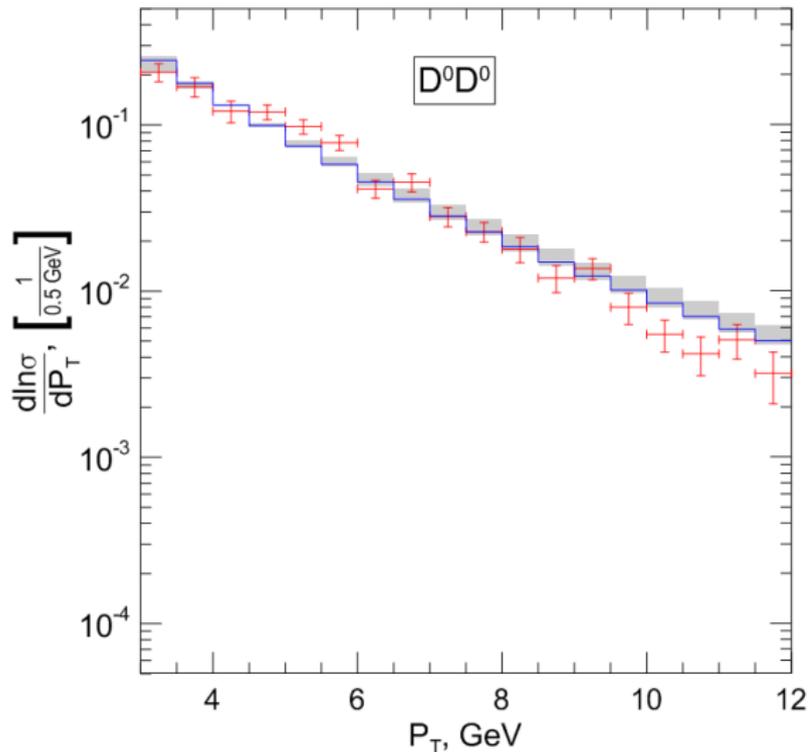
DD pair production at the LHCb

DD -pair azimuthal angle difference spectrum. The data are from LHCb Collaboration.



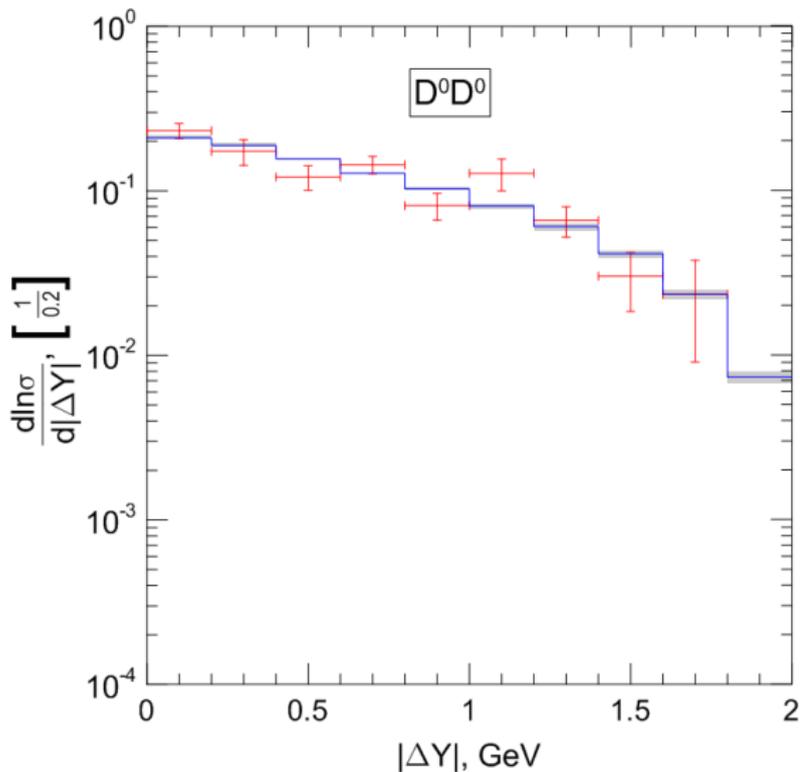
DD pair production at the LHCb

DD -pair transverse momentum difference spectrum. The data are from LHCb Collaboration.



DD pair production at the LHCb

DD -pair rapidity difference spectrum. The data are from LHCb Collaboration.



DD pair production in SPS and DPS

$$\sigma^{DPS}(pp \rightarrow DDX) = \frac{1}{2\sigma_{eff}} \sigma^{SPS}(pp \rightarrow DX) \cdot \sigma^{SPS}(pp \rightarrow DX)$$

$$\sigma_{eff} \simeq 15 \text{ mb} \quad (!!)$$

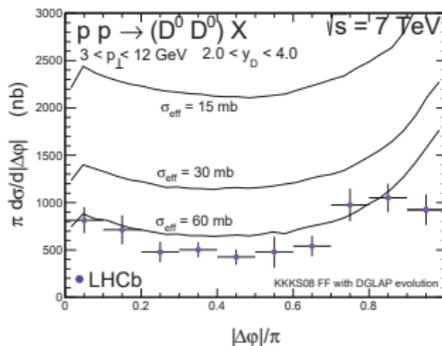
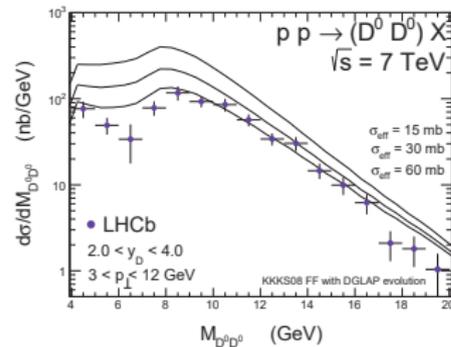
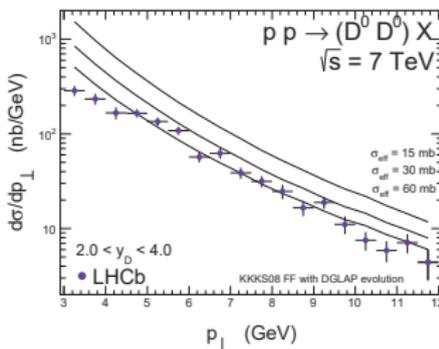
This value of σ_{eff} is extracted from comparison of data for different two-particle correlations and LO (mostly) and NLO calculation in CPM.

We perform calculation in LO PRA taking into account following partonic subprocesses,

For DPS: $R + R \rightarrow g(D)$ and $R + R \rightarrow c(D) + \bar{c}$

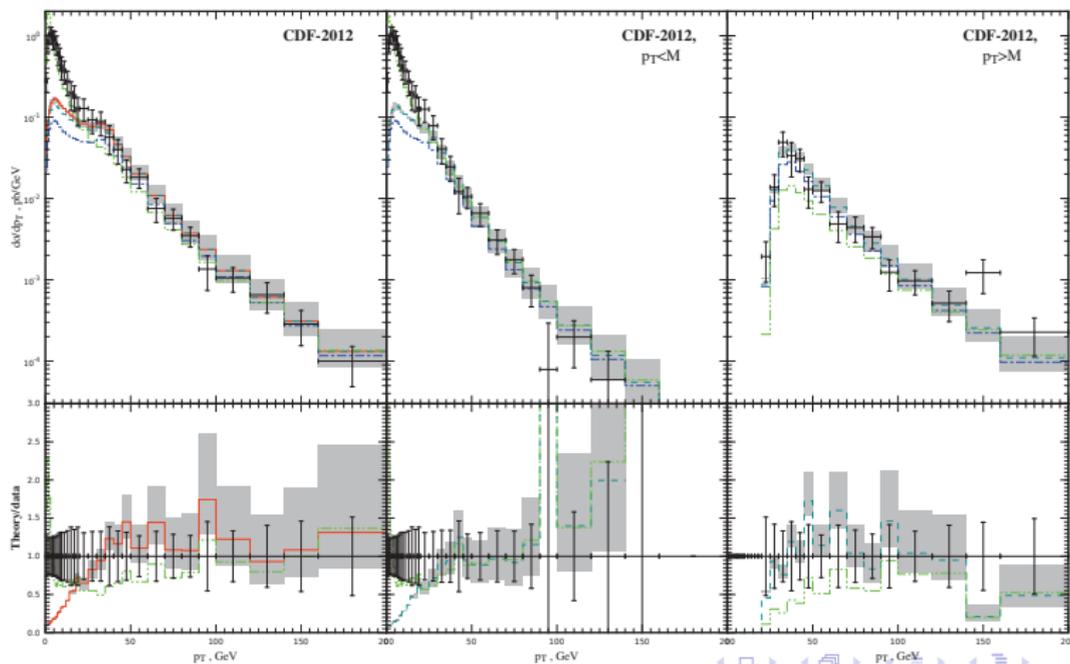
For SPS: $R + R \rightarrow g(D) + g(D)$

DD pair production in SPS and DPS



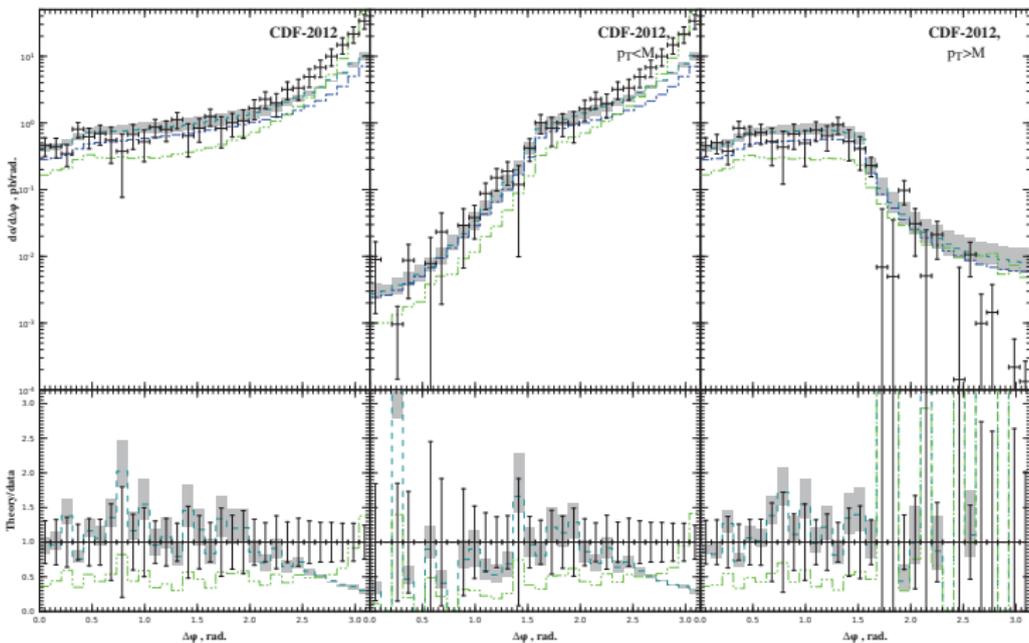
Remarks on NLO PRA

- Diphoton production in NLO* PRA
 - M. Nefedov and V. Saleev, "Diphoton production at the Tevatron and the LHC in the NLO approximation of the parton Reggeization approach," *Phys. Rev. D* **92** (2015) no.9, 094033



Remarks on NLO PRA

- Diphoton production in NLO* PRA



Conclusions

Conclusions

- PRA is hybrid approach based on high-energy factorization and Lipatov's effective theory of Reggeized partons
- LO PRA describe most part of inclusive data for heavy quark production (open charm and beauty production, b-jets, heavy quarkonia, ...)
- LO PRA describe two-particle correlations ($b\bar{b}$, $D\bar{D}$, DD)
- LO PRA can be proved to NLO PRA consistently and calculation of NLO corrections for heavy quark production look as good test of this program

Thank you for your attention!