Dynamics of quantum correlations in bipartite Gaussian open quantum systems

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MMCP'2017
International Conference
on Mathematical Modeling and Computational Physics
Dubna, Russia, 3 - 7 July 2017

Outline

- possibility of E + Gaussian q. discord generation in a s. of 2 coupled bosonic modes in a common thermal env.
- initial state of the subs is taken of Gaussian form and the evolution in the th. of OSs based on CP q. dyn. semig assures preservation in time of Gaussian form of the state
- evolution of QE and GQD in terms of covariance matrix for a Gaussian input state (logarithmic negativity - degree of QE)
- initial separable STS: E generation may take place, for definite values of squeezing parameter, average photon no., T, dissipation const. and of the strength of interaction between the 2 modes; after its generation - temporary suppressions and revivals of E
- initial entangled STS: ESD takes place for all T of the thermal bath; temporary revivals and suppressions of E
- initial uni-modal SS: GQD generation takes place
- limit of large times
- q. steering

Open systems

- the simplest dynamics for an OS which describes an irreversible process: semigroup of transformations introducing a preferred direction in time (characteristics for dissipative processes)
- in GKLS axiomatic formalism of introducing dissipation in quantum mechanics, the usual von Neumann-Liouville eq. ruling the time evolution of closed q. ss is replaced by the following Markovian master eq. (GKLS) for the density operator $\rho(t)$ in the Schrödinger rep.:

$$\frac{d\Phi_t(\rho)}{dt} = L(\Phi_t(\rho))$$

- Φ_t - the dynamical semigroup describing the irreversible time evolution of the open system and L is the infinitesimal generator of Φ_t
- fundamental properties are fulfilled (positivity, unitarity, Hermiticity)

Markovian master equation

- in axiomatic formalism based on CP q. dyn. semigs, irreversible time evolution of an OS (that incorporates the dissipative and noisy effects due to the environment) is described by Kossakowski-Lindblad Markovian master eq. for the density operator (Schrödinger rep.)

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar}\sum_j (2V_j\rho(t)V_j^\dagger - \{\rho(t), V_j^\dagger V_j\}_+)$$

- H - Hamiltonian of the q. OS
- $V_j, V_j^\dagger$ - operators defined on the Hilbert space of H (model the interaction of OS with the env.)

- the semigroup dynamics of the density operator which must hold for a quantum Markov process is valid only for the weak-coupling regime, with the damping λ typically obeying the inequality $\lambda \ll \omega_0$, where ω_0 is the lowest frequency typical of reversible motions

Complete positivity and entanglement

- positivity property guarantees the physical consistency of evolving states of single systems, while complete positivity prevents inconsistencies in entangled composite systems
- therefore the existence of entangled states makes the request of complete positivity necessary
- the positivity of the states of the compound system will be preserved only if the dyn. semig. of the subs. is completely positive

Operators

- q. dyn. semigs that preserve in time Gaussian form of the states: H - polyn. of second degree in coordinates x, y and momenta p_x, p_y of the 2 q. OS and V_j, $V_j^\dagger$ - polyns. of first degree in canonical observables ($j = 1, 2, 3, 4$):

$$V_j = a_{xj}p_x + a_{yj}p_y + b_{xj}x + b_{yj}y$$

- Hamiltonian of 2 identical coupled modes:

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{m\omega^2}{2}(x^2 + y^2) + qxy$$

- dyn. semig. implies positivity of the matrix formed by the scalar products of the vectors $\mathbf{a}_x, \mathbf{a}_y, \mathbf{b}_x, \mathbf{b}_y$ (their entries are the components $a_{xj}, a_{yj}, b_{xj}, b_{yj}$, resp.)

Equations of motion

- bimodal covariance matrix

$$\sigma(t) = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{xp_x} & \sigma_{p_xp_x} & \sigma_{yp_x} & \sigma_{p_xp_y} \\ \sigma_{xy} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{xp_y} & \sigma_{p_xp_y} & \sigma_{yp_y} & \sigma_{p_yp_y} \end{pmatrix}$$

$$\frac{d\sigma}{dt} = Y\sigma + \sigma Y^{\mathrm{T}} + 2D, \quad Y = \begin{pmatrix} -\lambda & 1/m & 0 & 0 \\ -m\omega^2 & -\lambda & -q & 0 \\ 0 & 0 & -\lambda & 1/m \\ -q & 0 & -m\omega^2 & -\lambda \end{pmatrix}$$

D - matrix of diffusion coefficients

$$D = \begin{pmatrix} D_{xx} & D_{xp_x} & D_{xy} & D_{xp_y} \\ D_{xp_x} & D_{p_xp_x} & D_{yp_x} & D_{p_xp_y} \\ D_{xy} & D_{yp_x} & D_{yy} & D_{yp_y} \\ D_{xp_y} & D_{p_xp_y} & D_{yp_y} & D_{p_yp_y} \end{pmatrix}$$

Time-dependent solution

$$\sigma(t) = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{xp_x} & \sigma_{p_xp_x} & \sigma_{yp_x} & \sigma_{p_xp_y} \\ \sigma_{xy} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{xp_y} & \sigma_{p_xp_y} & \sigma_{yp_y} & \sigma_{p_yp_y} \end{pmatrix}$$

$$\sigma(t) = M(t)(\sigma(0) - \sigma(\infty))M^{\mathrm{T}}(t) + \sigma(\infty),$$

$M(t) = \exp(tY)$, $\lim_{t\to\infty} M(t) = 0$ (Y must only have eigenvalues with negative real parts)

$$Y\sigma(\infty) + \sigma(\infty)Y^{\mathrm{T}} = -2D$$

Covariance matrix

Two-mode Gaussian state is entirely specified by its covariance matrix σ, which is a real, symmetric and positive matrix

$$\sigma = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}$$

(A, B and C are 2×2 matrices)

- for environments inducing asymptotic Gibbs state

$$m\omega D_{xx} = \frac{1}{m\omega} D_{p_x p_x} = m\omega D_{yy} = \frac{1}{m\omega} D_{p_y p_y} = \frac{\lambda}{2} \coth \frac{\hbar\omega}{2kT},$$
$$D_{xp_x} = D_{yp_y} = D_{xy} = D_{p_x p_y} = D_{xp_y} = D_{yp_x} = 0$$

- then we have equal unimodal covariance matrices $A = B$ and symmetric entanglement matrix C
- Gaussian states with $\det C \geq 0$ are separable states, but for $\det C < 0$, it may be possible that the states are entangled

Logarithmic negativity (1)

- for Gaussian states, the measures of entanglement of bipartite systems are based on some invariants constructed from the elements of the covariance matrix - logarithmic negativity
- for a Gaussian density operator, logarithmic negativity is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix
- $E_N = \max\{0, -\log_2 2\tilde{\nu}_-\}$, where $\tilde{\nu}_-$ is the smallest of the two symplectic eigenvalues of the partial transpose $\tilde{\sigma}$ of the 2-mode covariance matrix σ:

$$2\tilde{\nu}_{\mp}^2 = \tilde{\Delta} \mp \sqrt{\tilde{\Delta}^2 - 4 \det \sigma}$$

- symplectic invariant (seralian) $\tilde{\Delta} = \det A + \det B - 2 \det C$

Logarithmic negativity (2)

We apply the measure of entanglement based on negative eigenvalues of the partial transpose of the subsystem density matrix. In case of the Gaussian density operator, the negativity is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix.

- **Logarithmic negativity** $E_N = -\frac{1}{2}\log_2[4f(\sigma)]$,

$$f(\sigma) = \frac{1}{2}(\det A + \det B) - \det C - \sqrt{\left[\frac{1}{2}(\det A + \det B) - \det C\right]^2 - \det\sigma}$$

determines the strength of entanglement for $E_N > 0$; if $E_N \leq 0$, then the state is separable

Entangled initial states (1)

- initial Gaussian state: 2-mode STS, with CM

$$\sigma_{st}(0) = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & -c \\ c & 0 & b & 0 \\ 0 & -c & 0 & b \end{pmatrix},$$

$$a = n_1 \cosh^2 r + n_2 \sinh^2 r + \frac{1}{2} \cosh 2r,$$
$$b = n_1 \sinh^2 r + n_2 \cosh^2 r + \frac{1}{2} \cosh 2r,$$
$$c = \frac{1}{2}(n_1 + n_2 + 1) \sinh 2r,$$

  n_1, n_2 : average no. of thermal photons; r : squeezing parameter; $n_1 = 0$ and $n_2 = 0 \to$ CM of the 2-mode SVS

- a 2-mode STS is entangled when the $r > r_s$, where

$$\cosh^2 r_s = \frac{(n_1 + 1)(n_2 + 1)}{n_1 + n_2 + 1}$$

Entangled initial state (2)

- for all T, at certain finite moment of time, which depends on T, $E_N(t)$ becomes 0 and the state becomes separable - so-called phenomenon of entanglement sudden death; it is in contrast to the q. decoherence, during which the loss of q. coherence is usually gradual
- dissipation favorizes the phenomenon of entanglement sudden death – with increasing the dissipation parameter λ, entanglement suppression happens earlier
- dynamics of entanglement of the 2 os depends strongly on the initial states and the coefficients describing the interaction of the system with the thermal environment (dissipation constant and temperature)

Asymptotic covariance matrix

- while in the case of independent bosonic modes, the form of the coefficients would determine an asymptotic product Gibbs state describing a thermal equilibrium of the two modes with the thermal bath at temperature T, in the present model with coupled bosonic modes, the asymptotic state does not have anymore the form of a product state:

$$\sigma(\infty) = \frac{C}{4(L^2 - q^2)} \times$$

$$\times \begin{pmatrix} 2L^2 - q^2 & \lambda q^2 & -Lq & -\lambda Lq \\ \lambda q^2 & 2L^2 + (\lambda^2 - 2)q^2 & -\lambda Lq & q(L - q^2) \\ -Lq & -\lambda Lq & 2L^2 - q^2 & \lambda q^2 \\ -\lambda Lq & q(L - q^2) & \lambda q^2 & 2L^2 + (\lambda^2 - 2)q^2 \end{pmatrix}$$

$\omega_1 = \omega_2 \equiv \omega$, $C \equiv \coth(\omega/2kT)$, $L \equiv 1 + \lambda^2$

Asymptotic logarithmic negativity

- logarithmic negativity in the limit of large times:

$$E(\infty) = -\frac{1}{2}\log_2\{\frac{\coth^2\frac{\omega}{2kT}}{4}[4+\frac{3(1+\lambda^2)q^2}{(1+\lambda^2)^2-q^2}$$
$$-\frac{q}{(1+\lambda^2)^2-q^2}\sqrt{16(1+\lambda^2)^3+8(-1+\lambda^4)q^2+q^4}]\}$$

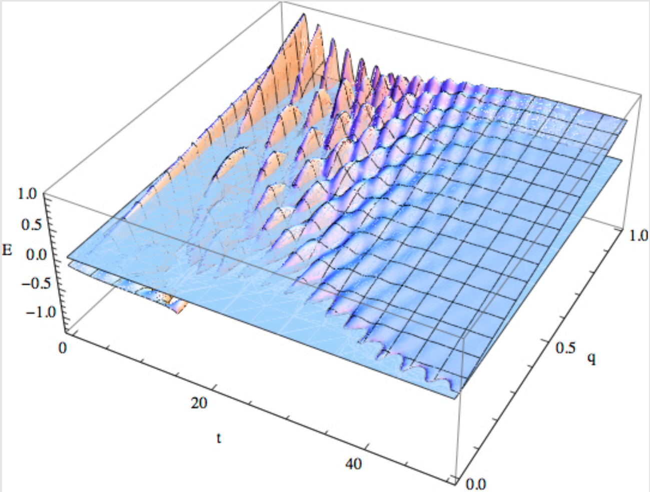

Figure: Logarithmic negativity E versus time t and interaction strength q for an initial separable squeezed thermal state with squeezing parameter $r = 0.5$, average photon numbers $n_1 = 0.5$, $n_2 = 1$, dissipation constant $\lambda = 0.08$, and temperature $T = 0$ of the thermal environment ($\hbar = 1$).

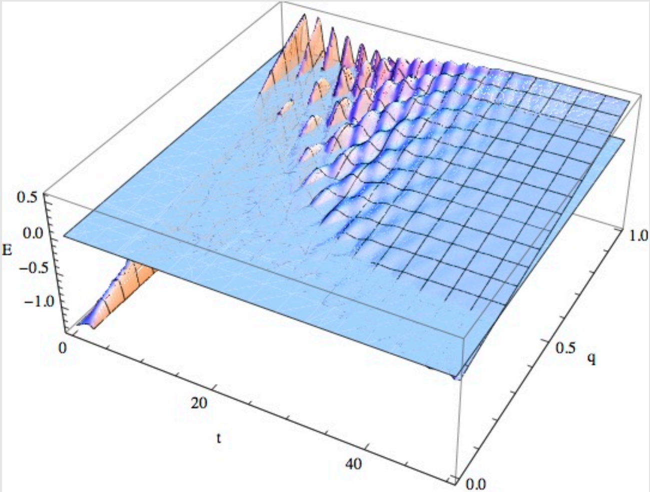

Figure: Same as in Fig. 1, for an initial separable squeezed thermal state with squeezing parameter $r = 0.3$, average photon numbers $n_1 = 1$, $n_2 = 1$, dissipation constant $\lambda = 0.08$, and $\frac{1}{2} \coth \frac{\omega}{2kT} = 1.1$.

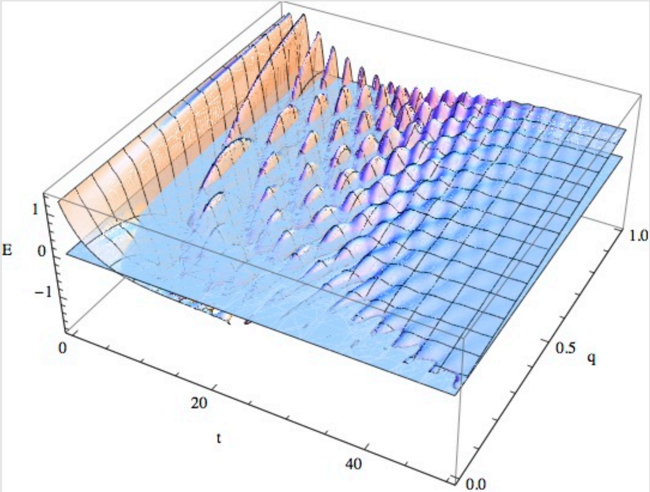

Figure: Same as in Fig. 1, for an initial entangled squeezed thermal state with squeezing parameter $r = 1$, average photon numbers $n_1 = 0.5$, $n_2 = 1$, dissipation constant $\lambda = 0.08$, and temperature $T = 0$ of the thermal environment.

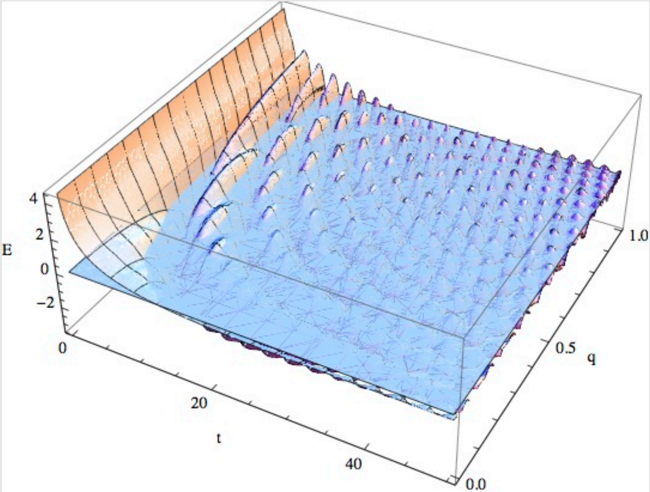

Figure: Same as in Fig. 1, for an initial entangled squeezed thermal state with squeezing parameter $r = 2$, average photon numbers $n_1 = 0.5$, $n_2 = 0.5$, dissipation constant $\lambda = 0.05$, and $\frac{1}{2}\coth\frac{\omega}{2kT} = 1.1$.

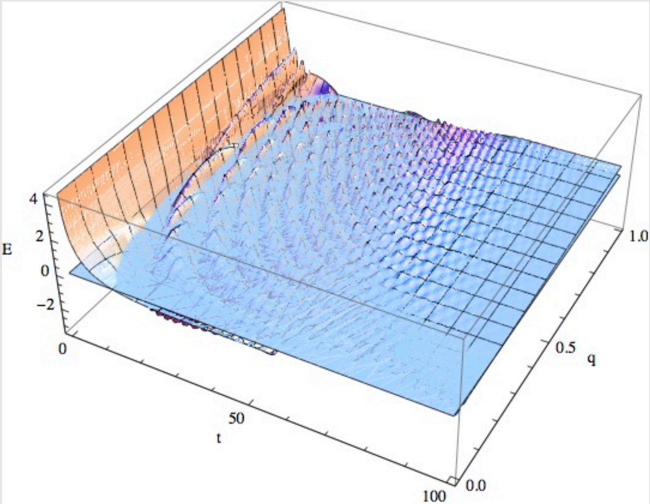

Figure: Same as in Fig. 4, for larger times.

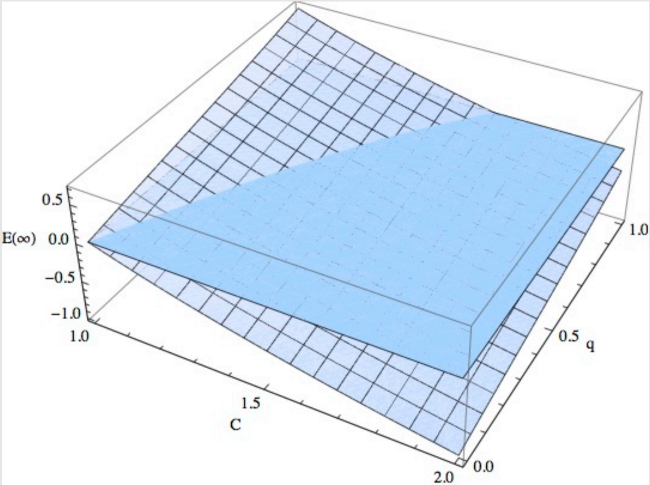

Figure: Asymptotic logarithmic negativity.

Quantum discord (1)

- QE does not describe all the non-classical properties of q. correlations – recent theoretical and experimental results indicate that some non-entangled mixed states can improve performance in some quantum computing tasks
- Zurek defined QD as a measure of q. correlations which includes entanglement of bipartite ss and it can also exist in separable states
- recently, an operational interpretation was given to QD in terms on consumption of entanglement in an extended quantum state merging protocol
- total amount of correlations contained in a q. state is given by the q. mutual information which is equal to the sum of the QD and classical correlations

Quantum discord (2)

- separability of q. states has often been described as a property synonymous with the classicality; however, recent studies have shown that separable states, usually considered as being classically correlated, might also contain q. correlations
- QD was introduced as a measure of all q. correlations in a bipartite state, including – but not restricted to – QE
- QD has been defined as the difference between 2 q. analogues of classically equivalent expression of the mutual information, which is a measure of total correlations in a q. state
- for pure entangled states QD coincides with the entropy of entanglement
- QD can be different from 0 also for some mixed separable states – correlations in such separable states with positive discord are an indicator of quantumness

Gaussian QD (1)

Gaussian QD of a general 2-mode Gaussian state ρ_{12} is QD where conditional entropy is restricted to generalized Gaussian positive operator valued measurements (POVM) on the mode 2; in terms of symplectic invariants (symmetry between modes 1 and 2 is broken) (Adesso)

$$D = f(\sqrt{\beta}) - f(\nu_-) - f(\nu_+) + f(\sqrt{\varepsilon})$$

$$f(x) = \frac{x+1}{2} \log \frac{x+1}{2} - \frac{x-1}{2} \log \frac{x-1}{2}$$

$$\varepsilon = \begin{cases} \dfrac{2\gamma^2 + (\beta-1)(\delta-\alpha) + 2|\gamma|\sqrt{\gamma^2 + (\beta-1)(\delta-\alpha)}}{(\beta-1)^2}, \\ \quad \text{if } \ (\delta - \alpha\beta)^2 \leq (\beta+1)\gamma^2(\alpha+\delta) \\ \\ \dfrac{\alpha\beta - \gamma^2 + \delta - \sqrt{\gamma^4 + (\delta-\alpha\beta)^2 - 2\gamma^2(\delta+\alpha\beta)}}{2\beta}, \\ \quad \text{otherwise} \end{cases}$$

Gaussian QD (2)

$$\alpha = 4 \det A, \quad \beta = 4 \det B, \quad \gamma = 4 \det C, \quad \delta = 16 \det \sigma$$

$\nu_{\mp}$ are the symplectic eigenvalues of the state, given by

$$2\nu_{\mp}^2 = \Delta \mp \sqrt{\Delta^2 - 4 \det \sigma}$$

$\Delta = \det A + \det B + 2 \det C$
Gaussian QD only depends on $|\det C|$, i.e., entangled ($\det C < 0$) and separable states are treated on equal footing

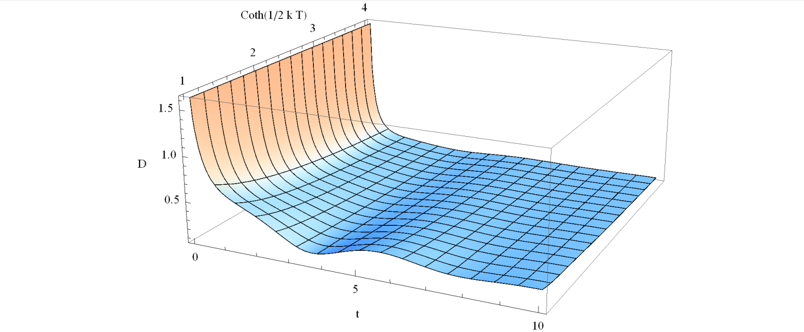

Figure: Gaussian Quantum Discord.

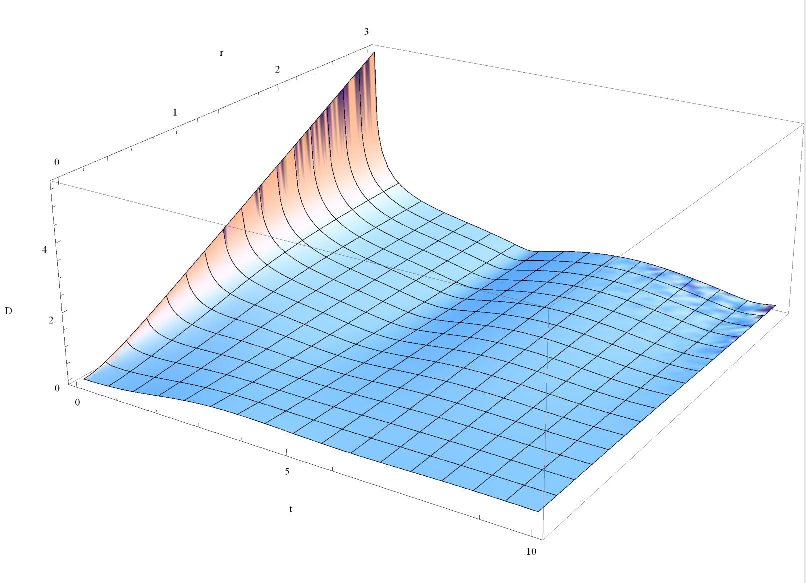

Figure: Gaussian Quantum Discord.

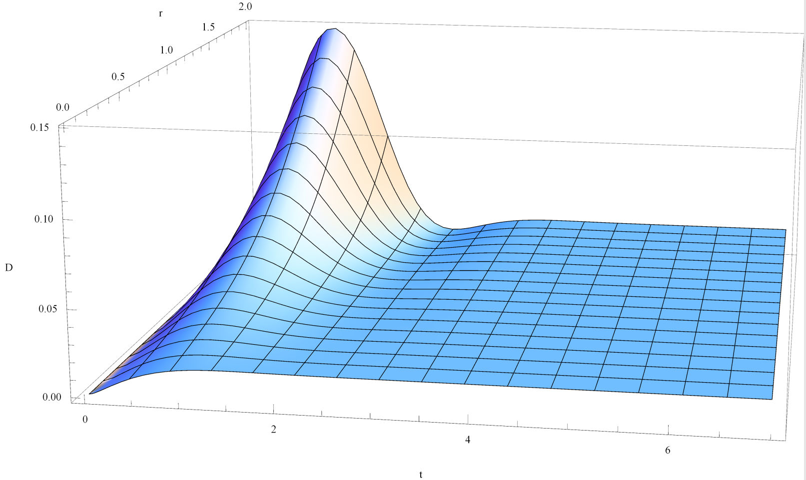

Figure: Gaussian Quantum Discord.

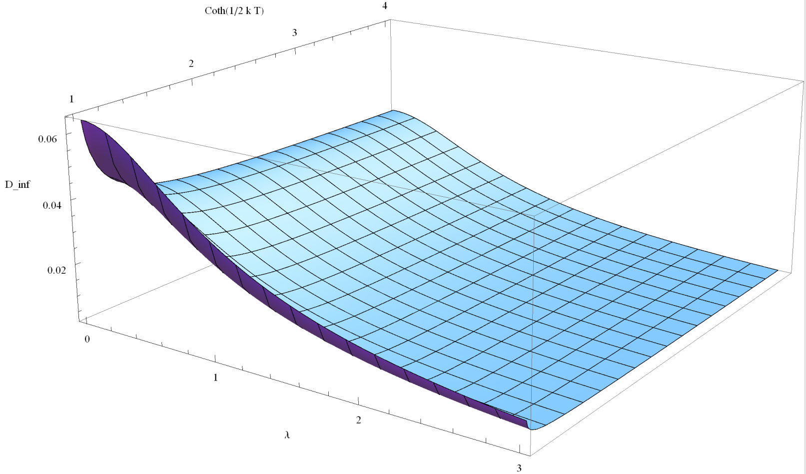

Figure: Gaussian Quantum Discord.

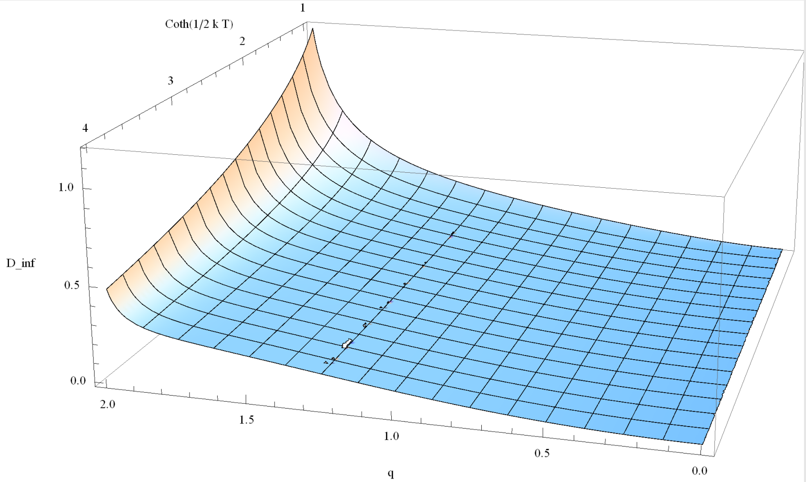

Figure: Gaussian Quantum Discord

EPR q. steering

- steering is a type of q. nonlocality first identified in the EPR paper, which is distinct from both nonseparability and Bell nonlocality (a type of q. corrs intermediate between E and nonlocality); to infer the steerability between two parties is equivalent with verifying the shared entanglement distribution by an untrusted party: Alice has to convince Bob (who does not trust Alice) that the state they share is entangled, by performing local measurements and classical communications;
- Alice performs a local measurement on her s., which makes it possible to steer Bob's local state depending on her choice of measurement settings (possibility for Alice to remotely prepare Bob's s. in different states depending on her own local measurements)
- behaviour of discord is strongly related to steering; for symmetric states, if the states are highly discordant, they are also highly steerable; a state is always steerable provided the discord exceeds a certain threshold

Gaussian q. steering

- captures the EPR paradox and quantifies to which extent Bob's mode can be steered by Alice Gaussian measurements on her mode in a 2-mode entangled Gaussian state
- in the case of bipartite Gaussian states a suitable measure of steering from Alice to Bob has been proposed using Gaussian measurements, which is easily computable for an arbitrary no. of modes, and has a particularly simple form when the steered party has one mode:
$G^{A->B}(\gamma) = \max\{0, \frac{1}{2}\ln\frac{\det A}{4\det\gamma}\} = \max\{0, S(A) - S(\gamma)\}$,
S : Renyi-2 entropy; for Gaussian states $S = \frac{1}{2}\ln(16\det\gamma)$

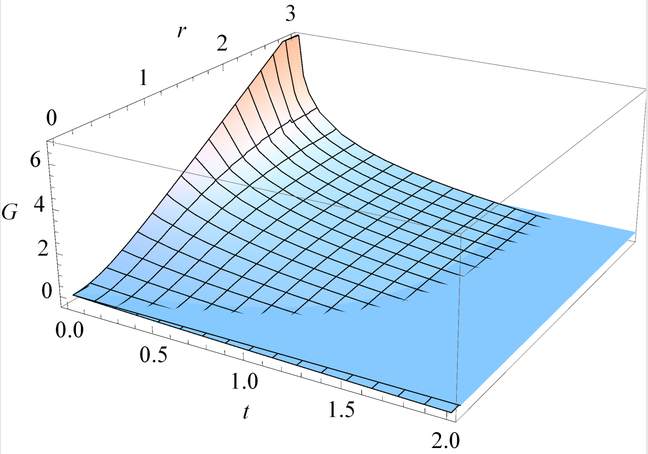

Figure: Gaussian quantum steering G versus time t and squeezing parameter r for an initial squeezed vacuum state in a thermal environment with temperature $C \equiv \coth(\omega/2kT) = 2$, dissipation parameter $\lambda = 0.1$ and $\omega = 1$ ($\omega_1 = \omega_2$).

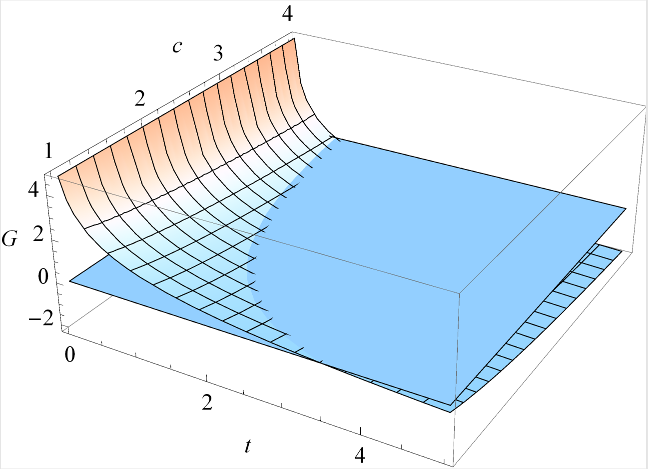

Figure: Gaussian quantum steering G versus time t and temperature $C \equiv \coth \frac{\omega}{2kT}$ for an initial squeezed vacuum state for $r = 2$, $\lambda = 0.1$ and $\omega = 1$.

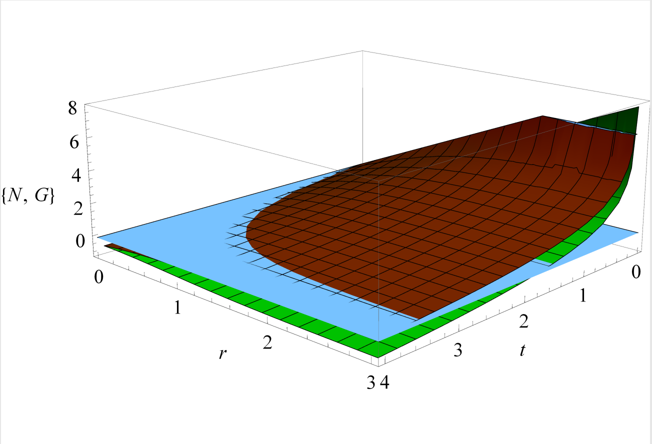

Figure: Gaussian quantum steering G (green plot) and logarithmic negativity N (red plot) versus time t and squeezing parameter r for an initial squeezed vacuum state, with $C \equiv \coth(\omega/2kT) = 2$, $\lambda = 0.1$ and $\omega = 1$.

Gaussian quantum steering

- an initial squeezed vacuum state ($n_1 = n_2 = 0$) is always steerable for $r > 0$; while Gaussian steering is increasing with squeezing parameter r, the thermal noise and dissipation destroy the steerability between the two parts
- compared to the GQD, which is decreasing asymptotically in time, the Gaussian quantum steering suffers a sudden death behaviour like quantum entanglement
- we described the time evolution of a measure that quantifies steerability for arbitrary bipartite Gaussian states in a system consisting of two bosonic modes embedded in a common thermal environment.
- we study Gaussian quantum steering in terms of the covariance matrix under the influence of noise and dissipation and find that the interaction with the environment destroys the steerability between the two parts

Conclusions (1)

- in the framework of th. of OS based on CP q. dyn. semigs - possibility of E + GQD generation in a s. consisting of 2 interacting bosonic modes embedded in a common thermal env.
- we solved master eq. for 2 interacting modes interacting also with an env.
- initial state of the subs is taken of Gaussian form and the evolution under the q. dyn. semig assures the preservation in time of the Gaussian form of the state
- evolution of QE and GQD in terms of covariance matrix for a Gaussian input state (logarithmic negativity - degree of QE)
- initial entangled STS - E suppression (ESD) takes place for all T of the thermal bath; one can also observe temporary revivals and suppressions of E

Conclusions (2)

- initial separable STS - E generation may take place, for definite values of the parameters characterizing the initial state of the s. (squeezing parameter, average photon no.), coeffs describing interaction of the s. with reservoir (T, dissipation const.) and of the strength of interaction between the 2 modes; after its generation one can observe temporary suppressions and revivals of E
- in the limit of large times the s. evolves asymptotically to an equilibrium state which can be entangled or separable; the direct interaction between the 2 modes favors the generation or the preservation of the created E, while T of the thermal bath acts towards preventing the generation of E or suppressing it once it was created - competition between these 2 factors determines the final state of being separable or entangled
- GQD + Gaussian q. steering

  1. A. Isar, Open Sys. Inf. Dynamics **23**, 1650007 (2016)
  2. T. Mihaescu, A. Isar, Eur. Phys. J. D **71**, 144 (2017)

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Thank You!