Covariant confined quark model and its application to heavy quark physics

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Contents

The study of heavy flavor physics: motivation

- **To determine the Cabibbo-Kobayashi-Maskawa matrix elements.**
- **To provide insights into the origin of flavor and CP-violation.**
- **To look for new physics beyond the standard model.**
- **The subject to study are heavy hadrons containing a b– or a c–quark and their weak decays.**

The study of heavy flavor physics: motivation

- **To determine the Cabibbo-Kobayashi-Maskawa matrix elements.**
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- **To look for new physics beyond the standard model.**
- **The subject to study are heavy hadrons containing a b– or a c–quark and their weak decays.**
- **The main idea in the theoretical studies of heavy-flavor decays is to separate short-distance (perturbative) QCD dynamics from long-distance (nonperturbative) hadronic effects.**
- **This task is accomplished by using the operator product expansion (OPE), renormalization group (RG) and matching the full theory to the effective one.**
- **The amplitude of some weak hadron processes is factorized into the Wilson coefficients and matrix elements of local operators.**
- **The Wilson coefficients characterize the short-distance dynamics and may be reliably evaluated by perturbative methods.**

The study of heavy flavor physics: motivation

- **The calculation of the hadronic matrix elements of local operators between initial and final states require nonperturbative methods. One needs to know how hadrons are constructed from quarks.**
- **Technically, any matrix element of a local operator may be expressed in terms of a set of scalar functions which are referred to as form factors.**
- **A variety of theoretical approaches have been used to evaluate the hadronic form factors:**

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- **The calculation of the hadronic matrix elements of local operators between initial and final states require nonperturbative methods. One needs to know how hadrons are constructed from quarks.**

- **Technically, any matrix element of a local operator may be expressed in terms of a set of scalar functions which are referred to as form factors.**

- **A variety of theoretical approaches have been used to evaluate the hadronic form factors:**

  - **The light-cone sum rule (LCSR) approach (Braun, Ball, Khodjamirian et al.)**

  - **Dyson-Schwinger equations in QCD (C.D. Roberts et al.)**

  - **A relativistic quark model (Faustov, Galkin et al.)**

  - **The constituent quark model with dispersion relations (Melikhov et al.)**

  - **A QCD relativistic potential model (Ladisa, et al.)**

  - **A QCD sum rule analysis (P. Colangelo et al.)**

Semileptonic flavor changing charged current decays

- $B \to D(D^*)\ell\bar{\nu}, \qquad \Lambda_b \to \Lambda_c\, \ell\bar{\nu}$

$$\text{matrix element} = \frac{G_F V_{bc}}{\sqrt{2}} \langle \text{out}|\bar{c}\Gamma_h b|\text{in}\rangle \langle \bar{\ell}\Gamma_\ell \nu\rangle$$

$$\text{SM:} \qquad \Gamma_h \times \Gamma_\ell = O^\mu \times O_\mu$$

- $O^\mu = \gamma^\mu(1-\gamma_5)$ is V-A weak matrix.
- If $D(D^*)$ and Λ_c are on mass-shell then two-fold distribution $(q^2, \cos\theta)$
- Cascade decays if $D^* \to D\pi$ and $\Lambda_c \to p\bar{K}^0$ then four-fold distribution $(q^2, \cos\theta, \cos\theta^*, \chi)$

Rare flavor changing neutral current decays

- $B \to K(K^*)\ell^+\ell^-, \qquad B_s \to \phi\ell^+\ell^-, \qquad \Lambda_b \to \Lambda\,\ell^+\ell^-$

$$\text{matrix element} = \frac{G_F V_{ts} V_{tb}^\dagger}{\sqrt{2}} \sum C_W \langle out|\bar{s}\Gamma_h b|in\rangle \langle \bar{\ell}\Gamma_\ell \ell\rangle$$

$$\text{SM:} \qquad \Gamma_h \times \Gamma_\ell = O^\mu \times \gamma_\mu, \qquad O^\mu \times \gamma_\mu\gamma_5, \qquad \sigma^{\mu q} \times \gamma_\mu$$

- If $K(K^*), \phi$ and Λ are on mass-shell then two-fold distribution $(q^2, \cos\theta)$
- Cascade decays if $K^* \to K\pi, \phi \to K^+K^-$ and $\Lambda \to p\bar{\pi}^-$ then four-fold distribution $(q^2, \cos\theta, \cos\theta^*, \chi)$

Nonleptonic decays

- $B_s \to D^{(*)}\bar{D}^{(*)}, \qquad B_s \to \phi J/\psi, \qquad \Lambda_b \to \Lambda\, J/\psi$

$$\text{matrix element} = \frac{G_F V_{cb} V_{cs}^{\dagger}}{\sqrt{2}} C_W f_V m_V \langle \text{out}|\bar{s}O^{\mu}b|\text{in}\rangle$$

- Cascade decay

$$\Lambda_b \to \Lambda(\to p\pi^-) + J/\psi(\to \ell^+\ell^-)$$

- Many other modes

Covariant quark model of hadrons

- **Main assumption: hadrons interact via quark exchange only**
- **Interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_H \cdot H(x) \cdot J_H(x)$$

Covariant quark model of hadrons

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$$\mathcal{L}_{\rm int} = g_H \cdot H(x) \cdot J_H(x)$$

- **Quark currents**

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1)\, \Gamma_M \, q^a_{f_2}(x_2) \qquad \text{Meson}$$

$$\begin{aligned} J_B(x) &= \int dx_1 \int dx_2 \int dx_3 \, F_B(x; x_1, x_2, x_3) \qquad \text{Baryon} \\ &\times \Gamma_1 \, q^{a_1}_{f_1}(x_1) \left[\varepsilon^{a_1 a_2 a_3} q^{T\,a_2}_{f_2}(x_2) C \, \Gamma_2 \, q^{a_3}_{f_3}(x_3) \right] \end{aligned}$$

$$\begin{aligned} J_T(x) &= \int dx_1 \ldots \int dx_4 \, F_T(x; x_1, \ldots, x_4) \qquad \text{Tetraquark} \\ &\times \left[\varepsilon^{a_1 a_2 c} q^{T\,a_1}_{f_1}(x_1) \, C\Gamma_1 \, q^{a_2}_{f_2}(x_2) \right] \cdot \left[\varepsilon^{a_3 a_4 c} \bar{q}^{T\,a_3}_{f_3}(x_3) \, \Gamma_2 C \, \bar{q}^{a_4}_{f_4}(x_4) \right] \end{aligned}$$

Compositeness condition $Z_H = 0$

Salam 1962; Weinberg 1963

- A composite field and its constituents are introduced as elementary particles
- The transition of a composite field to its constituents is provided by the interaction Lagrangian
- The renormalization constant $Z^{1/2}$ is the matrix element between a physical state and the corresponding bare state. If there is a stable bound state which we wish to represent by introducing a quasi-particle H, then elementary particle must have renormalization factor Z equal to zero

$$\boxed{Z_H^{1/2} = < H_{\rm bare} | H_{\rm dressed} > = 0}$$

We use the compositeness condition to determine the hadron-quark coupling constant, e.g. in the case of mesons

$$Z_M = 1 - \tilde{\Pi}'(m_M^2) = 0$$

where $\tilde{\Pi}(p^2)$ is the meson mass operator.

The vertex functions and quark propagators

- Translational invariance for the vertex function

$$F_H(x+a, x_1+a, x_2+a) = F_H(x, x_1, x_2)\,, \qquad \forall a\,.$$

- Our choice:

$$F_B(x, x_1, \ldots, x_n) \;=\; \delta^{(4)}\Big(x - \sum_{i=1}^{n} w_i x_i\Big)\;\Phi_H\Big(\sum_{i<j}(x_i - x_j)^2\Big)$$

where $w_i = m_i / \sum\limits_i m_i$.

- The quark propagators

$$S_q(x_1 - x_2) \;=\; \int \frac{d^4k}{(2\pi)^4 i}\,\frac{e^{-ik(x_1-x_2)}}{m_q - \not{k}}$$

The matrix elements

- The matrix elements are described by a set of the Feynman diagrams which are convolution of the quark propagators and vertex functions.
- Let Π be the matrix element corresponding to the Feynman diagram:

j external momenta;
n quark propagators;
ℓ loop integrations;
m vertices.
In the momentum space it will be represented as

$$\Pi(p_1, \ldots, p_j) = \int [d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}\left(-K^2_{i_1+n}\right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3})$$

$$K^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + \tilde{p}^{(i_2)}_{i_1+n})^2$$

$\tilde{k}_i$	are linear combinations of the loop momenta k_i
$\tilde{p}_i$	are linear combinations of the external momenta p_i

Infrared confinement

- Use the Schwinger representation of the propagator:

$$\frac{m + \not k}{m^2 - k^2} = (m + \not k) \int\limits_0^\infty d\alpha \, \exp[-\alpha(m^2 - k^2)]$$

- Choose a simple Gaissian form for the vertex function

$$\Phi(-K^2) = \exp\left(K^2/\Lambda^2\right)$$

where the parameter Λ characterizes the hadron size.

- We imply that the loop integration k proceed over Euclidean space:

$$k^0 \to e^{i\frac{\pi}{2}} k_4 = ik_4 \,, \qquad k^2 = (k^0)^2 - \vec{k}^2 \to -k_E^2 \le 0 \,.$$

- We also put all external momenta p to Euclidean space:

$$p^0 \to e^{i\frac{\pi}{2}} p_4 = ip_4 \,, \qquad p^2 = (p^0)^2 - \vec{p}^2 \to -p_E^2 \le 0$$

so that the quadratic momentum form in the exponent becomes negative-definite and the loop integrals are absolutely convergent.

Infrared confinement

- Convert the loop momemta in the numerator into derivatives over external momenta:

$$k_i^{\mu} e^{2kr} = \frac{1}{2}\frac{\partial}{\partial r_{i\,\mu}} e^{2kr},$$

- Move the derivatives outside of the loop integrals.
- Calculate the scalar loop integral:

$$\prod_{i=1}^{n} \int \frac{d^4 k_i}{i\pi^2} e^{kAk+2kr} = \prod_{i=1}^{n} \int \frac{d^4 k_i^E}{\pi^2} e^{-k_E A k_E - 2k_E r_E} = \frac{1}{|A|^2} e^{-rA^{-1}r}$$

  where a symmetric $n \times n$ real matrix A is positive-definite.
- Use the identity

$$P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) e^{-rA^{-1}r} = e^{-rA^{-1}r} P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1}r\right)$$

  to move the exponent to the left.

Infrared confinement

- **Employ the commutator**

$$[\frac{\partial}{\partial r_{i\,\mu}}, r_{j\,\nu}] = \delta_{ij}\, g_{\mu\nu}$$

to make differentiation in

$$P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1} r\right)$$

for any polynomial P. The necessary commutations of the differential operators are done by a FORM program.

- **One obtains**

$$\Pi = \int_0^\infty d^n\alpha \, F(\alpha_1, \ldots, \alpha_n)\,,$$

where F stands for the whole structure of a given diagram.

Infrared confinement

The set of Schwinger parameters α_i can be turned into a simplex by introducing an additional t–integration via the identity

$$1 = \int_0^\infty dt\,\delta(t - \sum_{i=1}^n \alpha_i)$$

leading to

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n\alpha\,\delta\Big(1 - \sum_{i=1}^n \alpha_i\Big)\,F(t\alpha_1, \ldots, t\alpha_n)\,.$$

Infrared confinement

- **Cut off the upper integration at $1/\lambda^2$**

$$\boxed{\Pi^c = \int\limits_0^{1/\lambda^2} dt t^{n-1} \int\limits_0^1 d^n\alpha\, \delta\Big(1 - \sum_{i=1}^{n} \alpha_i\Big)\, F(t\alpha_1, \ldots, t\alpha_n)}$$

- **The infrared cut-off has removed all possible thresholds in the quark loop diagram.**

- **We take the cut-off parameter λ to be the same in all physical processes.**

T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij,
Phys. Rev. D81, 034010 (2010)

Infrared confinement

- An example of a scalar one–loop two–point function:

$$\Pi_2(p^2) = \int \frac{d^4 k_E}{\pi^2} \frac{e^{-s\, k_E^2}}{[m^2 + (k_E + \frac{1}{2} p_E)^2][m^2 + (k_E - \frac{1}{2} p_E)^2]}$$

where the numerator factor $e^{-s\, k_E^2}$ comes from the product of nonlocal vertex form factors of Gaussian form. k_E, p_E are Euclidean momenta ($p_E^2 = -\, p^2$).

- Doing the loop integration one obtains

$$\Pi_2(p^2) = \int\limits_0^\infty dt \frac{t}{(s+t)^2} \int\limits_0^1 d\alpha \ \exp\left\{-\, t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2\right\}$$

A branch point at $p^2 = 4m^2$.

Infrared confinement

- By introducing a cut-off in the t–integration one obtains

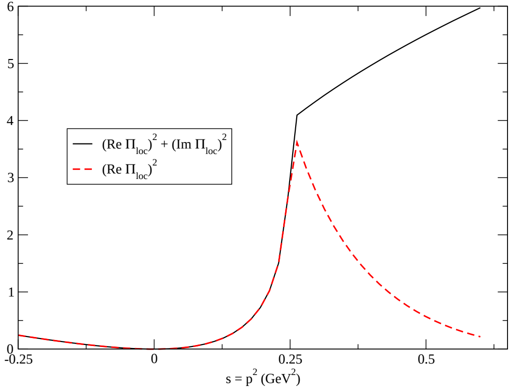
$$\Pi_2^c(p^2) = \int\limits_0^{1/\lambda^2} dt \frac{t}{(s+t)^2} \int\limits_0^1 d\alpha \, \exp\left\{ -t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2 \right\}$$

where the one–loop two–point function $\Pi_2^c(p^2)$ no longer has a branch point at $p^2 = 4m^2$.

- The confinement scenario also allows to include all possible both two-quark and multi-quark resonance states in our calculations.

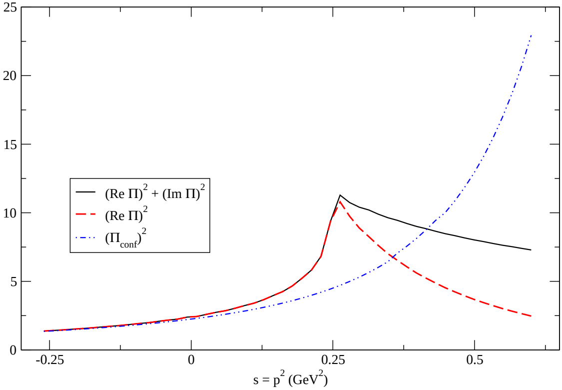
Infrared confinement

One-subtracted local loop without confinement



Infrared confinement

Nonlocal loop without and with infrared cutoff (confinement)



Subtleties: gauging

In order to guarantee local invariance of the strong interaction Lagrangian one multiplies each quark field $q(x_i)$ in nonlocal quark current $J_H(x)$ with a gauge field exponential:

$$q_i(x_i) \to e^{-ie_{q_1} I(x_i,x,P)}\, q_i(x_i) \quad \text{where} \quad I(x_i, x, P) = \int_x^{x_i} dz_\mu A^\mu(z).$$

The path P connects the end-points of the path integral.
We use the path-independent definition of the derivative of $I(x, y, P)$:

Mandelstam,1962,Terning,1991

$$\lim_{dx^\mu \to 0} dx^\mu \frac{\partial}{\partial x^\mu} I(x, y, P) = \lim_{dx^\mu \to 0} [I(x + dx, y, P') - I(x, y, P)]$$

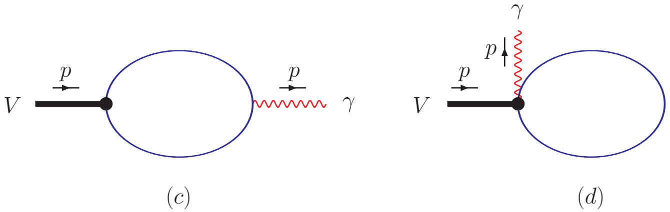
where the path P' is obtained from P by shifting the end-point x by dx.
The definition leads to the key rule

$$\frac{\partial}{\partial x^\mu} I(x, y, P) = A_\mu(x)$$

which in turn states that the derivative of the path integral $I(x, y, P)$ does not depend on the path P originally used in the definition.

Subtleties: gauging

Diagrams describing $\mathbf{V} \to \gamma$ transition:



$$\mathbf{M}_c^{\mu\nu}(\mathbf{p}) = \int \frac{d^4k}{4\pi^2 i} \Phi_V(-k^2)\, \mathrm{tr}\Big(\gamma^\mu S(k + \tfrac{1}{2}\,p)\gamma^\nu S(k - \tfrac{1}{2}\,p)\Big)$$

$$\mathbf{M}_d^{\mu\nu}(\mathbf{p}) = -\int \frac{d^4k}{4\pi^2 i}\,(2k + \tfrac{1}{2}\,p)^\mu \int_0^1 d\alpha \Phi_V'\Big(-\alpha(k+\tfrac{1}{2}p)^2 - (1-\alpha)k^2\Big)$$

$$\times \quad \mathrm{tr}\Big(\gamma^\nu S(k)\Big)$$

Subtleties: gauging

If $p = 0$ then the second diagram maybe transfered to the first one by using integration by parts

$$\int \frac{d^4k}{4\pi^2 i} \frac{\partial}{\partial k^\mu} \Big\{ \Phi_V\Big(-k^2\Big) \mathrm{tr}\Big(\gamma^\nu S(k)\Big) \Big\} =$$

$$= \int \frac{d^4k}{4\pi^2 i} \Big\{ -2k^\mu \Phi_V'\Big(-k^2\Big) \mathrm{tr}\Big(\gamma^\nu S(k)\Big)$$

$$+\Phi_V\Big(-k^2\Big) \mathrm{tr}\Big(\gamma^\mu S(k) \gamma^\nu S(k)\Big) \Big\} = 0.$$

Model parameters

M. A. I., J. G. Körner, S. G. Kovalenko, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D85, 034004 (2012)

Input values for the leptonic decay constants f_H (in MeV) and our least-squares fit values.

	Fit Values	PDG/LAT		This work	PDG/LAT
f_π	128.7	130.4 ± 0.2	f_ω	198.5	198 ± 2
f_K	156.1	156.1 ± 0.8	f_ϕ	228.2	227 ± 2
f_D	205.9	206.7 ± 8.9	$f_{J/\psi}$	415.0	415 ± 7
f_{D_s}	257.5	257.5 ± 6.1	f_{K^*}	213.7	217 ± 7
f_B	191.1	192.8 ± 9.9	f_{D^*}	243.3	245 ± 20
f_{B_s}	234.9	238.8 ± 9.5	$f_{D_s^*}$	272.0	272 ± 26
f_{B_c}	489.0	489 ± 5	f_{B^*}	196.0	196 ± 44
f_ρ	221.1	221 ± 1	$f_{B_s^*}$	229.0	229 ± 46

Model parameters

Input values for some basic electromagnetic decay widths and our least-squares fit values (in keV).

Process	Fit Values	PDG
$\pi^0 \to \gamma\gamma$	5.06×10^{-3}	$(7.7 \pm 0.4) \times 10^{-3}$
$\eta_c \to \gamma\gamma$	1.61	1.8 ± 0.8
$\rho^\pm \to \pi^\pm\gamma$	76.0	67 ± 7
$\omega \to \pi^0\gamma$	672	703 ± 25
$K^{*\pm} \to K^\pm\gamma$	55.1	50 ± 5
$K^{*0} \to K^0\gamma$	116	116 ± 10
$D^{*\pm} \to D^\pm\gamma$	1.22	1.5 ± 0.5
$J/\psi \to \eta_c\gamma$	1.43	1.58 ± 0.37

Model parameters

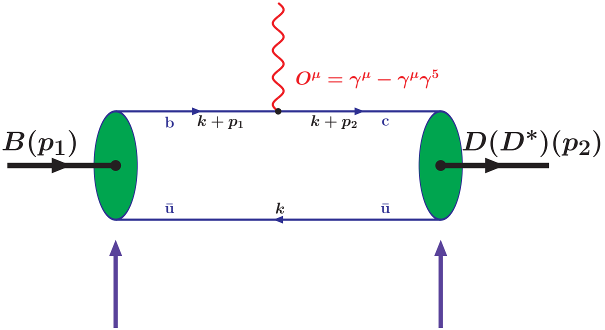
The results of the fit for the values of quark masses m_{q_i}, the infrared cutoff parameter λ and the size parameters Λ_{H_i} (all in GeV).

m_u	m_s	m_c	m_b	λ	
0.235	0.424	2.16	5.09	0.181	GeV

Λ_π	Λ_K	Λ_D	Λ_{D_s}	Λ_B	Λ_{B_s}	Λ_{B_c}	Λ_ρ
0.87	1.04	1.47	1.57	1.88	1.95	2.42	0.61

Λ_ω	Λ_ϕ	$\Lambda_{J/\psi}$	Λ_{K^*}	Λ_{D^*}	$\Lambda_{D_s^*}$	Λ_{B^*}	$\Lambda_{B_s^*}$
0.47	0.88	1.48	0.72	1.16	1.17	1.72	1.71

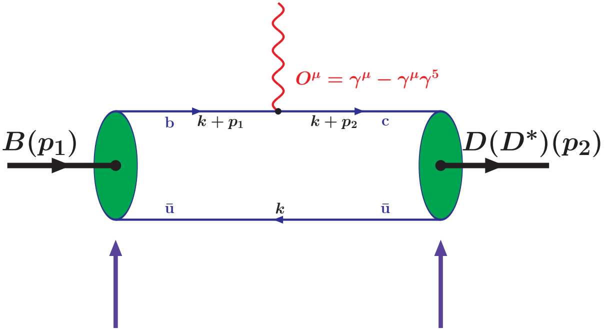
Heavy quark limit in B − D transition



$$\phi_B\left(-(k + w_u\, p_1)^2\right) \qquad \phi_{D(D^*)}\left(-(k + w'_u\, p_2)^2\right)$$

$$w_u = \frac{m_u}{m_u + m_b} \qquad w'_u = \frac{m_u}{m_u + m_c}$$

Heavy quark limit in B − D transition



$$\phi_B\left(-(k + w_u\, p_1)^2\right) \quad \phi_{D(D^*)}\left(-(k + w'_u\, p_2)^2\right)$$

$$w_u = \frac{m_u}{m_u + m_b} \qquad w'_u = \frac{m_u}{m_u + m_c}$$

Heavy quark limit: $m_H = m_Q + E$, $\quad m_Q \to \infty$; $\quad \Lambda_B = \Lambda_D = \Lambda_{D^*}$.

Isgur-Wise function

$$\frac{1}{m_i - \not{k} - \not{p}_i} \quad \to \quad -\frac{1 + \not{v}_i}{2} \cdot \frac{1}{k v_i + E}, \qquad v_i = \frac{p_i}{m_i}$$

Isgur-Wise function

$$\frac{1}{m_i - \not{k} - \not{p}_i} \quad \to \quad -\frac{1+\not{v}_i}{2} \cdot \frac{1}{kv_i + E}, \qquad v_i = \frac{p_i}{m_i}$$

$$M^{\mu}_{BD}(p_1, p_2) \quad = \quad f_+(q^2)(p_1 + p_2)^{\mu} + f_-(q^2)(p_1 - p_2)^{\mu}\,,$$

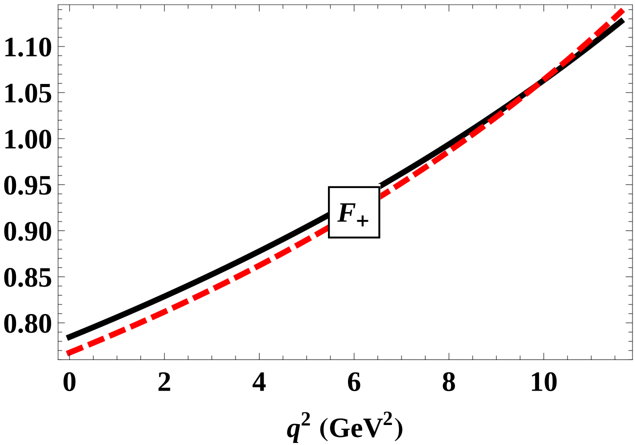
$$f_{\pm} \quad = \quad \pm\frac{m_1 \pm m_2}{2\sqrt{m_1 m_2}} \cdot \xi(w)\,, \qquad w = v_1 \cdot v_2.$$

the Isgur-Wise function is equal to

$$\xi(w) = \frac{J_3(E,w)}{J_3(E,1)}, \qquad J_3(E,w) = \int\limits_0^1 \frac{d\tau}{W} \int\limits_0^{\infty} du\, \widetilde{\Phi}^2(z)\, \frac{m_u + \sqrt{u/W}}{m_u^2 + z}$$

where $W = 1 + 2\tau(1-\tau)(w-1)$, $z = u - 2E\sqrt{u/W}$.

Isgur-Wise function



Nonleptonic $\mathbf{B_s}$ decays

M. A. I., J. G. Körner, S. G. Kovalenko, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D85, 034004 (2012)

- The modes $B_s \to D_s^- D_s^+,\ \ D_s^{*\,-} D_s^+ + D_s^- D_s^{*\,+},\ \ D_s^{*\,-} D_s^{*\,+}$ give the largest contribution to $\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H$ for the $B_s - \bar{B}_s$ system.
- The mode $B_s \to J/\psi\phi$ is color–suppressed but it is interesting for the search of possible CP-violating new physics effects in $B_s - \bar{B}_s$ mixing.
- Nonleptonic $B_s^0 \to J/\psi\eta(\eta')$ decays were observed by Belle Coll.:

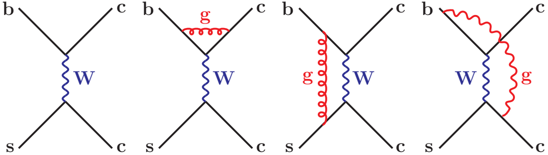
  J. Li *et al.* [Belle Collaboration], Phys. Rev. Lett. 108, 181808 (2012)

- Their decay widths were calculated in our approach by

  S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and A. Liptaj Phys. Rev. D 87, 074201 (2013)

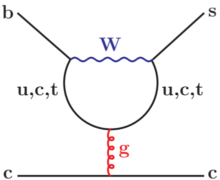
The effective Hamiltonian

Current-current diagrams



tree

OCD one-loop



OCD penguin

The effective Hamiltonian

- The effective Hamiltonian describing the B_s nonleptonic decays:

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^{\dagger} \sum_{i=1}^{6} C_i \, Q_i$$

- Current-current diagrams:

$$Q_1 = (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A} \qquad Q_2 = (\bar{c}_{a_1} \, b_{a_1})_{V-A}, (\bar{s}_{a_2} \, c_{a_2})_{V-A}$$

- QCD penguin diagram:

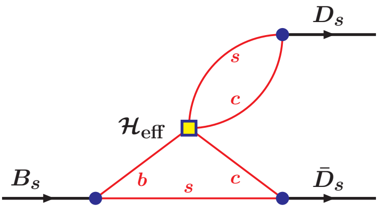
$$Q_3 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V-A} \qquad Q_4 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V-A}$$

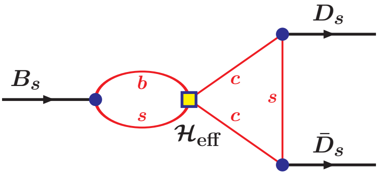
$$Q_5 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V+A} \qquad Q_6 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V+A}$$

$$(\bar{q}q)_{V-A} = \bar{q}\gamma^{\mu}(1-\gamma^5)q \qquad \text{left–chiral current}$$

$$(\bar{q}q)_{V+A} = \bar{q}\gamma^{\mu}(1+\gamma^5)q \qquad \text{right–chiral current}$$

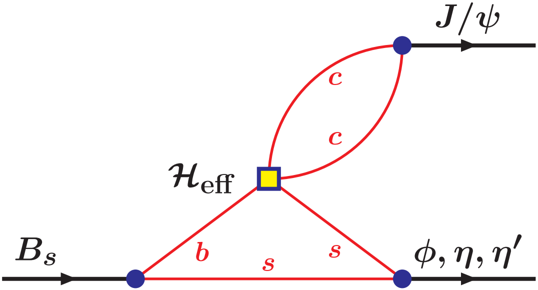
Nonleptonic B_s decays





Annihilation diagram

Nonleptonic B_s decays



Nonleptonic B_s decays

Calculated branching ratios (%) of the B_s nonleptonic decays.

Process	This work	PDG
$B_s \to D_s^- D_s^+$	1.65	$1.04^{+0.29}_{-0.26}$
$B_s \to D_s^- D_s^{*+} + D_s^{*-} D_s^+$	2.40	2.8 ± 1.0
$B_s \to D_s^{*-} D_s^{*+}$	3.18	3.1 ± 1.4
$B_s \to J/\psi\phi$	0.16	0.14 ± 0.05
$B_s \to J/\psi\eta$	$4.67 \cdot 10^{-2}$	$(5.10 \pm 1.12) \cdot 10^{-2}$
$B_s \to J/\psi\eta'$	$4.04 \cdot 10^{-2}$	$(3.71 \pm 0.95) \cdot 10^{-2}$

Nonleptonic B_s decays

$$\mathbf{R} \quad \equiv \quad \frac{\Gamma(B_s \to J/\psi + \eta')}{\Gamma(B_s \to J/\psi + \eta)} = \begin{cases} 0.73 \pm 0.14 \pm 0.02 & \text{Belle} \\ 0.90 \pm 0.09^{+0.06}_{-0.02} & \text{LHCb} \end{cases}$$

Our model

$$\mathbf{R}^{\,\mathrm{theor}} = \underbrace{\frac{|q_{\eta'}|^3}{|q_\eta|^3} \tan^2\delta}_{\approx 1.04} \times \underbrace{\left(\frac{F_+^{B_s\eta'}}{F_+^{B_s\eta}}\right)^2}_{\approx 0.83} \approx 0.86.$$

Lagrangian and 3-quark currents

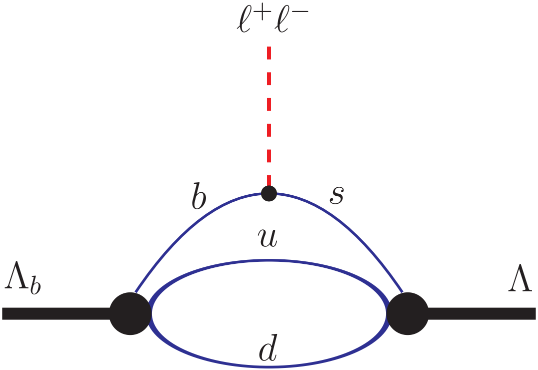
T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 87, 074031 (2013)

$$
\begin{aligned}
\mathcal{L}^{\Lambda}_{\rm int}(x) &= g_\Lambda\,\bar\Lambda(x)\cdot J_\Lambda(x) + g_\Lambda\,\bar J_\Lambda(x)\cdot\Lambda(x) \\
J_\Lambda(x) &= \int\! dx_1 \int\! dx_2 \int\! dx_3\, F_\Lambda(x;x_1,x_2,x_3)\, J^{(\Lambda)}_{3q}(x_1,x_2,x_3) \\
J^{(\Lambda)}_{3q}(x_1,x_2,x_3) &= Q^{a_1}(x_1)\cdot\epsilon^{a_1a_2a_3}\, u^{T\,a_2}(x_2)\, C\,\gamma^5\, d^{a_3}(x_3) \\
Q &= s,\, c,\, b
\end{aligned}
$$

The vertex function is chosen in the form

$$
F_\Lambda(x;x_1,x_2,x_3) = \delta^{(4)}\Big(x - \sum_{i=1}^{3} w_i x_i\Big)\,\Phi_\Lambda\Big(\sum_{i<j}(x_i-x_j)^2\Big) \qquad w_i = \frac{m_i}{m_1+m_2+m_3}
$$

Two-loop diagram



Form factors

Effective Hamiltonian leads to quark decay amplitude $b \to s\ell^+\ell^-$:

$$M(b \to s\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha\lambda_t}{2\pi}\Big\{C_9^{\text{eff}}\,(\bar{s}O^\mu b)\,(\bar{\ell}\gamma_\mu\ell) + C_{10}\,(\bar{s}O^\mu b)\,(\bar{\ell}\gamma_\mu\gamma_5\ell)$$
$$- \frac{2}{q^2}C_7^{\text{eff}}\left[m_b\left(\bar{s}\,i\sigma^{\mu q}(1+\gamma^5)\,b\right) + O(m_s)\right](\bar{\ell}\gamma_\mu\ell)\Big\}$$

Hadronic matrix elements are expanded in terms of dimensionless form factors:

$$\langle B_2\,|\,\bar{s}\,\gamma^\mu\,b\,|B_1\rangle = \bar{u}_2(p_2)\left[f_1^V(q^2)\gamma^\mu - f_2^V(q^2)i\sigma^{\mu q_1} + f_3^V(q^2)q_1^\mu\right]u_1(p_1)$$

$$\langle B_2\,|\,\bar{s}\,\gamma^\mu\gamma^5\,b\,|B_1\rangle = \bar{u}_2(p_2)\left[f_1^A(q^2)\gamma^\mu - f_2^A(q^2)i\sigma^{\mu q_1} + f_3^A(q^2)q_1^\mu\right]\gamma^5u_1(p_1)$$

$$\langle B_2\,|\,\bar{s}\,i\sigma^{\mu q}\,b\,|B_1\rangle = \bar{u}_2(p_2)\left[f_1^{TV}(q^2)(\gamma^\mu q_1^2 - q_1^\mu\;\not{q}_1) - f_2^{TV}(q^2)i\sigma^{\mu q_1}\right]u_1(p_1)$$

$$\langle B_2\,|\,\bar{s}\,i\sigma^{\mu q_1}\gamma^5\,b\,|B_1\rangle = \bar{u}_2(p_2)\left[f_1^{TA}(q^2)(\gamma^\mu q_1^2 - q_1^\mu\;\not{q}_1) - f_2^{TA}(q^2)i\sigma^{\mu q_1}\right]\gamma^5u_1(p_1)$$

where $q = p_1 - p_2$ and $q_1 = q/M_1$.

The fit of the size parameters

- We use the same values of the quark masses and the infrared cut-off as in meson sector.
- We determine the set of size parameters Λ_{Λ_s}, Λ_{Λ_c} and Λ_{Λ_b} by fitting data on the magnetic moment of the Λ-hyperon and the branching ratios of the semileptonic decays $\Lambda_c \to \Lambda \ell^+ \nu_\ell$ and $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$ by a one-parameter fit to these values.
- With the choice of dimensional parameters in GeV

$$\Lambda_{\Lambda_s} = 0.490 \qquad \Lambda_{\Lambda_c} = 0.864 \qquad \Lambda_{\Lambda_b} = 0.569$$

we get:

$$\begin{aligned} \mu_{\Lambda_s} &= -0.73 \qquad \mu_{\Lambda_s}^{\mathrm{expt}} = -0.613 \pm 0.004 \\ \mu_{\Lambda_c} &= +0.39 \\ \mu_{\Lambda_b} &= -0.06 \end{aligned}$$

The fit of the size parameters

Branching ratios of semileptonic decays of heavy baryons in %.

Mode	Our results	Data
$\Lambda_c \to \Lambda e^+ \nu_e$	2.0	2.1 ± 0.6
$\Lambda_c \to \Lambda \mu^+ \nu_\mu$	2.0	2.0 ± 0.7
$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$	6.6	$6.5^{+3.2}_{-2.5}$
$\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$	6.6	
$\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$	1.8	

Asymmetry parameter α in the semileptonic decays of heavy baryons.

Mode	Our results	Data
$\Lambda_c \to \Lambda e^+ \nu_e$	0.828	0.86 ± 0.04
$\Lambda_c \to \Lambda \mu^+ \nu_\mu$	0.825	
$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$	0.831	
$\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$	0.831	
$\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$	0.731	

Branching fractions of decays $\Lambda_b \to \Lambda + \ell^+\ell^-$ and $\Lambda_b \to \Lambda + \gamma$

Our results:

$$\mathcal{B}(\Lambda_b \to \Lambda\mu^+\mu^-) = 1.0 \cdot 10^{-6}$$

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 87 074031 (2013)

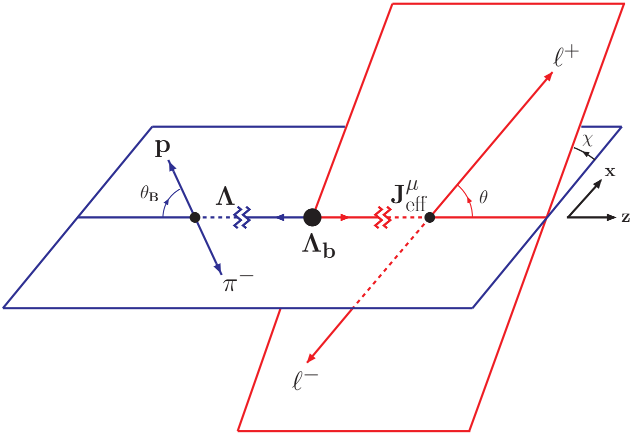
to be compared with the recent LHCb data:

$$\mathcal{B}(\Lambda_b \to \Lambda\mu^+\mu^-) = (0.96 \pm 0.16(\mathrm{stat}) \pm 0.13(\mathrm{syst}) \pm 0.21(\mathrm{norm})) \cdot 10^{-6}$$

RAaij *et al.* [LHCb Collaboration], arXiv:1306.2577 [hep-ex].

$\mathcal{B}(\Lambda_b \to \Lambda\gamma) = 0.4 \cdot 10^{-5}$ (experimental upper bound $< 130 \cdot 10^{-5}$)

The angular decay distribution for the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$



Joint four-fold angular decay distribution for the decay of an unpolarized Λ_b

$$W(\theta,\theta_B,\chi) \propto \quad A\,v^2 + B\,v + C\,\frac{2m_\ell^2}{q^2} \qquad v = \sqrt{1 - 4m_\ell^2/q^2}$$

$$\begin{aligned}
A &= \frac{9}{64}(1+\cos^2\theta)\left(U^{11}+U^{22}\right) + \frac{9}{32}\sin^2\theta\left(L^{11}+L^{22}\right) \\
&+ \frac{9}{32}\alpha_B\cos\theta_B\left[\sin^2\theta\left(L_P^{11}+L_P^{22}\right)+\frac{1}{2}(1+\cos^2\theta)\left(P^{11}+P^{22}\right)\right] \\
&+ \frac{9}{16\sqrt{2}}\alpha_B\sin 2\theta\,\sin\theta_B\left[\cos\chi\left(I1_P^{11}+I1_P^{22}\right)-\sin\chi\left(I2_P^{11}+I2_P^{22}\right)\right], \\
B &= -\frac{9}{16}\cos\theta\left[P^{12}+\alpha_B\cos\theta_B\,U^{12}\right] \\
&- \frac{9}{4\sqrt{2}}\alpha_B\sin\theta\,\sin\theta_B\left[\cos\chi\,I3_P^{12}-\sin\chi\,I4_P^{12}\right], \\
C &= \frac{9}{16}\left(U^{11}+L^{11}+S^{22}\right)+\frac{9}{16}\alpha_B\cos\theta_B\left(P^{11}+L_P^{11}+S_P^{22}\right),
\end{aligned}$$

Hadronic helicity amplitudes

The bilinear expressions

$$H_X^{mm'} \quad (X = U, L, S, P, L_P, S_P, I1_P, I2_P, I3_P, I4_P)$$

are defined by

$$H_U^{mm'} = \mathrm{Re}(H^m_{\frac{1}{2}1} H^{\dagger m'}_{\frac{1}{2}1}) + \mathrm{Re}(H^m_{-\frac{1}{2}-1} H^{\dagger m'}_{-\frac{1}{2}-1}) \qquad \text{transverse unpolarized}$$

etc.

The hadronic helicity amplitudes $H^m_{\lambda_\Lambda \lambda_j}(J)$ describe the full dynamics of the current–induced transitions $\Lambda_b \to \Lambda + j_{\mathrm{eff}}$ including the structure and the values of the short distance coefficients of the pertinent penguin operators.
The helicity labels on the helicity amplitudes take the values

$$\lambda_\Lambda = \pm\frac{1}{2}, \qquad \lambda_j = t \quad \text{for} \quad (J = 0) \quad \text{and} \quad \lambda_j = \pm 1, 0 \quad \text{for} \quad (J = 1)$$

The superscript m defines whether the hadronic helicity amplitude multiplies the lepton vector current $(m = 1)$ or the axial vector current $(m = 2)$.

The differential rate of the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

$$\frac{d\Gamma(\Lambda_b \to \Lambda\,\ell^+\ell^-)}{dq^2} = \frac{v^2}{2}\cdot\left(U^{11+22} + L^{11+22}\right) + \frac{2m_\ell^2}{q^2}\cdot\frac{3}{2}\cdot\left(U^{11} + L^{11} + S^{22}\right)$$

The total rate is obtained by q^2–integration in the range

$$4m_\ell^2 \;\le\; q^2 \;\le\; (M_1 - M_2)^2$$

The short notations:

$$X^{mm'} = \frac{1}{2}\,\frac{G_F^2}{(2\pi)^3}\left(\frac{\alpha|\lambda_t|}{2\pi}\right)^2\frac{|\vec{p}_2|\,q^2\,v}{12\,M_1^2}\,H_X^{mm'}\,,$$

where $\lambda_t = V_{ts}^\dagger V_{tb} = 0.041$ and $v = \sqrt{1 - 4m_\ell^2/q^2}$ is the lepton velocity in the $(\ell^+\ell^-)$ CM frame.

Lepton–side decay distribution for the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

$$\frac{d\Gamma(\Lambda_b \to \Lambda\,\ell^+\ell^-)}{dq^2 d\cos\theta} = v^2 \cdot \left[\frac{3}{8}(1+\cos^2\theta)\cdot\frac{1}{2}U^{11+22} + \frac{3}{4}\sin^2\theta\cdot\frac{1}{2}L^{11+22}\right] - v\cdot\frac{3}{4}\cos\theta\cdot P^{12} + \frac{2m_\ell^2}{q^2}\cdot\frac{3}{4}\cdot\left[U^{11}+L^{11}+S^{22}\right]$$

One can define a lepton–side forward–backward asymmetry A_{FB}^{ℓ} by $A_{FB}^{\ell} = (F-B)/(F+B)$ where F and B denote the rates in the forward and backward hemispheres.

$$A_{FB}^{\ell}(q^2) = -\frac{3}{2}\frac{v\cdot P^{12}}{v^2\cdot\left(U^{11+22}+L^{11+22}\right) + \frac{2m_\ell^2}{q^2}\cdot 3\cdot\left(U^{11}+L^{11}+S^{22}\right)}\,.$$

The integrated forward–backward asymmetry is defined as the ratio of the integrals of the numerator and denominator over q^2 in the full kinematical region.

Λ–polarization and hadron–side decay distribution for the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

$$\frac{d\Gamma(\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-)}{dq^2\, d\cos\theta_B} = \mathrm{Br}(\Lambda \to p\pi^-)\cdot\frac{1}{2}\,\frac{d\Gamma(\Lambda_b \to \Lambda\,\ell^+\ell^-)}{dq^2} \times \Big(1 + \alpha_B P_z^\Lambda \cos\theta_B\Big)$$

The z–component of the polarization of the daughter baryon Λ:

$$P_z^\Lambda = \frac{v^2 \cdot \left(P^{11+22} + L_P^{11+22}\right) + \frac{2m_\ell^2}{q^2}\cdot 3 \cdot \left(P^{11} + L_P^{11} + S_P^{22}\right)}{v^2 \cdot \left(U^{11+22} + L^{11+22}\right) + \frac{2m_\ell^2}{q^2}\cdot 3 \cdot \left(U^{11} + L^{11} + S^{22}\right)}$$

The forward–backward asymmetry is simply related to the polarization P_z^Λ via

$$A_{FB}^h(q^2) = \frac{\alpha_B}{2}\cdot P_z^\Lambda(q^2)$$

Asymmetries A^l_{FB} and A^h_{FB} with (without) long–distance contributions

Mode	A^l_{FB}	A^h_{FB}
$\Lambda_b \to \Lambda e^+ e^-$	3.2×10^{-10} (1.2×10^{-8})	-0.321 (-0.321)
$\Lambda_b \to \Lambda \mu^+ \mu^-$	1.7×10^{-4} (8.0×10^{-4})	-0.300 (-0.294)
$\Lambda_b \to \Lambda \tau^+ \tau^-$	5.9×10^{-4} (9.6×10^{-4})	-0.265 (-0.259)

Semileptonic and nonleptonic Λ_b-decays

- **Polarization effects in the cascade decay**

$$\boxed{\Lambda_b \to \Lambda(\to p\pi^-) + J/\psi(\to \ell^+\ell^-)}$$

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, Phys. Rev. D 88 114018 (2013)

- **Heavy-to-light semileptonic decays of Λ_b and Λ_c baryons**

$$\boxed{\Lambda_b \to p\ell^-\bar{\nu} \text{ and } \Lambda_c \to n\ell^+\bar{\nu}}$$

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij,P. Santorelli, Phys. Rev. D 90 114033 (2014)

- **Semileptonic decays**

$$\boxed{\Lambda_b \to \Lambda_c\tau^-\bar{\nu}_\tau \text{ and } \Lambda_c \to \Lambda\ell^+\bar{\nu}}$$

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli and N. Habyl, Phys. Rev. D 91 074001 (2015) Erratum: [Phys. Rev. D 91 119907 (2015)]

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, Phys. Rev. D 93 034008 (2016)

X(3872)-meson

- A narrow charmonium-like state X(3872) was observed in the exclusive decay process:

$$\boxed{B^+ \to K^+\pi^+\pi^- J/\psi}$$

S. K. Choi *et al.* [Belle Collaboration] Phys. Rev. Lett. 91, 262001 (2003)

- X-mass is close to $D^0 - D^{*\,0}$ mass threshold:

$$\begin{aligned} M_X &= 3871.68 \pm 0.17 \text{ MeV} \\ M_{D^0} + M_{D^{*\,0}} &= 3871.81 \pm 0.25 \text{ MeV} \end{aligned}$$

- Its width is rather small $\Gamma_X \leq 1.2$ MeV
- The quantum numbers of the X(3872) are

$$\boxed{J^{PC} = 1^{++}}$$

X(3872)-meson

- **An intepretation of the X(3872) as a tetraquark was suggested in**

  L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71, 014028 (2005)

$$\mathbf{X_q \Longrightarrow [cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}, \qquad (q = u, d)}$$

- **The physical states are the mixing of X_u and X_d**

$$\begin{aligned} X_l \equiv X_{low} &= X_u \cos\theta + X_d \sin\theta, \\ X_h \equiv X_{high} &= -X_u \sin\theta + X_d \cos\theta. \end{aligned}$$

- **The mixing angle θ is supposed to be found from the known ratio of the two-pion (via ρ) and three-pion (via ω) decay widths.**

X(3872)-meson as a tetraquark state: Lagrangian

S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and J. G. Körner, Phys. Rev. D 81, 114007 (2010)

- **An effective interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_X \, X_{q\,\mu}(x) \cdot J^{\mu}_{X_q}(x), \qquad (q = u, d).$$

- **The nonlocal version of the four-quark interpolating current**

$$J^{\mu}_{X_q}(x) = \int dx_1 \ldots \int dx_4 \, \delta\Big(x - \sum_{i=1}^{4} w_i x_i\Big) \, \Phi_X\Big(\sum_{i<j}(x_i - x_j)^2\Big) \, J^{\mu}_{4q}(x_1, \ldots, x_4)$$

$$J^{\mu}_{4q} = \frac{1}{\sqrt{2}} \, \varepsilon_{abc} \, [q_a(x_4) C \gamma^5 c_b(x_1)] \, \varepsilon_{dec} \, [\bar{q}_d(x_3) \gamma^{\mu} C \bar{c}_e(x_2)] + (\gamma^5 \leftrightarrow \gamma^{\mu}),$$

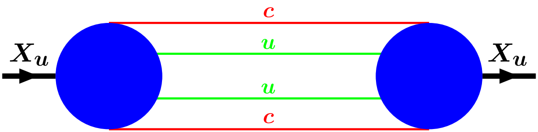
$$w_1 = w_2 = \frac{m_c}{2(m_q + m_c)} \equiv \frac{w_c}{2}, \qquad w_3 = w_4 = \frac{m_q}{2(m_q + m_c)} \equiv \frac{w_q}{2}.$$

Compositeness condition

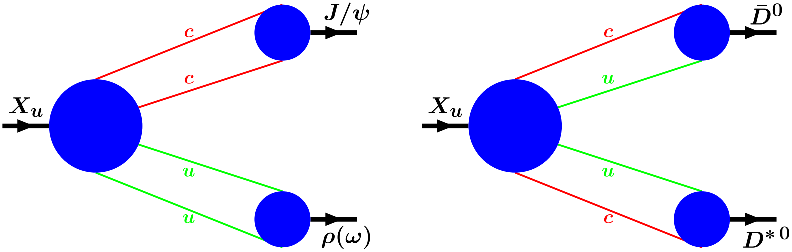
The coupling constant g_X is determined from the compositeness condition

$$\boxed{Z_X = 1 - \Pi'_X(M_X^2) = 0}$$

where $\Pi_X(p^2)$ is the scalar part of the vector-meson mass operator.



Strong off-shell decays



Since the X(3872) lies nearly the respective thresholds in both cases,

$$\begin{aligned} \mathbf{m_X - (m_{J/\psi} + m_\rho)} &= \mathbf{-0.90 \pm 0.41\,MeV,} \\ \mathbf{m_X - (m_{D^0} + m_{D^{*0}})} &= \mathbf{-0.30 \pm 0.34\,MeV} \end{aligned}$$

the intermediate $\rho(\omega)$ and $\mathbf{D^*}$ mesons should be taken off-shell.

The narrow width approximation

$$\frac{d\Gamma(X \to J/\psi + n\pi)}{dq^2} = \frac{1}{8\, m_X^2\, \pi} \cdot \frac{1}{3} |M(X \to J/\psi + v^0)|^2$$
$$\times \frac{\Gamma_{v^0}\, m_{v^0}}{\pi} \frac{p^*(q^2)}{(m_{v^0}^2 - q^2)^2 + \Gamma_{v^0}^2\, m_{v^0}^2} \mathrm{Br}(v^0 \to n\pi),$$

$$\frac{d\Gamma(X_u \to \bar{D}^0 D^0 \pi^0)}{dq^2} = \frac{1}{2\, m_X^2\, \pi} \cdot \frac{1}{3} |M(X_u \to \bar{D}^0 D^{*0})|^2$$
$$\times \frac{\Gamma_{D^{*0}}\, m_{D^{*0}}}{\pi} \frac{p^*(q^2)\, \mathcal{B}(D^{*0} \to D^0 \pi^0)}{(m_{D^{*0}}^2 - q^2)^2 + \Gamma_{D^{*0}}^2\, m_{D^{*0}}^2},$$

Strong decay widths

- Two new adjustable parameters: θ and Λ_X.
- The ratio

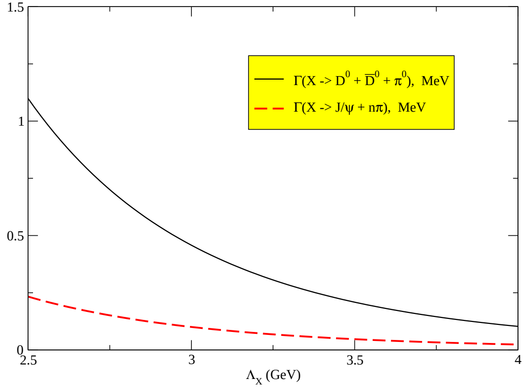
$$\frac{\Gamma(X_u \to J/\psi + 3\,\pi)}{\Gamma(X_u \to J/\psi + 2\,\pi)} \approx 0.25$$

  is very stable under variation of Λ_X.
- Using this result and the central value of the experimental data

$$\frac{\Gamma(X_{l,h} \to J/\psi + 3\,\pi)}{\Gamma(X_{l,h} \to J/\psi + 2\,\pi)} \approx 0.25 \cdot \left(\frac{1 \pm \tan\theta}{1 \mp \tan\theta}\right)^2 \approx 1$$

  gives $\theta \approx \pm 18.4^o$ for X_l $('' + '')$ and X_h $('' - '')$, respectively.
- This is in agreement with the results obtained by both Maiani: $\theta \approx \pm 20^o$ and Nielsen: $\theta \approx \pm 23.5^o$.

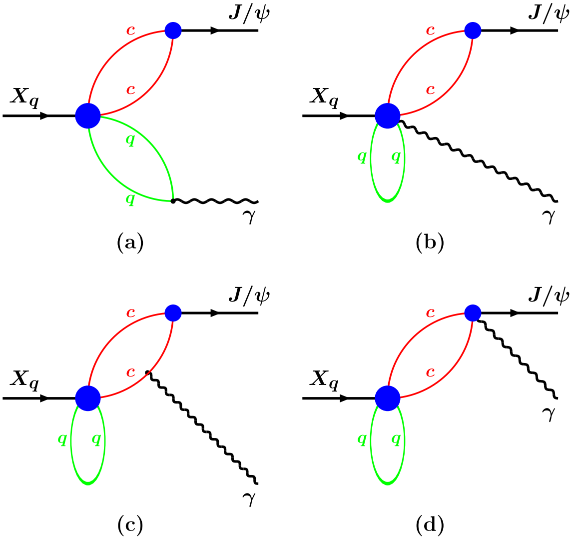
Strong decay widths



$$\frac{\Gamma(X\to D^0\bar{D}^0\pi^0)}{\Gamma(X\to J/\psi\pi^+\pi^-)} = \begin{cases} 4.5\ \pm 0.2 & \text{theor} \\ 10.5 \pm 4.7 & \text{expt} \end{cases}$$

Radiative X-decay

S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, J. G. Koerner, P. Santorelli and G. G. Saidullaeva,
Phys. Rev. D 84, 014006 (2011)



Radiative X-decay

The on-mass shell conditions

$$\varepsilon_{X}^{\mu} p_{\mu} = 0, \qquad \varepsilon_{J/\psi}^{\nu} q_{1\nu} = 0, \qquad \varepsilon_{\gamma}^{\rho} q_{2\rho} = 0$$

leave us five Lorentz structures:

$$\begin{aligned} T_{\mu\rho\nu}(q_1, q_2) &= \varepsilon_{q_2\mu\nu\rho}(q_1 \cdot q_2)\, W_1 + \varepsilon_{q_1 q_2 \nu\rho} q_{1\mu}\, W_2 + \varepsilon_{q_1 q_2 \mu\rho} q_{2\nu}\, W_3 \\ &+ \varepsilon_{q_1 q_2 \mu\nu} q_{1\rho}\, W_4 + \varepsilon_{q_1 \mu\nu\rho}(q_1 \cdot q_2)\, W_5 \,. \end{aligned}$$

Using the gauge invariance condition

$$q_2^{\rho} T_{\mu\rho\nu} = (q_1 \cdot q_2)\varepsilon_{q_1 q_2 \mu\nu}(W_4 + W_5) = 0$$

one has $W_4 = -W_5$ which reduces the set of independent covariants to four. However, there are two nontrivial relations among the four covariants which can be derived by noting that the tensor

$$T_{\mu[\nu_1\nu_2\nu_3\nu_4\nu_5]} = g_{\mu\nu_1}\varepsilon_{\nu_2\nu_3\nu_4\nu_5} + \mathrm{cycl.}(\nu_1\nu_2\nu_3\nu_4\nu_5)$$

vanishes in four dimensions since it is totally antisymmetric in the five indices $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5)$.

Radiative X-decay

The two conditions reduce the set of independent covariants to two. This is the appropriate number of independent covariants since the photon transition is described by two independent amplitudes as e.g. by the **E1** and **M2** transition amplitudes. One has

$$\Gamma(X \to \gamma\, J/\psi) = \frac{1}{12\pi}\,\frac{|\vec{q}_2|}{m_X^2}\left(|H_L|^2 + |H_T|^2\right) = \frac{1}{12\pi}\,\frac{|\vec{q}_2|}{m_X^2}\left(|A_{E1}|^2 + |A_{M2}|^2\right),$$

where the helicity amplitudes H_L and H_T are expressed in terms of the Lorentz amplitudes as

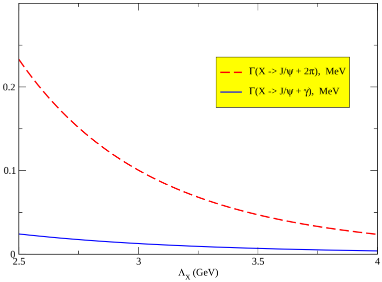
$$\begin{aligned} H_L &= i\frac{m_X^2}{m_{J/\psi}}|\vec{q}_2|^2\left[W_1 + W_3 - \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}W_4\right], \\ H_T &= -im_X|\vec{q}_2|^2\left[W_1 + W_2 - \left(1 + \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}\right)W_4\right], \end{aligned}$$

$$|\vec{q}_2| = \frac{m_X^2 - m_{J/\psi}^2}{2m_X}.$$

The **E1** and **M2** multipole amplitudes are obtained via

$$A_{E1/M2} = (H_L \mp H_T)/\sqrt{2}.$$

Radiative X-decay



If one takes $\Lambda_X \in (3,4)$ GeV with the central value $\Lambda_X = 3.5$ GeV then our prediction for the ratio of widths reads

$$\left.\frac{\Gamma(X_l \to \gamma + J/\psi)}{\Gamma(X_l \to J/\psi + 2\pi)}\right|_{\rm theor} = 0.15 \pm 0.03$$

which fits very well the experimental data from the Belle Collaboration

$$\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to J/\psi\, 2\pi)} = \begin{cases} 0.14 \pm 0.05 & \textbf{Belle} \\ 0.22 \pm 0.06 & \textbf{BaBar} \end{cases}$$

Summary

- **Covariant covariant quark model**
  - **effective lagrangian = hadron×interpolating quark current**
  - **compositeness condition**
  - **interaction via quark exchange (Feynman diagrams)**
  - **infrared confinement**
- **Its applications**
  - **hadronic form factors in heavy flavor physics in the full kinematical range of momentum transfer**
  - **multiquark sates (baryons, tetraquarks, pentaquarks, etc.)**