# Electron-phonon heat exchange in layered nanostructures

**Dragos-Victor Anghel** and Sergiu Cojocaru

Institutul National de Fizica si Inginerie Nucleara—
Horia Hulubei

# Summary

- Motivation
- Model
- Results
- Conclusions

DVA and S. Cojocaru, Solid State Communications 227, 56 (2016).
S. Cojocaru and DVA, Phys. Rev. B **93**, 115405 (2016).

# Motivation

# Higher and higher resolution

Astronomical observations:

- Details of star formation.
- High resolution images of remote objects.
- Asymmetries in the cosmic background radiation.
- Early universe.

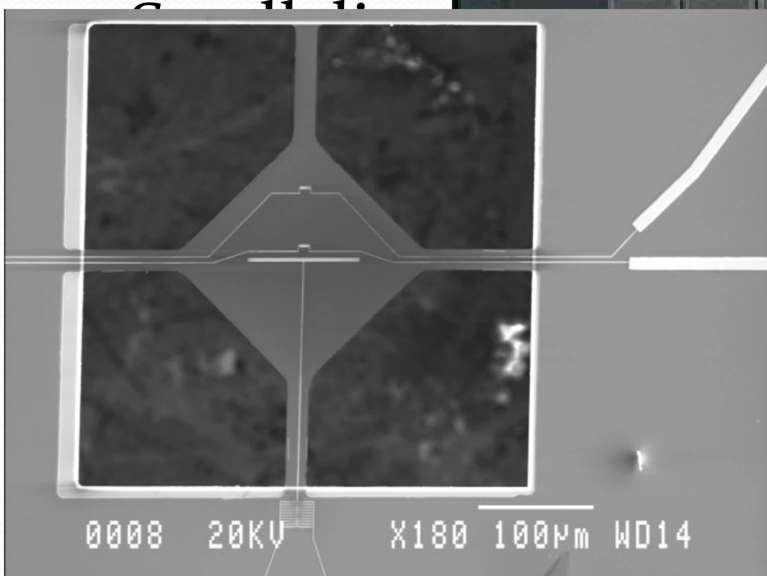
Terrestrial applications:

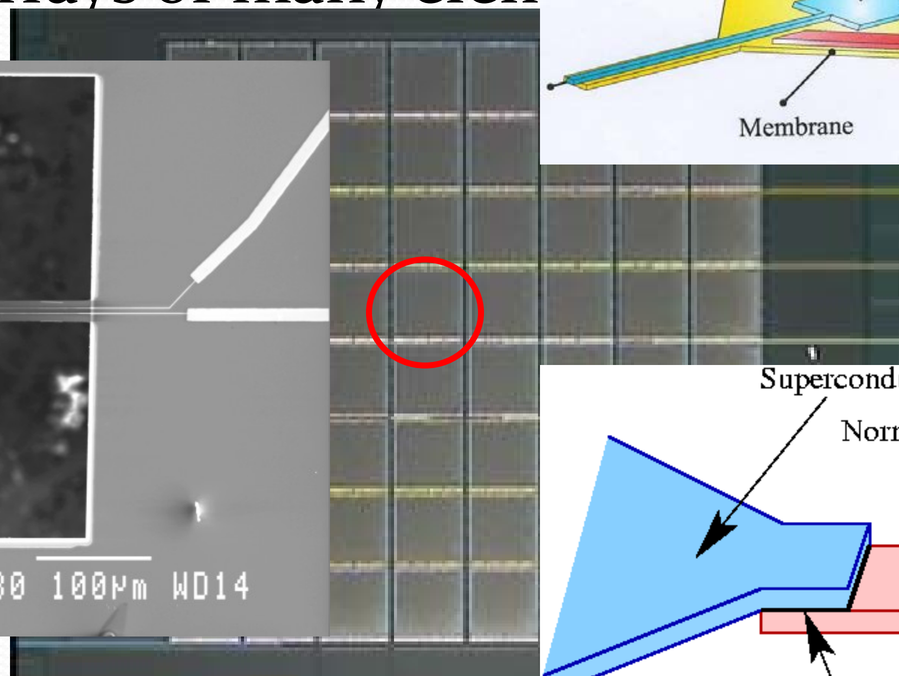
- Sensing ultra-weak electroluminescence from large scale integrated circuits

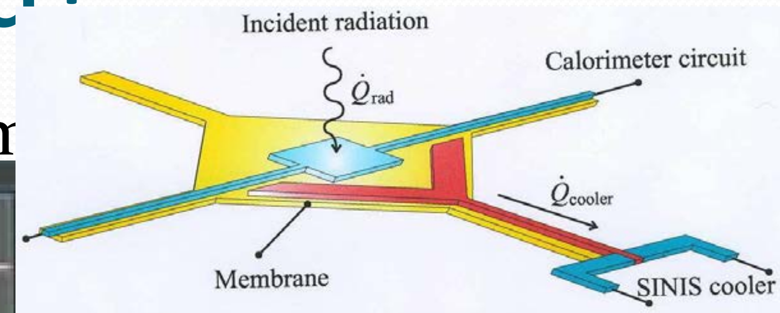
# Model

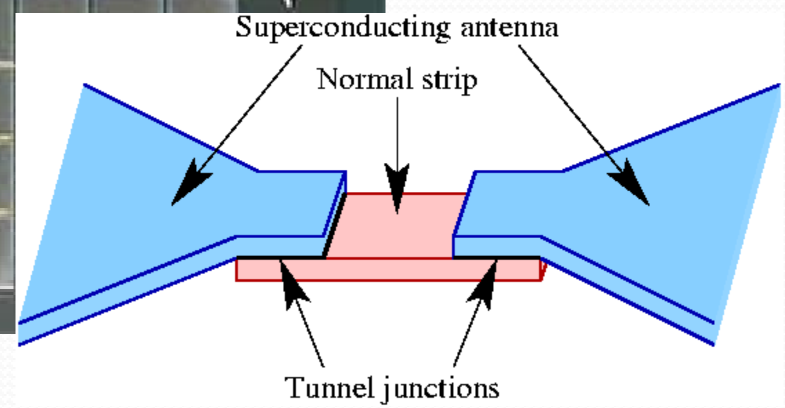
# What do we need?

- Detector arrays of many elem

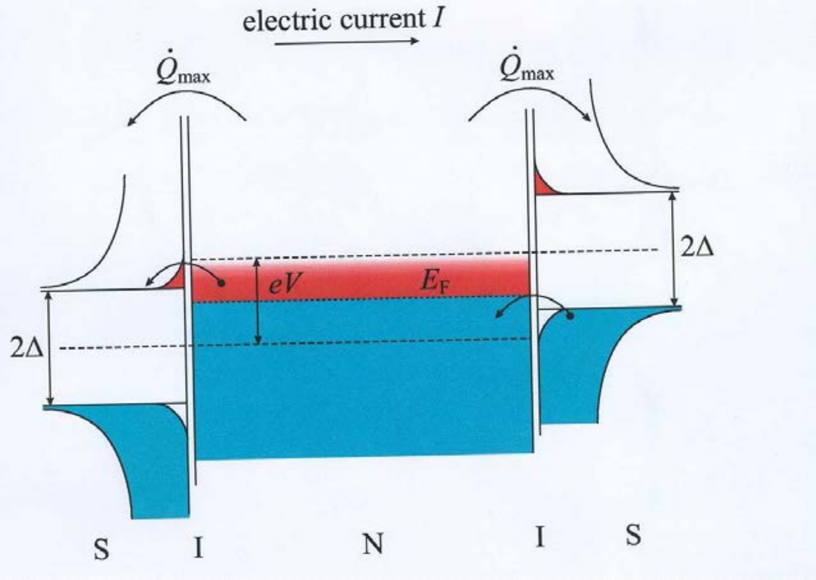


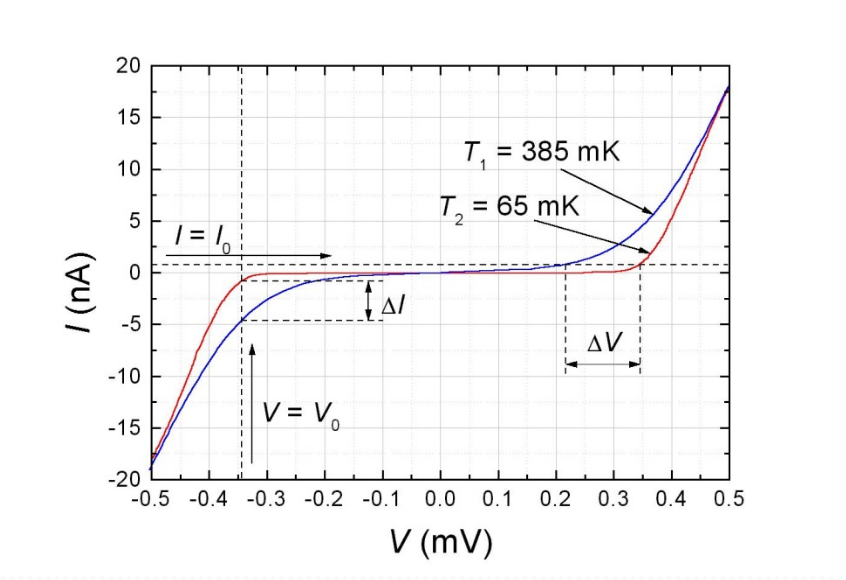





# Tunneling through NIS junctions

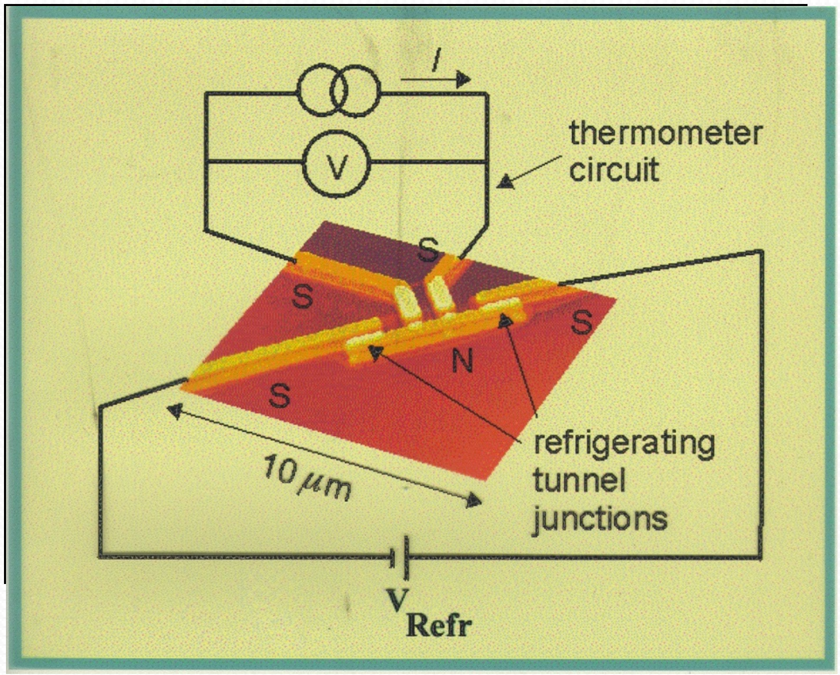


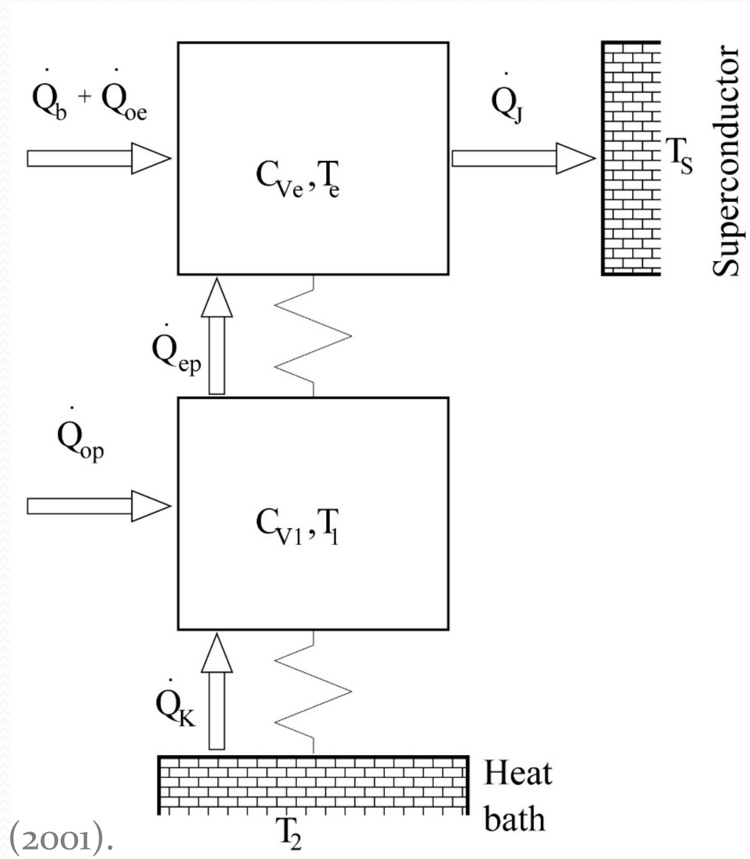





M. Nahum at al., Appl. Phys. Lett. **65**, 3123 (1994).
M. M. Leivo et al., Appl. Phys. Lett. **68**, 1996 (1996).
J. P. Pekola, DVA, et al., Appl. Phys. Lett. **76**, 2782 (2000).

# NIS detectors

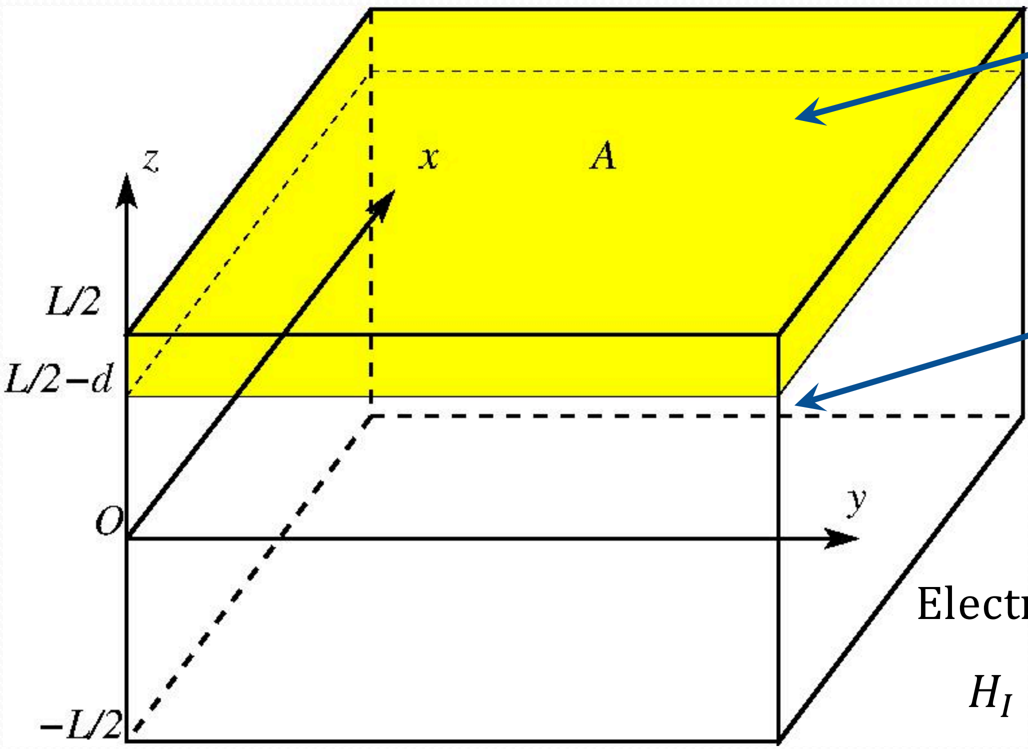



Calculations of the noise equivalent power (NEP) requires $T_e \sim 100$ mK.

DVA, A. Luukanen, J. P. Pekola, Appl. Phys. Lett. 78, 556 (2001).
DVA, J. P. Pekola, J. Low Temp. Phys. 123, 197 (2001).

# Model for the electron-phonon interaction




Metallic layer
Ψ=electronic wavefunction

Dielectric membrane
**u**=elastic displacement field

Electron-phonon interaction Hamiltonian:

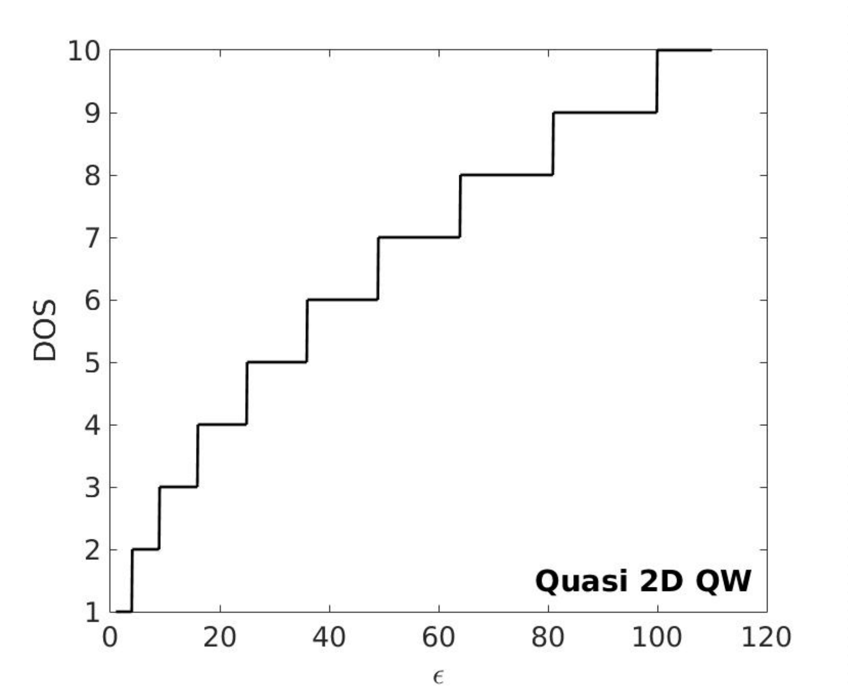
$$H_I = \frac{2}{3} E_F \int_{V_{el}} \Psi^+(\mathbf{r})\Psi(\mathbf{r})\, \nabla \cdot \mathbf{u}(\mathbf{r})\, d^3\mathbf{r}$$

In a 3D model the electron-phonon heat power is: $P \propto T_e^5 - T_{ph}^5$

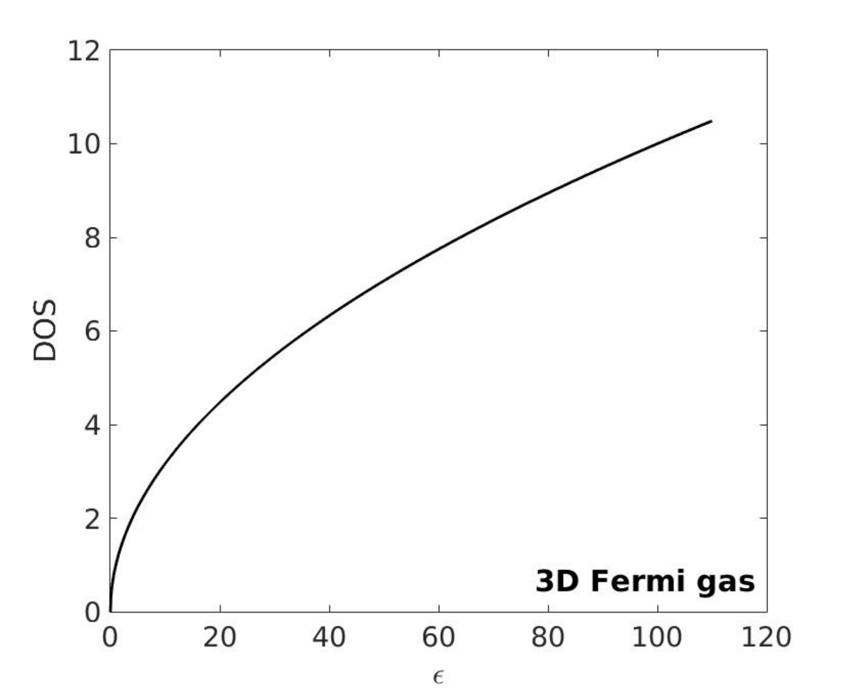
F. C. Wellstood, C. Urbina, and J. Clarke, Phys. Rev. B 49, 5942 (1994).

# Metallic layer over free standing membrane: the electron system

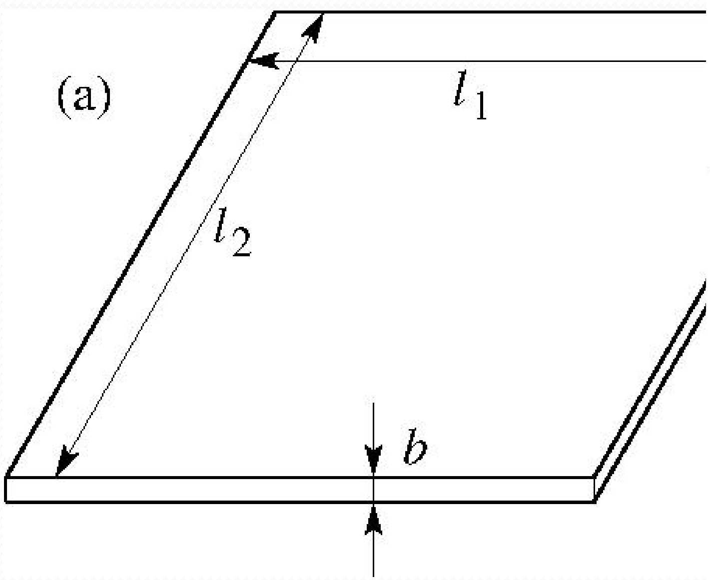
## Metallic wire as a quantum well (QW)



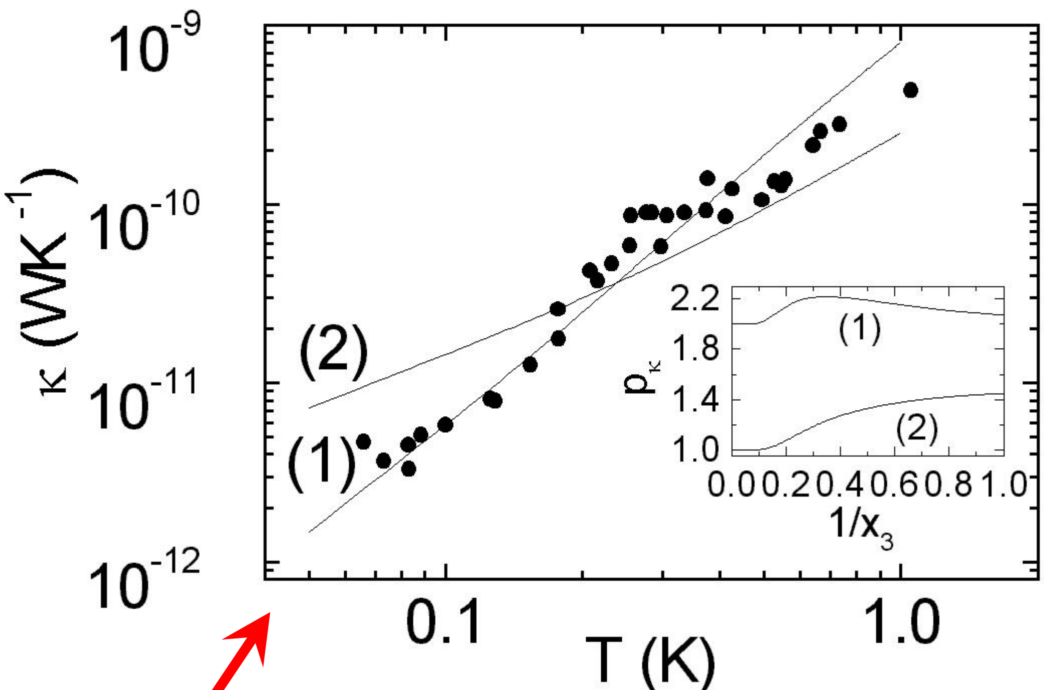

## Metallic wire as a 3D Fermi liquid (FL)



# Thermal properties of the supporting membrane



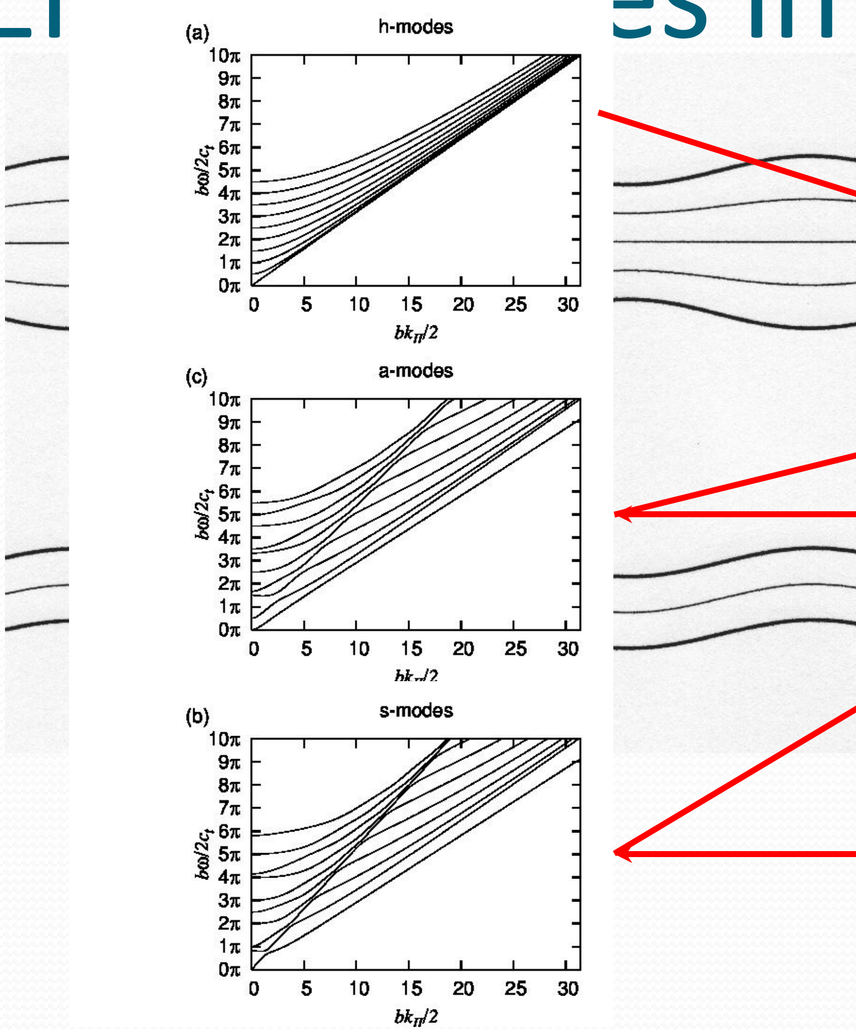

$b \approx 100$ nm
$l_1, l_2 \approx 400$ µm
$w \approx 10$ µm

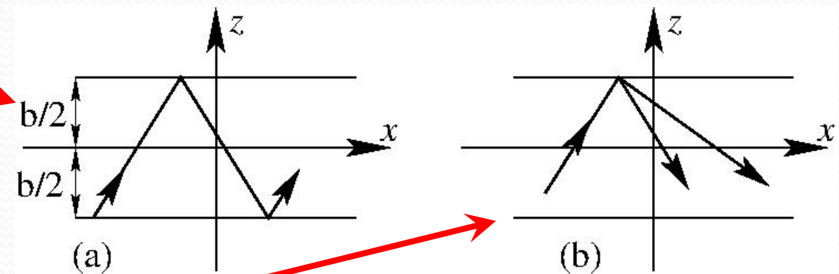



Dimensionality cross-over between 3D and 2D phonon gas distributions.

DVA et al., Phys. Rev. Lett. **81**, 2958 (1998).

# Elastic modes in the membrane



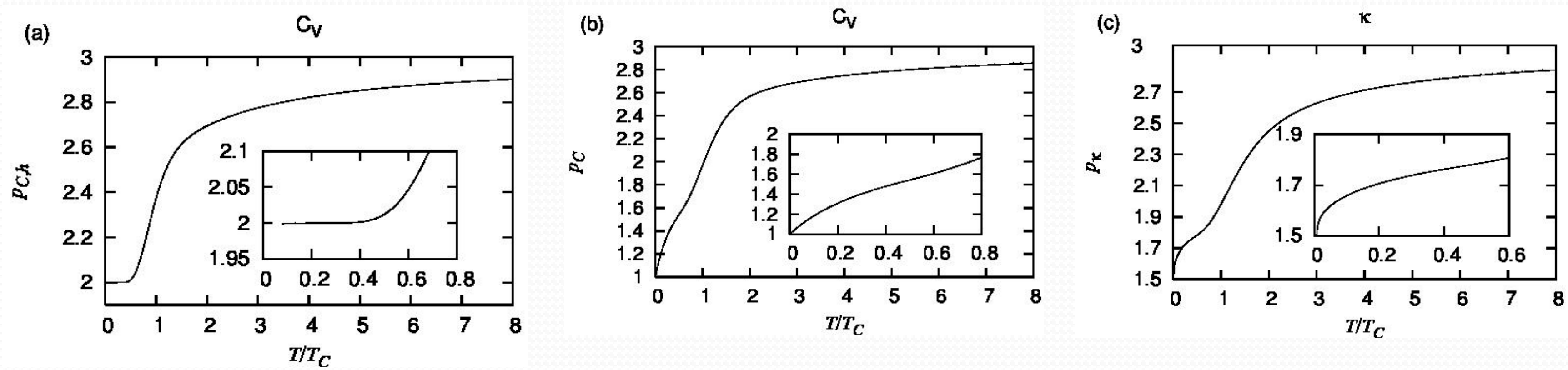

$$\frac{\tan\left(\frac{bk^{l}{}_{\perp}}{2}\right)}{\tan\left(\frac{bk^{t}{}_{\perp}}{2}\right)} = -\frac{4k^{l}{}_{\perp}\,k^{t}{}_{\perp}\,k^{2}{}_{\parallel}}{\left[\left(k^{t}{}_{\perp}\right)^{2} - k^{2}{}_{\parallel}\right]}$$

horizontal share modes (polarization perp.to the screen)

Lamb modes (polarization in the screen plane)

$$\frac{\tan\left(\frac{bk^{t}{}_{\perp}}{2}\right)}{\tan\left(\frac{bk^{l}{}_{\perp}}{2}\right)} = -\frac{4k^{l}{}_{\perp}\,k^{t}{}_{\perp}\,k^{2}{}_{\parallel}}{\left[\left(k^{t}{}_{\perp}\right)^{2} - k^{2}{}_{\parallel}\right]}$$

# Thermal properties of membranes and bridges




Simple longitudinal and transversal waves

Membrane waves
$C_V \propto T^{p_C}$

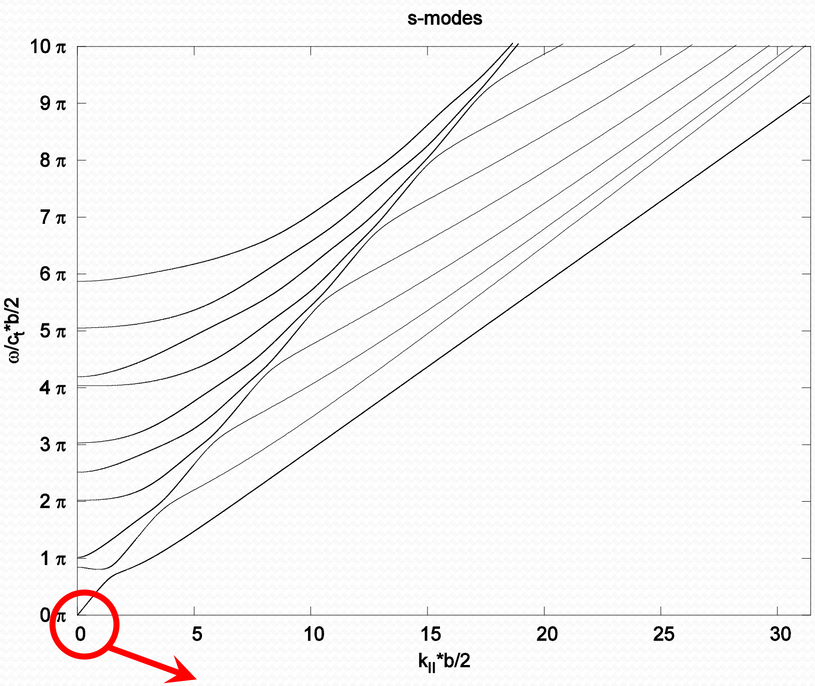
Membrane waves
$\kappa \propto T^{p_\kappa}$

$T_C = \frac{c_l \hbar}{2 k_B b} \approx 237$ mK for $SiN_x$ membrane of thickness $b = 100$ nm.

T. Kuhn, DVA et al., Phys. Rev. B **70**, 125425 (2004).
T. Kuhn, DVA et al., Phys. Rev. B **76**, 165425 (2007).

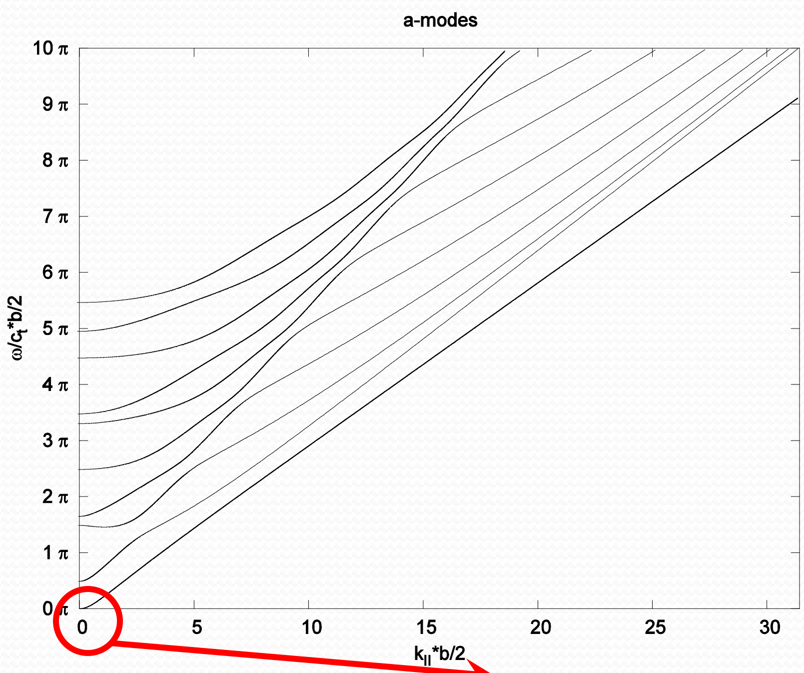
# Metallic layer over free standing membrane: the phonon system

## Symmetric modes




$$\omega^s \approx 2q_{\parallel}c_t\sqrt{1-\frac{c_t^2}{c_l^2}}$$

## Antisymmetric modes




$$\omega^a \approx c_t b\sqrt{\frac{1}{3}\left(1-\frac{c_t^2}{c_l^2}\right)q_{\parallel}^2}$$

# Results

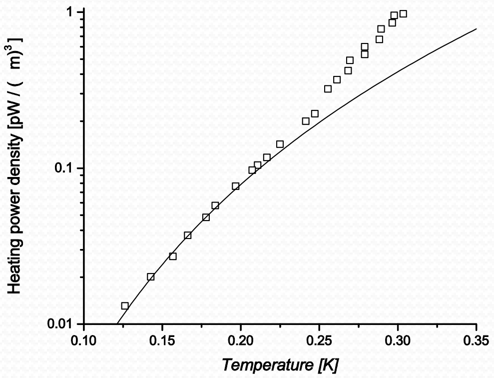
DVA, S. Cojocaru, Sol. State Commun. **277**, 56 (2016).

S. Cojocaru, DVA, Phys. Rev. B **93**, 115405 (2016).

# Low temperature expansion of $P$

- In general it is expected that $P \propto T_e^x - T_{ph}^x$
- For 3D systems, $x = 5$ (PRB **49**, 5942, 1994)
- In monolayer and bilayer graphene, $x = 4$ (PRB **81**, 245404, 2010)
- For quasi 1D geometry, $x = 3$ (PRB **77**, 033401, 2008).
- In general, $P \propto T_e^{s+2} - T_{ph}^{s+2}$, where $s$ is the smaller dimension of the system (PRB **81**, 245404, 2010).
- There are also claims that x>5 (thin metal films deposited on semi-infinite dielectrics, Phys. Rev. B **72**, 224301, 2005).
- Experimental results are inconclusive.

# 3D Fermi gas model




Comparison of the theory (line) with experimental results (dots) from
J. Karvonen and I. Maasilta, Phys. Rev. Lett. **99**, 145503 (2007).

$$P = \Sigma_{2D} A\left(T_e^4 - T_p^4\right)$$

$$\Sigma_{2D} \approx \frac{0.013 k_B^4 k_F^4 J^2}{\rho \hbar^2 c_t^3 (1-J)^{\frac{3}{2}}}, \qquad J \equiv \frac{c_t^2}{c_l^2}$$

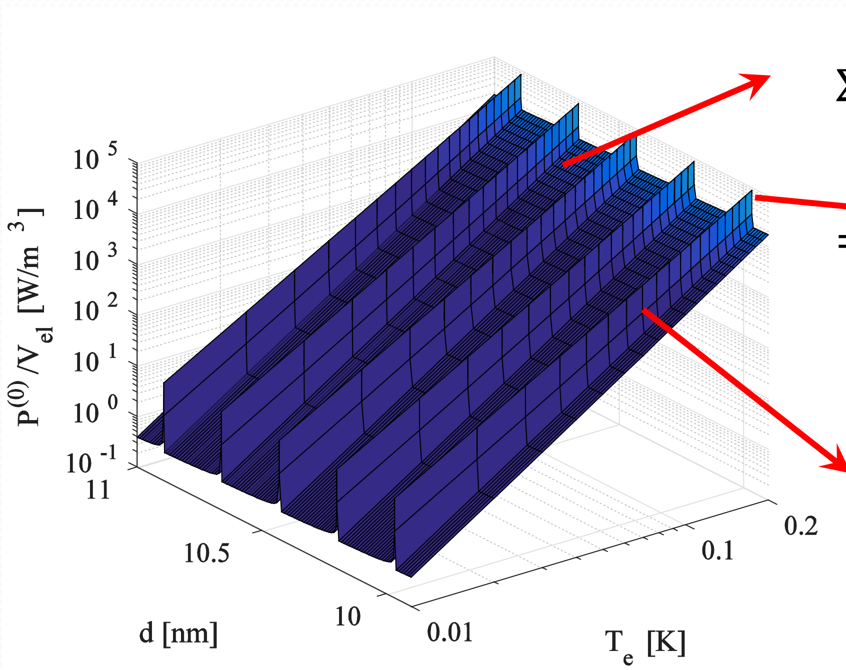
S. Cojocaru, DVA, submitted.

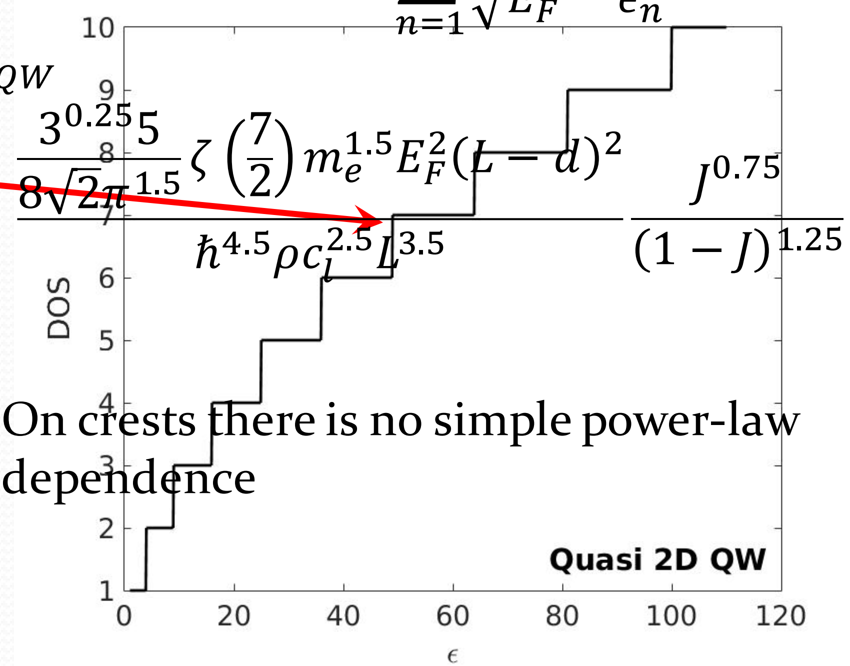
# Quantum well model $e \rightarrow ph$

In the valleys

$$P^{(0)} = \Sigma_{QW}(k_B T_e)^{3.5} \sum_{n=1}^{n_F} \frac{1}{\sqrt{E_F - \epsilon_n}}$$

$$\Sigma_{QW} = \frac{\frac{3^{0.25}5}{8\sqrt{2}\pi^{1.5}}\zeta\left(\frac{7}{2}\right) m_e^{1.5} E_F^2 (L-d)^2}{\hbar^{4.5}\rho c_l^{2.5} L^{3.5}} \frac{J^{0.75}}{(1-J)^{1.25}}$$




On crests there is no simple power-law dependence

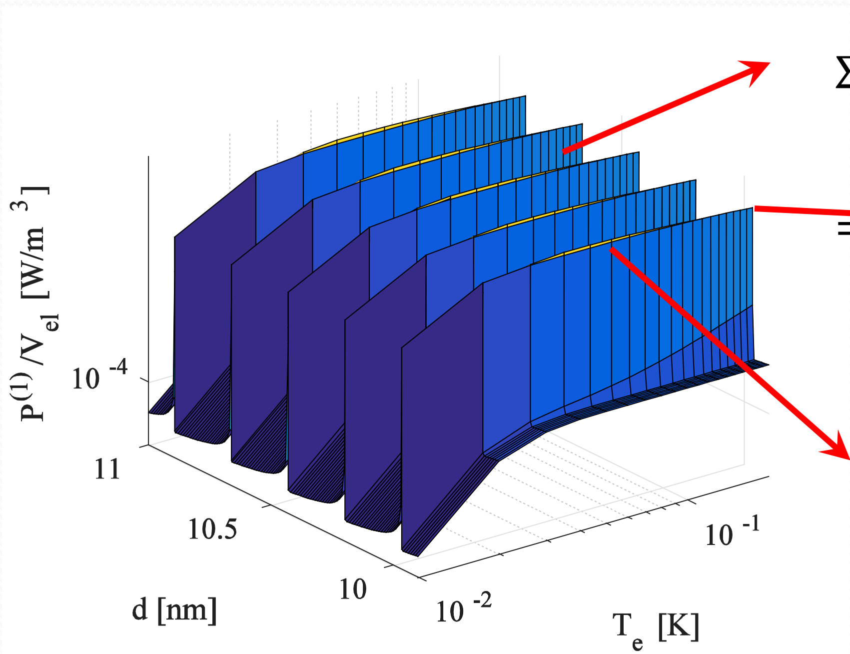
$P^{(0)}$ =electrons to phonons heat flux

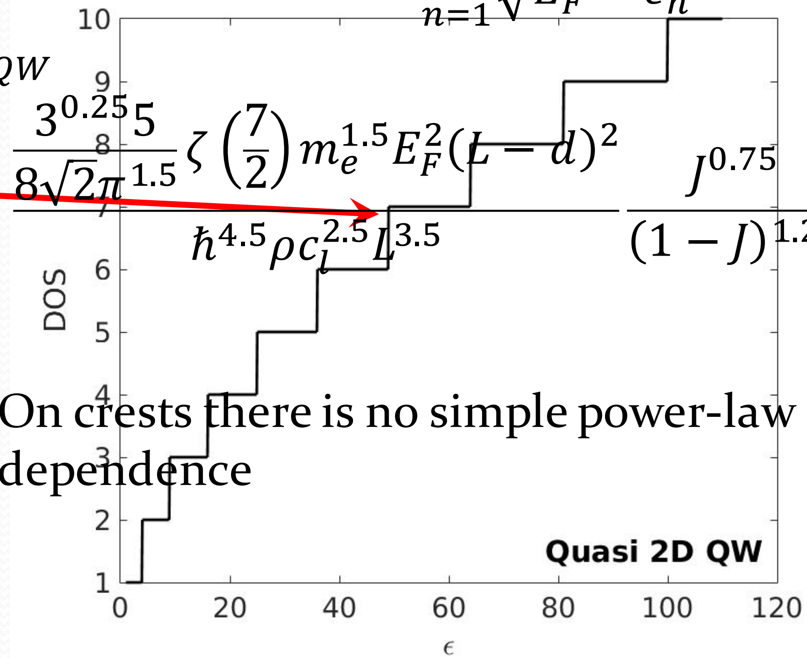
# Quantum well model $ph \rightarrow e$

In the valleys

$$P^{(1)} = \Sigma_{QW}(k_B T_{ph})^{3.5} \sum_{n=1}^{n_F} \frac{1}{\sqrt{E_F - \epsilon_n}}$$

$$\Sigma_{QW} = \frac{\frac{3^{0.25}5}{8\sqrt{2}\pi^{1.5}}\zeta\left(\frac{7}{2}\right) m_e^{1.5} E_F^2 (L-d)^2}{\hbar^{4.5}\rho c_l^{2.5} L^{3.5}} \frac{J^{0.75}}{(1-J)^{1.25}}$$




On crests there is no simple power-law dependence

$P^{(1)}$ =phonons to electrons heat flux

DVA, S. Cojocaru, submitted.

# Conclusions

- We analyzed the electron-phonon heat flux $P$ in layered nano-systems, employing two models
- In the 3D Fermi gas model we obtained $P \propto T_e^4 - T_{ph}^4$, in accordance with the general assumption, that $P \propto T_e^{s+2} - T_{ph}^{s+2}$, $s$ being the smaller dimension
- In the QW model we observed very strong oscillations of $P$ with the thickness of the metallic film.
- P forms crests where it increases by more than one order of magnitude.
- Between the crests, $P \propto T_e^{3.5} - T_{ph}^{3.5}$
- The crests (if observable) may provide opportunities for applications

# Acknowledgements


- Financial support from the Romnian ANCS project PN 16420101
- Financial support from the Romania-JINR collaboration projects, JINR Order 220/10.04.2017, positions 18, 19, 21, 22.