“Triangular” dilaton charged black holes

Coupling quantization and integrability

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Based on arXiv:1712.06570 [hep-th]
A. Zadora, D.V. Galtsov, C.-M. Chen, 2017

Outline

- Motivation
- Previous results
- New analytic solutions

Type equation here.

- Entropy at extremality
- Discussion

Why to study dilaton BH?

- Dilaton comes from SUGRA and String Theory
- Dilaton comes from KK reduction

APPLICATIONS

- Holography: condensed matter (strange metals etc.)
- Holography: Quark-gluon plasma
- Asymptotically AdS spaces

Charged black holes

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{2(D-2)!} F^2_{[D-2]} - \frac{1}{4} F^2_{[2]} \right)$$

- Two horizons
- Extreme limit: two horizons coincide, T = 0
- Residual entropy at T = 0:

$$S \sim |PQ|$$

↑

CFT (Cardy formula)

Charged dilatonic black holes

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{4} e^{a\phi} F_{[2]}^2 - \frac{1}{2(D-2)!} e^{a\phi} F_{[D-2]}^2 \right)$$

One charge (electric or magnetic) → Entropy vanishes at the extremality **APPEALING FOR HOLOGRAPHY**

Two charges → non-vanishing entropy again (as in RN – charged black holes without dilaton case) *probably not so interesting…*

Previous studies

- 1989: 2 analytic solutions were known (Gibbons, Maeda; Dobiasch, Maison)

$$a = 1, \sqrt{3}$$

from Liouville and sl(3, R) Toda integrable systems

- 1995: triangular quantization conjecture (Wiltshire, Poletti)

$$a^2 = \frac{n(n+1)}{2}$$

- 2013: analytic proof by Taylor expansion from dilaton analyticity:

$$\phi(x) = \phi_0 + \mu x^n + O(x^{n+1}) \,, \quad n \in \mathbb{Z}$$

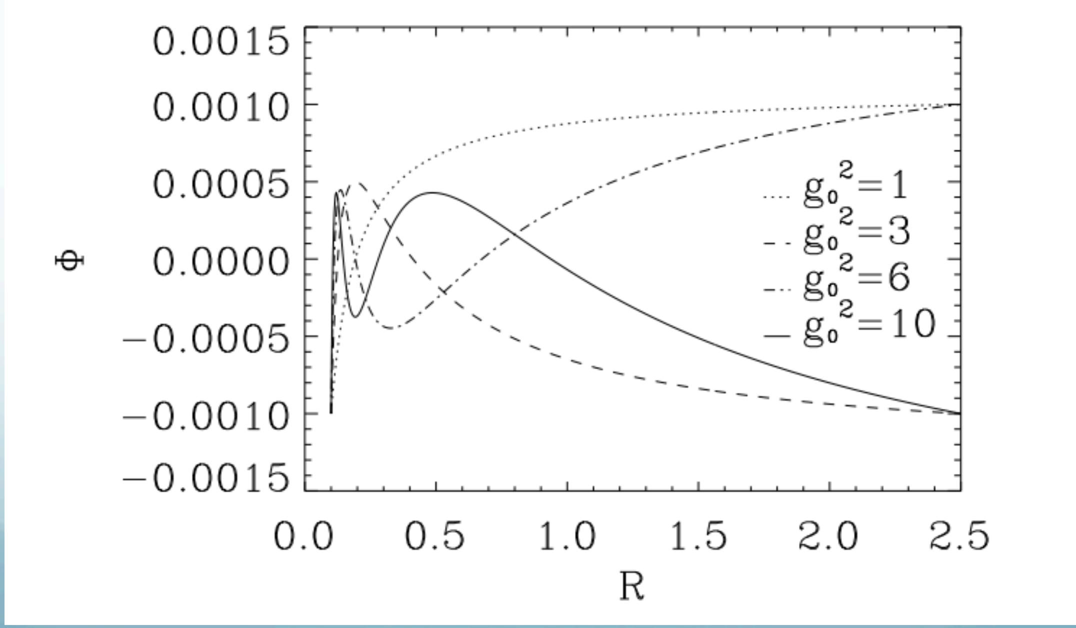
- 2017: 2 new analytic solutions are obtained

Properties (previous results)

- Analyticity of dilaton on the extreme (degenerate) horizon

$$\phi(x) = \phi_0 + \mu x^n + O(x^{n+1}) \,, \quad n \in \mathbb{Z}$$

- Bound states of dilaton between two horizons



$$\Phi = \varphi - \varphi_0$$

New analytic solutions

- **TODA CHAINS** with underlying Lie algebraic structure → integrable systems

$$a \to (a, b)$$

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2(D-2)!} e^{a\phi} F^2_{[D-2]} - \frac{1}{4} e^{b\phi} F^2_{[2]} \right)$$

$$ds^2 = -e^{2B} dt^2 + e^{-\frac{2B}{D-3}} f^{\frac{1}{D-3}} \left(f^{-1} dr^2 + r^2 d\Omega^2_{D-2} \right)$$

$$f = 1 - \frac{2\mu}{r^{D-3}}$$

$$\mathcal{L} = \dot{B}^2 + \frac{\lambda^2}{4}\dot{\phi}^2 + \frac{\lambda^2}{4} e^{2B} \left(P^2 e^{a\phi} + Q^2 e^{-b\phi} \right)$$

$$\lambda^2 = \frac{2(D-3)}{D-2}$$

Toda lattices

$$\mathcal{L} = \dot{B}^2 + \frac{\lambda^2}{4}\dot{\phi}^2 + \frac{\lambda^2}{4}e^{2B}\left(P^2 e^{a\phi} + Q^2 e^{-b\phi}\right)$$

$$H = \frac{1}{2} B_{ij}\dot{p}_i\dot{p}_j + g_i^2 e^{C_{ij}\chi_j}$$

(p_i, χ_i) – canonical variables

$B^{-1}C^T$ is diagonal

C – is Cartan matrix for given Lie algebra G

Toda lattices

$$A_1 \oplus A_1\colon\ ab = 1; \quad A_2\colon\ a = b = \sqrt{3};$$
$$B_2\colon\ a = \pm 2, b = \pm 3; \quad G_2\colon\ a = \frac{5}{\sqrt{3}}, b = 3\sqrt{3}$$

$$\phi(x) = \phi_0 + \mu x^n + O(x^{n+1})\ , \quad n \in \mathbb{Z}$$

$$ab = \frac{n(n+1)}{2}$$

$$H_1 = 1 + \frac{P_1}{r^{D-3}} + \frac{P_2}{r^{2(D-3)}} + \cdots + \frac{P_p}{r^{p(D-3)}}, \quad p = 2\gamma_1,$$
$$H_2 = 1 + \frac{Q_1}{r^{D-3}} + \frac{Q_2}{r^{2(D-3)}} + \cdots + \frac{Q_q}{r^{q(D-3)}}, \quad q = 2\gamma_2.$$

$$\gamma = (\gamma_1, \gamma_2)$$

DUAL WEYL

Entropy at extremality

- **B_2**

$$S_{\text{ext}} = \frac{5\pi}{4 \cdot 2^{2/5} 3^{3/5}} P^{6/5} Q^{4/5}$$

- **G_2**

$$S_{\text{ext}} = \frac{7\pi}{6(3^{2/7})(5^{5/14})} |P|^{9/7} |Q|^{5/7}.$$

$$S_- S_+ = S_{ext}^2$$

What CFT?

Discussion

Non-integrability

$$S_{ext} = \frac{\pi^{\frac{D-1}{2}}}{2\Gamma\left(\frac{D-1}{2}\right)} R_h^2 = \frac{\pi^{\frac{D-1}{2}}}{2\Gamma\left(\frac{D-1}{2}\right)} \left[\left(\frac{a}{b}\right)^{-\frac{a}{a+b}} \frac{a+b}{2b(D-2)(D-3)} P^{\frac{2b}{a+b}} Q^{\frac{2a}{a+b}} \right]^{\frac{1}{D-3}}$$

(if they exist)

CFT?

Strong dependence on system parameters

Bound states between horizons