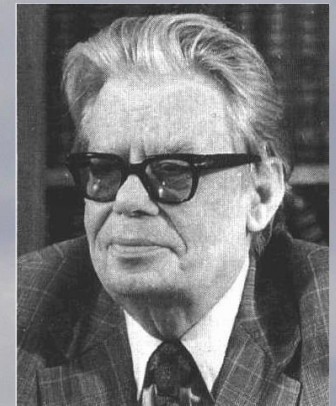
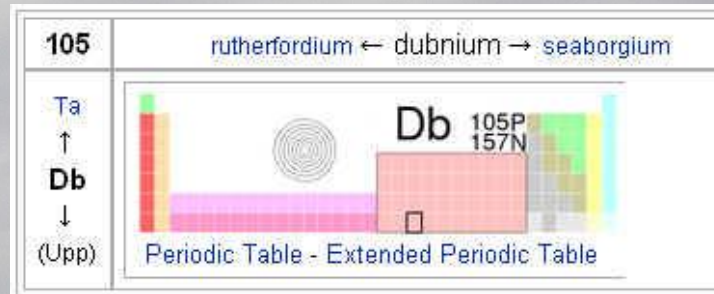


# JINR MEMBER STATES



Agreements are signed on the governmental level with (associated members)





# Физические явления в фи-ноль переходе. Маятник Капицы и переворот магнитного момента

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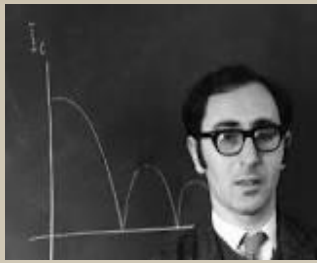
A. Buzdin (LOMA UMR-CNRS, France)

K. Sengupta (IACS, India)

# Outline

1. Introduction
2. Numerical methods
3.  $\Phi$ -0 junctions
4. Kapitza pendulum
5. Magnetization reversal
6. Ferromagnetic resonance
7. Dynamics of magnetization along IVC
8. Devil's staircase in SFS structure
9. Kapitza pendulum features in other models



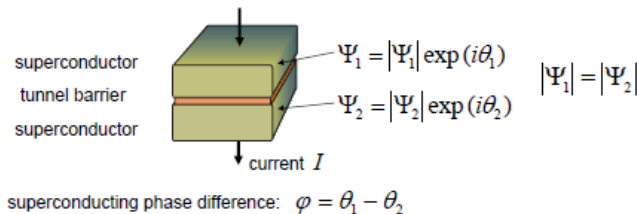


Brian Josephson



Ivar Giaever

### Josephson effect



$$I = \frac{dV}{d\tau} + \beta V + \sin \varphi$$

$$V = \frac{d\varphi}{d\tau}, \quad \text{where } \tau = \omega_p t,$$

$$\omega_p = \frac{2eI_c}{\hbar C} \quad \text{and} \quad \beta = \frac{1}{\omega_p RC}$$

**dc** Josephson effect:

$$I_s(\varphi) = I_c \sin \varphi \quad (1)$$

**ac** Josephson effect:

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \quad (2)$$

From (2) it follows that  $\varphi = \frac{2e}{\hbar} Vt + \varphi_0$ .

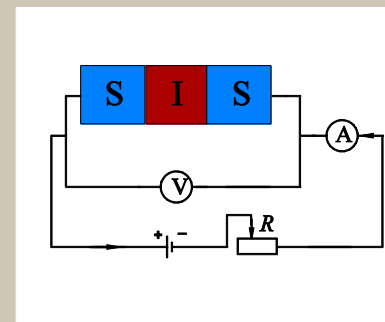
Thus,  $I_s$  oscillates with frequency

$$f = \frac{2e}{2\pi\hbar} V = \frac{1}{\Phi_0} V, \quad (3)$$

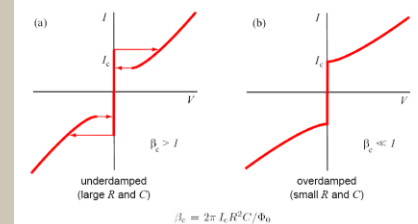
where  $\Phi_0 = 2.068 \times 10^{-15} \text{ Wb}$  is the magnetic flux quantum.

Josephson junction is a **quantum dc voltage - to - frequency converter**

**$1 \mu\text{V} \leftrightarrow 483.59767 \text{ MHz}$**



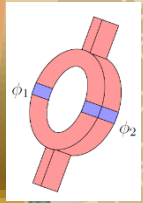
### Effect of damping



Applications in the superconducting electronics, quantum metrology, medicine.  
Particularly, the system is a source of coherent electromagnetic radiation.

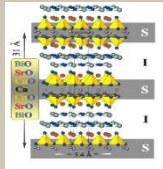


# Superconducting quantum interference device (SQUID)

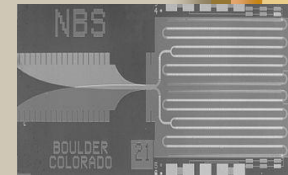


## Nonequilibrium effects in Josephson junctions

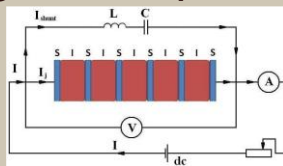
## Voltage standard



Models of JJ systems:  
CCJJ, CCJJ+DC, CIB, SBP

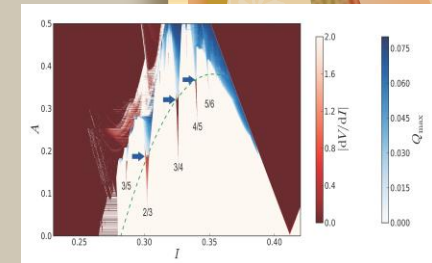


## Shunting of Josephson junctions



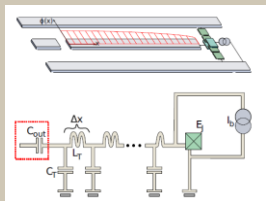
Intrinsic Josephson effect  
R. Kleiner, P. Muller, 1992

Radiation effects  
**Chaos in Josephson Junctions**



## Phase Dynamics of Long Josephson Junctions

### Charge Density Waves



JJ as a single photon detector

## Josephson junction as generator of coherent electromagnetic radiation

## Detector of Majorana fermions

## Quantum computers

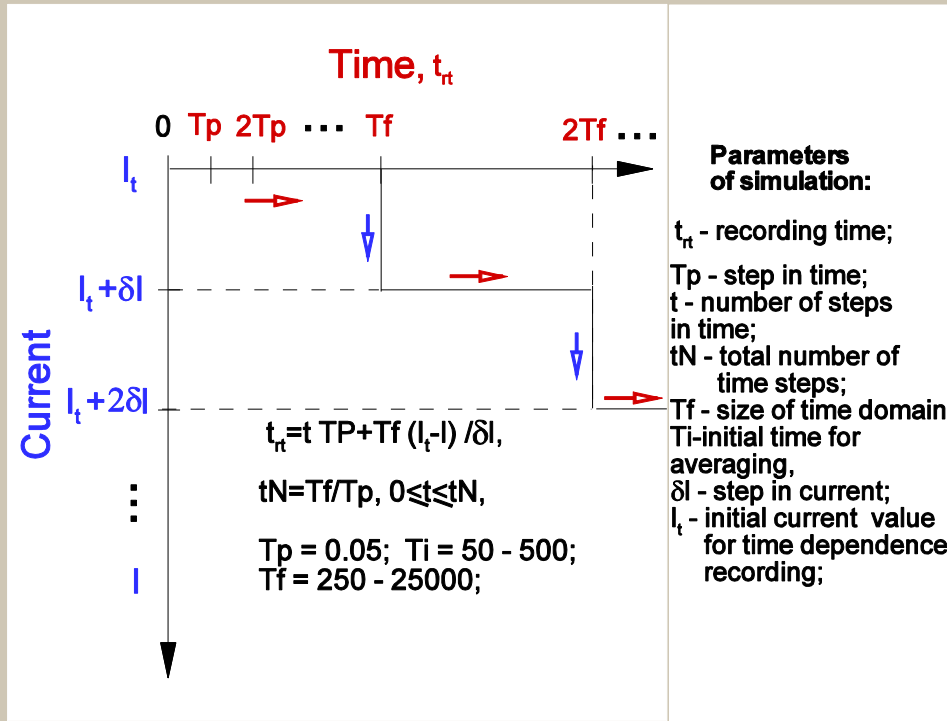
Spintronics (interaction of superconducting current and magnetic moment)



Figure 1 The terahertz gap. The gap, lying roughly between 300 GHz (0.3 THz) and 30 THz, exists because the frequencies generated by transistors and lasers, typical semiconductor devices, don't overlap. No current semiconductor technology can efficiently convert electrical power into electromagnetic radiation in that range. But the 'heterostructure laser' produced by Köhler *et al.* might, in due course, meet the demand for radiation sources at these terahertz wavelengths.



# Simulation procedure for phase dynamics and IV-characteristics

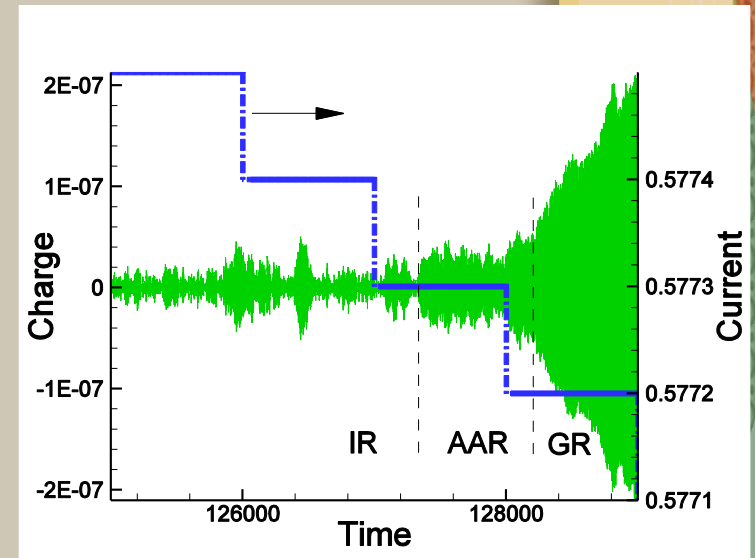


$$\begin{cases} \frac{d\varphi_l}{dt} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l) \\ \frac{dV_l}{dt} = I - \sin \varphi_l - \beta \varphi_l + A \sin(\omega t) \end{cases}$$

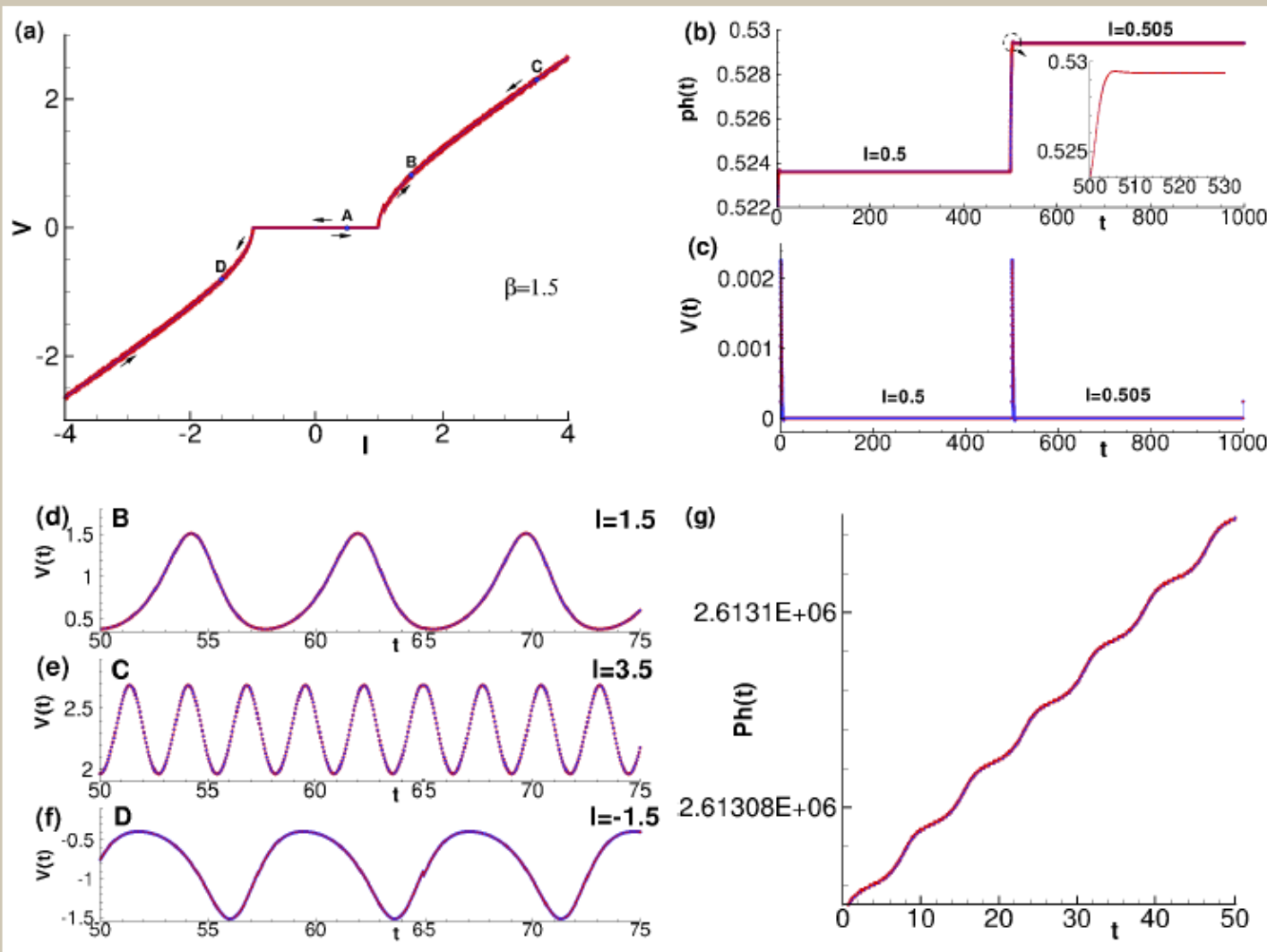
$$\text{div} (\epsilon \epsilon_0 \mathbf{E}) = Q$$

$$Q_l = Q_0 \propto (V_{l+1} - V_l)$$

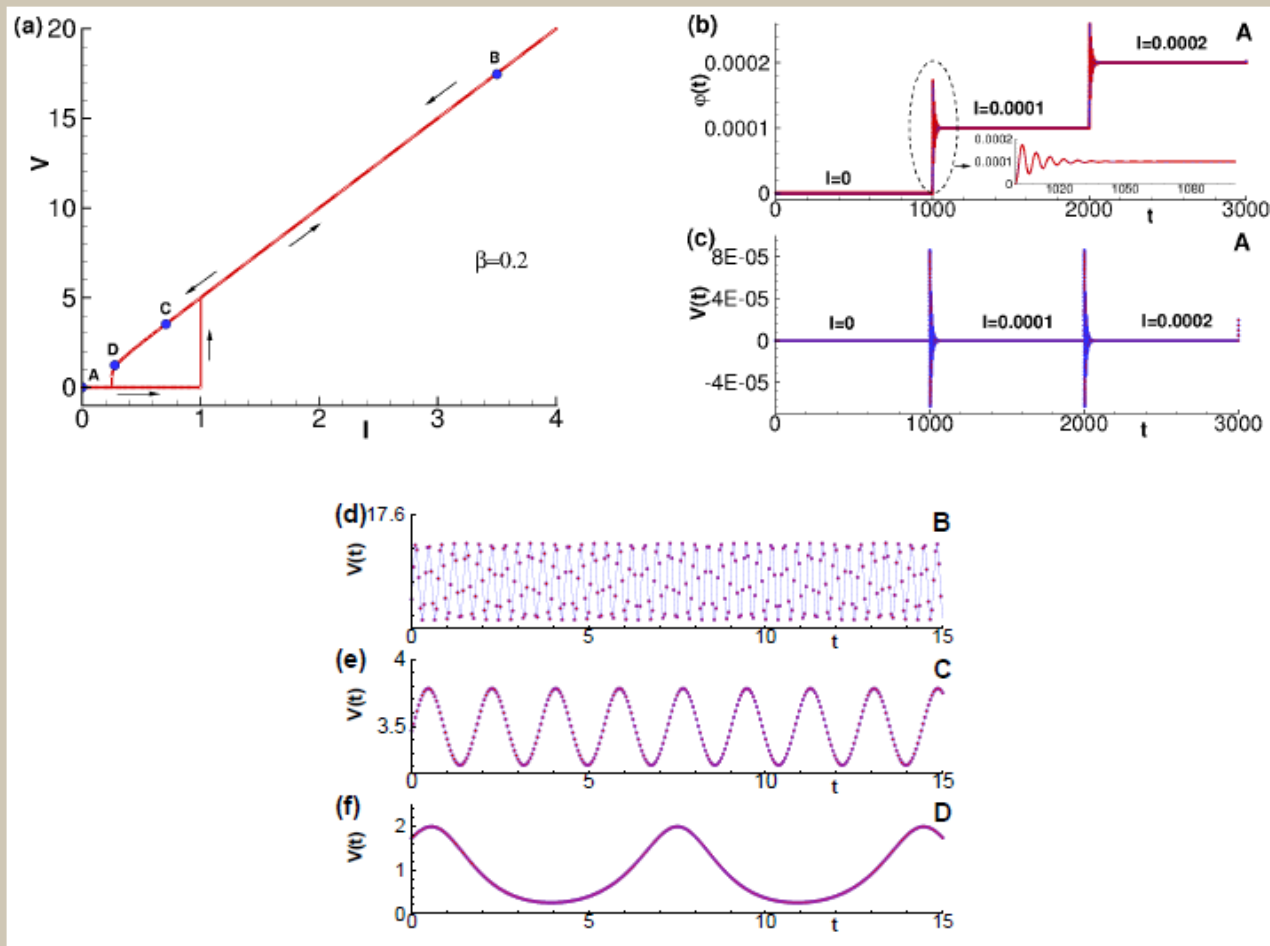
$$Q_0 = \epsilon \epsilon_0 V_0 / r_D^2$$



# Phase dynamics and IV-characteristics of the overdamped JJ

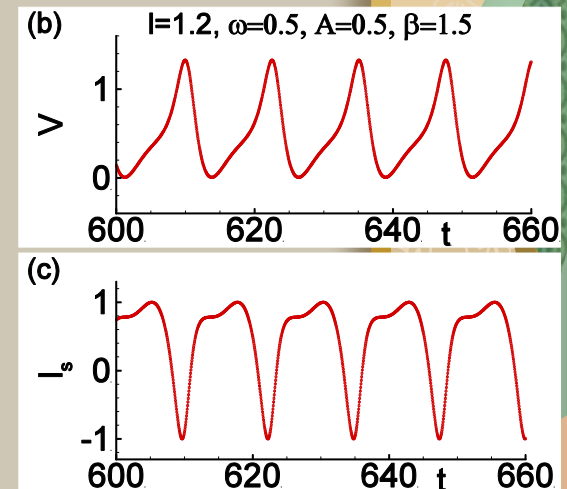
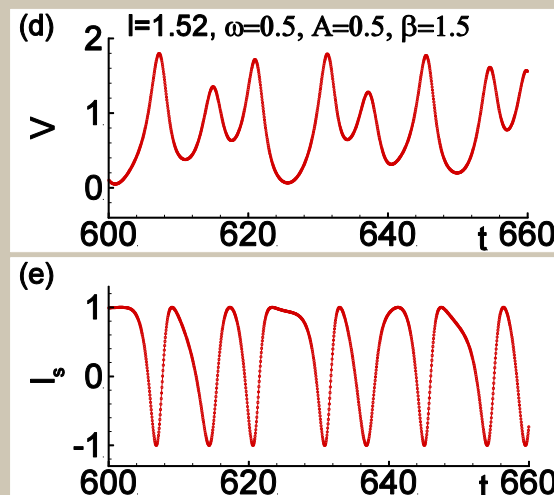
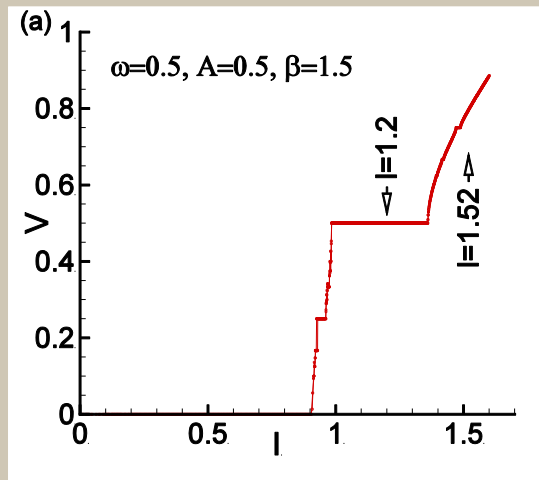
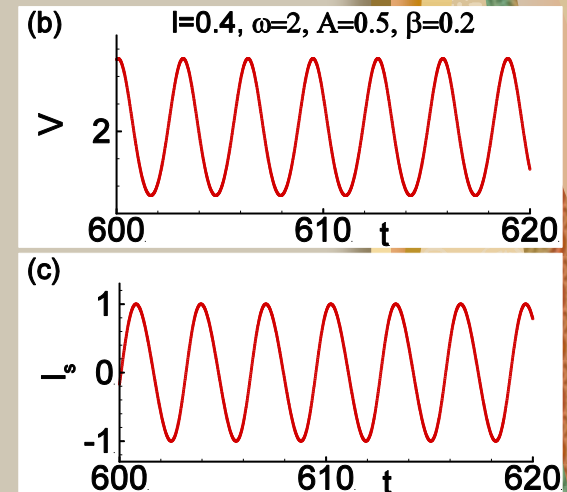
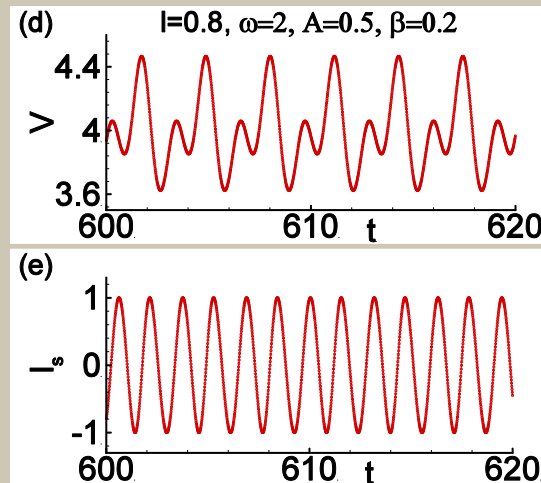
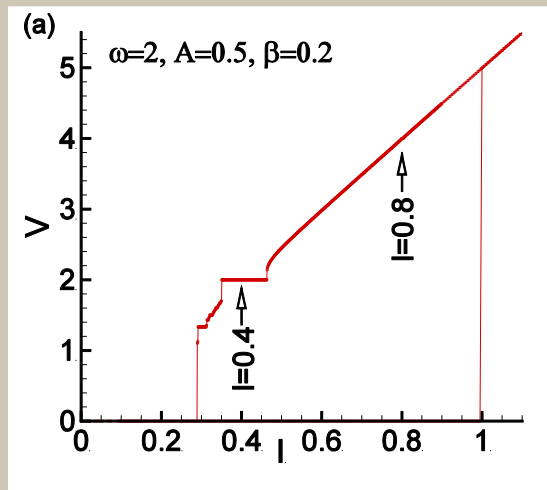


# Phase dynamics and IV-characteristics of the underdamped JJ





# IV-characteristics and dynamics under radiation in the underdamped and overdamped cases



# Phi-0 Junction



# Mechanism for the $\varphi_0$ - Josephson junction realization.

Very special situation is possible when the weak link in Josephson junction is a non-centrosymmetric magnetic metal with broken inversion symmetry !

Suitable candidates : MnSi, FeGe  
of thin ferromagnetic films on the substrate.

Josephson junctions with time reversal symmetry:  $j(-\varphi) = -j(\varphi)$ ;  
i.e. higher harmonics can be observed  $\sim j_n \sin(n\varphi)$  –the case of the  $\pi$  junctions.

Without this restriction a more general dependence is possible  
 $j(\varphi) = j_0 \sin(\varphi - \varphi_0)$ .

**Rashba-type** spin-orbit coupling

$$\alpha(\vec{\sigma} \times \vec{p}) \cdot \vec{n}$$

$\vec{n}$  is the unit vector along the asymmetric potential gradient.

A. Buzdin, PRL, 101,107005 (2008)

## Phi-0 Josephson Junctions

$$F = a|\psi|^2 + \gamma|\vec{D}\psi|^2 + \frac{b}{2}|\psi|^4 - \varepsilon\vec{n} \cdot \{\vec{h} \times [\psi(\vec{D}\psi)^* + \psi^*(\vec{D}\psi)]\}$$

$$D_i = -i\partial_i - 2eA_i$$

$$a\psi - \gamma \frac{d^2\psi}{dt^2} + 2i\varepsilon h \frac{d\psi}{dt} = 0$$

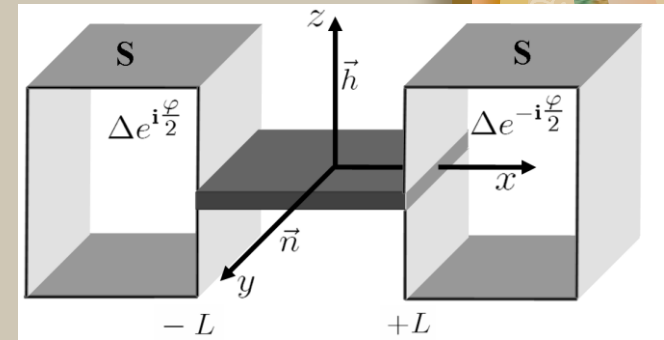
$$\psi = A \exp(q_1 x) + B \exp(q_2 x)$$

$$j = 4e\gamma|\Delta|^2 \sqrt{\frac{a}{\gamma} - \varepsilon^2} \sin(\varphi + 2\varepsilon L) \exp\left(-2 \sqrt{\frac{a}{\gamma} - \varepsilon^2} L\right)$$

$$j(\varphi) = j_c \sin(\varphi + \varphi_0)$$

$$E_J \sim -j_c \cos(\varphi + \varphi_0)$$

A. Buzdin, PRL, 101,107005 (2008)



## Energy of the system:

$$E_{tot} = -\frac{\phi_0}{2\pi} \varphi I + E_s(\varphi, \varphi_0) + E_M(\varphi_0)$$

$$E_s(\varphi, \varphi_0) = E_J [1 - \cos(\varphi - \varphi_0)]$$

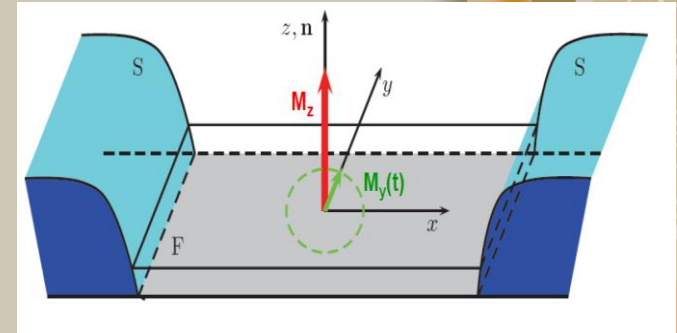
$$E_M = \frac{-KV}{2} \left( \frac{M_z}{M_0} \right)^2 \quad E_J = \frac{\phi_0 I_c}{2\pi} \quad \varphi_0 = l \frac{v_0}{v_F} \frac{M_y}{M_0},$$

Landau-Lifshitz-Gilbert equation:

$$\frac{d\mathbf{M}}{dt} = -\gamma [\mathbf{M} \times \mathbf{H}_{eff}] + \frac{\alpha}{M_0} [\mathbf{M} \times \frac{d\mathbf{M}}{dt}]$$

$$\mathbf{H}_{eff} = \frac{K}{M_0} \left[ \Gamma \sin \left( \varphi - r \frac{M_y}{M_0} \right) \hat{y} + \frac{M_z}{M_0} \hat{z} \right]$$

$$\Gamma = Gr = \frac{E_J}{KV} l \frac{v_0}{v_F}, \quad G = \frac{E_J}{KV}, \quad r = l \frac{v_0}{v_F}, \quad l = 4 \frac{\hbar L}{\hbar v_F}$$



A. Buzdin.  
Phys. Rev. Lett. 101, 107005 (2008).  
F. Konschelle, A. Buzdin.  
Phys. Rev. Lett. 102, 017001 (2009).



# System of equations for magnetization dynamics

$$\begin{cases} \dot{m}_x = \frac{1}{1+\alpha^2} \{-m_y m_z + Grm_z \sin(\omega t - rm_y) - \alpha[m_x m_z^2 + Grm_x m_y \sin(\omega t - rm_y)]\} \\ \dot{m}_y = \frac{1}{1+\alpha^2} \{m_x m_z - \alpha[m_y m_z^2 - Gr(m_z^2 + m_x^2) \sin(\omega t - rm_y)]\} \\ \dot{m}_z = \frac{1}{1+\alpha^2} \{-Grm_x \sin(\omega t - rm_y) - \alpha[Grm_y m_z \sin(\omega t - rm_y) - m_z(m_x^2 + m_y^2)]\} \end{cases}$$

Naturally, we may expect that the most interesting situation corresponds to the case when the magnetic anisotropy energy does not exceed too much the Josephson energy. From the measurements [18] on permalloy with very weak anisotropy, we may estimate  $K \sim 4 \times 10^{-5} \text{ K} \cdot \text{\AA}^{-3}$ . On the other hand, typical value of  $L$  in S/F/S junction is  $L \sim 10 \text{ nm}$  and  $\sin \ell / \ell \sim 1$ . Then, the ratio of the Josephson over magnetic energy would be  $E_J/E_M \sim 100$  for  $T_c \sim 10 \text{ K}$ . Naturally, in the more realistic case of stronger anisotropy, this ratio would be smaller, but it is plausible to expect a great variety of regimes from  $E_J/E_M \ll 1$  to  $E_J/E_M \gg 1$ .

F . Konschelle, A. Buzdin.  
Phys. Rev. Lett . 102,  
017001 ( 2009) .

# Phi-0 Josephson Junction: Analytical Results I

- Magnetic moment precession around z-axis

$$\begin{cases} \dot{m}_x = \frac{1}{1+\alpha^2} \{-m_y m_z + Gr m_z \sin(\omega t - r m_y) - \alpha[m_x m_z^2 + Gr m_x m_y \sin(\omega t - r m_y)]\} \\ \dot{m}_y = \frac{1}{1+\alpha^2} \{m_x m_z - \alpha[m_y m_z^2 - Gr(m_z^2 + m_x^2) \sin(\omega t - r m_y)]\} \\ \dot{m}_z = \frac{1}{1+\alpha^2} \{-Gr m_x \sin(\omega t - r m_y) - \alpha[Gr m_y m_z \sin(\omega t - r m_y) - m_z(m_x^2 + m_y^2)]\} \end{cases}$$

At  $G \ll 1$ ,  $(m_x, m_y) \ll 1$

$$\begin{cases} \dot{m}_x = -m_y + Gr \sin(\omega t) \\ \dot{m}_y = m_x \end{cases}$$

$$m_x = \frac{Gr \omega \cos \omega t}{1 - \omega^2} \quad \text{and} \quad m_y = \frac{Gr \sin \omega t}{1 - \omega^2}$$

- Second harmonic :  
its monitoring would reveal the dynamics of magnetic system

$$\begin{aligned} \frac{I}{I_c} = \sin(\omega t - r m_y) &= \sin\left(\omega t - r \frac{Gr \sin \omega t}{1 - \omega^2}\right) = \\ &= \sin \omega t + \frac{Gr^2}{2} \frac{1}{\omega^2 - 1} \sin 2\omega t + \dots \end{aligned}$$

## Phi-0 Josephson Junction: Analytical Results II

$$\begin{cases} \dot{m}_x = \frac{1}{1+\alpha^2} \{-m_y m_z + Gr m_z \sin(\omega t - r m_y) - \alpha[m_x m_z^2 + Gr m_x m_y \sin(\omega t - r m_y)]\} \\ \dot{m}_y = \frac{1}{1+\alpha^2} \{m_x m_z - \alpha[m_y m_z^2 - Gr(m_z^2 + m_x^2) \sin(\omega t - r m_y)]\} \\ \dot{m}_z = \frac{1}{1+\alpha^2} \{-Gr m_x \sin(\omega t - r m_y) - \alpha[Gr m_y m_z \sin(\omega t - r m_y) - m_z(m_x^2 + m_y^2)]\} \end{cases}$$

- Effect of damping: a damped resonance at  $\omega \rightarrow 1$

$$\begin{cases} \dot{m}_x = \frac{1}{1+\alpha^2} \{-m_y + Gr \sin \omega t - \alpha m_x\} \\ \dot{m}_y = \frac{1}{1+\alpha^2} \{m_x - \alpha[m_y - Gr \sin \omega t]\} \end{cases} \quad m_y(t) = \frac{\omega_+ - \omega_-}{r} \sin \omega t - \frac{\alpha_+ + \alpha_-}{r} \cos \omega t$$

$$\omega_{\pm} = \frac{Gr^2}{2} \frac{\omega \pm 1}{\Omega_{\pm}} \quad \text{and} \quad \alpha_{\pm} = \frac{Gr^2}{2} \frac{\alpha \omega}{\Omega_{\pm}} \quad \Omega_{\pm} = (\omega \pm 1)^2 + \alpha^2 \omega^2$$

- Effect of damping: a dc contribution to the Josephson current

$$I(t) \approx I_c \sin \omega t + I_c \frac{\omega_+ - \omega_-}{2} \sin 2\omega t - I_c \frac{\alpha_+ + \alpha_-}{2} \cos 2\omega t + I_0(\alpha)$$

$$I_0(\alpha) = \frac{\alpha Gr^2 \omega}{4} \left( \frac{1}{\Omega_-} + \frac{1}{\Omega_+} \right)$$

# Generation of magnetic precession at fixed Josephson frequency

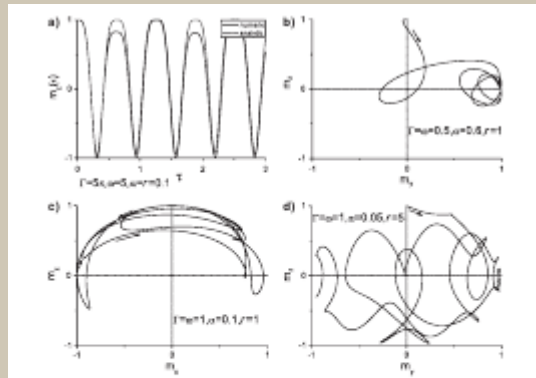
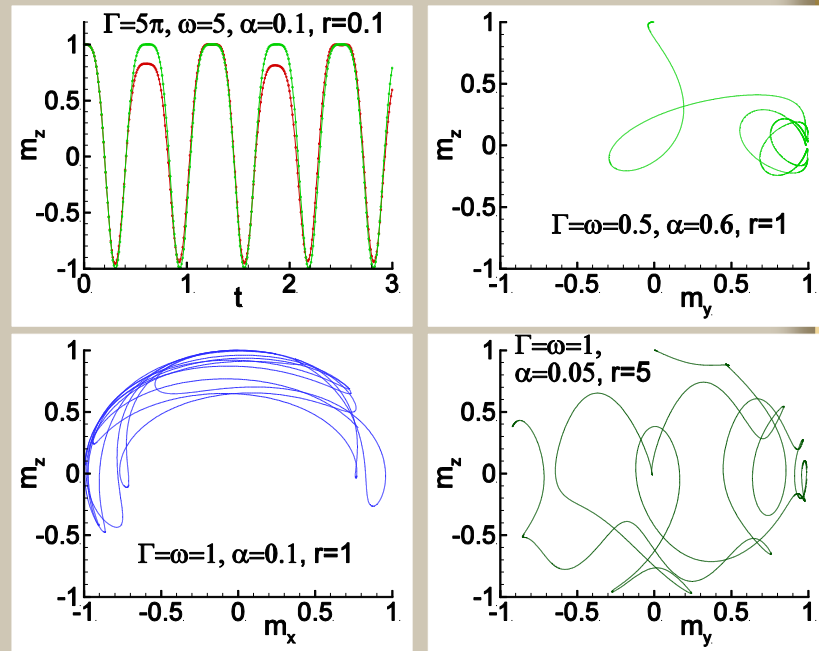


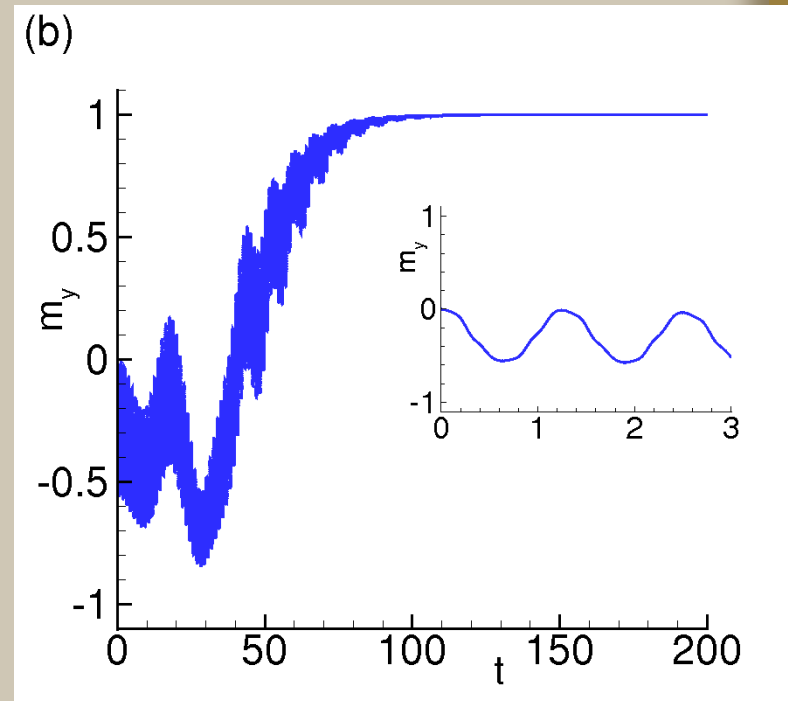
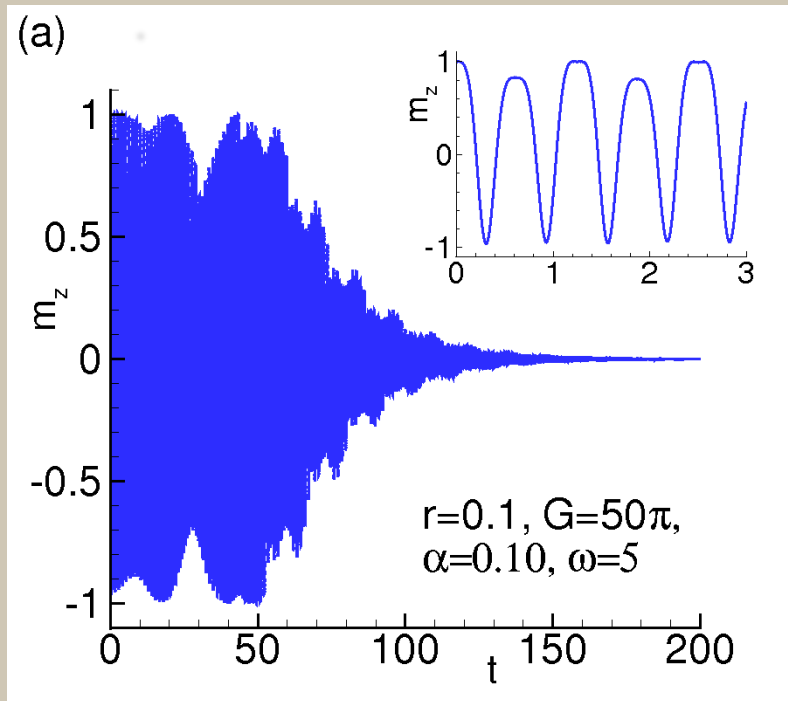
FIG. 2. Results of numerical analysis of the magnetic moment dynamics of the  $\phi_0$ -junction. (a) Reversal of  $m_z$  from analytical expression Eq. (15) (dashed curve) and numerical one (normal curve). The other curves are related to the  $\mathbf{M}$  trajectory: (b) in strong damping case (c) and (d) in the strong coupling regime revealing incomplete and complete magnetic moment reversal, respectively.



F. Konschelle, A. Buzdin.  
Phys. Rev. Lett. 102,  
017001 (2009).

Our simulations

# Dynamics of magnetization components without signal



Yu.M. Shukrinov, I.R. Rahmonov, K. Sengupta, A.Buzdin,  
APL, 110, 182407, 2017

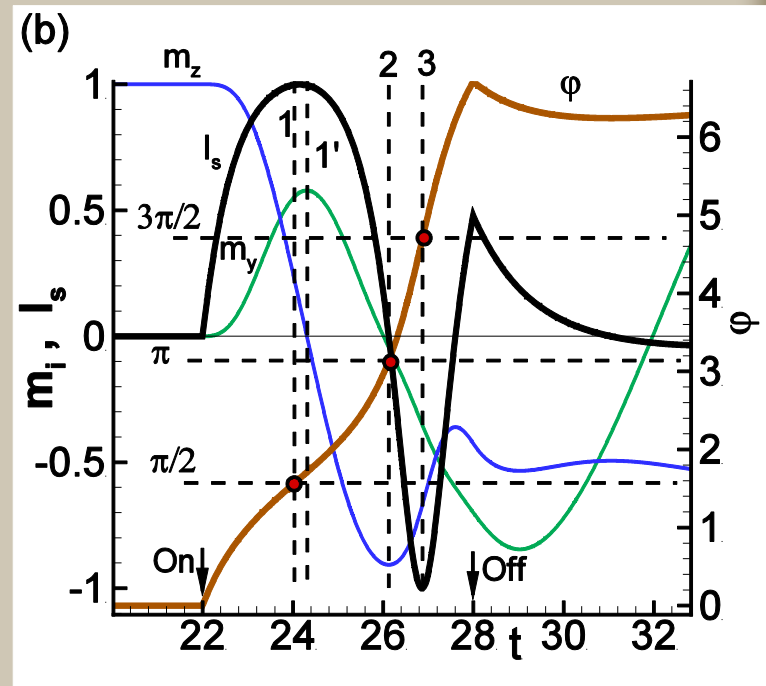
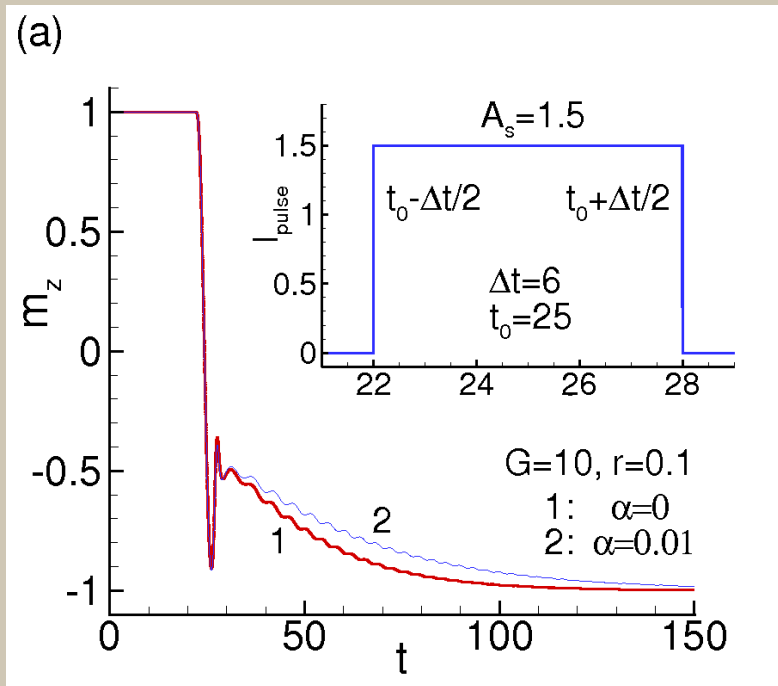


# Magnetization reversal



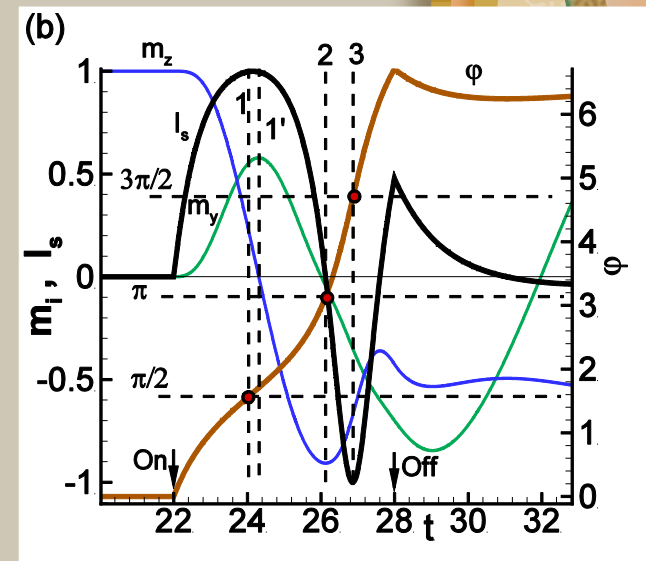
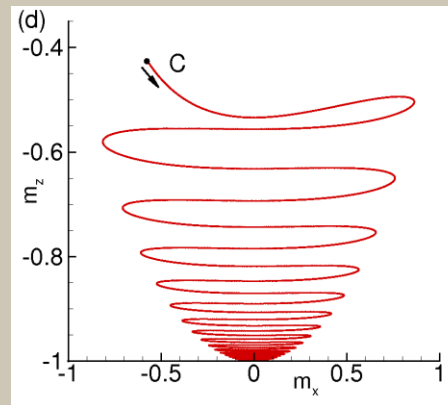
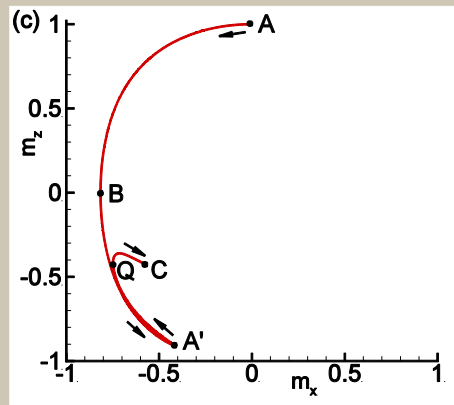
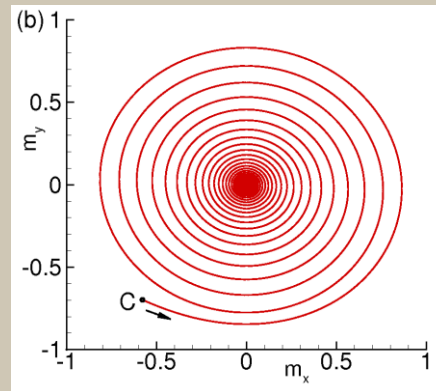
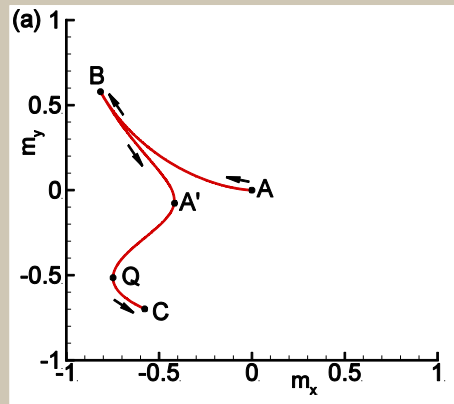
# Transition dynamics in the system under rectangular current pulse

$$I_{pulse} = w \frac{d\varphi}{dt} + \sin(\varphi - rm_y)$$



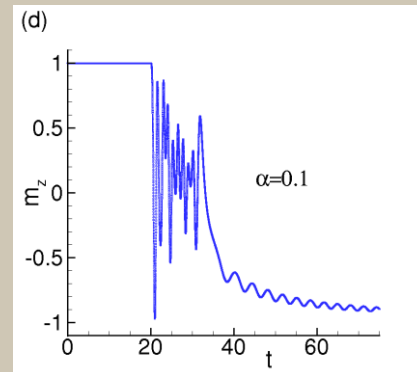
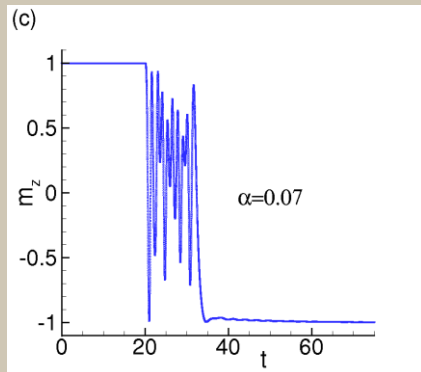
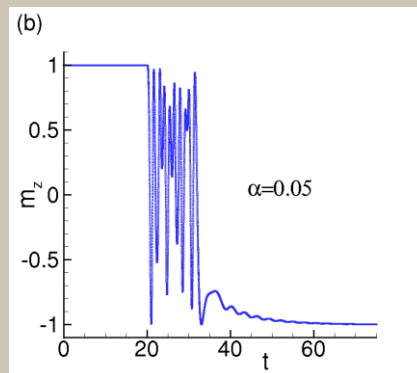
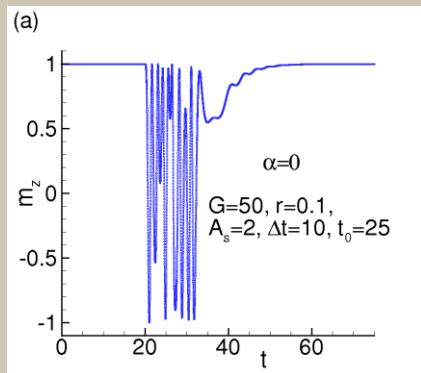
$$\sin \frac{1}{2} \left[ \int_{t_{min}}^{t_{max}} V(t') dt' - r [m_y(t_{max}) - m_y(t_{min})] \right] = 1$$

# Trajectories of magnetization in transition region

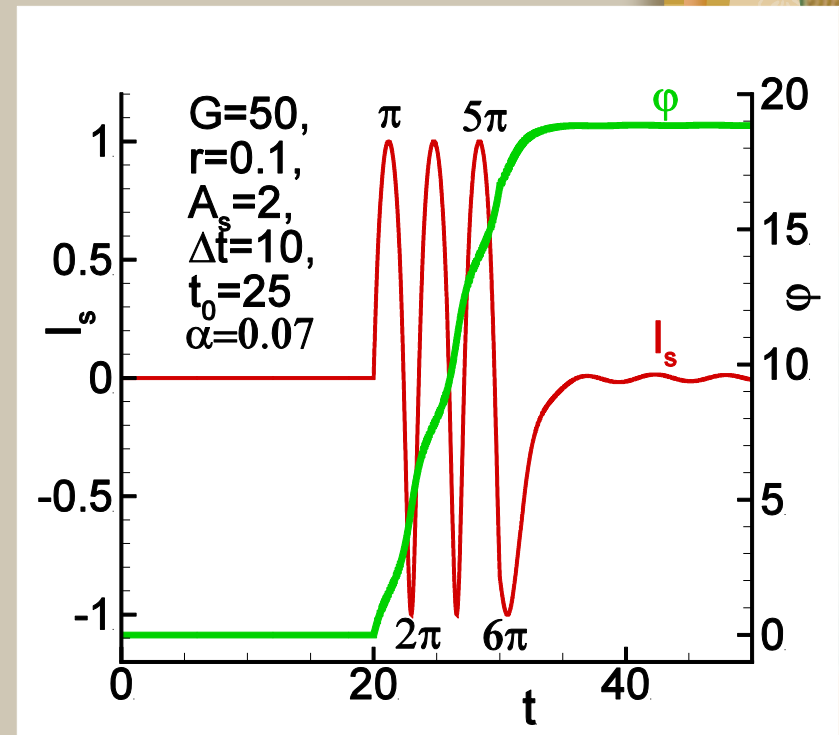


Yu.M. Shukrinov, I.R. Rahmonov, K. Sengupta, A.Buzdin,  
APL, 110, 182407, 2017

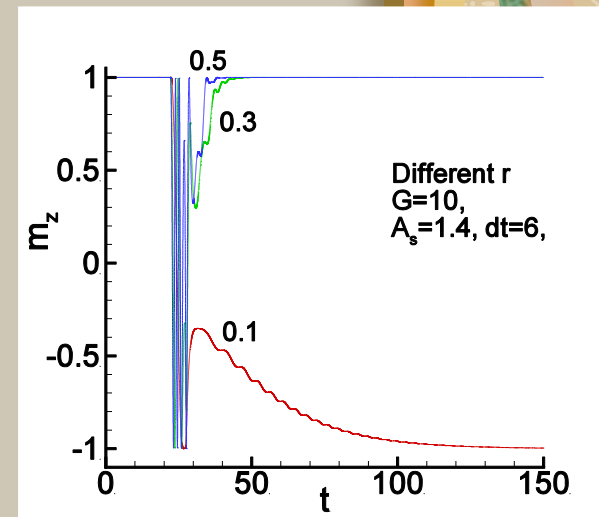
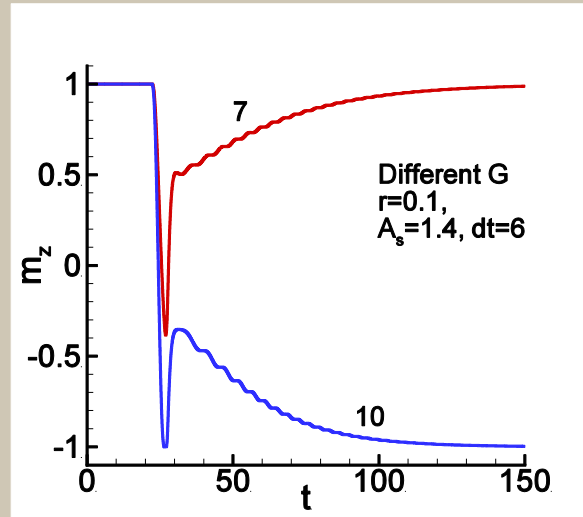
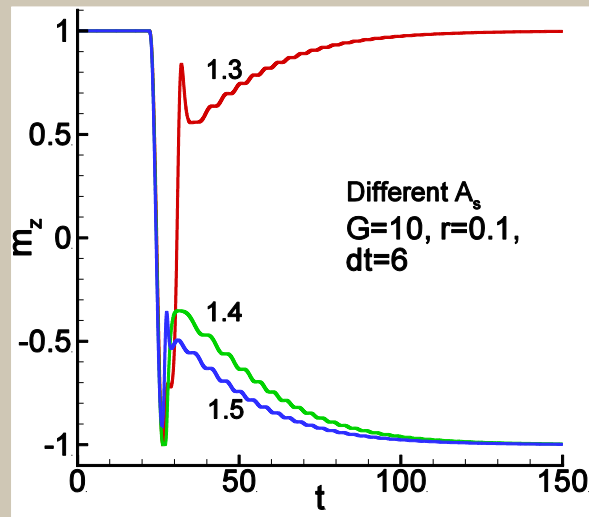
# Magnetization dynamics under rectangular pulse signal in the system at different values of dissipation parameter



## Transition dynamics of the superconducting current



# Different protocols for magnetization reversal

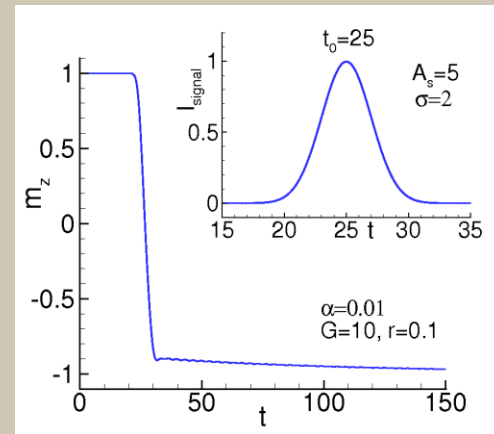


Yu.M. Shukrinov, I.R. Rahmonov, K. Sengupta, A.Buzdin,  
APL, 110, 182407, 2017



# Transition dynamics under electric current pulse of the Gaussian form

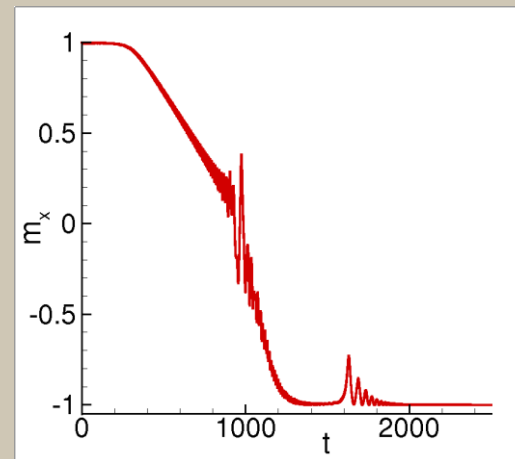
$$I_{pulse} = A \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right)$$



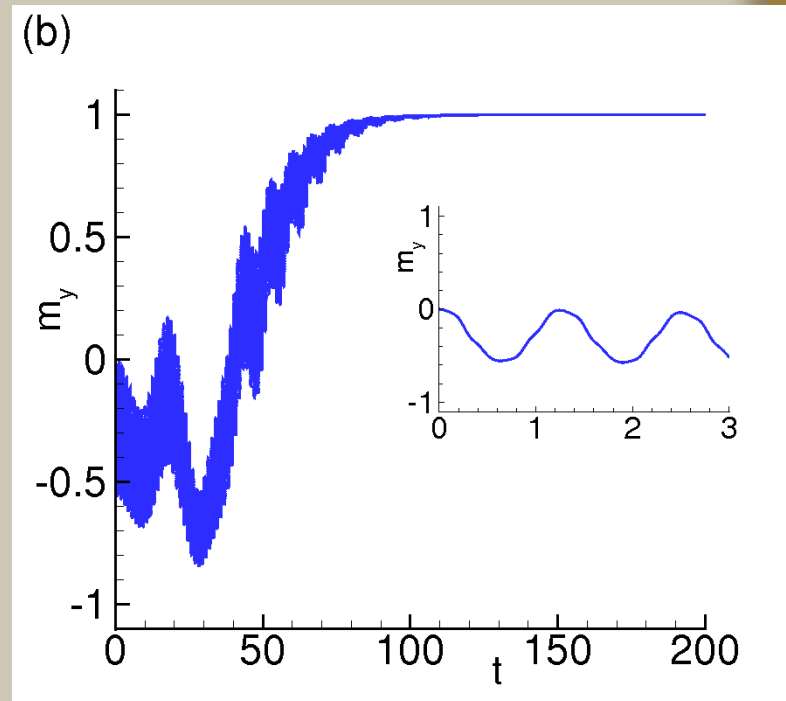
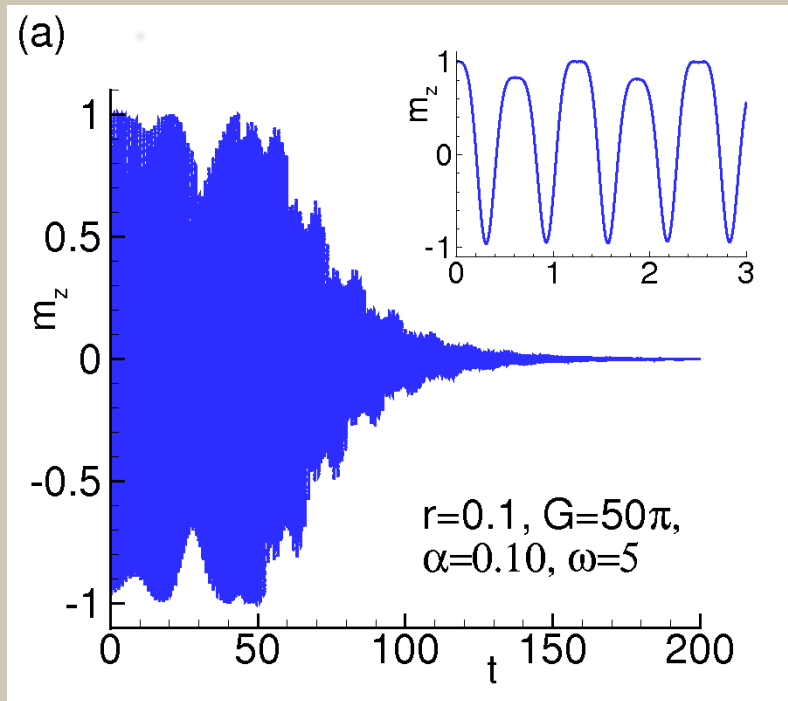
$$\bar{V} = 1.5 - 0.00075\bar{t},$$

$$\varepsilon = \bar{k} = 0.05, \eta = 0.01$$

L.Kai, E.M.Chudnovsky,  
PRB, 82,104429 (2010)



# Dynamics of magnetization components without signal



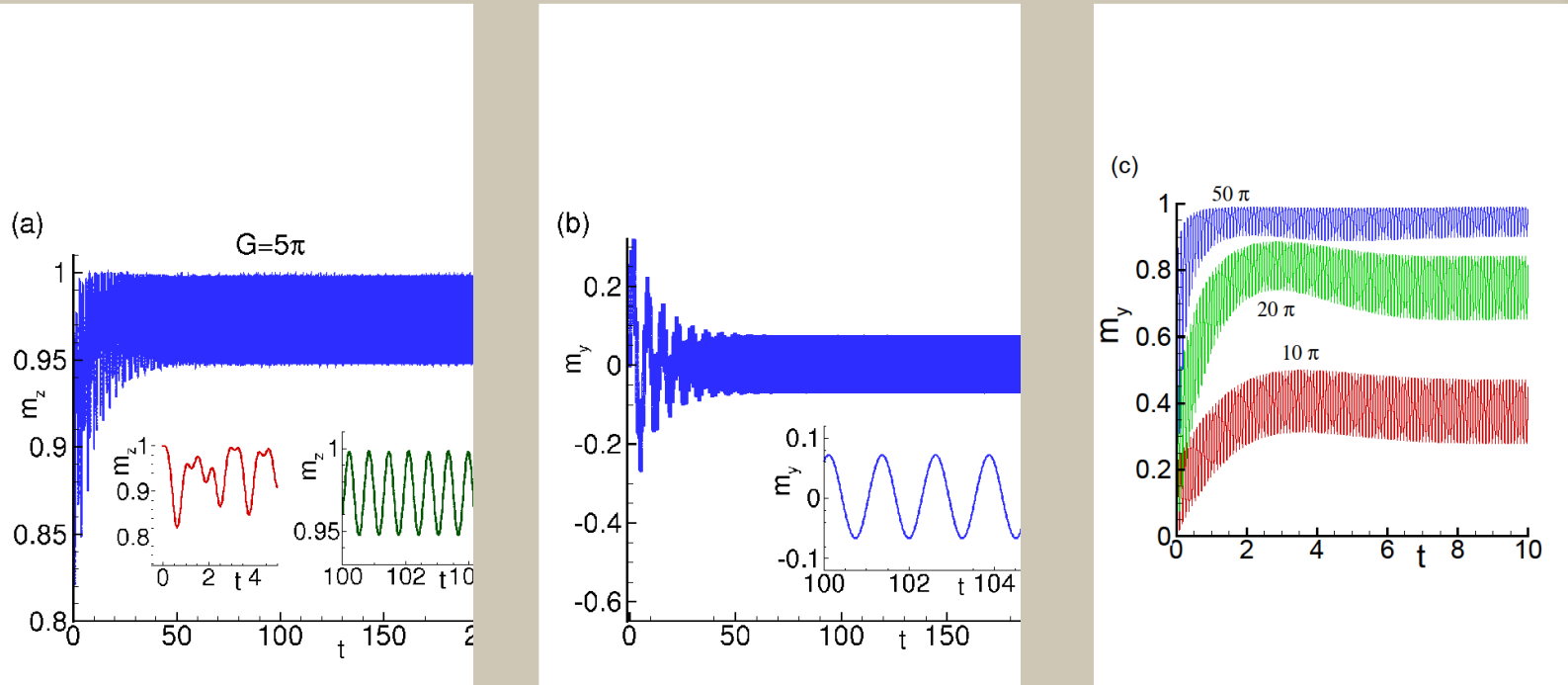
Yu.M. Shukrinov, I.R. Rahmonov, K. Sengupta, A.Buzdin,  
APL, 110, 182407, 2017

# Reminiscent Kapitza Pendulum Features of $\Phi_0$ Josephson Junction





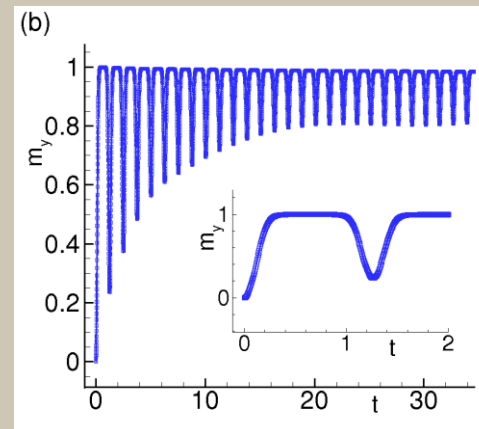
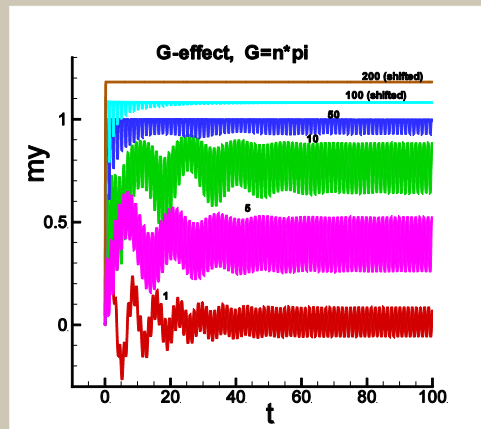
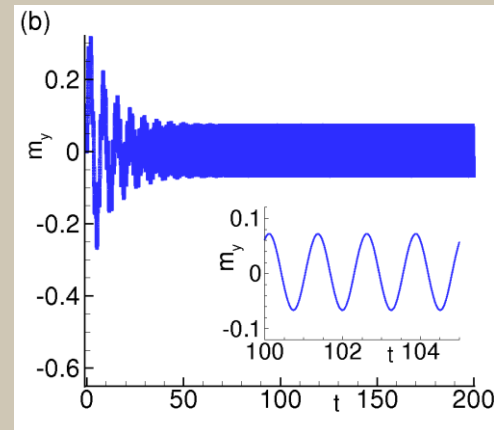
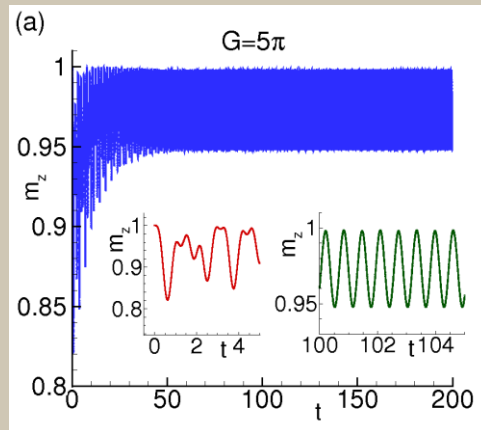
# Dynamics of magnetization components, $r = 0.1$



Yu. M. Shukrinov, A. Mazanik, I. R. Rahmonov, A. E. Botha and A. Buzdin.  
Re-orientation of easy axis in  $\Phi$ -0-junction,  
Europhysics Letters, EPL, 122 (2018) 37001

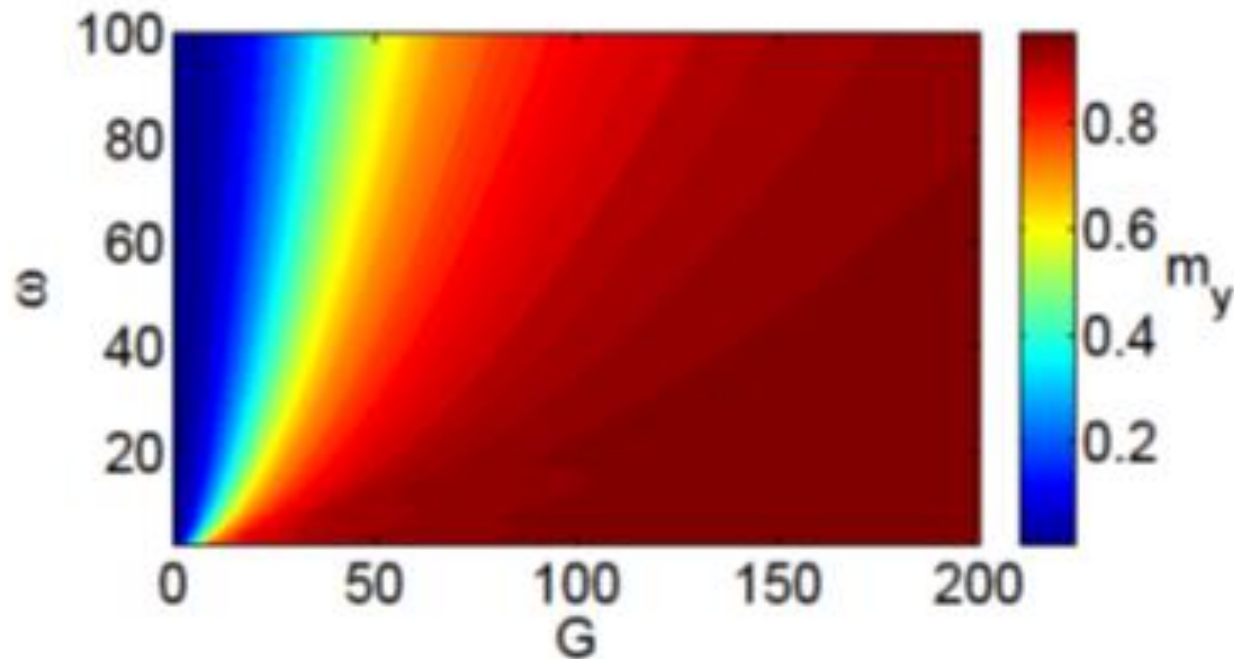


# Dynamics of magnetization components



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The frequency - G-diagram  
for averaged  $m_y$  shown by color.



An increase in  $G$  makes orientation of  $m_y$  along y-axis stable, but that the frequency dependence differs from characteristic Kapitza pendulum behavior.

# Kapitza method

The system of equations, describing the dynamics of the magnetic moment in the angular variables  $m_z = \cos \theta$ ,  $m_x = \sin \theta \cos \phi$ ,  $m_y = \sin \theta \sin \phi$ , can be written as

$$\begin{cases} \dot{\phi} = \frac{\cos \theta}{1+\alpha^2} - \\ \frac{Gr}{1+\alpha^2} \frac{1}{\sin \theta} [\cos \theta \sin \phi - \alpha \cos \phi] \sin [\omega\tau - r \sin \theta \sin \phi], \\ \dot{\theta} = -\frac{\alpha \sin 2\theta}{2(1+\alpha^2)} + \\ \frac{Gr}{1+\alpha^2} [\alpha \cos \theta \sin \phi + \cos \phi] \sin [\omega\tau - r \sin \theta \sin \phi]. \end{cases}$$

Let us make the replacements

$$\begin{aligned} \theta &\rightarrow \Theta + \xi, \\ \phi &\rightarrow \Phi + \eta. \end{aligned}$$

Here  $\Theta$  and  $\Phi$  describe slow movement, while  $\xi$  and  $\eta$  are coordinates for fast-varying movement. Conditions for  $\xi$  and  $\eta$  are discussed in the Supplement. We consider  $\bar{\theta} = \Theta$ ,  $\bar{\phi} = \Phi$ , where averaging is taken over the period of the fast-varying force  $T = \frac{2\pi}{\omega}$ .

# Kapitza method

The system of equations for slow movement has a form

$$\begin{cases} \dot{\Phi} = \frac{\cos \Theta}{1+\alpha^2} - \frac{(Gr)^2 r \alpha}{2\omega(1+\alpha^2)^2} \cdot \frac{1}{\sin \Theta} [\cos \Theta \sin \Phi \\ - \alpha \cos \Phi] \{1 - \sin^2 \Theta \sin^2 \Phi\}, \\ \dot{\Theta} = -\frac{\alpha \sin 2\Theta}{2(1+\alpha^2)} + \frac{(Gr)^2 r \alpha}{2\omega(1+\alpha^2)^2} [\alpha \cos \Theta \sin \Phi \\ + \cos \Phi] \{1 - \sin^2 \Theta \sin^2 \Phi\} \end{cases}$$

There are some equilibrium points of the system following from  $\dot{\Theta} = 0, \dot{\Phi} = 0$ .

The first pair is located on the equator

$$\Theta_0 = \pi/2, \Phi_0 = \pi/2 \text{ and } \Theta_0 = \pi/2, \Phi_0 = 3\pi/2$$

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# Kapitza method

Linearization of after substitution  $\Phi = \Phi_0 + \delta\phi$ ,  $\Theta = \Theta_0 + \delta\theta$  gives

$$\begin{cases} \delta\dot{\phi} = \frac{1}{1+\alpha^2}\delta\theta, \\ \delta\dot{\theta} = \frac{\alpha}{1+\alpha^2}\delta\theta. \end{cases} \quad (1)$$

It's clear that  $\delta\theta \sim e^{\frac{\alpha\tau}{1+\alpha^2}}$ , so these points are unstable.

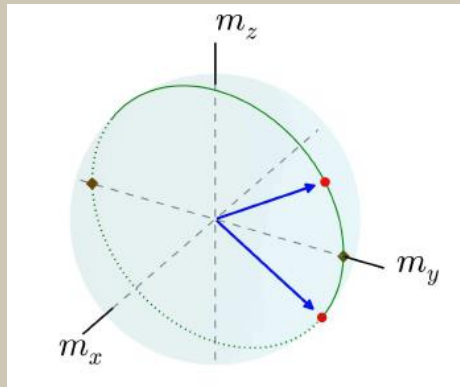
We find that the equilibrium points at  $\Phi_0 = \frac{\pi}{2}$  are described by equation

$$\sin \Theta_0 = \frac{-1 + \sqrt{1 + 4\beta^2}}{2\beta}, \quad (2)$$

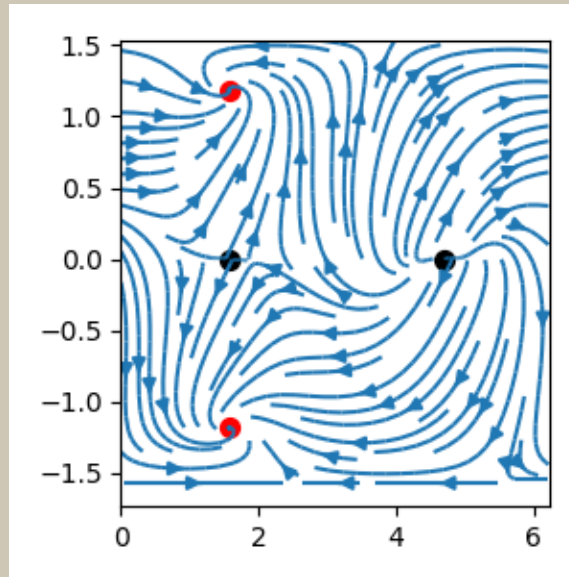
where  $\beta = \frac{(Gr)^2 r \alpha}{2\omega(1+\alpha^2)}$ , which can be approximated as  $\sin \Theta_0 = \beta$  at small  $\beta$ . It has two solutions in the interval  $0 \leq \Theta_0 \leq \pi$ .

Note that at  $\Phi_0 = \frac{3\pi}{2}$  we have  $\sin \Theta_0 = \frac{1 - \sqrt{1 + 4\beta^2}}{2\beta}$  which leads to the negative  $\sin \Theta_0$ , but  $\Theta$  is always positive. So, there are not any stable points at  $\Phi_0 = \frac{3\pi}{2}$ .

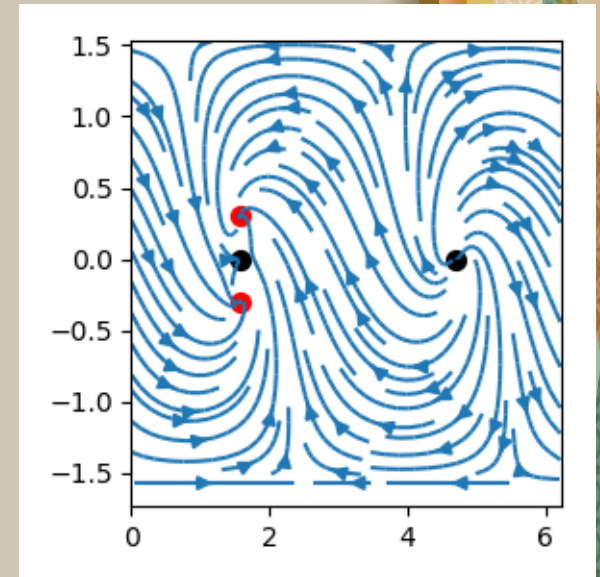
# Demonstration of equilibrium points



Arrows show stable points, other two are unstable.



$G=31.4$

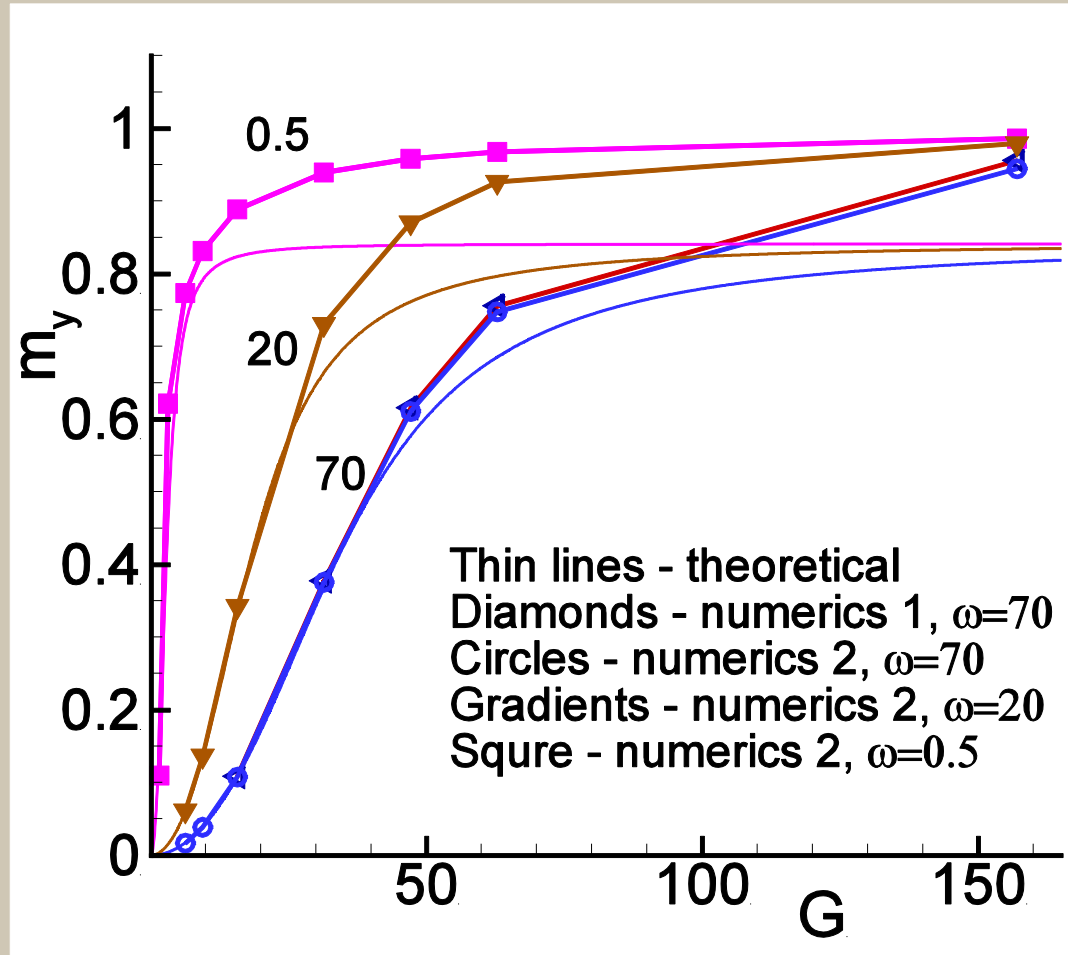


$G=157.0$

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# Comparison of theoretical and numerical calculations



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# Conclusions - I

- Reversal magnetization at different parameters of electric current pulse and JJ is shown.
- Reminiscent of Kapitza pendulum features in  $\phi_0$  Josephson junction is demonstrated.

Thank you  
for your attention!

