dS vacua and inflation in string theory

Lecture notes

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Contents

We will not discuss string compactifications but rather start from the resulting 4d $\mathcal{N} = 1$ supergravities that one obtains from compactifying type IIB string theory from 10d to 4d. Please see the lectures from Edvard Musaev for the details of string compactifications.

Please see the lectures by Dmitry S. Gorbunov for an introduction to standard cosmology. Here we review a few important points that motivate a study of inflation in string theory.

For a detailed introduction to string theory and inflation see [1] and references therein.

1 Introduction/Motivation

1.1 Slow-roll inflation

Inflation in the early universe can be described by a scalar field ϕ coupled to general relativity (GR). (We can neglect the familiar particles in the standard model of particle physics. These can be added but only become important after inflation.) The action is

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) , \qquad \mu = 0, 1, 2, 3, \tag{1.1}$$

where $M_P = \sqrt{\frac{\hbar c}{8\pi G}} \approx 2.4\times 10^{18} GeV$ is the reduced Planck mass and we will set $\hbar = c = 1$ in these lectures. Pictorial for slow-roll inflation we have

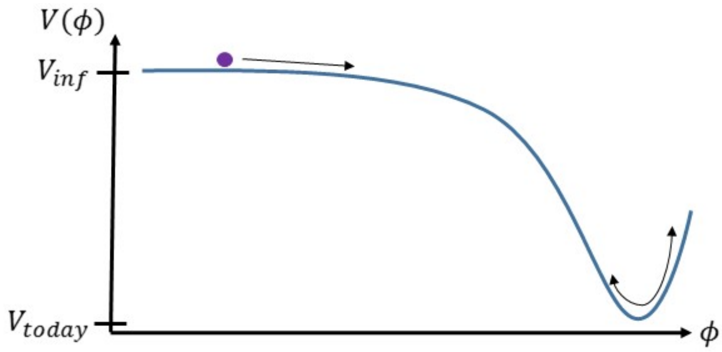

Figure 1: The scalar potential for natural inflation.

The above leads to slow-roll inflation, if

$$\epsilon_V \equiv \frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1 , \tag{1.2}$$

$$|\eta_V| \equiv M_P^2\left|\frac{V''(\phi)}{V(\phi)}\right| \ll 1 . \tag{1.3}$$

[This means, contrary to what the picture seems to imply, that for example $V(\phi) = \frac{1}{2}m^2\phi^2$ works for $\phi \gg M_P$.]

During inflation space gets stretched exponentially so that spatial variations $\partial_i\phi$, $i = 1, 2, 3$ die off quickly

$$\Rightarrow \quad \partial_\mu\phi\partial^\mu\phi \approx \partial_t\phi\partial^t\phi . \tag{1.4}$$

Slow-roll inflation happens when $\partial_t\phi\partial^t\phi \ll V(\phi)$. In this case the above action becomes

$$S \approx \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - V(\phi)\right) . \tag{1.5}$$

Since ϕ is changing slowly, $V(\phi)$ is approximately constant for a while and therefore the scalar field part behaves effectively as GR with a non-vanishing cosmological constant $\Lambda M_P^4 = -V(\phi)$. So during inflation, as well as at the end of inflation when the scalar field settles into the minimum of its potential, the scalar field behaves simply like a cosmological constant.

The observed accelerated expansion of our universe today can be explained by $V_{today} \approx 10^{-120} M_P^4$. The inflationary energy is not very constrained and the current upper bound is roughly $V_{inf} \lesssim (10^{16} GeV)^4 \approx 10^{-9} M_P^4$. [The lower bound is model dependent. A value that is large enough to avoid any potential conflicts with particle physics is $(10^5 GeV)^4 < V_{inf}$.]

1.2 Planck suppressed operators

GR is expected to break down near the Planck scale since the action contains unknown Planck suppressed operators

$$S = \int d^4x \sqrt{-g} M_P^4 \left(\Lambda + \frac{R}{2M_P^2} + \sum_{i=2}^{\infty} c_i \left(\frac{R}{M_P^2} \right)^i + \ldots \right) . \tag{1.6}$$

Here ... denotes other curvature invariants and since the coefficient in front of the Einstein-Hilbert term R is 1, one expects that the c_i could be order 1 as well. So for $R \approx M_P^2$ the usual Einstein equation derived from the Einstein-Hilbert term get corrected.

During inflation we have $V \lesssim 10^{-9} M_P^4$ so it should be ok to neglect Planck suppressed corrections to the Einstein-Hilbert action. However, the Planck suppressed corrections to the scalar potential $V(\phi)$ cannot be neglected: There are two classes of inflationary models, so called small field and large field models. Their names are due to the distance in field space that the inflaton travels during inflation. For small field models we have $\Delta\phi \equiv |\phi_{initial} - \phi_{final}| \ll M_P$ and for large field models we have $\Delta\phi \gtrsim M_P$.

There is a common, so called η-problem in supergravity, as we will review below and sketch here: Let us look at the scalar potential $V(\phi)$. If there are Planck suppressed corrections of the form $\delta V = cV(\phi)\phi^2/M_P^2$, then they can spoil inflation and lead to a large η_V parameter, unless the coefficient c is very small. This should be obvious for large field models of inflation for which $\phi^2 \gtrsim M_P^2$ at one point during inflation.[1] However, the same is true for small field models of inflation with $\phi \ll M_P$ since

$$\begin{aligned} |\eta_{V_{cor}}| &\equiv M_P^2 \left| \frac{V''_{cor}(\phi)}{V_{cor}(\phi)} \right| \\ &= M_P^2 \left| \frac{V''(\phi)(1 + c\phi^2/M_P^2) + 4cV'(\phi)\phi/M_P^2 + 2V(\phi)c/M_P^2}{V(\phi)(1 + c\phi^2/M_P^2)} \right| \approx |\eta_V + 2c| . \end{aligned} \tag{1.7}$$

So if we had initially $|\eta_V| \ll 1$ in order to have slow-roll inflation, then this corrections will spoil it, unless $c \ll 1$. So it seems that inflation requires the knowledge of Planck suppressed operators!

1.3 Inflationary models in string theory

String theory is a UV complete theory of quantum gravity, i.e. it combines GR with quantum mechanics and it does not break down at high energies. We certainly don't know that string theory is the correct theory of quantum gravity that describes our universe, but since inflation is so UV sensitive it seems very worthwhile to study it in a UV complete theory of quantum gravity. Fortunately, string theory is well enough understood to do that.

[1] The large field models of inflation are actually sensitive to an infinite number of Planck suppressed operators.

Before we lay the groundwork for this endeavor, let us ask whether we can suppress somehow all Planck suppressed operators by imposing a symmetry. This seems very difficult since the above correction $\delta V = cV(\phi)\phi^2/M_P^2$ involves the potential $V(\phi)$ which we don't want to forbid and ϕ^2 that transform for example under a $U(1)$ symmetry $\phi \to e^{i\theta}\phi$ for $\theta \in \{0, 2\pi\}$ in the same way as the kinetic term. However, one way to forbid higher order corrections, while maintaining a non-trivial scalar potential and a kinetic term, seems to be a discrete shift symmetry $\phi \to \phi + 2\pi f$, $f \in \mathbb{R}$. In this case the potential should be proportional to a trigonometric function

$$V(\phi) = \lambda^4(1 + \cos(\phi/f))\,. \tag{1.8}$$

This model is called natural inflation and the scalar potential is show in the following figure

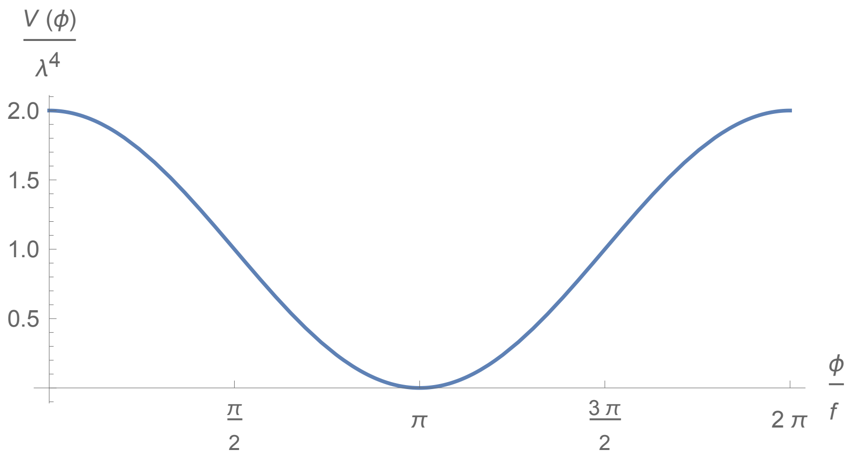

Figure 2: The scalar potential for natural inflation.

Consistency with observation requires that $\lambda \approx 10^{16}GeV$ and $f > M_P$ and this is a large field model of inflation.

In this case quantum gravity also has something to say and it is not yet clear whether we can construct models with $f > M_P$ in string theory or any other theory of quantum gravity. The simplest string theory models all have $f < M_P$ and we have no really trustworthy models at all with $f \gg M_p$. So it seems very worthwhile to study inflation in string theory! This is even more so since the current experiments are testing large field models of inflation with $V_{inf} \approx 10^{16}GeV$ and $\Delta\phi \gtrsim M_p$ and we are expecting a lot of data to come in in the next 10-15 years.

1.4 A reasonable goal for string models of inflation

String theory is rather complicated and to construct a model that includes the standard model of particle physics (SM) plus a period of inflation that ends with $V_{today} > 0$ is beyond what we can currently do. We will therefore try to derive a model of the form

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)\,, \tag{1.9}$$

from string theory, such that $V(\phi)$ can give rise to a period of inflation and has a minimum with $V_{today} > 0$. [So we forget about the reheating, which describes the energy transfer from the inflaton ϕ to the SM particles and we do not discuss dark matter.]

This is already a very challenging goal! We need to:

1. Pick a string theory and compactify all but three of its spatial dimensions. Usually one takes as starting point the low energy limit of one of the superstring theories: type IIA/IIB or heterotic (or M-theory). In this low energy limit one restricts to energies much smaller than the string scale $E \ll M_s = 1/\sqrt{\alpha'}$, so that strings become point particles. Their action is given by a 10D (or 11D) supergravity theory. Then we compactify this supergravity theory to 4d by taking 6 (or 7) spatial dimensions to be compact with small radius. The details of this procedure are discussed in Edvard Musaev's lectures at this school.

2. Then we need to analyze the resulting 4d theory. Ideally we want, as described above, one light scalar field ϕ (the inflaton) that has a potential suitable for inflation and whose potential has a minimum with $V(\phi_{min}) = V_{today} > 0$.

The second point will occupy the rest of these lectures. Since the resulting 4D theories in the most studied and best understood string compactifications will be 4d $\mathcal{N} = 1$ supergravities, we will study these first.

2 Introduction to 4d $\mathcal{N} = 1$ SUSY/SUGRA

This section contains a brief overview of a few important points of supersymmetry (SUSY) and supergravity (SUGRA) that will play a role later. For a thorough introduction to supersymmetry and supergravity please look at for example [2, 3].

2.1 Supersymmetry

Supersymmetry is a fermionic symmetry that maps fermions (with half-integer spin) to bosons (with integer spin) and vice versa. It was discovered in the 1970's and provides a unique extension of the standard symmetries in relativistic quantum field theories.

You might have heard about the Poincare group that consist of the Lorentz group and translations in space and time. The generator for translations in space and time is usually called P_μ and the generator for the Lorentz group $SO(3,1)$ is usually called $M_{\mu\nu}$. The 3-vector P_i, $i = 1,2,3$ generates space translations, while P_t generates time translations. The six generators of the anti-symmetric $M_{\mu\nu}$ can be grouped into three generators for rotations M_{ij} and three generators of so called boosts M_{ti}. The Poincare group is the symmetry group of $\mathbb{R}^{3,1}$ which describes our world very well. Therefore, we write down physical models that are invariant under the Poincare group, i.e. for example we require our physical theories to not depend on the time or place where we are located or the direction we are facing. Additionally, we often require the invariance under so called internal symmetries like for example a $U(1)$ symmetry under which complex scalar fields transform as $\phi \to e^{iq\theta}\phi$. Such a symmetry corresponds to electro-magnetism, where a scalar particle with charge q transforms as above. For $\theta \in \mathbb{R}$, the symmetry is called a global symmetry. If we take $\theta(x^\mu)$ to be spacetime dependent, then we need to "gauge" the symmetry. We need to introduce a dynamical spin-1 gauge field, usually called $A_\mu(x^\mu)$. This gauge field transforms as $A_\mu(x^\mu) \to A_\mu(x^\mu) + \partial_\mu\theta(x^\mu)$. [This allows us to write down gauge covariant derivatives $\partial_\mu\phi \to D_\mu\phi = (\partial_\mu - \mathrm{i}qA_\mu)\phi$ and actions that are invariant under local gauge transformations, like $S = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\phi D^\mu\bar{\phi}\right)$.] These internal (usually gauged) symmetries can also be non-abelian and the SM of particle physics is based on an internal $SU(3) \times SU(2) \times U(1)$ gauge symmetry, where the first factor corresponds to the strong force and the last two to the electro-weak force. In 1974 Haag, Lopuszanski and Sohnius [4] proved that the only extension of the above symmetry group, namely the Poincare group plus internal symmetries, involves anti-commuting generators Q^A_α that satisfy the following algebra[2]

$$\begin{aligned}
\{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} &= 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A{}_B \, , \\
\{Q^A_\alpha, Q^B_\beta\} &= \{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\} = 0 \, , \\
[P_\mu, Q^A_\alpha] &= [P_\mu, \bar{Q}_{\dot{\alpha}A}] = 0 \, , \\
[P_\mu, P_\nu] &= 0 \, ,
\end{aligned} \tag{2.1}$$

where $\sigma^0 = -\mathbb{1}_{2\times 2}$, σ^i are the Pauli matrices and $A, B = 1, 2, \ldots, \mathcal{N}$. $\alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2$ are spinor indices and the generators Q^A_α are therefore spin-1/2 generators.[3]

Supersymmetry also exists in other spacetime dimension smaller and larger than 4 and it also exists for other spacetimes different from $\mathbb{R}^{3,1}$. [The low energy limit of M-theory is an 11 dimensional theory with $\mathcal{N} = 1$ supersymmetry and the low energy limits of the various string theories are 10 dimensional with $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetry.]

Supersymmetry is very appealing since it provides a unique extension of the symmetry groups and it would therefore be a pity if it weren't realized in nature. Some additional advantages of supersymmetry are

+ Divergences in quantum field theories with supersymmetry are usually milder in supersymmetric theories.

+ Supersymmetry might (could have?) explained the smallness of the Higgs mass.

+ The minimal supersymmetric standard model leads to gauge coupling unification, i.e. it seems to hint at a unification of the strong, weak and electro-magnetic force at energies near $10^{16} GeV$.

+ Supersymmetric theories can give naturally rise to dark matter particles.

+ Supersymmetric theories are constrained by a larger symmetry group, which leads to a more constrained form of the action, which simplifies many calculations.

On the other hand there are also downsides to having supersymmetry

− There is no experimental evidence for supersymmetry so far, so supersymmetry would have to be broken at an energy scale above what we can experimentally access (approx. $10^4 GeV$). [Symmetry breaking is not that unusual. For example, the $SU(2)\times U(1)$ symmetry group in the SM is broken to $U(1)$ below the electroweak scale ($\approx 246 GeV$).]

− Supersymmetry requires us to introduce a new, so called superpartner, for every single particle we know (like electrons, quarks, photons etc.).

[2] We use that $[A, B] = AB - BA$ and $\{A, B\} = AB + BA$. This algebra was studied for $\mathcal{N} = 1, 2$ in the paper [5]. See [6] for a detailed historical account.

[3] We will often suppress the spinor indices and write for example Q^A.

2.2 Representations and extended supersymmetry

In these lectures we will restrict to supersymmetry in four dimension and after reviewing some important features we will discuss supersymmetric models that are of interest for cosmology. Here we sketch the relevant representation of our algebra that include the Poincare algebra, internal symmetries and now also supersymmetry. [We will mostly and implicitly restrict to massless particles.] Under the Poincare algebra particles are classified by their helicity (or spin), so we have scalars like the Higgs particle, fermions like the electrons and vectors like the photon. Under the internal symmetry they carry specific charges, like for example the electric charge. The standard model also contains for example the $SU(2)$ internal symmetry under which the left-handed Weyl spinors, the electron χ_e and the electron neutrino ν_e, form a doublet $(\chi_e, \nu_e)_L$. The right-handed Weyl spinor $(\chi_e)_R$ is an $SU(2)$ singlet.

As we have seen above, the supersymmetry generators are fermionic operators, so they map states with different helicity (or spin) into each other. We can make this very precise. Let us look at a massless particle with $P_\mu = (-E, 0, 0, E)$ so that $P_\mu P^\mu = 0$, then we have

$$\begin{aligned} \{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} &= 2\begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix} \delta^A{}_B \,, \\ \{Q^A_\alpha, Q^B_\beta\} &= \{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\} = 0 \,. \end{aligned} \tag{2.2}$$

We can define $\mathcal{N}$ creation and annihilation operators

$$a^A = \frac{1}{2\sqrt{E}} Q^A_1 \,, \qquad a^\dagger_A = \frac{1}{2\sqrt{E}} \bar{Q}^A_{\dot{1}} = (a^A)^\dagger \,. \tag{2.3}$$

that satisfy the standard algebra

$$\begin{aligned} \{a^A, a^\dagger_A\} &= \delta^A{}_B \,, \\ \{a^A, a^B\} &= \{a^\dagger_A, a^\dagger_B\} = 0 \,. \end{aligned} \tag{2.4}$$

The Q^A_2 and $\bar{Q}_{\dot{2}A}$ are totally anti-commuting and must therefore be represented by zero.

We can start with the lowest helicity state Ω that is defined by $a^A\Omega = 0$, $\forall A$. When acting with creation operators $a^\dagger_A$ we obtain states with higher helicity. The simplest case where $\mathcal{N} = 1$ and therefore $A, B = 1$ is the one of interest to us. Here we can for example start with a scalar ϕ and obtain a left-handed Weyl fermion $\chi = a^\dagger_1 \phi$.[4] So we obtain what is called a chiral (or matter) multiplet (ϕ, χ). Likewise, if we start with a left-handed Weyl fermion λ as lowest helicity state, we obtain a vector field A_μ. They form a so called vector multiplet (λ, A_μ). This $\mathcal{N} = 1$ case is the most relevant for phenomenology. The reason is that for $\mathcal{N} = 2$ any matter multiplet (i.e. a multiplet that doesn't involve gauge fields) would start with a right-handed Weyl fermion λ_R that gets mapped to two scalars $a^\dagger_1\lambda_R$ and $a^\dagger_2\lambda_R$ and these get then mapped to a single left-handed fermion $\lambda_L = a^\dagger_1 a^\dagger_2 \lambda_R$. This is inconsistent with the standard model since, as discussed above, the right-handed electron is an $SU(2)$ singlet and the left-handed electron sits in an $SU(2)$ doublet. If the left- and right-handed electrons would sit in an $\mathcal{N} = 2$ multiplet, then they would have to carry the same charges under the internal symmetries. Thus the most studied compactifications of string theory give rise to 4d $\mathcal{N} = 1$ supersymmetric theories.

[4] We will often drop the subscript L for left-handed fermions. All fermions we work with are Weyl fermions (or Majorana) fermions.

As mentioned above, the 4d $\mathcal{N}=1$ multiplets like the chiral multiplet and the vector multiplet involve a scalar and a left-handed fermion or a left-handed fermion and a vector. The fields have the same number of states, as can be shown from the supersymmetry algebra: If we define a fermion number operator $(-1)^F$ that has eigenvalues $+1$ on bosons with integer spin and -1 on fermions with half integer spin, then it follows that $(-1)^F Q^A_\alpha = -Q^A_\alpha(-1)^F$.

Then we can take the following trace over any finite dimensional representation

$$\begin{aligned} Tr\left[(-1)^F\{Q^A_\alpha,\bar{Q}_{\dot\beta B}\}\right] &= Tr\left[(-1)^F\left(Q^A_\alpha\bar{Q}_{\dot\beta B}+\bar{Q}_{\dot\beta B}Q^A_\alpha\right)\right] \\ &= Tr\left[-Q^A_\alpha(-1)^F\bar{Q}_{\dot\beta B}+Q^A_\alpha(-1)^F\bar{Q}_{\dot\beta B}\right]=0\,, \end{aligned} \tag{2.5}$$

where we used the cyclic property of the trace for the second term. From the supersymmetry algebra we then find

$$Tr\left[(-1)^F\{Q^A_\alpha,\bar{Q}_{\dot\beta B}\}\right] = 2\sigma^\mu_{\alpha\dot\beta}\delta^A{}_B Tr\left[(-1)^F P_\mu\right]=0\,. \tag{2.6}$$

For any given non-vanishing momentum P_μ this then gives

$$Tr\left[(-1)^F\right]=0\,, \tag{2.7}$$

so that any finite dimensional representation has the same number of fermionic and bosonic states. Let us check this for the chiral multiplet: A Dirac fermion in 4d has four complex components. A Weyl or Majorana spinor has four real components that satisfy first order equations of motion $\gamma^\mu\partial_\mu\chi=0$. Thus we have to specify four initial conditions. For a massless scalar ϕ the equation of motion $\partial_\mu\partial^\mu\phi=0$ are second order and we need to specify the value of the scalar and its first derivative. If the scalar ϕ is complex, then these are four initial conditions which is the same as for a Weyl or Majorana fermion. So a chiral multiplet contains necessarily a complex scalar field ϕ and a Weyl (or Majorana) fermion χ. [One can likewise check that the equations of motion of a gauge field A_μ requires us to specify four initial conditions so that its supersymmetric partner is likewise a Weyl (or Majorana) spinor.] So here we just encountered our first restriction arising from supersymmetry: We will not be able to simply write down an action for a single real scalar field that serves as the inflaton. We necessarily need at least two real scalar fields in our action. [String theory will usually give rise to a very large number of scalar fields, so that the resulting 4d action will usually be rather complicated to analyze.]

2.3 Local/gauged supersymmetry = supergravity

As in the case of internal symmetries, we can allow the parameter that appears in the supersymmetry transformation to depend on spacetime. Since supersymmetry is a fermionic symmetry this parameter is a fermion ϵ_α and a supersymmetry transformation is obtained by acting with $\epsilon^\alpha Q_\alpha+\bar\epsilon_{\dot\alpha}\bar{Q}^{\dot\alpha}$. This parameter ϵ^α allows us to rewrite the supersymmetry algebra in terms of commutators only[5]

$$\begin{aligned} [\epsilon Q,\bar\epsilon\bar Q] &= 2\epsilon\sigma^\mu\bar\epsilon P_\mu\,, \\ [\epsilon Q,\epsilon Q] &= [\bar\epsilon\bar Q,\bar\epsilon\bar Q]=0\,, \\ [P_\mu,\epsilon Q] &= [P_\mu,\bar\epsilon\bar Q]=0\,, \\ [P_\mu,P_\nu] &= 0\,. \end{aligned} \tag{2.8}$$

[5]We restrict to $A=B=1=\mathcal{N}$ and drop this index from now on.

Now if we gauge supersymmetry and allow $\epsilon(x^\mu)$ to depend on spacetime, then we see from the algebra above, that the commutator of two SUSY transformations give a translation with spacetime dependent coefficient $2\epsilon(x^\mu)\sigma^\mu\bar{\epsilon}(x^\mu)$. So our theory must be invariant under diffeomorphisms and requires gravity, i.e. it requires the metric $g_{\mu\nu}(x^\mu) = e^a_\mu(x^\mu)e^b_\nu(x^\mu)\eta_{ab}$ and the vielbein $e^a_\mu(x^\mu)$ to be dynamical fields. [The reverse is also true: gravity is inconsistent with global supersymmetry.] So local supersymmetries requires general relativity but this cannot be all in a supersymmetric theory. As for gauge symmetries, where we need to introduce a spin-1 gauge field A_μ that transforms as $A_\mu(x^\mu) \to A_\mu(x^\mu) + \partial_\mu\theta(x^\mu)$ once we gauge an internal symmetry, we need to introduce another field here in order to write down a supersymmetric action. This gauge field needs to transform as A_μ above

$$\Psi_{\mu\alpha} \to \Psi_{\mu\alpha} + \partial_\mu\epsilon_\alpha(x^\mu)\,. \tag{2.9}$$

So we see that, since the supersymmetry transformation parameter ϵ_α is a spinor, the gauge field $\Psi_{\mu\alpha}$ needs to likewise carry a spinor index so it is not a spin-1 gauge field but rather a spin-$\frac{3}{2}$ field. This field is called the gravitino and it forms an $\mathcal{N}=1$ multiplet together with the vielbein (e^a_μ, Ψ_μ).[6] The so called pure supergravity contains only the multiplet (e^a_μ, Ψ_μ) and its action is somewhat lengthy. To leading order in Ψ_μ it is given by

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\left[R - \bar{\Psi}_\mu\gamma^{\mu\nu\rho}(\partial_\nu + \frac{1}{4}\omega_{\nu ab}\gamma^{ab})\Psi_\rho + \mathcal{O}(\Psi_\mu^4)\right]\,, \tag{2.10}$$

where $\omega_{\nu ab}$ is the spin connection and $\gamma^{\mu\nu\rho}$ denotes antisymmetrized gamma matrices.

2.4 The bosonic action of 4d N=1 supergravity

We can add to the above action an arbitrary number of chiral multiplets (ϕ^I, χ^I), $I = 1, 2, \ldots, N_c$ and vector multiplets (λ^A, A^A_μ), $A = 1, 2, \ldots, N_v$. Here the vector multiplets can transform under the adjoint representation of abelian or non-abelian gauge groups and the chiral multiplets transform in trivial or non-trivial representations of the gauge groups. We discuss part of the resulting action below and refer the interested reader to for example chapter 18 of [3], where the full component action is spelled out in equations (18.6) to (18.19).

One important feature of supersymmetry is that the action is invariant under a symmetry that relates bosons and fermions. In particular, this allows us to restrict to the bosonic action since the fermionic part of the action follows from the bosonic action plus the invariance under supersymmetry. Therefore here and in most of the literature about string compactifications you will usually only see the bosonic action for the fields. Furthermore, since the bosonic and fermionic actions are related via supersymmetry we find that the bosonic (and fermionic) actions are somewhat restricted. In particular, the action cannot contain any arbitrary bosonic term since under SUSY transformations this term must transform into something that is canceled by the supersymmetry transformation of another term. This leads to the simplification that the action of 4d $\mathcal{N}=1$ supergravity can be determined in terms of four different functions:

Our field content is (e^a_μ, Ψ_μ), an arbitrary matter content with N_c chiral multiplets (ϕ^I, χ^I) and N_v vector multiplets (λ^A, A^A_μ) corresponding to an unspecified gauge group

[6] We will again drop the spinor index.

$G = G_1 \times G_2 \times \ldots \times G_{N_v}$. Then the bosonic action takes the relatively simple form

$$\begin{aligned} S \;\; = \;\; & \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}R - K_{I\bar{J}}\hat{\partial}_\mu\phi^I\hat{\partial}^\mu\bar{\phi}^{\bar{J}} - V_F - V_D \\ & -\frac{\mathrm{Re}(f_{AB})}{4}F^A_{\mu\nu}F^{\mu\nu\,B} + \mathrm{i}\frac{\mathrm{Im}(f_{AB})}{4}F^A_{\mu\nu}\tilde{F}^{\mu\nu\,B}\Big] \,. \end{aligned} \tag{2.11}$$

Let us discuss the different part in detail:

The first term is simply the Einstein-Hilbert term that also appears in GR.

The second line contains the kinetic terms for the gauge field $F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu$ and $\tilde{F}^{\mu\nu\,A} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^A_{\rho\sigma}$. The coefficients in the second line are determine by a holomorphic function of the scalar fields $f_{AB}(\phi^I) = f_{BA}(\phi^I)$. This function (or matrix of functions) is often called the gauge-kinetic function since it determines the kinetic terms for the gauge fields.

The kinetic terms for the scalar fields ϕ^I are determined by a real valued function of the complex scalar fields $K(\phi^I, \bar{\phi}^{\bar{J}})$. Its second derivatives $K_{I\bar{J}} = \partial_{\phi^I}\partial_{\bar{\phi}^{\bar{J}}}K$ gives rise to a positive definite metric that determines the kinetic terms of the scalar fields. The gauge covariant derivatives in the second term are defined as $\hat{\partial}_\mu\phi^I = \partial_\mu\phi^I + \mathrm{i}A^A_\mu K^{I\bar{J}}\partial_{\bar{\phi}^{\bar{J}}}\mathcal{D}_A$, where $K^{I\bar{J}}$ is the inverse of $K_{I\bar{J}}$ and the so called D-term $\mathcal{D}_A(\phi^I, \bar{\phi}^{\bar{J}})$ is a real valued function of the scalar fields. Under a gauge transformation of the gauge group G_A with parameter $\theta^A(x^\mu)$ the scalar fields transform as $\phi^I \to \phi^I - \mathrm{i}\theta^A K^{I\bar{J}}\partial_{\bar{\phi}^{\bar{J}}}\mathcal{D}_A$.

The first part of the scalar potential V_F is called the F-term potential and it is determined in terms of a holomorphic function of the scalar fields, the so called superpotential $W(\phi^I)$, as

$$V_F = e^{\frac{K}{M_P^2}}\left(K^{I\bar{J}}D_IW\overline{D_JW} - 3\frac{|W|^2}{M_P^2}\right) \,, \tag{2.12}$$

where the Kähler covariant derivative is given by $D_IW \equiv \partial_{\phi^I}W + W\partial_{\phi^I}K/M_P^2$.

Lastly, the D-term scalar potential is given by

$$V_D = \frac{1}{2}\left(\mathrm{Re}(f)\right)^{-1\,AB}\mathcal{D}_A\mathcal{D}_B \,, \tag{2.13}$$

where $\left(\mathrm{Re}(f)\right)^{-1\,AB}$ is the inverse matrix of $\mathrm{Re}(f_{AB})$. Note, that since $\mathrm{Re}(f_{AB})$ determines the kinetic terms for the vector fields, it has to be positive definite, which implies $V_D \geq 0$.

So to summarize, the most general two derivative 4d $\mathcal{N} = 1$ supergravity action is determined in terms of two real valued functions $K(\phi^I, \bar{\phi}^{\bar{J}})$, $\mathcal{D}_A(\phi^I, \bar{\phi}^{\bar{J}})$ and two holomorphic functions $W(\phi^I)$, $f_{AB}(\phi^I)$. [7]

In the rest of these lectures we will restrict ourselves to string compactifications that do not give rise to gauge fields. Therefore the action is determined in terms of the Kähler potential K and the superpotential W and it reduces to

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - K_{I\bar{J}}\partial_\mu\phi^I\partial^\mu\bar{\phi}^{\bar{J}} - V_F(\phi^I, \bar{\phi}^{\bar{J}})\right] \,. \tag{2.14}$$

Note, that this action looks already very much like the minimal model, we wanted to obtain above in equation (1.9).

If we combine the action with the fermionic action, then we get an action that is invariant under $\mathcal{N} = 1$ supersymmetry. For any given minimum of the scalar potential

[7] Gauge invariance leads to some restrictions on these functions that will not be relevant for us.

supersymmetry can be preserved at the minimum or it can be spontaneously broken. This is similar to the SM of particle physics, where the Higgs field at the minimum breaks the $SU(2) \times U(1)$ symmetry to a $U(1)$ symmetry group. An important feature, that we won't prove here, is that supersymmetry in the case of an F-term potential is preserved if and only if the Kähler covariant derivative vanishes at the minimum $D_I W = \partial_{\phi^I} W + W \partial_{\phi^I} K / M_P^2 = 0$. In this case we find that

$$V_F \Big|_{D_I W = 0} = -3 e^{\frac{K}{M_P^2}} \frac{|W|^2}{M_P^2} \leq 0 \,, \tag{2.15}$$

so that any supersymmetry preserving minimum gives rise to a negative (or zero) cosmological constant. The same holds true in the presence of a D-term potential that breaks supersymmetry unless $\mathcal{D}_A = 0$ at the minimum which implies $V_D = 0$. Since in our universe we have $V_{today} > 0$, we necessarily need to break supersymmetry. However, the scales involved are very different. We are currently probing particle physics at energies of roughly $10^4 GeV$ at the LHC and we have so far found no sign of supersymmetry. So we would need $e^{\frac{K}{M_P^2}} K^{I\bar{J}} D_I W \overline{D_J W} > (10^4 GeV)^4$ and for an F-term potential that explains our observed cosmological constant we need

$$V_{today} \approx 10^{-120} M_P^4 \approx (2.4 \times 10^{-12} GeV)^4 \approx e^{\frac{K}{M_P^2}} \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3 \frac{|W|^2}{M_P^2} \right) \,, \tag{2.16}$$

so that in the minimum $e^{\frac{K}{M_P^2}} K^{I\bar{J}} D_I W \overline{D_J W}$ and $-3 e^{\frac{K}{M_P^2}} \frac{|W|^2}{M_P^2}$ must cancel very precisely.

Another simple feature of supersymmetric vacua, that you are invited to check, is that

$$D_I W = 0 \qquad \Rightarrow \qquad \partial_{\phi^I} V_F = 0 \,. \tag{2.17}$$

This means that if we solve the simpler equation $D_I W = 0$, we find critical points of V_F. Furthermore, these supersymmetric critical points are stable. [If at the critical point $V = 0$, then the masses of all scalar fields are positive semi-definite. If $V < 0$ at the minimum, then we are in AdS and scalar masses can be negative without causing an instability as long as they are above the Breitenlohner-Freedman bound, which is the case, whenever $D_I W = 0$. This follows from the non-canonical kinetic term in AdS: $-\frac{1}{2} \partial_\mu \phi g^{\mu\nu} \partial_\nu \bar{\phi}$ with $g_{\mu\nu} \neq \eta_{\mu\nu}$.]

2.5 Two simple examples

Example 1:

Let us work out explicitly a simple example of a 4d $\mathcal{N} = 1$ supergravity theory to get a feel for the equations involved. We restrict to a single chiral multiplet with complex scalar ϕ and take

$$K = \frac{1}{2} \phi \bar{\phi} \,, \qquad W = M_P^2 \, m \,, \tag{2.18}$$

where $m \in \mathbb{R}$. This leads to $K_{\phi\bar{\phi}} = \frac{1}{2}$ and $D_\phi = \partial_\phi W + W \partial_\phi K / M_P^2 = \frac{1}{2} m \bar{\phi}$. This gives the scalar potential

$$V_F = e^{\frac{K}{M_P^2}} \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3 \frac{|W|^2}{M_P^2} \right) = e^{\frac{|\phi|^2}{2 M_P^2}} \left(\frac{1}{2} m^2 |\phi|^2 - 3 m^2 \, M_P^2 \right) \,. \tag{2.19}$$

From the discussion above, we know that $D_I W = \frac{1}{2} m\bar{\phi} = 0$, i.e. $\phi = 0$, is a minimum of the scalar potential that preserves supersymmetry. Expanding the potential for small ϕ we find

$$V_F = -3m^2\, M_P^2 - m^2|\phi|^2 + \dots\,. \tag{2.20}$$

So the minimum of the scalar potential has $V_F = -3m^2 M_P^2$ and the mass of the complex scalar field is $-2m^2$. While this is a maximum of the scalar potential, the scalar field does actually not roll away from the critical point since its mass is above the Breitenlohner-Freedman bound

$$m_\phi^2 = -2m^2 \geq \frac{3}{4}\frac{V_F}{M_P^2} = -\frac{9}{4}m^2. \tag{2.21}$$

[Note, that in Minkowski space we have $V_F = 0$ and therefore the Breitenlohner-Freedman bound in this limits becomes the simple statement that scalars have to have positive semi-definite masses squared.]

Example 2:

Let us now look at another example with a single scalar field ϕ and the following Kähler and superpotential

$$K = \frac{1}{2}\phi\bar{\phi} - \frac{c}{8}\frac{(\phi\bar{\phi})^2}{M_P^2}\,, \qquad W = M_P m\phi\,, \tag{2.22}$$

with $c, m \in \mathbb{R}$. This leads to $K_{\phi\bar{\phi}} = \frac{1}{2}(1 - c|\phi|^2/M_P^2)$ and $D_\phi = \partial_\phi W + W\partial_\phi K/M_P^2 = M_P m\left(1 + \frac{1}{2}\left(1 - \frac{1}{4}c|\phi|^2/M_P^2\right)|\phi|^2/M_P^2\right)$. This gives the scalar potential

$$V_F = e^{\frac{\frac{1}{2}|\phi|^2 - \frac{c|\phi|^4}{8M_P^2}}{M_P^2}} \left(\frac{\left[M_P m\left(1 + \frac{1}{2}\left(1 - \frac{1}{4}\frac{c|\phi|^2}{M_P^2}\right)\frac{|\phi|^2}{M_P^2}\right)\right]^2}{\frac{1}{2}\left(1 - \frac{c|\phi|^2}{M_P^2}\right)} - 3m^2|\phi|^2 \right). \tag{2.23}$$

Since the scalar potential is a function of $|\phi|^2 = \phi\bar{\phi}$, it has a critical point $\partial_\phi V = \partial_{\bar{\phi}} V = 0$ at $\phi = \bar{\phi} = 0$. To study this critical point we can again expand the potential for small values of ϕ to get

$$V_F = 2M_P^2 m^2 + \left(m^2 + 2m^2 + 2cm^2 - 3m^2\right)|\phi|^2 + \ldots = 2M_P^2 m^2 + 2cm^2|\phi|^2 + \ldots \tag{2.24}$$

So we see that this scalar potential has a minimum at $\phi = 0$ with $V_{min} = 2M_P^2 m^2 > 0$. The mass of the complex scalar field is given by $4cm^2 > 0$. Reversing the logic from above, we know that $V_{min} > 0$ implies that supersymmetry is spontaneously broken in the minimum and indeed we have $D_\phi W|_{\phi=0} = M_P m$. Note that the supersymmetry breaking scale $e^{\frac{K}{M_P^2}} K^{I\bar{J}} D_I W \overline{D_J W} = 2M_P^2 m^2$ is equal to the value of the potential and hence to the cosmological constant in this minimum.

2.6 The eta-problem in supergravity

In the above example, we have included a Planck suppressed operator in the Kähler potential, i.e. we have started to expanded in powers of $1/M_P^2$

$$K = \frac{1}{2}\phi\bar{\phi} - \frac{c}{8}\frac{(\phi\bar{\phi})^2}{M_P^2}\,. \tag{2.25}$$

In general, since the first term determines the kinetic term, which in this case is canonical, and has to be non-vanishing, it is very difficult to forbid higher order corrections to the Kähler potential that are of the form $c_n \frac{(\phi\bar{\phi})^n}{M_P^{2n-2}}$. Since the scalar potential is multiplied by $e^{\frac{K}{M_P^2}}$, such corrections to K therefore lead to corrections to the scalar potential of the form

$$V \to V + \sum_n \delta V_n \,, \qquad \text{with} \qquad \delta V_n \propto c_n \frac{(\phi\bar{\phi})^n}{M_P^{2n-2}} V \,. \tag{2.26}$$

So in particular for $n = 2$ this correction is of the type discussed above and leads to a correction to η_V of the order of c_2. So we need to know the size of these Planck suppressed operators.

The modification of K due to Planck suppressed operators does also modify the inverse Kähler metric $K^{\phi\bar{\phi}}$ and the derivative K_ϕ appearing in $D_\phi W$, so there are a variety of Planck suppressed corrections to the scalar potential at each order in $1/M_P$. For these not to spoil inflation, they either have to be all very small or, if they are all order 1, then they have to (almost) cancel each other.

3 dS vacua in string theory (GKP, KKLT and LVS)

Now that we have learned some basic concepts related to supergravity, we will discuss and analyze concrete models that arise in string compactifications. We will focus on a very simple toy model, whenever we discuss an explicit example, to keep things tractable.

As discussed above, for cosmological models we are mostly interested in the scalar fields of the theory. In this section we will discuss what kind of scalar fields arise from string compactifications. Many important details will be skipped, since they will be covered in the lectures by Edvard Musaev.

3.1 The string scale and the KK-scale

String theory is unfortunately rather complicated and we can't really solve full-fledged string theories in non-trivial backgrounds that give interesting models for cosmology. However, what we can do is to take a low energy limit of string theory in which we restrict to energies below the string scale $E \ll M_s = 1/\sqrt{\alpha'}$, where $\sqrt{\alpha'}$ is the string length, the only dimensionfull parameter in string theory, that sets the length scale of the strings. In this limit the various string theories reduce to a 10d supergravity theory, i.e. a supersymmetric theory of particles instead of strings. The particles in these theories are massless and correspond to the lowest excitation of the string.

Another important scale enters when we compactify such a 10D supergravity theory to 4D. This scale is called the Kaluzha-Klein (KK) scale and it is given by the inverse size (or the inverse radius) of the compact space $M_{KK} = 1/R$.[8]

To see the relevance of this KK-scale let us look at a very simple example: a real scalar field $\phi(x^\mu, y)$ in a 5d spacetime $\mathbb{R}^{3,1} \times S^1$, where the circle S^1 has radius R, so that we have $y = y + 2\pi R$. We do a Fourier expansion

$$\phi(x^\mu, y) = \sum_{k\in\mathbb{Z}} \phi_k(x^\mu) e^{\mathrm{i}ky/R} \,. \tag{3.1}$$

[8]If we compactify six dimensions then we can in principles have different radii associated with the different directions.

Since we took ϕ to be real, the Fourier coefficients have to satisfy $\bar{\phi}_k = \phi_{-k}$. Now let us reduce a 5D action for a massless real scalar field to four dimensions by integrating over the circle S^1

$$\begin{aligned} S &= -\int d^4xdy\partial_M\phi\partial^M\phi = -\int d^4xdy(\partial_\mu\phi\partial^\mu\phi + \partial_y\phi\partial^y\phi) \\ &= -\int d^4xdy\sum_{k,l}\left(\partial_\mu\phi_k\partial^\mu\phi_l - \frac{kl}{R^2}\phi_k\phi_l\right)e^{iy(k+l)/R} \\ &= -\int d^4x(2\pi R)\sum_k\left(\partial_\mu\phi_k\partial^\mu\phi_{-k} + \frac{k^2}{R^2}\phi_k\phi_{-k}\right)\,. \end{aligned} \tag{3.2}$$

So in 4D we now have a massless scalar field ϕ_0 and an infinite number of massive scalar fields ϕ_k with masses set by the size of the inverse radius $M_{KK} = 1/R$. These massive fields are called the KK-tower and in string compactifications we usually neglect them. This means that we need to restrict ourselves to energies below the KK-scale $E \ll M_{KK}$.

The Planck scale in four dimensions can be obtained by reducing the 10D Einstein Hilbert action to 4D. Doing this one finds for an internal space with volume $vol_6 = \mathcal{V}(\alpha')^3$ that $M_P = g_s^{-1/4}\mathcal{V}^{1/2}M_s$, where g_s is the string coupling, that sets the interaction strength for the strings in string theory. This string coupling is not a free parameter but it is rather determined via a real scalar field ϕ as $g_s = e^\phi$, where ϕ is called the dilaton.

The 10D supergravity theories that we started with are derived in the limit of large volume[9] $\mathcal{V} \gg 1$ and weak string coupling $g_s \ll 1$. So in order to trust our supergravity we have to satisfy

$$E \ll M_{KK} \ll M_s \ll M_P \approx 2.4\times 10^{18}GeV\,, \tag{3.3}$$

where we used that $M_{KK} = 1/R \approx 1/(vol_6)^{1/6} = M_s/\mathcal{V}^{1/6}$.

Note, that the above are generic restrictions that we need to satisfy in compactification of 10d SUGRA theories that follow from string theory. These conditions need to be met also during inflation. In particular, we can usually neglect (integrate out) heavy particles with masses above the Hubble scale $H \approx V_{inf}^2/M_P$. For large field models of inflation we have $V_{inf} \lesssim 10^{16}GeV$ and therefore $H \lesssim 10^{14}GeV$ so that there is no real parametric control. However, for example string loop corrections are suppressed by a factor of $16\pi^2$, so that $g_s \lesssim .5$ might be considered as weak coupling region. If we take for example $g_s = .1$ and $\mathcal{V} = 10^3 \gg 1$, then we find $M_{KK} \approx 1.3\times 10^{16}GeV$ and $M_s \approx 4.3\times 10^{16}GeV$. This shows that it is difficult, if not impossible to get parametric control in large field models of inflation in string theory. However, this does not mean that one cannot build models of large field inflation in string theory. We often understand the leading string loop corrections and α' corrections that modify our theory. We can calculate those and check explicitly how we can ensure that they don't modify our results in any significant way. So we don't really need to have $1/g_s$ and $\mathcal{V}$ to be parametrically larger than 1.

3.2 The moduli problem

An important feature of dimensional reductions is that not only higher dimensional scalar fields can give rise to 4D scalar fields. For example, if we have a 5D vector field A_M then it will reduce to a 4D vector field A_μ and a 4D scalar A_y. The 4D scalar field arises since

[9] The internal dimension in string compactifications are usually tiny, although gravitational experiments on extra dimensions require only $R \lesssim 10^{-4}m$ or so. When we say large extra dimensions or large volume we mean that the radius $R \approx (vol_6)^{1/6} \gg 1.6\times 10^{-35}m \approx l_P$ with l_P the Planck length.

the one free index of A_M extends along the internal direction. Now, if we have six internal circle directions y^I, $I = 1, 2, \ldots, 6$, then the internal index of A_M can extend along six different directions giving rise to six scalar fields A_{y^I}. The 10D supergravities that are the low-energy limits of string theory contain fields with more than one index like B_{MN} and C_{MNOP}. These indices can likewise extend along the internal directions and give rise to scalars. So string theory compactifications usually give rise to a lot of massless scalar fields.

While this might not immediately seem like a problem, it actually is the so called "moduli problem". The term modulus here refers to a massless scalar field. These appear abundantly in the simplest string compactifications but so far, besides the Higgs field, we haven't observed any scalar fields.

There are several reasons why one should worry about light scalar fields: They would lead to a 5th force between SM particles, if they couple to them, they could also be abundantly produced in the early universe and lead to unobserved effects and lastly from the theoretical point of view, they can cause problems. As mentioned above, we have to stay in the 'large volume' and weak coupling regime. The string coupling is set by a scalar field, the dilaton, as $g_s = \langle e^{\phi} \rangle$. If this dilaton field is massless at leading order and there are some tiny perturbative or non-perturbative corrections to its potential, then it will start to roll. If it roles to the strong coupling region $\langle e^{\phi} \rangle \gtrsim 1$, then we can't trust our theory anymore. Similarly, as we will discuss below, the size of internal cycles also correspond to vacuum expectation values of scalar fields (that sit in the metric g_{MN}), so that for example $R = \langle \phi \rangle$. If such a massless scalar field starts to roll either to 0 or to ∞ due to the leading correction $\delta V \propto \pm \phi^p$, then either we couldn't trust our theory since the volume becomes small for $R \to 0$ or it would become 5D for $R \to \infty$. So it seems very important that we stabilize all scalar fields by generating a non-trivial potential that gives them a mass at a fixed acceptable value $\langle \phi \rangle$. Before we do that, we will discuss in a little bit more detail what kind of scalar fields we expect to get from string compactifications on so called Calabi-Yau (CY) manifolds.

3.3 The moduli of type IIB compactifications

We will for concreteness restrict ourselves to type IIB string theory and its low energy limit, which is a 10d $\mathcal{N} = 2$ supergravity. This theory contains two 10D scalars, the dilaton that sets the string coupling and the axion C_0.[10] These two can be combined into a complex scalar, the axio-dilaton,[11]

$$S = C_0 + \frac{\mathrm{i}}{g_s} = C_0 + \mathrm{i}e^{-\phi} \,. \tag{3.4}$$

Additionally, there are two real 2-form (objects with two antisymmetric indices) B_{MN} and C_{MN}. These can in principle combine to give rise to complex 4D scalars, if the two indices extend along the internal directions. However, we will restrict to compactifications where there are no appropriate 2-cycles along which the two indices of B_{MN} and C_{MN} can extend. So we will restrict to models where these two fields do not give rise to 4d scalar fields. There is one more, so called 4-form, C_{MNOP} that has four indices and that will give rise to scalar fields in 4d. These real scalar fields will combine with scalar fields from the metric (so called geometric moduli) to form complex 4d scalar fields.

[10] An axion is a scalar field whose action is invariant under a (discrete) shift-symmetry.
[11] The axio-dilaton is also often denoted by τ.

Additionally to the above fields we only have the 10D metric g_{MN} that will give rise to scalar fields. How this is happening is discussed in Edvard Musaev lectures for the case of interest, which are CY-manifolds. Here we discuss the toy example of two circles, i.e. a torus $T^2 = S^1 \times S^1$. We can think of a torus as a sheet of paper on which we identify the opossite sides, as shown in figure 3.

Figure 3: Identifying two opposite sides of a sheet of paper leads to a cylinder. Identifying the two ends of the cylinder, we find a torus T^2. If this torus is sufficiently small, then the existence of extra dimensions compactified on a space with the shape of such a torus, is consistent with all current experimental bounds.

When looking at the torus as a parallelogram, then it is clear that it is describe by three real parameters: The length of the side at the bottom R_1, the height of the parallelogram R_2 and the angle θ between the two sides. The overall volume of the torus is given by R_1R_2, while the shape of the torus is determined by R_1/R_2 and θ. If we write down a theory with dynamical gravity $g_{MN}(x^\mu, y^I)$, i.e. GR, and compactify it on this torus then these size and shape parameters of the compact space are part of the internal metric $g_{y^I y^J}(x^\mu, y^I)$ and they give rise to dynamical 4d scalar fields.

The same is true for more complicated string compactifications on six real dimensional CY-manifolds. We can actually construct a (singular) limit of a CY-manifold by taking three copies of the above T^2. If we take three identical T^2, which can be enforced by a $\mathbb{Z}_3$ symmetry that maps the T^2's into each other, then we have only the three real scalar fields discussed above.[12]

As mentioned above, in string theory we also have the 4-form field C_{MNOP}. In the case of the above compactification it gives rise to one additional real scalar field, which combines with the volume modulus R_1R_2 to give a complex modulus T. The two real scalar fields R_1/R_2 and θ combine to give one complex scalar U. This model, is the most simple string compactification that gives rise to a 4d $\mathcal{N} = 1$ supergravity and it is often called STU-model. Doing a compactification of the 10D $\mathcal{N} = 2$ supergravity action on this compact space, leads to a 4d $\mathcal{N} = 1$ supergravity theory that can be described by K and W. In particular, in this concrete case we find

$$\begin{aligned} K &= -\ln\left(-\mathrm{i}(S-\bar{S})\right) - 3\ln\left(-\mathrm{i}(T-\bar{T})\right) - 3\ln\left(-\mathrm{i}(U-\bar{U})\right) , \\ W &= 0 . \end{aligned} \tag{3.5}$$

As mentioned above, to trust out theory we have to ensure that the string coupling is small and the volume of the internal space is large. This is equivalent to demanding that $\mathrm{Im}(S) \gg 1$ and $\mathrm{Im}(T) \gg 1$. [13]

[12]To make sure that the internal space is the direct product of three T^2's we can mod out by $\mathbb{Z}_2$ symmetries that inverts the coordinates on all but one of the T^2's. The resulting space is $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$.

[13]We can redefine fields by multiplying them by i. Sometimes people use $\mathcal{S} = \mathrm{i}S$ and $\mathcal{T} = \mathrm{i}T$ instead.

More generic string compactifications of type IIB theory on CY-manifolds give rise to

- the axion-dilaton modulus S, that has to satisfy $\mathrm{Im}(S) \gg 1$ to ensure that we are at weak string coupling,
- multiple, so called *Kähler moduli*, T^k whose imaginary parts control the volumes of the internal cycles and that have to satisfy $\mathrm{Im}(T^k) \gg 1$,
- and multiple, so called *complex structure moduli*, U^i.[14]

3.4 The Gukov-Vafa-Witten flux superpotential

One classic reference for flux compactifications is the paper by Giddings, Kachru and Polchinski (GKP) [7]. Here we will however, just be able to discuss some small parts of their results, including the so called Gukov-Vafa-Witten superpotential [8].

As we have seen above in our simple STU-model we can compactify string theory to obtain a rather simple 4d $\mathcal{N} = 1$ theory. In fact it is too simple: the three scalar fields have no scalar potential since $W = 0 \Rightarrow V = 0$. So the question is how we can modify our compactification so that it gives rise to a non-trivial scalar potential. One important idea here is the use of fluxes. In QED (or EM) we have the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \equiv \partial_{[\mu} A_{\nu]}$ that encodes the electic and magnetic fields. If we are in higher dimensions in a spacetime with non-trivial topology, like for example our T^2 above, then we can turn on non-trivial electric or magnetic fluxes that thread the torus and give a non-zero contribution to the scalar potential

$$\begin{aligned} S &\supset -\frac{1}{4}\int d^4x dy^1 dy^2 \sqrt{-g} F_{MN}F^{MN} \\ &= -\frac{1}{4}\int d^4x dy^1 dy^2 \sqrt{-g}\left(F_{\mu\nu}F^{\mu\nu} + F_{y^1y^2}F^{y^1y^2}\right) . \end{aligned} \tag{3.6}$$

If the flux $F_{y^1y^2}$ is not vanishing, then the second term in the above action will give a contribution to the scalar potential. This contribution will depend on the moduli that are in the metric since the indices on $F^{y^1y^2}$ are raised with the inverse metric $g^{y^Iy^J}$.

In string theory compactifications this was pioneered by Gukov-Vafa-Witten and used for example in GKP. In type IIB string theory we have the two 2-form potentials B_{MN} and C_{MN} that do not give rise to scalar fields in our particular compactifications but we can nevertheless have a non-zero field strength for them. In particular, we can have a non-zero value for the complex combination

$$G_{MNO}(S) = \partial_{[M}B_{NO]} - S\,\partial_{[M}C_{NO]} . \tag{3.7}$$

If we turn on a non-trivial flux $G_{I_1I_2I_3}(S)$ along the internal directions, then we should get a contribution to the scalar potential that depends on S and maybe other moduli. As you will learn in Edvard Musaev lectures, the complex structure moduli U^i are packaged in a holomorphic 3-form $\Omega_{J_1J_2J_3}(U^i)$, i.e. in another object with three indices. The contraction of these two 3-forms, integrated over the internal CY-manifold, is actually the resulting superpotential that one finds from a proper reduction of the higher dimensional theory[15]

$$K = -\ln\left(-\mathrm{i}(S-\bar{S})\right) - 3\ln\left(-\mathrm{i}(T-\bar{T})\right) - 3\ln\left(F(U^i,\bar{U}^{\bar{\imath}})\right) ,$$

[14] For generic CY compactifications we can only derive the scalar potential in the so called large complex structure limit, which requires us to also demand that $\mathrm{Im}(U^i) \gg 1$.

[15] We are from now on setting $M_P = 1$.

$$W = \int_{CY} d^6y\sqrt{g_{CY}}G_{I_1I_2I_3}(S)g_{CY}^{I_1J_1}g_{CY}^{I_2J_2}g_{CY}^{I_3J_3}\Omega_{J_1J_2J_3}(U^i) \equiv W_{GVW}(S,U^i)\,, \quad (3.8)$$

where the Kähler potential in the case of multiple U^i moduli is given by $F(U^i,\bar{U}^{\bar{\imath}}) = \int_{CY} d^6y\sqrt{g_{CY}}\Omega_{I_1I_2I_3}(U^i)g^{I_1J_1}g^{I_2J_2}g^{I_3J_3}\bar{\Omega}_{J_1J_2J_3}(\bar{U}^{\bar{\imath}})$. Thus we have a non-trivial scalar potential that involves the complex structure moduli U^i and the axio-dilaton S. Let us study the resulting scalar potential in more detail and restrict for simplicity to the case of single Kähler modulus T:

The superpotenial W above does not depend on T [16] so we calculate

$$\begin{aligned} D_TW &= \partial_TW + W\partial_TK = 0 - \frac{3W}{T-\bar{T}}\,, \\ K_{T\bar{T}} &= \partial_T\partial_{\bar{T}}K = -\frac{3}{(T-\bar{T})^2}\,. \end{aligned} \quad (3.9)$$

Using this we find that

$$K^{T\bar{T}}D_TW\overline{D_TW} = -\frac{(T-\bar{T})^2}{3}\left(-\frac{3W}{T-\bar{T}}\right)\left(\frac{3\bar{W}}{T-\bar{T}}\right) = 3|W|^2\,. \quad (3.10)$$

This leads then to

$$\begin{aligned} V &= e^K\left(K^{T\bar{T}}D_TW\overline{D_TW} + K^{S\bar{S}}D_SW\overline{D_SW} + K^{U^i\bar{U}^{\bar{\jmath}}}D_{U^i}W\overline{D_{U^j}W} - 3|W|^2\right) \\ &= e^K\left(K^{S\bar{S}}D_SW\overline{D_SW} + K^{U^i\bar{U}^{\bar{\jmath}}}D_{U^i}W\overline{D_{U^j}W}\right)\,. \end{aligned} \quad (3.11)$$

There are several important points to make about the above form of the potential:

- The modulus T satisfies the so called *no-scale property* since its contribution inside the parenthesis cancels the $-3|W|^2$ term.
- The Kähler metric controls the kinetic terms and therefore has to be positive definite. This means that the above scalar potential is the sum of two positive definite terms.
- The modulus T only enters the above scalar potential via the prefactor $e^K \propto \frac{1}{\mathrm{i}(T-\bar{T})^3} = \frac{1}{8\,\mathrm{Im}(T)^3}$. So unless $D_SW = D_{U^i}W = 0$, $\forall i$, the volume modulus $\mathrm{Im}(T)$ will be minimized at $\mathrm{Im}(T) = \infty$ which means that our theory is decompactified and 10 dimensional, since the internal space has infinite volume. For an extremum with $D_SW = D_{U^i}W = 0$, we have $(\partial_T)^nV = 0$, $\forall n$ so that the T modulus remains a flat direction and in particular massless.

So based on the argument above, we know that the above scalar potential only has critical points when $D_SW = D_{U^i}W = 0$. For a sufficiently general choice of fluxes $D_SW = D_{U^i}W = 0$ provide the same number of independent complex equations as we have complex moduli S and U^i. Thus we expect to find solutions at which all fields S and U^i take a fixed value. For an appropriate choice of fluxes we can also satisfy the requirement $\mathrm{Im}(S) \gg 1$.

[16] T controls the volume of the internal space and therefore it corresponds to an overall rescaling of the internal metric g_{CY}. It is easy to see that $g_{CY\,IJ} \to c\,g_{CY\,IJ}$, $c > 0$, leaves the superpotential invariant. Hence W is independent of T.

Next we have to ask whether the masses of the fields S and U^i are positive, i.e. whether we are dealing with a minimum, saddle point or maximum. In the case that $D_T W = -3W/(T-\bar{T}) = 0$, i.e. when $W = 0$, then we satisfy $D_S W = D_{U^i} W = D_T W = 0$ and we have a supersymmetric solution that, by supersymmetry, is guaranteed to be stable. However, in the general case with $W \neq 0$ supersymmetry is broken. Nevertheless, we find in this particular case that the critical point is a minimum and the fields S and U^i all have positive masses. This does not require a calculation, since we saw from the form of the scalar potential that it is positive definite $V \geq 0$. So any critical point with $V = 0$ is a global minimum and the masses of the fields S and U^i have to be likewise positive definite, since moving along a negative mass eigenvalue would lower the value of the scalar potential below zero which is not possible.

So to summarize, using fluxes that threat the internal directions we have been able to find solutions in which the value of the scalar potential vanishes and in which the fields S and U^i have generically positive masses.

3.5 The KKLT construction of dS vacua

In 1998 experiments discovered that the cosmological constant in our universe is non-zero and positive. Since string theory has no free-parameters we cannot just add or turn on a cosmological constant but we rather have to find a minimum of the scalar potential with $V_{min} > 0$. This was first achieved in a seminal paper by Kachru, Kallosh, Linde and Trivedi in 2003 [9].

Their starting point is the scalar potential above, in which we have a single T modulus and fluxes that give a non-zero potential to S and the U^i. As mentioned above, the T modulus remains a flat direction, which is actually forbidden in string theory. More concretely, in string theory we cannot have continuous global symmetry. However, the above Kähler and superpotential in eqn. (3.8) are invariant under the continuous shift symmetry $\mathrm{Re}(T) \to \mathrm{Re}(T) + c$, $c \in \mathbb{R}$. In string theory such continuous symmetries are broken to discrete symmetries via non-perturbative effects and the superpotential receives the following correction[17]

$$
\begin{aligned}
K &= -\ln\left(-\mathrm{i}(S-\bar{S})\right) - 3\ln\left(-\mathrm{i}(T-\bar{T})\right) - \ln\left(-\mathrm{i}F(U^i,\bar{U}^{\bar{\imath}})\right) , \\
W &= W_{GVW}(S,U^i) + A e^{iaT} ,
\end{aligned}
\tag{3.12}
$$

where $A = A(S, U^i)$ can in principle be a function of S and U^i and $a \in \mathbb{R}$ is a model dependent constant. We see that in the regime of large volume with $\mathrm{Im}(T) \gg 1$, where we can trust our supergravity, the non-perturbative term is exponentially suppressed. Since the non-perturbative term is suppressed compared to the terms that appear in the superpotential from the fluxes, we can actually simplify our life and set S and U^i to their minimum values from above (these don't really change much since the non-perturbative corrections are small). In this case, where we essentially integrate out the heavy fields S and U^i, we find that $A(U^i, S) = A(U^i_{min}, S_{min}) = const.$ and

$$
W_0 \equiv W_{GVW}(S_{min}, U^i_{min}) = const. \tag{3.13}
$$

So we are left with the single field model

$$
K = -3\ln\left(-\mathrm{i}(T-\bar{T})\right) ,
$$

[17] The non-perturbative effects can be either Euclidean D3-branes with $a = 2\pi$ or a stack of N D7-branes that undergoes gaugino condensation which leads to $a = 2\pi/N$.

$$W = W_0 + Ae^{iaT}\,, \tag{3.14}$$

Usually one expects A to be an order one number and the same is true for W_0. However, W_0 receives many different contributions and they can in certain cases cancel and lead to $|W_0| \ll 1$.

Let us calculate

$$D_T W = \partial_T W + W \partial_T K = \mathrm{i}aAe^{\mathrm{i}aT} - 3\frac{W_0 + Ae^{\mathrm{i}aT}}{T - \bar{T}}\,. \tag{3.15}$$

Let us take for simplicity $W_0, A \in \mathbb{R}$. Then we can solve $D_T W = 0$ by solving its real and imaginary part. To that end we write $T = b + \mathrm{i}\rho$ and find

$$0 = \mathrm{Re}(D_T W) = -aAe^{-a\rho}\mathrm{Im}(e^{\mathrm{i}ab}) - 3\frac{Ae^{-a\rho}\mathrm{Im}(e^{\mathrm{i}ab})}{2\rho}\,, \tag{3.16}$$

which can be simply solved by setting $\mathrm{Re}(T) = b = 0$. Next we have to solve

$$0 = \mathrm{Im}(D_T W) = aAe^{-a\rho} + 3\frac{W_0 + Ae^{-a\rho}}{2\rho}\,. \tag{3.17}$$

An implicit solution is given by

$$W_0 = -Ae^{-a\rho_{min}}\left(1 + \frac{2}{3}a\rho_{min}\right) \neq 0\,. \tag{3.18}$$

The above implies that W_0 cannot be order 1 since we need $\mathrm{Im}(T) = \rho \gg 1$ in order to trust our low energy effective action (that would otherwise get further corrections proportional to $(e^{\mathrm{i}aT})^n$ for $n > 1$). While $|W_0| \ll 1$ is non-generic, it can certainly happen or be arranged for by an appropriate choice of $G_{I_1 I_2 I_3}(S)$.

So we have now succeeded in finding a supersymmetric minimum of the scalar potential. The potential for S and the U^i is essentially unchanged so these fields still have a positive mass. One can additionally show that b and ρ likewise both have positive masses squared. However, the value of the scalar potential at the minimum is negative

$$\begin{aligned} V_{min} &= e^K(K^{T\bar{T}} D_T W \overline{D_T W} - 3|W|^2) \\ &= -3e^K|W|^2 = -3\frac{1}{8\rho_{min}^3}|W_0 + Ae^{-a\rho_{min}}|^2 \\ &= -3\frac{1}{8\rho_{min}^3}\left|\frac{2}{3}\rho_{min} aAe^{-a\rho_{min}}\right|^2 = -\frac{a^2A^2e^{-2a\rho_{min}}}{6\rho_{min}} < 0\,. \end{aligned} \tag{3.19}$$

One can show that the scalar potential does not have any other minima (that could in principle have a positive value $V_{min} > 0$). For $b = 0$, this can be seen by simply plotting the scalar potential, as is done in figure 4.

In order to obtain a dS vacuum KKLT added a so called uplift term to their model that for appropriately chosen coefficients lifts the minimum to a positive value without destabilizing any moduli. Such an uplift term can arise from higher dimensional objects in string theory that are called anti-D3-branes. In the low energy effective action this uplift can be described by a new chiral multiplet N that however has no scalar degrees of freedom since its scalar part is actually given by a fermion bilinear $\bar{\chi}\chi$.[18] The presence of this new field modifies W and K such that

$$K = -3\ln\left(-\mathrm{i}(T - \bar{T})\right) + N\bar{N}\,,$$

[18] χ is the Goldstino, the fermionic goldstone particle that arises when we break supersymmetry. χ gets eaten by the gravitino Ψ_μ which then becomes massive.

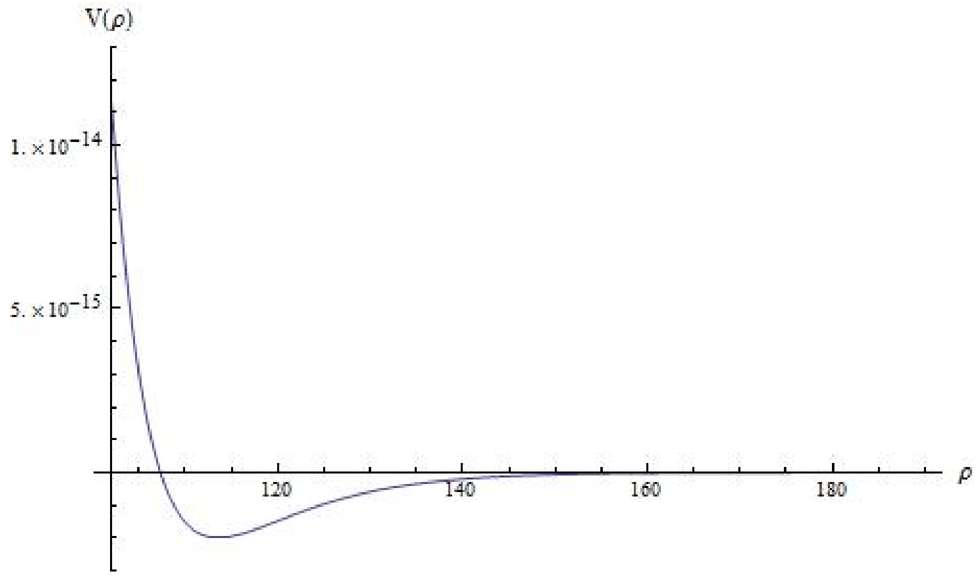

Figure 4: The scalar potential for $\mathrm{Re}(T) = 0$ as a function of $\mathrm{Im}(T) = \rho$ for the values $W_0 = -10^{-4}$, $A = 1$ and $a = 0.1$.

$$W = W_0 + Ae^{iaT} + \mu N \equiv W_{KKLT} + \mu N \,, \tag{3.20}$$

and since the scalar N is actually a fermion bilinear, we can use all our formulas from above but at the end we have to set $N = 0$ to get the bosonic answer. So for example, we can calculate

$$\begin{aligned} D_N W &= \partial_N W + W \partial_N K = \mu + W \bar{N} = \mu \,, \\ D_T W &= \partial_T W + W \partial_T K = \partial_T W_{KKLT} - \frac{3}{T - \bar{T}} W_{KKLT} \equiv D_T W_{KKLT} \,. \end{aligned} \tag{3.21}$$

So we see that whenever $\mu \neq 0$, then there are no supersymmetric solutions. We also see that the new field N does not change $D_T W$ since we have to set $N = 0$.

We can now calculate the scalar potential

$$\begin{aligned} V &= e^K (K^{T\bar{T}} D_T W \overline{D_T W} + K^{N\bar{N}} D_N W \overline{D_N W} - 3|W|^2) \\ &= \frac{1}{8\rho^3} (K^{T\bar{T}} D_T W_{KKLT} \overline{D_T W_{KKLT}} + |\mu|^2 - 3|W_{KKLT}|^2) \\ &= V_{KKLT} + \frac{|\mu|^2}{8\rho^3} \,. \end{aligned} \tag{3.22}$$

So we see that the only effect of the new field is the addition of a positive definite term $|\mu|^2/(8\rho^3)$ to the scalar potential. For an appropriate choice of $|\mu|^2$ we now find dS vacua, as is shown in figure 5.

These dS vacua require $|\mu|^2 \ll 1$, which can be natural in string theory model that can give rise to exponentially small $|\mu|^2$. Since we are now canceling a positive and a negative term to obtain dS vacua, we can fine tune our parameters so that $V_{min} > 0$ and $V_{min} \ll 1$. The supersymmetry breaking scale set by $|\mu|^2$ is in these models independent of the value of V_{min}. So these string models, although they have been the first dS vacua ever constructed, have many appealing features.

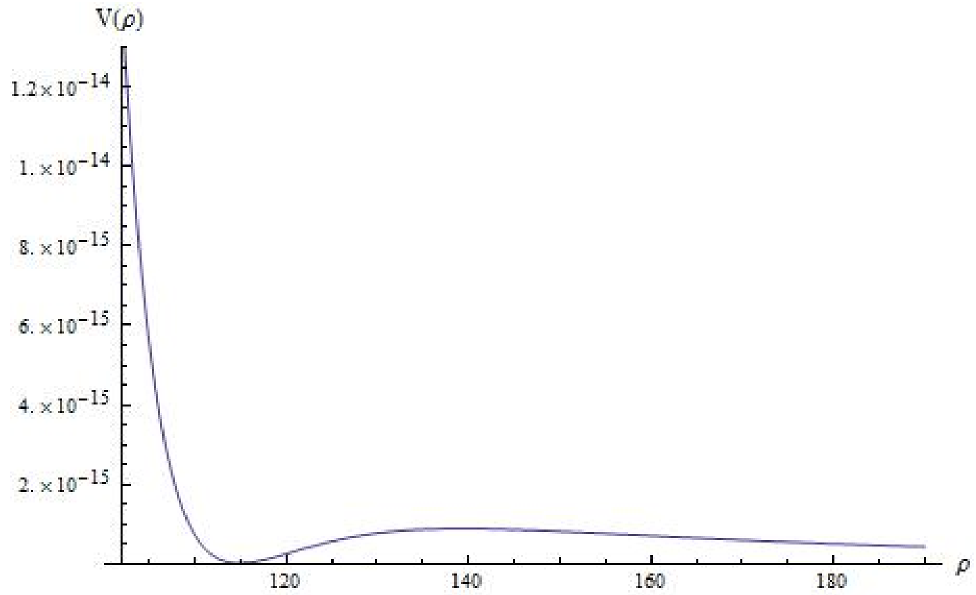

Figure 5: The scalar potential for $\mathrm{Re}(T) = 0$ as a function of $\mathrm{Im}(T) = \rho$ for the values $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$ and $|\mu|^2 = 2.4 \times 10^{-8}$.

3.6 The LVS construction of dS vacua

Another way of stabilizing the flat Kähler moduli T^k was proposed by Balasubramanian, Berglund, Conlon and Quevedo [10]. This is the so called Large Volume Scenario (LVS). The author included in their model perturbative (so called α') corrections. These are Planck suppressed corrections that we discussed above. They modify the Kähler potential but the superpotential (due to its holomorphicity) is actually protected from perturbative corrections and it still only receives non-perturbative corrections of which we again include the leading term. Allowing for multiple Kähler moduli T^k, we have the following Kähler and superpotential

$$\begin{aligned} K &= -\ln\left(-\mathrm{i}(S-\bar{S})\right) - 2\ln\left(\mathcal{V}(T^k) + \frac{\xi}{2}\left(\frac{-\mathrm{i}(S-\bar{S})}{2}\right)^{\frac{3}{2}}\right) - \ln\left(-\mathrm{i}F(U^i,\bar{U}^{\bar{\imath}})\right) , \\ W &= W_{GVW}(S,U^i) + \sum_k A_k e^{ia_k T^k} . \end{aligned} \tag{3.23}$$

Here $\mathcal{V}$ is the dimensionless volume and in the case of a single modulus we have $\mathcal{V} = \mathrm{Im}(T)^{\frac{3}{2}}$ so we have $-3\ln\left(-\mathrm{i}(T-\bar{T})\right) = -2\ln(\mathcal{V}) - \ln(8)$. So up to an irrelevant factor, the first term in the above Kähler potential is the usual one from above. The second term is the perturbative correction that depends on a numerical prefactor ξ and the axio-dilaton S.

As above, we can argue that the perturbative and non-perturbative correction should be small compared to the leading tree-level term that is the first term in W. This means that we can again stabilize S and the U^i as above (i.e. without taking into account the new term in K and the potential dependence of A on S and the U^i). This leads then to the simplified form

$$K = -2\ln\left(\mathcal{V}(T^k) + \zeta\right) ,$$

$$W = W_0 + \sum_k A_k e^{ia_k T^k} , \tag{3.24}$$

where ζ is a model dependent constant of order 1.

The simplest explicit LVS construction has two Kähler moduli that we will call T_l and T_s where the subscripts stand for large and small. The volume is given by $\mathcal{V} = \text{Im}(T_l)^{\frac{3}{2}} - \text{Im}(T_s)^{\frac{3}{2}}$. Calabi-Yau manifolds with such a volume are of so called "Swiss cheese type" since T_s controlls the size of a hole inside the manifold. In order to be in a large volume regime with $\mathcal{V} \gg 1$ we need $\text{Im}(T_l) \gg \text{Im}(T_s) \gg 1$. This then implies that we can neglect the term $A_l e^{\text{i}a_l T_l}$ in the the superpotential which leads to

$$\begin{aligned} K &= -2\ln\left(\text{Im}(T_l)^{\frac{3}{2}} - \text{Im}(T_s)^{\frac{3}{2}} + \zeta\right) , \\ W &= W_0 + A_s e^{ia_s T_s} . \end{aligned} \tag{3.25}$$

While this model doesn't look to complicated, it is actually non-trivial to minimize the scalar potential. The reason is that in the LVS case, the scalar potential has an AdS minimum that is non-supersymmtric so that we cannot find it by solving $D_{T_l}W = D_{T_s}W = 0$.

One can show that the real parts of T_l and T_s vanish in this minimum. Then it is actually easiest to calculate the scalar potential in terms of $\mathcal{V}$ and $\text{Im}(T_s) \equiv t$ and do an expansion in terms of $\mathcal{V} \gg 1$. Taking furthermore into account that $a_s t \gg 1$ to neglect higher order non-perturbative corrections we get

$$V = \frac{8a_s^2 A_s^2 \sqrt{t} e^{-2a_s t}}{3\mathcal{V}} + \frac{4a_s A_s t W_0 e^{-a_s t}}{\mathcal{V}^2} + \frac{3\zeta |W_0|^2}{2\mathcal{V}^3} + \ldots \tag{3.26}$$

In order for this scalar potential to have a minimum with respect to the overall volume $\mathcal{V}$ we need all three terms to be relevant. This is only possible (for reasonable values of the parameters), if the small volume modulus is related to the overall volume as $e^{a_s t} \propto \mathcal{V}$. Note that this implies that the volume $\mathcal{V}$ is actually exponentially large since $a_s t \gg 1$, hence the name Large Volume Scenario. One can check that for such a scaling, the above potential is the leading contribution and the other terms are suppressed by (fractional) powers of the exponentially large volume.

Taking the parameters to be real one can check that $\partial_{\mathcal{V}} V = \partial_t V = 0$ for the above leading potential is solve for $at \gg 1$ by

$$\mathcal{V} = -\frac{3}{4}\frac{W_0}{a_s A_s}\sqrt{t} e^{a_s t} , \qquad t = \zeta^{\frac{2}{3}} . \tag{3.27}$$

So in order to have a positive volume we need $A_s W_0 < 0$ and we also need $\zeta > 0$ so that $\text{Im}(T_s) = t$ is real. One can show that $D_{T_l}W \neq 0$ and $D_{T_s}W \neq 0$ for the above solution, so supersymmetry is broken.

The above critical point corresponds to a minimum of the scalar potential in which all scalar fields have a positive mass squared. However, the value of the potential at the minimum is $V_{min} < 0$, so that we again have to uplift it with an anti-D3-brane as was done in the KKLT construction above.

4 Models of inflation in string theory

Full-fledged string theory models of inflation are rather complicated and many of them make explicitly use of stringy ingredients like D-branes and the different internal dimensions. However, there are a few simple ideas for models that we can discuss easily from a purely 4d $\mathcal{N}=1$ supergravity point of view. We will work in the KKLT or LVS setups discussed above and ask which scalar fields could be potential inflaton candidates and what kind of inflationary models they can give rise to.

4.1 Axions as inflaton

From the above Kähler potential in the STU model

$$K = -\ln\left(-\mathrm{i}(S-\bar{S})\right) - 3\ln\left(-\mathrm{i}(T-\bar{T})\right) - 3\ln\left(-\mathrm{i}(U-\bar{U})\right)\,, \tag{4.1}$$

we see that the real parts of the three fields S, T and U do not appear in the Kähler potential. This is not a coincidence but in the absence of fluxes and non-perturbative effects, these real parts actually enjoy a continuous shift symmetry, like for example $\mathrm{Re}(T) \to \mathrm{Re}(T) + c$, $c \in \mathbb{R}$. Such symmetries can either be explicitly broken by fluxes as is the case for S and U or they are broken by non-perturbative effects as is the case for T. However, one can show that they are not broken by perturbative corrections to the Kähler potential. This means that Planck suppressed corrections to the Kähler potential lead to (recall that we have set $M_P = 1$)

$$K_{cor} = K + \sum_n \left(F_n(S-\bar{S}) + G_n(T-\bar{T}) + H_n(U-\bar{U})\right)\,, \tag{4.2}$$

i.e. the corrections do not depend on the real parts of the moduli. These real parts of the moduli are called axions. They are fields that, at least perturbatively, can only couple via derivatives since nothing else would be invariant under the shift symmetry. Due to this shift symmetry and the corresponding protection from Planck suppressed corrections to the inflaton potential, these axions are excellent inflaton candidates and we will discuss some inflationary models based on these axions in the next subsections. These models are not full-fledged working models of string inflation since they are too simple, but they allow us to discuss many important features and ideas.

4.2 Natural inflation from the Kähler modulus axion?

As we have argued above, the complex structure moduli U^i and the axion-dilaton S are generically stabilized at a much higher scale and can be integrated out from the low-energy effective action leading to

$$\begin{aligned} K &= -3\ln\left(-\mathrm{i}(T-\bar{T})\right) + N\bar{N}\,, \\ W &= W_0 + Ae^{iaT} + \mu N \equiv W_{KKLT} + \mu N\,, \end{aligned} \tag{4.3}$$

and the full scalar potential is given by

$$V(\rho, b) = \frac{4aA^2\rho e^{-2a\rho}(a\rho+3) + 12aA\rho W_0 e^{-a\rho}\cos(ab) + 3\mu^2}{24\rho^3}\,. \tag{4.4}$$

Now we want to use the dS vacuum of the above potential as end point of a period of inflation. The shift symmetry for the $\mathrm{Re}(T) = b$ modulus is broken by non-perturbative effects Ae^{iaT} to a discrete shift-symmetry (hence we expected and got a cosine potential). The non-perturbative potential Ae^{iaT} is the leading term in an infinite series of non-perturbative corrections of the form $(Ae^{iaT})^n$ and in order to only keep the leading term we need to demand that

$$|Ae^{iaT}| \;\gg\; |(Ae^{iaT})|^2$$

$$
\begin{aligned}
1 &\gg |(Ae^{iaT})| \\
1 &\gg |A|e^{-a\rho} .
\end{aligned}
\tag{4.5}
$$

Since A is of order one this amounts to $a\rho \gtrsim 1$, which is satisfied for our numerical values above in figure 5, that lead to $a\rho \approx 11.5$. Note that this condition is essentially equivalent to the requirement of being in a large volume region, i.e. having $\mathrm{Im}(T) = \rho \gg 1$.

Now we want to use one of the fields as inflaton. As argued above, ρ appears in K and should get Planck suppressed corrections to its scalar potential that can spoil inflation. In string theory one can calculate the leading order corrections to the potential and, even if the uncorrected potential seems to have a flat direction, inflation does usually not happen since the corrections lead to a large η-parameter. However, for the shift symmetric field b, we argued that perturbative corrections are absent and further non-perturbative corrections are small for $a\rho \gtrsim 1$. These arguments could have been done in a low energy effective field theory and thus it seems that we don't really need a string theory description for these models of inflation. However, as we will see in a moment, we cannot make this model compatible with observations and this is a generic problem in any simple string theory construction.[19]

Let us look at the scalar potential for ρ sitting at its minimum. In this case we have the simplified form

$$
V(b) = \lambda_1^4 - \lambda_2^4 \cos(ab) \,, \tag{4.6}
$$

with $\lambda_1^4 - \lambda_2^4 = V_{min} > 0$. This potential is plottet in figure 6.

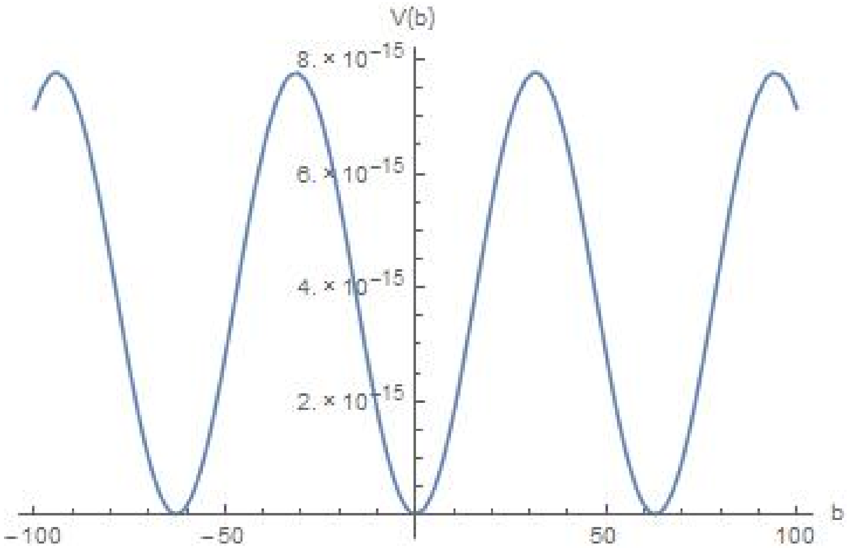

Figure 6: The scalar potential for $\mathrm{Im}(T) = \rho_{min}$ as a function of $\mathrm{Re}(T) = b$ for the values $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$ and $|\mu|^2 = 2.4 \times 10^{-8}$.

However, the field b does not have a canonical kinetic term, rather we have

$$
\mathcal{L}_{kin} = -K_{T\bar{T}}(\partial_\mu b \partial^\mu b + \partial_\mu \rho \partial^\mu \rho) = -\frac{3}{4\rho^2}(\partial_\mu b \partial^\mu b + \partial_\mu \rho \partial^\mu \rho) \,. \tag{4.7}
$$

[19] More complicated models are harder to analyze and concrete no-go theorems forbidding such models also have not been established so the current status of these models of natural inflation is unclear.

So the field with a canonical kinetic term $-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$ is given by $\varphi = \sqrt{3/2}b/\rho_{min}$. In terms of this field the potential is given by

$$V(\varphi) = \lambda_1^4 - \lambda_2^4 \cos\left(\sqrt{\frac{2}{3}}\rho_{min} a\,\varphi\right) \equiv \lambda_1^4 - \lambda_2^4 \cos\left(\frac{\varphi}{f}\right)\,. \tag{4.8}$$

This potential looks like a beautiful candidate for so called natural inflation. However, current observations have already constrained such models substantially and one of the clear experimental requirements to match the data is that the so called axion decay constant f is larger than $M_P = 1$, i.e.

$$f = \sqrt{\frac{3}{2}}\frac{1}{\rho_{min} a} \gtrsim 1 \qquad \Leftrightarrow \qquad 1 \gtrsim \rho_{\min} a\,. \tag{4.9}$$

This requirement is clearly contradicting the requirement of being in a controlled regime of our theory, i.e. of having suppressed higher order non-perturbative corrections and a large volume, which as we showed above amounts to $\rho_{\min} a \gg 1$.

Another problem with this model is that the mass of ρ, the imaginary part of T, is not much larger than the mass of the inflaton b. This means that we can't really neglect ρ during inflation since its mass is below the Hubble scale $H \sim V_{inf}^2/M_P \ll V_{inf}$.

There have been ideas how one can get large axion decay constants in string theory models with more than one axion and it was widely believed that this is possible and semi-explicit models exist. However, recently a discussion started based on the weak-gravity-conjecture that conjectures that gravity is always the weakest force in any theory of quantum gravity. This conjecture can be recast in such a form that it becomes a bound on the axion decay constant f. There are a variety of different forms of this conjecture that might or might not forbid natural inflation with $f \gtrsim 1$. Currently this discussion is still ongoing and some people question whether the semi-explicit string theory models with large axion decay constant are correct. However, due to their complexity this is often not easy to decide. Hopefully string theorist can find a definite answer before experiments confirm or exclude these models.

4.3 Axion monodromy inflation

Based on the arguments above the real parts of S and the U^i are also potential candidates for the inflaton.[20] For these moduli we break the shift symmetry by turning on fluxes and these give a mass to them that is usually much larger than the mass of the T^k fields. However, in string compactifications we often have hundreds of complex structure moduli U^i and it is imaginable that one of them turns out to be much lighter than the other ones and much lighter than the T^k fields. We can accomplish this by fine tuning the flux parameters $G_{I_1 I_2 I_3}(S)$. [21]

For these constructions it is important to work in the LVS scenario. The reason is that we are interested in moving the real part of let's say a complex structure modulus U over a

[20]For general CY compactifications, $\mathrm{Re}(U^i)$ only enjoys an approximate shift symmetry in the so called large complex structure limit $\mathrm{Im}(U^i) \gg 1$.

[21]The flux parameters are actually quantized and take integer values so that they can't be fine-tuned in an actual string compactification. However, the large number of fields U^i and integer quantized flux numbers can in principle lead to a cancellation among certain terms so we can imagine our non-quantized parameters in the model below to arise from a full-fledged string compactification after integrating out all but one of the U^i.

large distant (for large field models of inflation). The holomorphic superpotential usually is a polynomial in U (up to the third power). So if we take $\mathrm{Re}(U) \gg 1$, then we expect the term with the highest power of $\mathrm{Re}(U)$ to dominate. This is generically the term $-3|W|^2$. This means our potential during our would-be inflation is $V \approx -e^K|W|^2 < 0$, so this does not work. This problem can be circumvented in the LVS construction since there $D_{T^k}W \neq 0$ and one actually finds to leading order in the large volume that $K^{T\bar{T}}D_TW\overline{D_TW} = 3|W|^2$. The leading order potential for S and the U^i is therefore

$$V = e^K\left(K^{S\bar{S}}D_SW\overline{D_SW} + K^{U^i\bar{U}^{\bar{j}}}D_{U^i}W\overline{D_{U^j}W}\right) . \tag{4.10}$$

Note, that since $e^K \propto 1/\mathcal{V}^2$, this potential is the leading term and it is less suppressed than the potential for the Kähler moduli that scales as $1/\mathcal{V}^3$ (see eqn. (3.26) and the discussion below it). Above we argued that we can minimize S and the U^i by setting $D_SW = D_{U^i}W = 0$, which is clearly consistent with the expression for V above. Now however, we would like one of the U^i to be much lighter than the rest so that its real part can give rise to inflation.[22]

So let us consider the case where we integrate out the heavy S and all but one of the U^i. We call the remaining light field U and its potential is given by

$$V \propto \frac{1}{\mathcal{V}^2}e^{K(U,\bar{U})}K^{U\bar{U}}D_UW\overline{D_UW} . \tag{4.11}$$

If we now displace $\mathrm{Re}(U)$ from its minimum value at which $D_UW = 0$, then we generate a non-trivial potential for the volume that could destabilize it, since it enters at a smaller power of $1/\mathcal{V}$ compared to the stabilizing potential given above in eqn. (3.26). Thus we need to ensure that during inflation $e^{K(U,\bar{U})}K^{U\bar{U}}D_UW\overline{D_UW} \lesssim 1/\mathcal{V}$ in order to ensure that the overall volume remains stabilized. Furthermore, we now have to worry about a potential U dependence of the A_s above. We will assume here that such a U dependence is absent (which might be arranged in string theory).

With these assumptions it is now straight forward to study the shape of the potential for U. In the large complex structure limit $\mathrm{Im}(U) \gtrsim 1$, the superpotential is simply an up to cubic polynomial in U, while the Kähler potential after integrating out the other U^i becomes[23]

$$\begin{aligned} K &= -p\ln\left(-\mathrm{i}(U-\bar{U})\right) , \quad p \in \{1,2,3\} \\ W &= a_0 + a_1U + a_2U^2 + a_3U^3 . \end{aligned} \tag{4.12}$$

Now we calculate

$$D_UW = \partial_UW + W\partial_UK = a_1 + 2a_2U + 3a_3U^2 - p\frac{a_0 + a_1U + a_2U^2 + a_3U^3}{U-\bar{U}} . \tag{4.13}$$

This is one complex equation for one complex variable and it has a solution that is a global minimum since V as given in eqn. (4.11) is positive semi-definite. We write $U = u + \mathrm{i}v$.

[22] S appears only linear in W which can be seen from the general form in eqn. (3.8) since $G_{I_1I_2I_3}(S)$ is linear in S. In the large complex structure limit $\mathrm{Im}(U^j) \gg 1$, U^j appears with up to cubic powers and therefore we have more freedom in obtaining an interesting potential for inflation so we consider the case where one of the U^i is the inflaton (and not S).

[23] The uplift and the filed N don't really modify the story so we neglect it.

Since e^K and $K^{U\bar{U}}$ only depend on $\mathrm{Im}(U) = v$ we can also already see that the scalar potential for u takes the following form

$$V(u) \propto \sum_{n=0}^{6} c_n u^n \,. \tag{4.14}$$

Thus in this case we find so called models of chaotic inflation with $V(\phi) \propto \phi^p$. These models are likewise currently being tested in experiments and current observations place a bound on p that is roughly $p \lesssim 1$ at the 2σ level. Now when expanding our potential V around the minimum, then we expect no linear term and for large field models with $u > 1$ we naively also expect the highest power of u to dominate during inflation. This seems to make these kind of models inconsistent with the data. However, this is not quite right. While we don't have enough time to work out a full-fledged model, let us at least sketch the reason why such models can be consistent with the data.

Let us restrict to a very simple explicit example

$$\begin{aligned} K &= -\ln\left(-\mathrm{i}(U - \bar{U})\right) \,, \\ W &= a_2(48 + U^2) \,. \end{aligned} \tag{4.15}$$

This leads to

$$D_U W = \partial_U W + W \partial_U K = -2a_2 U - \frac{a_2(48 - U^2)}{U - \bar{U}} \,, \tag{4.16}$$

and

$$\mathrm{Re}(D_U W) = -2a_2 u + \frac{2a_2 uv}{2v} = -a_2 u = 0 \,. \tag{4.17}$$

Setting $\mathrm{Re}(U) = u = 0$ we then find

$$\mathrm{Im}(D_U W) = -2a_2 v + a_2 \frac{48 + v^2}{2v} = 24\frac{a_2}{v} - \frac{3}{2}a_2 v \,. \tag{4.18}$$

This vanishes for $v = 4$ so that at the minimum $u = 0, v = 4$. The scalar potential for $v = 4$, as a function of u, then takes the simple form

$$V(u) \propto \frac{1}{8} a_2^2 \left(64u^2 + u^4\right) \,. \tag{4.19}$$

This is the expected answer, since the highest power of U in W and in $D_U W$ is U^2. However, in solving the equations above we have assumed that we are at the minimum $u = 0$, which is not true during inflation. Rather what we should do, is solve the equations of motion for $u \neq 0$, i.e. we have to solve

$$\mathrm{Im}(D_U W) = -2a_2 v + a_2 \frac{48 - u^2 + v^2}{2v} = 24\frac{a_2}{v} - \frac{3}{2}a_2 v - a_2 \frac{u^2}{2v} = 0 \,. \tag{4.20}$$

The solution is $v = \sqrt{16 - u^2/3}$ which leads to the scalar potential

$$V(u) \propto 2a_2^2 u^2 \sqrt{16 - \frac{u^2}{3}} \,. \tag{4.21}$$

So for largish $u \gtrsim 1$ we have now $V(u) \propto u^3$ instead of the naively expected $V(u) \propto u^4$. This is something that very often happens in string theory constructions, where we have many scalar fields that can (adiabatically) track their minimum during inflation and thereby modify the highest power of the inflaton that appears in the scalar potential. This flattening can actually lead to highest powers of the inflaton that are non-integers and string models have been constructed with $V(\phi) \propto \phi^p$ for $p = \frac{4}{3}, 1, \frac{2}{3}$.

In these kind of models we have broken the shift symmetry of the axionic particle $\mathrm{Re}(U) = u$ explicitly by turning on fluxes. We hence have to check explicitly that Planck suppressed operators do not spoil inflation. This can be done in string theory and while we usually can't explicitly calculate all these corrections, we know their scaling with the string coupling g_s and the volume $\mathcal{V}$. This allows one to argue that even for coefficients of order unity we can neglect these Planck suppressed operators in certain constructions.

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