Moving mirrors in 2d QFT

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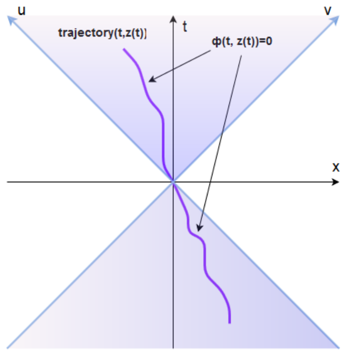
Ideal mirror

K-G equation and boundary condition

$$(\partial_t^2 - \partial_x^2 + m^2)\,\phi(t,x) = 0,\ \phi\big(t,z(t)\big) = 0$$



$\big(t,z(t)\big)$-trajectory of the ideal mirror

Set up of the problem

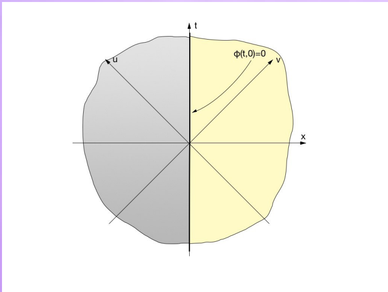
- Expand field ϕ(t,x) in terms of space-time harmonics according to the mirror trajectory z(t)
- Quantize the field, satisfy the canonical commutation relations
- Investigate the vacuum average of tx-component of the stress-energy tensor $\langle T_{tx} \rangle$, responsible for the flux of energy density
- Obtain the Hamiltonian, the system evolution operator

Mirror at rest

In this case available only area for $x \geq 0$,

Field and boundary condition

$$\phi(t,x) = i \int_0^\infty \frac{dk}{2\pi} \sqrt{\frac{2}{\omega}} \sin(kx) \left[a_k e^{-ikt} - a_k^\dagger e^{ikt}\right], \phi(t,0) = 0$$



$$[a_k, a_{k'}^\dagger] = 2\pi\delta(k - k')$$

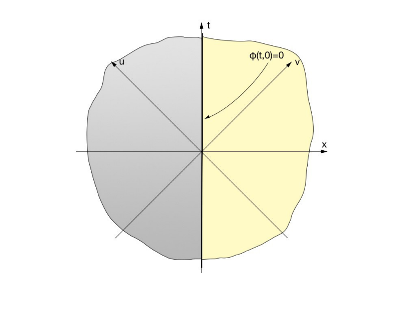
$$[\phi(t,x), \pi(t,y)] = i\left[\delta(x-y) - \delta(x+y)\right]$$

where $\pi(t,y) = \partial_t \phi(t,y)$
- canonical momentum

Mirror at rest

Symmetrized stress-energy tensor

$$T_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\phi\,\partial_\nu\phi + \partial_\nu\phi\,\partial_\mu\phi\right) - \frac{1}{2}g_{\mu\nu}\left(\partial_\alpha\phi\,\partial^\alpha\phi + m^2\phi^2\right), \partial^\mu T_{\mu\nu} = 0$$



$$H = \int_0^\infty T_{tt}\,dx$$

$$T_{tx} = \frac{1}{2}\left(\partial_t\phi\,\partial_x\phi + \partial_x\phi\,\partial_t\phi\right).$$

$$\langle T_{tx}\rangle = 0$$

$$H = \int_0^{+\infty}\frac{dk}{2\pi}\,\frac{\omega}{2}\left(a_k a_k^\dagger + a_k^\dagger a_k\right)$$

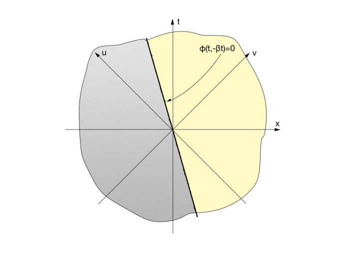
Mirror with constant velocity

In this case consider the velocity $0 < \beta < 1$, $\phi(t, -\beta t) = 0$

Field

$$\phi(t,x) = i\int_{\gamma\beta m}^{+\infty} \frac{dk}{2\pi}\frac{1}{\sqrt{2\omega}} a_k \left(e^{-i\omega t - ikx} - e^{-i\omega_r t + ik_r x}\right) + h.c.$$

$$[\phi(t,x),\pi(t,y)] = i\left[\delta(x-y) - \delta\left(2\beta\gamma^2 t + (1+\beta^2)\gamma^2 x + y\right)\right]$$



$$\omega_r = (1+\beta^2)\gamma^2\omega - 2\beta\gamma^2 k$$

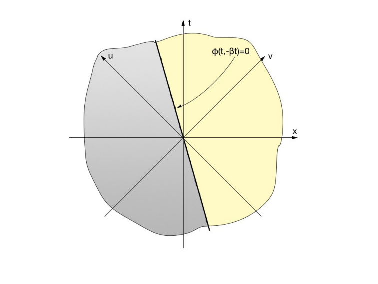
$$k_r = 2\beta\gamma^2\omega + (1+\beta^2)\gamma^2 k$$

Mirror with constant velocity

$$\langle T_{tx}\rangle = \lim_{\varepsilon\to 0}\frac{1}{2}\langle \partial_t\phi(t,x)\partial_x\phi(t+i\varepsilon,x)+\partial_x\phi(t,x)\partial_t\phi(t+i\varepsilon,x)\rangle$$

Vacuum average of tx-component

$\langle T_{tx}\rangle = -\frac{1}{2\pi}\gamma^2\beta m^2 K_0(2m\gamma(x+\beta t))$,$K_0$-McDonald function



For each fixed x, as $t\to+\infty$, $\langle T_{tx}\rangle\to 0$

Boost the mirror at rest

$\langle T_{tx}\rangle = \beta\gamma^2(\langle T_{t't'}\rangle + \langle T_{x'x'}\rangle - - \langle T_{t't'}\rangle_{0,M} - \langle T_{x'x'}\rangle_{0,M}) = -\frac{1}{2\pi}m^2\beta\gamma^2 K_0(2mx')$

Necessary to subtract vacuum average or to do big mass regularization

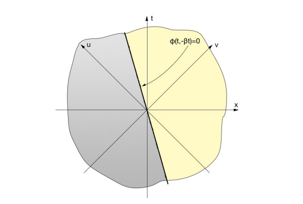
Mirror with constant velocity

$$H = \int_{-\beta t}^{\infty} T_{tt}\, dx = \frac{1}{2}\int_{-\beta t}^{+\infty}[(\partial_t\phi)^2 - \phi\partial_t^2\phi]dx,\ P = \int_{-\beta t}^{\infty} dx\, T_{tx}$$

Translation operator

$$H - \beta P = \int_{\gamma\beta m}^{\infty} \frac{dk}{2\pi}\frac{\gamma^2(\omega-\beta k)(\omega-\beta k-\beta(1-\beta)\omega)}{2\omega_r}\left[a_k a_k^\dagger + a_k^\dagger a_k\right]$$

Operator of the translations along the mirror is diagonal unlike the Hamiltonian and Momentum separately



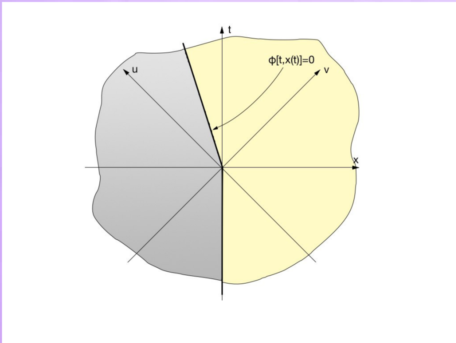
For massless field

$$H - \beta P = (1-\beta)\int_0^{\infty}\frac{dk}{2\pi}\frac{k}{2}\left[a_k a_k^\dagger + a_k^\dagger a_k\right]$$

Conclusions

- Moving mirror violate the homogeneity along the time axis, thus Hamiltonian has non-diagonal terms
- Hamiltonian - operator of translations along the wall
- Chosen method of regularization has a physical sense
- During the Lorenz transformations it is necessary to subtract vacuum terms or to do big mass regularization
- In canonical commutation relation appear the boundary delta-function, need to consider non-ideal mirror.

Plans



- Consider massive case for "broken" non-ideal mirror case
- Consider the interaction $\lambda\phi^4$, obtain the corrections to Keldysh propagator

Thank you for attention!