# Magnetic fields in Heavy Ion Collisions at NICA energies

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# Ultra relativistic Quantum Molecular Dynamics

The UrQMD model is a microscopic transport theory based on the covariant propagation of all hadrons in classical trajectories.
It represents a solution to a system of partial integral-differential equations using the Monte Carlo method for time evolution for various phase densities $f_i(\mathbf{x},\mathbf{p})$ as a solution of the relativistic Boltzman's equation $[1]$:

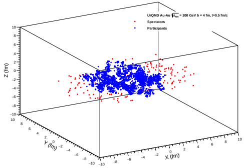
$$\frac{df_i(\mathbf{x},\mathbf{p})}{dt} = \frac{\partial p}{\partial t}\frac{\partial f_i(\mathbf{x},\mathbf{p})}{\partial p} + \frac{\partial x}{\partial t}\frac{\partial f_i(\mathbf{x},\mathbf{p})}{\partial x} + \frac{\partial f_i(\mathbf{x},\mathbf{p})}{\partial t} = Stf_i(\mathbf{x},\mathbf{p}) \quad (1)$$

The Hamiltonian of UrQMD that calculates the wave functions is $[2]$:

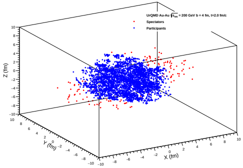
$$H_{UrQMD} = \sum_{j=1}^{N} E_j^{kin} + \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(E_{jk}^{Sk2} + E_{jk}^{Yukawa} + E_{jk}^{Coulomb} + E_{jk}^{Pauli}\right) + \frac{1}{6}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} E_{jkl}^{Sk3} \quad (2)$$

We obtain parameters such as:

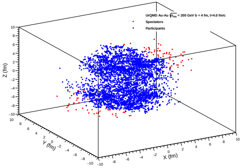
- Time
- $\vec{x}$
- Energy
- $\vec{p}$
- Mass
- Particle ID
- Charge
- Number of collisions
- Strangeness
- Freeze-out's Time, $\vec{x}$, E, $\vec{p}$
- Cross section $\sigma$
- We can calculate $\eta$, $P_t$, $|\vec{P}|$, $|\vec{R}|$.



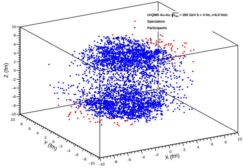

(a) t=0.5 fm



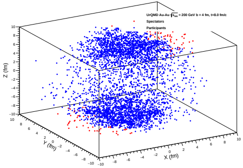

(b) t=2 fm



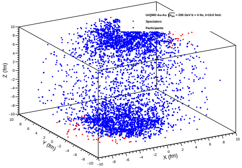

(c) t=4 fm




(d) t=6 fm




(e) t=8 fm




(f) t=10 fm

Figure: Temporal evolution of the collision of two gold ions at $\sqrt{s_{NN}} = 200$ GeV, generated in UrQMD. The spectator particles are shown in red and the participants in blue [3].

# Glauber Model

This model is used to calculate "geometric" quantities, which are typically expressed as impact parameter (b), number of participating nucleons (Npart) and number of binary nucleon-nucleon collisions (Ncoll) [4].
The nuclear charge density is parameterized with the Fermi distribution:

$$\rho(r) = \rho_0 \frac{1 + w(r/R)^2}{1 + exp(\frac{r-R}{a})} \tag{3}$$

where $\rho_0$ corresponds to the nucleon density in the center of the nucleus, $R$ corresponds to the nuclear radius, $a$ to the “skin depth” and $w$ characterizes deviations from a spherical shape.

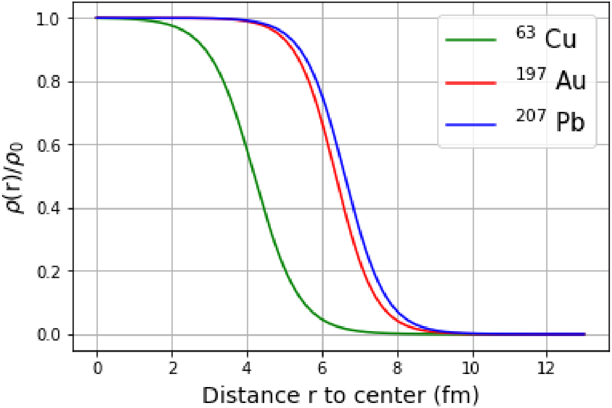



Figure: Density distributions for nuclei used at RHIC.

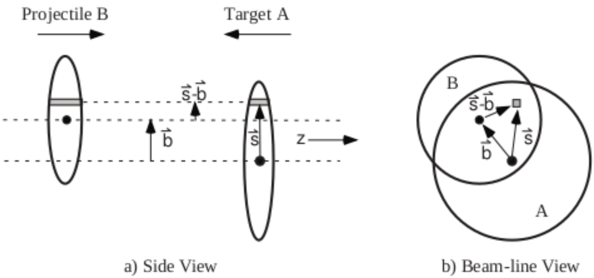



Figure: Schematic representation of the Optical Glauber Model geometry, with transverse (a) and longitudinal (b) views.

The probability per unit transverse area of a given nucleon being located in the target flux tube is:

$$\hat{T}_A(\mathbf{s}) = \int \hat{\rho}_A(\mathbf{s}, Z_A) dZ_A \tag{4}$$

Then the effective overlap area for which a specific nucleon in A can interact with a given nucleon in B is:

$$\hat{T}_{AB}(\mathbf{b}) = \int \hat{T}_A(\mathbf{s})\hat{T}_B(\mathbf{s}-\mathbf{b})d^2s \tag{5}$$

The probability of an interaction occurring is then:

$$\hat{T}_{AB}(\mathbf{b})\sigma_{inel}^{NN} \tag{6}$$

The probability of having n interactions between type A nuclei (denoting $A$ as the number of nucleons) and B (denoting $B$ as the number of type B nucleons) is given by the binomial distribution:

$$P(n, \mathbf{b}) = \binom{AB}{n} \left(\hat{T}_{AB}(\mathbf{b})\sigma_{inel}^{NN}\right)^n \left(1 - \hat{T}_{AB}(\mathbf{b})\sigma_{inel}^{NN}\right)^{AB-n} \quad (7)$$

The total probability of an interaction between A and B is:

$$\frac{d^2\sigma_{inel}^{A+B}}{db^2} = P_{inel}^{A+B}(b) = \sum_{n=1}^{AB} P(n, \mathbf{b}) = 1 - \left(1 - \hat{T}_{AB}(\mathbf{b})\sigma_{inel}^{NN}\right)^{AB} \quad (8)$$

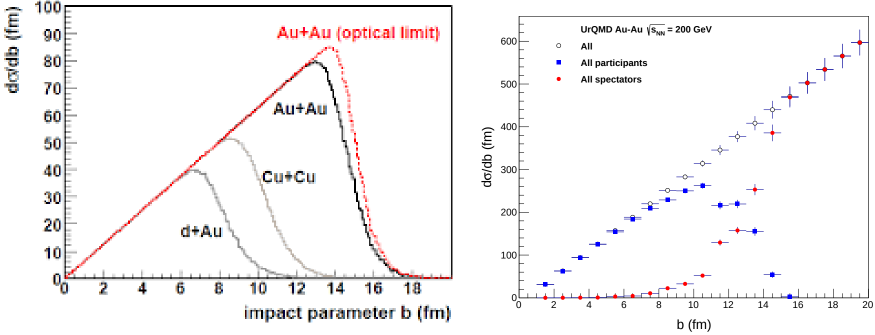



Figure: (Left) Inelastic geometrical cross section from Glauber Monte Carlo calculations (d+Au, Cu+Cu, and Au+Au at $\sqrt{s_{NN}} = 200$ GeV). (Right) Total geometrical cross section calculated in the simulator UrQMD at $\sqrt{s_{NN}} = 200$ GeV.

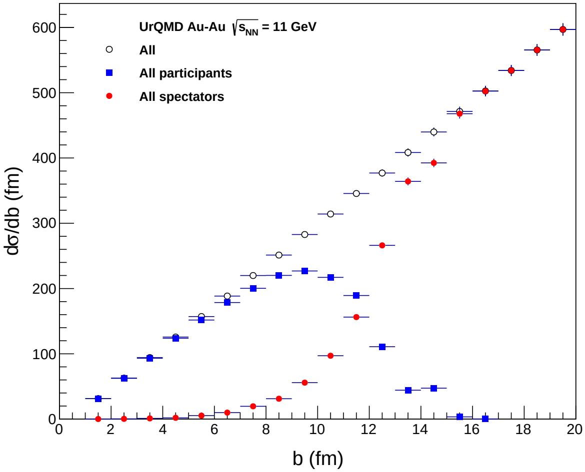



Figure: Total geometrical cross section calculated in the simulator UrQMD at $\sqrt{s_{NN}} = 11$ GeV.

The total number of nucleon-nucleon collisions is:

$$N_{coll}(b) = \sum_{n=1}^{AB} nP(n,b) = AB\hat{T}_{AB}(b)\sigma_{inel}^{NN} \tag{9}$$

The number of participants (or wounded nucleons) at impact parameter b is given by:

$$\begin{aligned} N_{part}(\mathbf{b}) = A \int \hat{T}_A(\mathbf{s}) \left[1 - \left(1 - \hat{T}_B(\mathbf{s}-\mathbf{b})\sigma_{inel}^{NN}\right)^B\right] d^2s \\ + B \int \hat{T}_B(\mathbf{s}-\mathbf{b}) \left[1 - \left(1 - \hat{T}_A(\mathbf{s})\sigma_{inel}^{NN}\right)^A\right] d^2s \end{aligned} \tag{10}$$

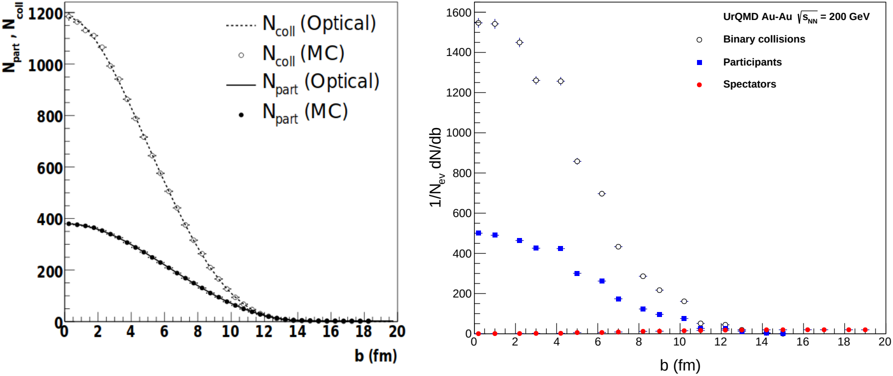



Figure: (Left) $N_{coll}$ and $N_{part}$ as a function of impact parameter, calculated in the optical approximation (lines) and with a Glauber Monte Carlo (symbols). (Right) $N_{coll}$ and $N_{part}$ as a function of impact parameter, simulated in UrQMD.

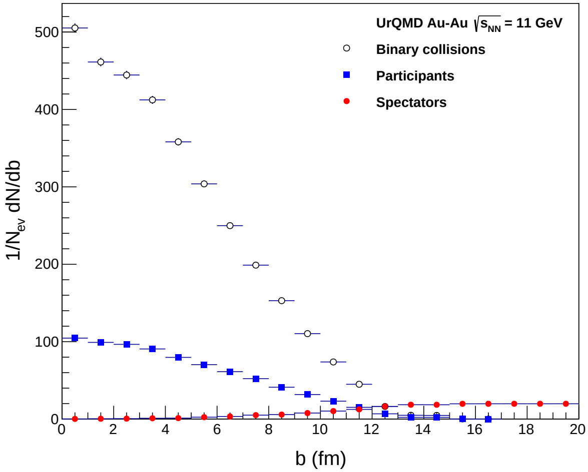



Figure: $N_{coll}$ and $N_{part}$ as a function of impact parameter, calculated in the simulator UrQMD at $\sqrt{s_{NN}} = 11$ GeV.

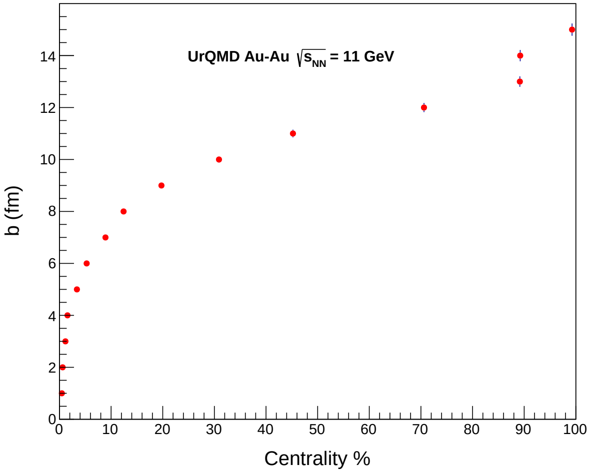



Figure: Impact parameter versus centrality at $\sqrt{s_{NN}} = 11$ GeV..

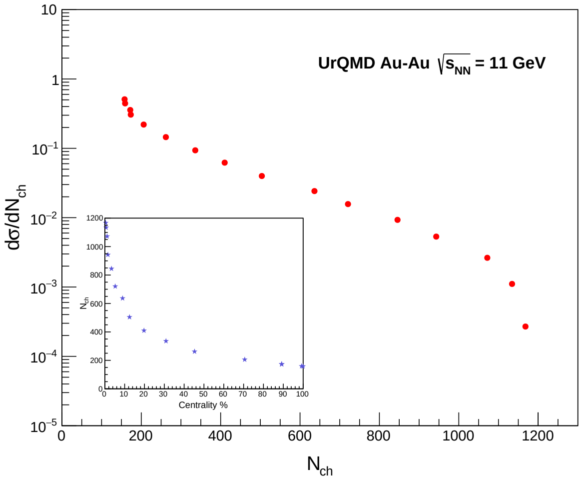



Figure: Correlation of the final state observable $N_{ch}$ with the cross section calculated in UrQMD at $\sqrt{s_{NN}} = 11$ GeV. (Inside) Relation between the centrality classes and the observable $N_{ch}$.

# Electromagnetic fields at heavy ion collisions

The Liénard-Wiechert potentials describe electromagnetic fields of a distribution of moving charges in terms of the vector potential $(\mathbf{A})$ and the scalar potential $(\phi)$.

$$\phi(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R - \mathbf{R}\cdot\mathbf{v}(t)}\right]_{t=t_{ret}} \tag{11}$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\left[\frac{q\mathbf{v}(t)}{R - \mathbf{R}\cdot\mathbf{v}(t)}\right]_{t=t_{ret}} \tag{12}$$

It is considered a new parameter called time delay. Which is related to the time it takes to interact this potentials from the charged moving particle with the observer.

$$t_{ret} = t - \frac{|\mathbf{r} - \mathbf{r}'|}{c} = t - \frac{|\mathbf{R}|}{c} \tag{13}$$

Obtained directly from Maxwell's equations, these potentials fully and relativistically describe the electromagnetic field that varies over time from a point charge in arbitrary motion but without considering mechanical-quantum phenomena [5].

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{(\hat{\mathbf{n}} - \mathbf{v})(1 - v^2)}{g^3 R^2} + \frac{\hat{\mathbf{n}} \times \{(\hat{\mathbf{n}} - \mathbf{v}) \times \dot{\mathbf{v}}\}}{g^3 R}\right]_{ret} = \mathbf{E_v} + \mathbf{E_a} \quad (14)$$

$$\mathbf{B} = \frac{\mu_0 q}{4\pi}\left[\frac{(\mathbf{v} \times \hat{\mathbf{n}})(1 - v^2)}{g^3 R^2} + \frac{(\mathbf{v} \times \hat{\mathbf{n}})(\dot{\mathbf{v}} \cdot \hat{\mathbf{n}}) + g\dot{\mathbf{v}} \times \hat{\mathbf{n}}}{g^3 R}\right]_{ret} = \mathbf{B_v} + \mathbf{B_a} \quad (15)$$

Where $g = 1 - \mathbf{R} \cdot \mathbf{v}$

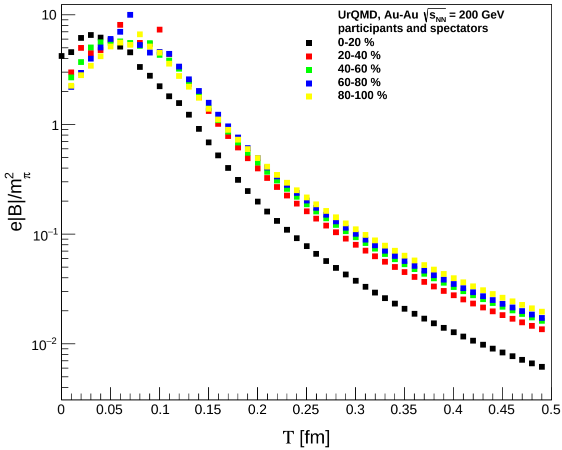



Figure: Mean magnetic field strength produced by spectators and participants at the middle of the interaction region as a function of time for all centrality classes $0-20\%$, $20-40\%$, $40-60\%$, $60-80\%$ and $80-100\%$ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

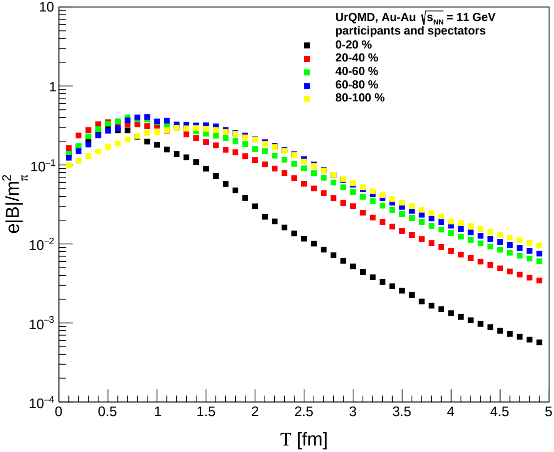



Figure: Mean magnetic field strength produced by spectators and participants at the middle of the interaction region as a function of time for all centrality classes $0 - 20\%$, $20 - 40\%$, $40 - 60\%$, $60 - 80\%$ and $80 - 100\%$ in Au+Au collisions at $\sqrt{s_{NN}} = 11$ GeV.

# Conclusions

- UrQMD gives a more complete view of physics in heavy ion collisions from the point of view of the Glauber model.
- Heavy ion collisions generate magnetic fields with a magnitude of $10^{18}$ gauss at RHIC energies and $10^{17}$ gauss at NICA energies.
- The pulse of the magnetic field produced at RHIC energies is of an order of magnitude smaller compared to the produced at NICA energies since the system evolves more slowly because the center of mass energy is smaller.
- The value of the magnitude of the magnetic field and the pulse evolution time is a variable that can be used to calculate certain physical observables in the experiment [6].

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