Малонуклонные системы в формализме Бете-Солпитера

к 70-летию професора В.В.Бурова

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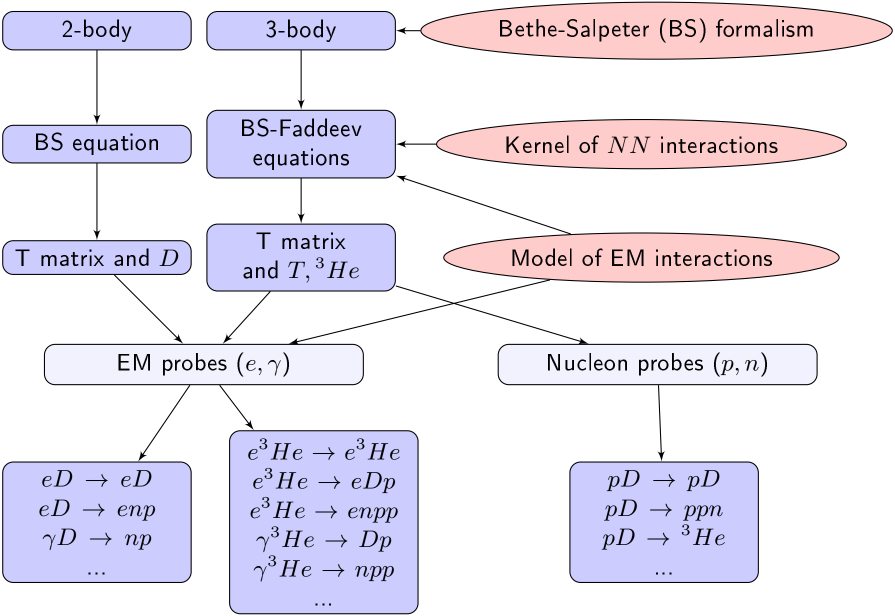
Bethe-Salpeter approach with separable kernel:

- N/D:-presentation for T matrix, dispersion relations: S.Bondarenko, V.Burov, S.Dorkin
- elastic eD-scattering: S.Bondarenko, V.Burov, S.Dorkin, A.Bekzhanov, M.Beyer, H.Toki, A.Hosaka, N.Hamamoto, Y.Manabe
- deep-inelastic scattering: V.Burov, A.Molochkov, G.Smirnov
- deuteron photodisintegration at threshold: S.Bondarenko, V.Burov, K.Kazakov, D.Shulga
- partial-wave analysis of NN-scattering: S.Bondarenko, V.Burov, E.Rogochaya, P.Hwang
- exclusive deuteron electrodisintegration: S.Bondarenko, V.Burov, E.Rogochaya
- three-nucleon systems: S.Bondarenko, V.Burov, S.Yurev

Reactions in the BS approach



Bethe-Salpeter equation for the nucleon-nucleon T matrix

$$T(p',p;P) = V(p',p;P) + \frac{i}{4\pi^3}\int d^4k\, V(p',k;P)\, S_2(k;P)\, T(k,p;P)$$

$p',\ p$ - the relative four-momenta
P - the total four-momentum

$V(p',p;P)$ - the interaction kernel

$S_2^{-1}(k;P) = \left(\frac{1}{2}\, P\cdot\gamma + k\cdot\gamma - m\right)^{(1)}\left(\frac{1}{2}\, P\cdot\gamma - k\cdot\gamma - m\right)^{(2)}$
free two-particle Green function

Partial-wave decomposition

T matrix and V in the c.m.s. frame

$$T_{\alpha\beta,\gamma\delta}(p',p;P_{(0)}) = \sum_{JMab} (\mathcal{Y}_{aM}(-\mathbf{p}')U_C)_{\alpha\beta} \otimes (U_C \mathcal{Y}^{\dagger}_{bM}(\mathbf{p}))_{\delta\gamma}\; T_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s)$$

$$V_{\alpha\beta,\gamma\delta}(p',p;P_{(0)}) = \sum_{JMab} (\mathcal{Y}_{aM}(-\mathbf{p}')U_C)_{\alpha\beta} \otimes (U_C \mathcal{Y}^{\dagger}_{bM}(\mathbf{p}))_{\delta\gamma}\; V_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s)$$

The partial-wave decomposed equation

$$T_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s) = V_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s) + \frac{i}{4\pi^3}\sum_{cd}\int_{-\infty}^{+\infty} dk_0 \int_0^{\infty} \mathbf{k}^2 d|\mathbf{k}|$$

$$V_{ac}(p'_0,|\mathbf{p}'|;k_0,|\mathbf{k}|;s) S_{cd}(k_0,|\mathbf{k}|;s)\, T_{db}(k_0,|\mathbf{k}|;p_0,|\mathbf{p}|;s)$$

$T_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s)$ - T matrix radial parts
$V_{ac}(p'_0,|\mathbf{p}'|;k_0,|\mathbf{k}|;s)$ - V kernel radial parts

Separable kernels of the NN interaction

The separable kernels of the nucleon-nucleon interaction are widely used in the calculations. The separable kernel as a *nonlocal* covariant interaction representing complex nature of the space-time continuum.

Separable ansatz for the kernel

$$V_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \lambda_{mn}^{[a'a]}(s) g_m^{[a']}(p_0', |\mathbf{p}'|) g_n^{[a]}(p_0, |\mathbf{p}|)$$

Solution for the T matrix

$$T_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{i,j=1}^{N} \tau_{ij}(s)\, g_i^{[a']}(p_0', |\mathbf{p}'|)\, g_j^{[a]}(p_0, |\mathbf{p}|)$$

where

$$\left[\tau_{ij}(s)\right]^{-1} = \left[\lambda_{mn}^{[a'a]}(s)\right]^{-1} + h_{ij}(s),$$

$$h_{ij}(s) = -\frac{i}{4\pi^3} \sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0, |\mathbf{k}|) g_j^{[a]}(k_0, |\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2},$$

$g_j^{[a]}$ - the model functions, $\lambda_{ij}^{[a'a]}(s)$ - a matrix of model parameters.

What is separable kernel?

The integral equations in the nuclear physics (Lippmann-Schwinger, Bethe-Salpeter) can be reduced to the Fredholm (first or second) type of equations. The separable kernel of the integral equation is the degenerated kernel. Fredholm integral equation of the second type:

$$\phi(x) = f(x) + \lambda \int dy \; K(x,y)\phi(y)$$

Degenerated kernel of the equation:

$$K(x,y) = \sum_i a_i(x) b_i(y)$$

Solution of the equation:

$$\phi(x) = f(x) + \lambda \sum_i c_i a_i(x)$$

Constants c_i can be found by solving the system of linear equations

$$c_i - \lambda \sum_j k_{ij} c_j = f_i$$

Matrix k_{ij} and f_i are:

$$k_{ij} = \int dy \; b_i(y) a_j(y), \qquad f_i = \int dy \; f(y) b_i(y)$$

Historical remarks

Separable kernel for Schrodinger (Lippmann-Schwinger) equation with separable potential
Yoshio Yamaguchi "Two-Nucleon Problem When the Potential Is Nonlocal but Separable. I" Phys.Rev.95, 1628 (1954)
Yoshio Yamaguchi, Yoriko Yamaguchi "Two-Nucleon Problem When the Potential Is Nonlocal but Separable. II" Phys.Rev.95, 1635 (1954)

Nonlocal: $\langle \mathbf{r}|V|\mathbf{r}'\rangle = V(\mathbf{r},\mathbf{r}')$
in configuration space

$$\langle \mathbf{r}|V|\mathbf{r}'\rangle = -(\lambda/m_N)v^*(\mathbf{r})v^*(\mathbf{r}')$$

in momentum space

$$\langle \mathbf{p}|V|\mathbf{p}'\rangle = (\lambda/m_N)g^*(\mathbf{p})g^*(\mathbf{p}')$$

for S-state: $g(p) = 1/(\mathbf{p}^2+\beta^2)$
for D-state: $g(p) = \mathbf{p}^2/(\mathbf{p}^2+\beta^2)^2$
for the deuteron and scattering problem (invariant to Galilean transformation)

Separable nucleon-nucleon potential was widely used for the two- and three-nucleon calculations in nonrelativistic nuclear physics

Willibald Plessas et al. Graz, Graz-II potentials, separable representation of the popular Bonn and Paris potentials

K. Schwarz, Willibald Plessas, L. Mathelitsch "Deuteron Form-factors And E D Polarization Observables For The Paris And Graz-II Potentials" Nuovo Cim. A76 (1983) 322-329

Separabilization of the one-meson exchange model (using the Ernst, Shakin, and Thaler (EST) method)

J. Haidenbauer, Willibald Plessas "Separable Representation Of The Paris Nucleon Nucleon Potential" Phys.Rev. C30 (1984) 1822-1839

Johann Haidenbauer, Y. Koike, Willibald Plessas "Separable representation of the Bonn nucleon-nucleon potential" Phys.Rev. C33 (1986) 439-446

$$g(p) \sim \frac{|\mathbf{p}|^l}{(\mathbf{p}^2 + \beta_l^2)^{l+1}}$$

l corresponds to the angular momentum

Why a relativistic approach?

- Elastic electron-deuteron scattering experiments
  "Large Momentum Transfer Measurements of the Deuteron Elastic Structure Function A(Q^2) at Jefferson Laboratory"
  JLab Hall A Collaboration, Phys.Rev.Lett.82:1374-1378,1999
  Q^2=0.7-6.0 (GeV/c)^2

  Lorentz transformation: $\eta_{LOR} = -Q^2/4M_d^2 \sim 0.43$, $\sqrt{1+\eta_{LOR}} \sim 1.19$, $\sqrt{\eta_{LOR}} \sim 0.65$
- Exclusive disintegration of the deuteron experiments
  JLab Hall C Deuteron Electro-Disintegration at Very High Missing Momenta (E12-10-003) proposal
  https://www.jlab.org/exp_prog/proposals/10/PR12-10-003.pdf:
  "We propose to measure the D(e,e'p)n cross section at Q^2 = 4.25 (GeV/c)^2 and xbj = 1.35 for missing momenta ranging from pm = 0.5 GeV/c to pm = 1.0 GeV/c expanding the range of missing momenta explored in the Hall A experiment (E01-020)"

  Lorentz transformation: $\eta_{LOR} = -Q^2/4s_{np} \sim 0.30$, $\sqrt{1+\eta_{LOR}} \sim 1.14$, $\sqrt{\eta_{LOR}} \sim 0.55$

Separable NN kernels for BS equation

- NN scattering with spinor nucleon propagators

  G. Rupp and J. A. Tjon "Relativistic contributions to the deuteron electromagnetic form factors" Phys. Rev. C41. 472 (1990)
  Relativistic Graz-II (only $^3S_1 -^3 D_1$ partial-wave states)

- NN scattering with scalar nucleon propagators

  K. Schwarz, J. Haidenbauer, J. Frohlich "A Separable Approximation of the NN Paris Potential in the Framework of the Bethe-Salpeter Equation" Phys.Rev. C33 456-466 (1986)
  partial-wave states with $J = 0, 1$ for Paris meson-exchange potentials

  G. Rupp, J.A. Tjon "Bethe-Salpeter calculation of three-nucleon with multirank observables separable interactions" Phys.Rev. C45 2133 (1991)
  $^1S_0$ and $^3S_1 -^3 D_1$ partial-wave states for Paris and Bonn meson-exchange potentials

Relativistic Graz-II kernel

Graz-II covariant kernel, rank III ($J = 1 :^3 S_1 -^3 D_1$ partial-wave states)

$$g_1^{(\mathrm{S})}(p_0, |\mathbf{p}|) = \frac{1 - \gamma_1(p_0^2 - \mathbf{p}^2)}{(p_0^2 - \mathbf{p}^2 - \beta_{11}^2)^2},$$

$$g_2^{(\mathrm{S})}(p_0, \mathbf{p}) = -\frac{(p_0^2 - \mathbf{p}^2)}{(p_0^2 - \mathbf{p}^2 - \beta_{12}^2)^2},$$

$$g_3^{(\mathrm{D})}(p_0, |\mathbf{p}|) = \frac{(p_0^2 - \mathbf{p}^2)(1 - \gamma_2(p_0^2 - \mathbf{p}^2))}{(p_0^2 - \mathbf{p}^2 - \beta_{21}^2)(p_0^2 - \mathbf{p}^2 - \beta_{22}^2)^2},$$

$$g_1^{(\mathrm{D})}(p_0, |\mathbf{p}|) = g_2^{(\mathrm{D})}(p_0, |\mathbf{p}|) = g_3^{(\mathrm{S})}(p_0, |\mathbf{p}|) \equiv 0.$$

Deuteron and low-energy scattering properties

	$p_{\mathrm{D}}(\%)$	ϵ_{D} (MeV)	Q_{D} (Fm^{-2})	μ_{D} $(e/2m)$	$\rho_{\mathrm{D/S}}$	r_0 (Fm)	a (Fm)
Covariant Graz-II	4	2.225	0.2484	0.8279	0.02408	1.7861	5.4188
Experimental data		2.2246	0.286	0.8574	0.0263	1.759	5.424

Elastic eD scattering cross section

$$\frac{d\sigma}{d\Omega_{\rm e}'} = \left(\frac{d\sigma}{d\Omega_{\rm e}'}\right)_{\rm Mott}\left[A(q^2) + B(q^2)\tan^2\frac{\theta_{\rm e}}{2}\right],$$

$$\left(\frac{d\sigma}{d\Omega_{\rm e}'}\right)_{\rm Mott} = \frac{\alpha^2\cos^2\theta_{\rm e}/2}{4E_{\rm e}^2(1+2E_{\rm e}/M_d\sin^4\theta_{\rm e}/2)},$$

where $\theta_{\rm e}$ is the electron scattering angle, M_d is the deuteron mass, E_e is the incident electron energy.

Deuteron structure functions $A(q^2)$ and $B(q^2)$

$$A(q^2) = F_{\rm C}^2(q^2) + \frac{8}{9}\eta^2 F_{\rm Q}^2(q^2) + \frac{2}{3}\eta F_{\rm M}^2(q^2)$$

$$B(q^2) = \frac{4}{3}\eta(1+\eta)F_{\rm M}^2(q^2)$$

where $\eta = -q^2/4M_d^2 = Q^2/4M_d^2$

Relativistic impulse approximation (RIA)

Deuteron current matrix element

$$\langle D'\mathcal{M}'|J_\mu^{RIA}|D\mathcal{M}\rangle =$$

$$ie\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{\bar{\chi}^{1\mathcal{M}'}(P',k')\Gamma_\mu^{(\mathrm{S})}(q)\chi^{1\mathcal{M}}(P,k)(P\cdot\gamma/2-k\cdot\gamma+m)\right\}$$

$\chi^{1\mathcal{M}}(P,k)$ - the BS amplitude of the deuteron, $P'=P+q$ and $k'=k+q/2$.

The vertex of γNN interaction

$$\Gamma_\mu^{(\mathrm{S})}(q)=\gamma_\mu F_1^{(\mathrm{S})}(q^2)-\frac{\gamma_\mu q\cdot\gamma-q\cdot\gamma\gamma_\mu}{4m}F_2^{(\mathrm{S})}(q^2)$$

is chosen to be the form factor on mass shell.

The isoscalar form factors of the nucleon

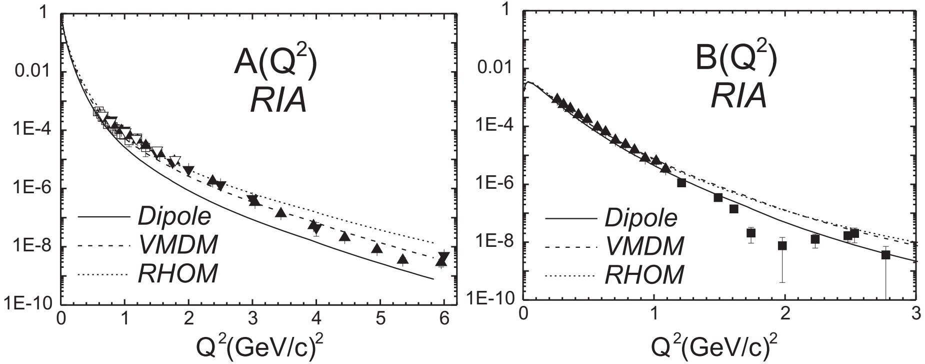
$$F_{1,2}^{(\mathrm{S})}(q^2)=(F_{1,2}^{(\mathrm{p})}(q^2)+F_{1,2}^{(\mathrm{n})}(q^2))/2$$

with normalization condition

$$F_1^{(\mathrm{S})}(0)=1/2,\quad F_2^{(\mathrm{S})}(0)=(\varkappa_p+\varkappa_n)/2$$

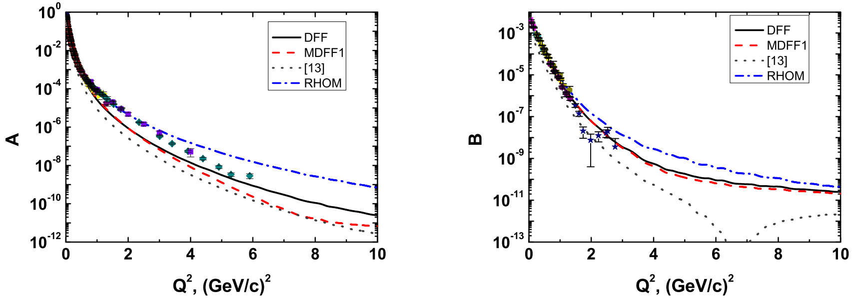
with $\varkappa_p=\mu_p-1$ and $\varkappa_n=\mu_n$ being anomalous parts of the proton μ_p and neutron μ_n magnetic moments, respectively.

Structure functions $A(q^2)$ and $B(q^2)$



Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Structure functions $A(q^2)$ and $B(q^2)$ with relativistic Graz-II kernel



[13] C. Adamuscin, E. Bartos, S. Dubnicka, A.Z. Dubnickova, Nucl. Phys. Proc. Suppl. 245, 69 (2013)

Towards to inelastic reactions: motivation for MY(I)N kernel functions

Consider integral

$$\int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{[g(k_0, |\mathbf{k}|)]^2}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2}$$

Simple poles of propagators

$$k_0^{(1,2)} = \pm\sqrt{s}/2 \mp E_{\mathbf{k}} \pm i\epsilon$$

Taking into account pole and Yamaguchi-like type of $g(k_0, |\mathbf{k}|) = 1/(k_0^2 - \mathbf{k}^2 - \beta^2)$

$$(2\pi i) \int \mathbf{k}^2 d|\mathbf{k}| \frac{1}{(s/4 - \sqrt{s}E_{\mathbf{k}} + m^2 - \beta^2)^2} \frac{1}{\sqrt{s} - 2E_{\mathbf{k}} + i\epsilon}$$

Motivation for MY(I)N kernel functions

Consider numerator $f = s/4 - \sqrt{s}E_{\mathbf{k}} + m^2 - \beta^2$

- If $4(m-\beta)^2 < s < 4(m+\beta)^2$ then function $1/f^n$ is integrable
- For bound state $s = M_d^2 = (2m - \epsilon_D)^2$ then function $1/f^n$ is integrable
- If $4(m-\beta)^2 > s > 4(m+\beta)^2$ then $1/f^n$ can be non-integrable

To avoid the non-integrable one needs to modify function $g(k_0, |\mathbf{k}|)$

$$g_{\mathrm{Y}}(k_0, |\mathbf{k}|) = 1/(k_0^2 - \mathbf{k}^2 - \beta^2) \to g_{\mathrm{MY}}(k_0, |\mathbf{k}|) = 1/((k_0^2 - \mathbf{k}^2 - \beta^2)^2 + \alpha^4)$$

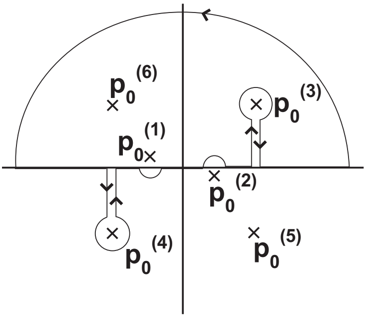
here Y stands for Yamaguchi-like and MY - for Modified Yamaguchi

Modified Yamaguchi functions

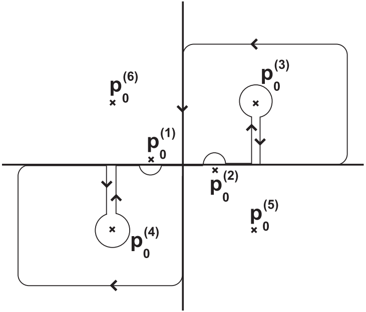
$$g_i^{[a]}(p_0,|\boldsymbol{p}|)=\frac{(p_{ci}-p_0^2+\mathbf{p}^2)^{n_i}(p_0^2-\mathbf{p}^2)^{m_i}}{((p_0^2-\mathbf{p}^2-\beta_{1i}^2)^2+\alpha_{1i}^4)^{k_i}((p_0^2-\mathbf{p}^2-\beta_{1i}^2)^2+\alpha_{1i}^4)^{l_i}}$$

All parameters - n_i, m_i, k_i, l_i (integer), $p_{ci}, \beta_{1i}, \beta_{2i}, \alpha_{1i}, \alpha_{2i}$ (real) - depend on channel $[a]$.

Contour of p_0 integration



Cauchy theorem



The Wick rotation

S matrix (Arndt-Roper parametrization) for uncoupled partial-wave state L

$$S = \frac{1 - K_i + iK_r}{1 + K_i - iK_r} = \eta \exp(2i\delta)$$

$$K = K_r + iK_i$$

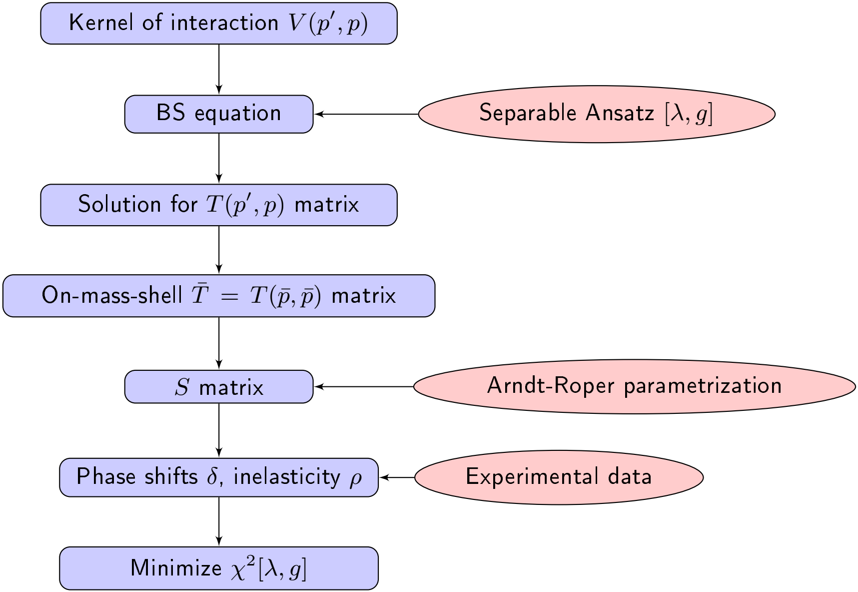
$$K_r = \tan\delta, \quad K_i = \tan^2\rho$$

δ - the phase shifts, ρ - the inelasticity parameter

$$\eta^2 = \frac{1 + K^2 - 2K_i}{1 + K^2 + 2K_i} = |S|^2$$

$$K^2 = K_r^2 + K_i^2$$

If there are no inelastic channels: $(\rho = 0)$, $\delta = \delta_e$, $\eta = 1$ and $S = S_e = \exp(2i\delta_e)$.


Kernel of interaction $V(p', p)$
BS equation
Separable Ansatz $[\lambda, g]$
Solution for $T(p', p)$ matrix
On-mass-shell $\bar{T} = T(\bar{p}, \bar{p})$ matrix
S matrix
Arndt-Roper parametrization
Phase shifts δ, inelasticity ρ
Experimental data
Minimize $\chi^2[\lambda, g]$

Procedure ($J = 0 - 3$)

calculate the kernel parameters – $\lambda_{ij}(s)$-matrix and parameter of the g-functions – to minimize the function χ^2:

$\chi^2 =$

$\sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2/(\Delta\delta^{\exp}(s_i))^2$ – for all partial-wave states

$\sum_{i=1}^{n} (\rho^{\exp}(s_i) - \rho(s_i))^2/(\Delta\rho^{\exp}(s_i))^2$ – for all partial-wave states

$+(a_0^{\exp} - a_0)^2/(\Delta a_0^{\exp})^2$ – for the ${}^1S_0^+$ and ${}^3S_1^+$ partial-wave states

$+(E_d^{\exp} - E_d)^2/(\Delta E_d^{\exp})^2$ – for the ${}^3S_1^+$-${}^3D_1^+$ partial-wave states

$\{+...\}$

δ - the phase shifts, a_0, r_0 - the low-energy parameters (the scattering length, the effective range), E_d - the deuteron binding energy.

Experimental data for analysis

R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, R.L. Workman, PRC76, 025209 (2007) [SP07_3.0 GeV]

SAID: http://gwdac.phys.gwu.edu/analysis/nn_analysis.html

for uncoupled states $l = J$: $\delta_l,\ \rho_l$

for coupled states $l_< = J-1, l_> = J+1$: $\delta_{l_<}, \delta_{l_>}, \epsilon_J,\ \rho_{l_<}, \rho_{l_>}, \mu_J$

All results are presented as a function of the kinetic energy $T_{\rm lab} = s/2m_N - 2m_N$

Partial-wave states classification

$$s_1 = s_2 = 1/2 : \quad S = 0, 1$$

$$L : \; S(L=0), \; P(L=1), \; D(L=2), \; F(L=3), \; G(L=4)$$

$$\vec{J} = \vec{L} + \vec{S} : \quad {}^{2S+1}L_J$$

$$J = 0 \; \begin{cases} {}^1S_0 \\ {}^3P_0 \end{cases}$$

$$J = 1 \; \begin{cases} {}^1P_1 \\ {}^3P_1 \\ {}^3S_1 -{}^3D_1 \end{cases}$$

$$J = 2 \; \begin{cases} {}^1D_2 \\ {}^3D_2 \\ {}^3P_2 -{}^3F_2 \end{cases}$$

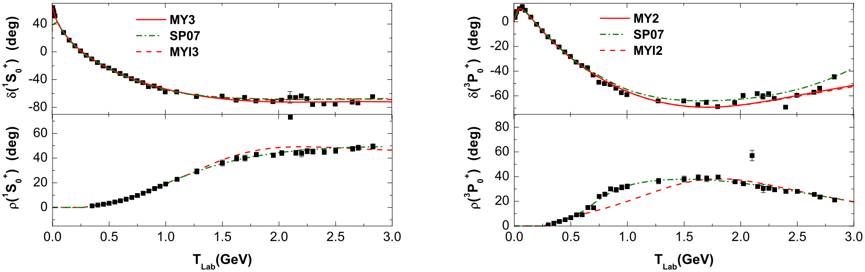
$$J = 3 \; \begin{cases} {}^1F_3 \\ {}^3F_3 \\ {}^3D_3 -{}^3G_3 \end{cases}$$

Other groups results:

- Nonrelativistic (or semi-relativistic) (realistic) potentials – BONN, CD-BONN, PARIS, Reid, Argone v_{18}, ... – and their separable forms some of them
- Covariant spectator theory (Gross, Stadler ...)
- Equation with LF dynamics (Karmanov, Carbonell)
- BS equation with OME kernel (Tjon, Fleischer)
- BSLT equation with two-pion exchange kernel (Tjon, Cozma, Scholten, Timmermans) and Δ (Tjon, Van Faassen)
- BS equation for deuteron and ${}^1S_0$-state with OME kernel (Dorkin, Kaptari, Semikh)

$$J = 0 \begin{cases} {}^1S_0 \\ {}^3P_0 \end{cases}$$

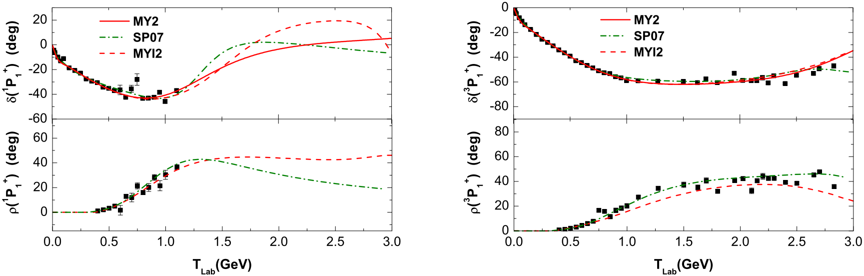
${}^1S_0$ and ${}^3P_0$ partial-wave state



Phase shifts δ and inelasticity parameter ρ

$$J = 1 \begin{cases} {}^1P_1 \\ {}^3P_1 \\ {}^3S_1 - {}^3D_1 \end{cases}$$

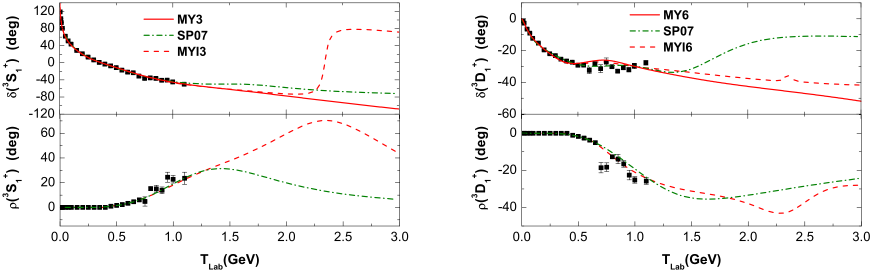
${}^1P_1$ and ${}^3P_1$ partial-wave state



Phase shifts δ and inelasticity parameter ρ

$$J = 1 \begin{cases} {}^1P_1 \\ {}^3P_1 \\ {}^3S_1 - {}^3D_1 \end{cases}$$

$^3S_1 - ^3D_1$ coupled partial-wave states



Phase shifts δ and inelasticity parameter ρ

$$J = 1 \begin{cases} {}^1P_1 \\ {}^3P_1 \\ {}^3S_1 - {}^3D_1 \end{cases}$$

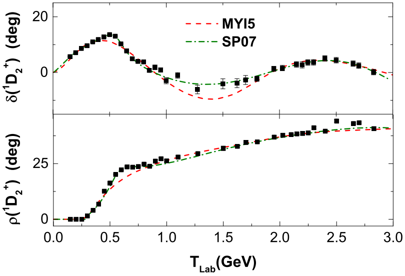
${}^3S_1 - {}^3D_1$ coupled partial-wave states

Deuteron and low-energy scattering properties

	a_{0t} (fm)	r_{0t} (fm)	p_D (%)	E_d (MeV)	$\rho_{D/S}$	μ_d $(e/2m)$
MYI6	5.42	1.800	4.92	2.2246	0.0255	0.8500
Graz-II	5.42	1.779	5	2.2254	0.0269	0.8512
Paris	5.43	1.770	5.77	2.2249	0.0261	0.8469
CD-Bonn	5.4196	1.751	4.85	2.2246	0.0256	0.8522
Exp.	5.424(4)	1.759(5)	4-7	2.224644(46)	0.0256(4)	0.8574

$$J = 2 \begin{cases} {}^1\boldsymbol{D_2} \\ {}^3D_2 \\ {}^3P_2 - {}^3F_2 \end{cases}$$

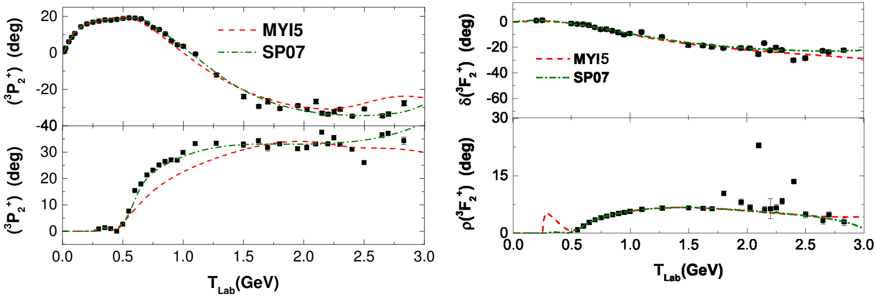
${}^1\boldsymbol{D_2}$ partial-wave state



Phase shifts δ and inelasticity parameter ρ

$$J = 2 \begin{cases} {}^1D_2 \\ {}^3D_2 \\ {}^3P_2 - {}^3F_2 \end{cases}$$

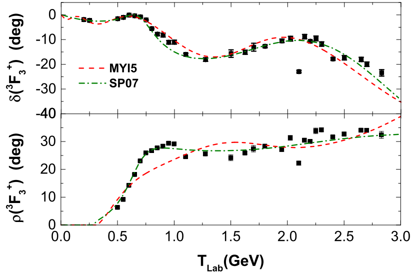
${}^3P_2 - {}^3F_2$ partial-wave state



Phase shifts δ and inelasticity parameter ρ

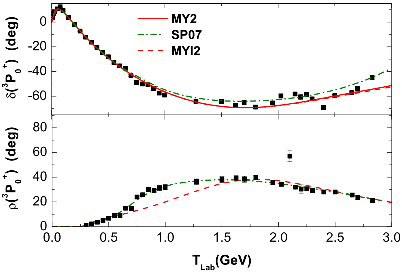
$$J = 3 \begin{cases} {}^1F_3 \\ {}^3\boldsymbol{F_3} \\ {}^3D_3 - {}^3G_3 \end{cases}$$

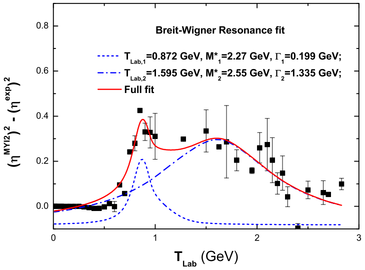
${}^3F_3$ partial-wave state



Phase shifts δ and inelasticity parameter ρ

$^3P_0$ partial-wave state



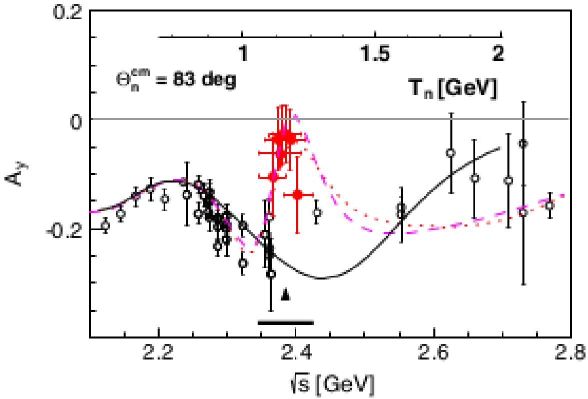


Phase shifts δ and inelasticity parameter ρ

$$\Delta\eta^2 = (\eta^{exp})^2 \text{-} (\eta^{MYI})^2$$

Parameters of wide dibaryons (PRELIMINARY!)

T_{Lab} (GeV)	M^* (GeV)	Γ^* (GeV)	Partial-wave state	J	S	T
0.856	2.26	0.200	$^3P_2$	2	1	0
0.872	2.27	0.199	$^3P_0$	0	1	0
0.874	2.27	0.206	$^3F_2$	2	1	0
1.161	2.39	0.245	$^1D_2$	2	0	1
1.505	2.52	0.639	$^3F_3$	3	1	0
1.555	2.54	0.238	$^1D_2$	2	0	1
1.595	2.55	1.335	$^3P_0$	0	1	0
1.760	2.61	2.264	$^3F_2$	2	1	0
2.100	2.73	0.118	$^3F_2$	2	1	0
2.381	2.83	0.543	$^1S_0$	0	0	1
2.700	2.93	0.374	$^3F_3$	3	1	0



Narrow resonance-like structure $I(J^P) = 0(3^+)$
$M^* = 2380$ MeV, $\Gamma = 70$ MeV observed by COSY

"Evidence for a New Resonance from Polarized Neutron-Proton Scattering",
P. Adlarson et al. (WASA-at-COSY Collaboration, SAID Data Analysis Center)
PRL **112** 202301 (2014)

Bethe-Salpeter-Faddeev equation

$$\begin{bmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} - \begin{bmatrix} 0 & T_1 G_1 & T_1 G_1 \\ T_2 G_2 & 0 & T_2 G_2 \\ T_3 G_3 & T_3 G_3 & 0 \end{bmatrix} \begin{bmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{bmatrix},$$

where full three-particles T matrix $T = \sum_i T^{(i)}$, G_i is the free two-particles (j and n) Green function (ijn is cyclic permutation of (1,2,3)):

$$G_i(k_j, k_n) = 1/(k_j^2 - m_N^2 + i\epsilon)/(k_n^2 - m_N^2 + i\epsilon),$$

and T_i is the two-particles T matrix which can be written as following

$$T_i(k_1, k_2, k_3; k_1', k_2', k_3') = (2\pi)^4 \delta^{(4)}(k_i - k_i') T_i(k_j, k_n; k_j', k_n').$$

with $s_i = (k_j + k_n)^2 = (k_j' + k_n')^2$.

Partial-wave three-nucleon functions for rank-one separable NN-kernel

$$\Psi^{(a)}_{\lambda L}(p_0,|\mathbf{p}|,q_0,|\mathbf{q}|;s) = g^{(a)}(p_0,|\mathbf{p}|)\tau^{(a)}[(\frac{2}{3}\sqrt{s}+q_0)^2-\mathbf{q}^2]\,\Phi^{(a)}_{\lambda L}(q_0,|\mathbf{q}|;s)$$

System of the integral equations

$$\Phi^{(a)}_{\lambda L}(q_0,|\mathbf{q}|;s) = \frac{i}{4\pi^3}\sum_{a'\lambda'}\int_{-\infty}^{\infty}dq_0'\int_0^{\infty}\mathbf{q}'^2 d|\mathbf{q}'|\,Z^{(aa')}_{\lambda\lambda'}(q_0,q;q_0',|\mathbf{q}'|;s)$$

$$\frac{\tau^{(a')}[(\frac{2}{3}\sqrt{s}+q_0')^2-\mathbf{q}'^2]}{(\frac{1}{3}\sqrt{s}-q_0')^2-\mathbf{q}'^2-m^2+i\epsilon}\Phi^{(a')}_{\lambda' L}(q_0',|\mathbf{q}'|;s)$$

with effective kernels of equation

$$Z^{(aa')}_{\lambda\lambda'}(q_0,|\mathbf{q}|;q_0',|\mathbf{q}'|;s) = C_{(aa')}\int d\cos\vartheta_{\mathbf{q}\mathbf{q}'}K^{(aa')}_{\lambda\lambda' L}(|\mathbf{q}|,|\mathbf{q}'|,\cos\vartheta_{\mathbf{q}\mathbf{q}'})$$

$$\frac{g^{(a)}(-q_0/2-q_0',|\mathbf{q}/2+\mathbf{q}'|)g^{(a')}(q_0+q_0'/2,|\mathbf{q}+\mathbf{q}'/2|)}{(\frac{1}{3}\sqrt{s}+q_0+q_0')^2-(\mathbf{q}+\mathbf{q}')^2-m_N^2+i\epsilon}$$

Method of solution

- Wick-rotation procedure: $q_0 \to iq_4$
- The Gaussian quadrature with $N_1 \times N_2[q_4 \times |\mathbf{q}|]$ grid

$$q_4 = (1+x)/(1-x)$$

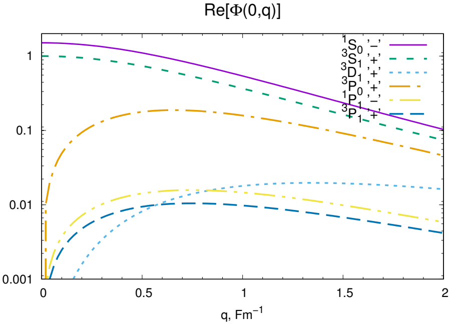
$$|\mathbf{q}| = (1+y)/(1-y)$$

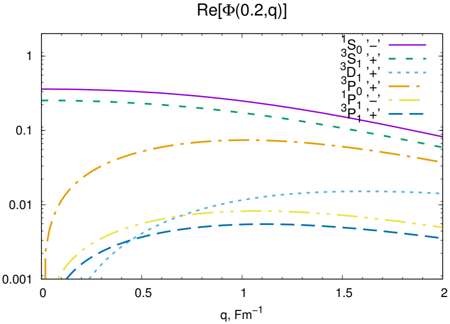
- Iteration method to obtain the triton binding energy

$$\lim_{n\to\infty} \frac{\Phi_n(s)}{\Phi_{n-1}(s)}\Big|_{s=M_B^2} = 1$$

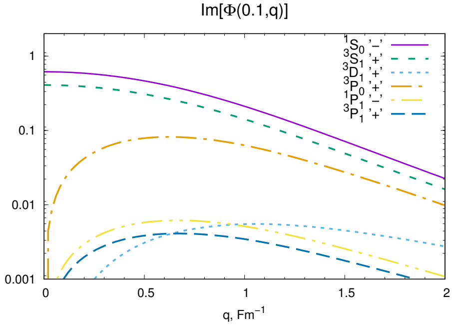
The convergence was investigated and $N_1 = 96$, $N_2 = 15$ was used in calculations

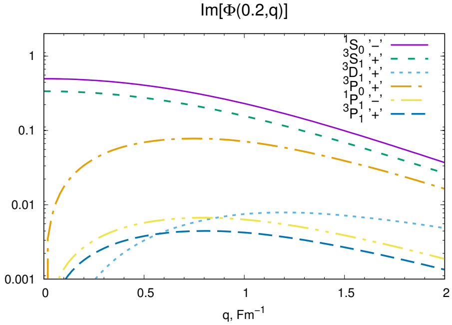
Real part at $q_4 = 0$ Fm^{-1} and $q_4 = 0.2$ Fm^{-1}





Imaginary part at $q_4 = 0.1$ Fm^{-1} and $q_4 = 0.2$ Fm^{-1}





Triton binding energy (MeV)

p_D	$^1S_0 - ^3S_1$	$^3D_1$	$^3P_0$	$^1P_1$	$^3P_1$
4	9.221	9.294	9.314	9.287	9.271
5	8.819	8.909	8.928	8.903	8.889
6	8.442	8.545	8.562	8.540	8.527
Exp.		8.48			

- the main contribution is from S-states
- the D-state contribution is about 0.8 – 1.2 % depending on D-wave (pseudo)probability in deuteron
- the P-state contributions are alternating and give about -0.2%

Future tasks

- To use amplitudes from phase-shifts analysis to estimate final-state interaction photo- ($\gamma d \to np$) and exclusive electrodisintegration ($ed \to enp$) of deuteron for JLab experiments
- To calculate three-nucleon form EM form factors ($e^3He \to e^3He$) for JLab experiments
- To extend Bethe-Salpeter-Faddeev formalism to scattering processes for @COSY and @NICA experiments ($pd \to pd$, $pd \to ppn$ ...)