QED corrections to PDFs

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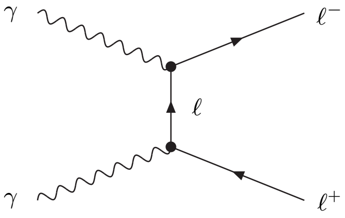
Motivation



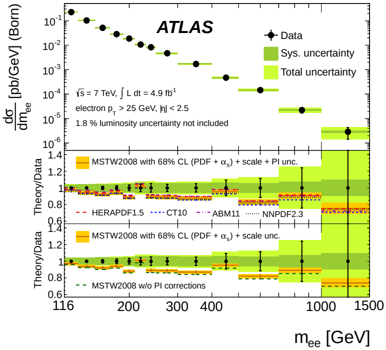
Photon-induced $\gamma\gamma \to \ell^+\ell^-$ process is an irreducable background to neutral current Drell-Yan-like process $q\bar{q} \to \gamma/Z \to \ell^+\ell^-$ and should be considered as part of NLO EW contribution to the production of charged lepton pair.

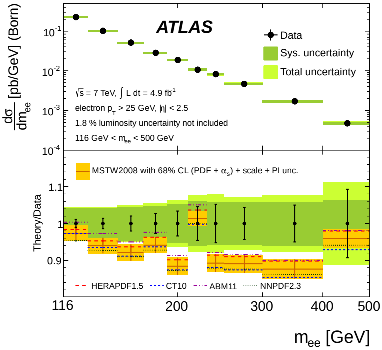
The cross-section of the process $p[\gamma]p[\gamma] \to \ell^+\ell^- + X$ at LO:

$$\sigma^{\mathrm{LO}}_{\gamma\gamma}(P_1, P_2) = \int_0^1 dx_1 \int_0^1 dx_2 \; f_\gamma(x_1, \mu_F^2) f_\gamma(x_2, \mu_F^2) \int d\hat{\sigma}^{\mathrm{LO}}_{\gamma\gamma}(x_1 P_1, x_2 P_2)$$

Motivation

- Recent studies on high mass Drell-Yan production in ATLAS [arXiv:1305.4192] has shown that the size of the photon-induced contribution ($\gamma\gamma \to e^+e^-$) is as large as the uncertainties arising from the different choice of PDF set





- It become important to address the impact of QED corrections to PDFs
- Currently only two PDF sets are available which incorporate photon contribution. These are MRST2004QED [arXiv:0411040] (which are based on old data) and most recent NNPDF2.3QED [arXiv:1308.0598]

DGLAP evolution equations

If we know the PDFs at initial scale μ_0^2, then the evaluation of PDFs at scale μ^2 in pQCD is performed using DGLAP evolution equations:

$$\frac{\partial q_i}{\partial \ln \mu^2} = \sum_{j=1}^{n_f} P_{q_i q_j} \otimes q_j + \sum_{j=1}^{n_f} P_{q_i \bar{q}_j} \otimes \bar{q}_j + P_{q_i g} \otimes g,$$

$$\frac{\partial \bar{q}_i}{\partial \ln \mu^2} = \sum_{j=1}^{n_f} P_{\bar{q}_i q_j} \otimes q_j + \sum_{j=1}^{n_f} P_{\bar{q}_i \bar{q}_j} \otimes \bar{q}_j + P_{\bar{q}_i g} \otimes g,$$

$$\frac{\partial g}{\partial \ln \mu^2} = \sum_{j=1}^{n_f} P_{g q_j} \otimes q_j + \sum_{j=1}^{n_f} P_{g \bar{q}_j} \otimes \bar{q}_j + P_{gg} \otimes g,$$

where $\otimes$-operation denotes the Mellin convolution defined as

$$\left[f \otimes g\right](x) = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z) = \int_x^1 \frac{dz}{z} f(z) g\left(\frac{x}{z}\right).$$

QED-modified evolution

QED-modified DGLAP evolution equations for Parton Distribution Functions of quarks $q_i(x, \mu_F^2)$, anti-quarks $q_i(x, \mu_F^2)$, gluon $g(x, \mu_F^2)$ and photon $\gamma(x, \mu_F^2)$ can be written as:

$$\frac{\partial q_i}{\partial \ln \mu^2} = \sum_{j=1}^{n_f} P_{q_i q_j} \otimes q_j + \sum_{j=1}^{n_f} P_{q_i \bar{q}_j} \otimes \bar{q}_j + P_{q_i g} \otimes g + P_{q_i \gamma} \otimes \gamma,$$

$$\frac{\partial \bar{q}_i}{\partial \ln \mu^2} = \sum_{j=1}^{n_f} P_{\bar{q}_i q_j} \otimes q_j + \sum_{j=1}^{n_f} P_{\bar{q}_i \bar{q}_j} \otimes \bar{q}_j + P_{\bar{q}_i g} \otimes g + P_{\bar{q}_i \gamma} \otimes \gamma,$$

$$\frac{\partial g}{\partial \ln \mu^2} = \sum_{j=1}^{n_f} P_{g q_j} \otimes q_j + \sum_{j=1}^{n_f} P_{g \bar{q}_j} \otimes \bar{q}_j + P_{gg} \otimes g,$$

$$\frac{\partial \gamma}{\partial \ln \mu^2} = \sum_{j=1}^{n_f} P_{\gamma q_j} \otimes q_j + \sum_{j=1}^{n_f} P_{\gamma \bar{q}_j} \otimes \bar{q}_j + P_{\gamma\gamma} \otimes \gamma.$$

QED-modified evolution

The expressions for splitting functions at NLO QCD and LO QED

$$P_{q_iq_j} = P_{\bar{q}_i\bar{q}_j} = a_s\delta_{ij}P_{qq}^{(0)} + a_s^2\left(\delta_{ij}\frac{P_+^{(1)} + P_-^{(1)}}{2} + \frac{P_{qq}^{(1)} - P_+^{(1)}}{2n_f}\right) + a\delta_{ij}e_ie_j\tilde{P}_{qq}^{(0)},$$

$$P_{q_i\bar{q}_j} = P_{\bar{q}_iq_j} = a_s^2\left(\delta_{ij}\frac{P_+^{(1)} - P_-^{(1)}}{2} + \frac{P_{qq}^{(1)} - P_+^{(1)}}{2n_f}\right),$$

$$P_{q_ig} = P_{\bar{q}_ig} = a_s\frac{P_{qg}^{(0)}}{2n_f} + a_s^2\frac{P_{qg}^{(1)}}{2n_f},$$

$$P_{q_i\gamma} = P_{\bar{q}_i\gamma} = ae_i^2\frac{P_{q\gamma}^{(0)}}{2n_f},$$

$$P_{gq_i} = P_{g\bar{q}_i} = a_sP_{gq}^{(0)} + a_s^2P_{gq}^{(1)},$$

$$P_{gg} = a_sP_{gg}^{(0)} + a_s^2P_{gg}^{(1)},$$

$$P_{\gamma q_i} = P_{\gamma\bar{q}_i} = ae_i^2P_{\gamma q}^{(0)},$$

$$P_{\gamma\gamma} = aP_{\gamma\gamma}^{(0)}.$$

QED-modified evolution

In the case of QCD evolution equations can be simplified using singlet and non-singlet combinations of quark densities. Then the singlet quark density obeys the evolution equation coupled to gluon density and non-singlet combinations evolve independently. But this decomposition is not suitable for QED-modified evolution since up- and down-quarks have different electric charges.

For QED-modified DGLAP evolution it is convenient to use the following basis of distribution functions

$$
\begin{aligned}
f_1 &= \Delta = u + \bar{u} + c + \bar{c} - d - \bar{d} - s - \bar{s} - b - \bar{b},\\
f_2 &= \Sigma = u + \bar{u} + c + \bar{c} + d + \bar{d} + s + \bar{s} + b + \bar{b},\\
f_3 &= g,\\
f_4 &= \gamma,\\
f_5 &= d_v = d - \bar{d},\\
f_6 &= u_v = u - \bar{u},\\
f_7 &= \Delta_{ds} = d + \bar{d} - s - \bar{s},\\
f_8 &= \Delta_{uc} = u + \bar{u} - c - \bar{c},\\
f_9 &= \Delta_{sb} = s + \bar{s} - b - \bar{b}.
\end{aligned}
$$

QED-modified evolution

In this basis we have 4 coupled and 5 uncoupled evolution equations:

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} \otimes \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix},$$

$$\frac{\partial f_i}{\partial \ln \mu^2} = P_{ii} \otimes f_i, \quad i = 5, \ldots, 9.$$

The expressions for splitting kernels P_{ii} at NLO QCD and LO QED are given by

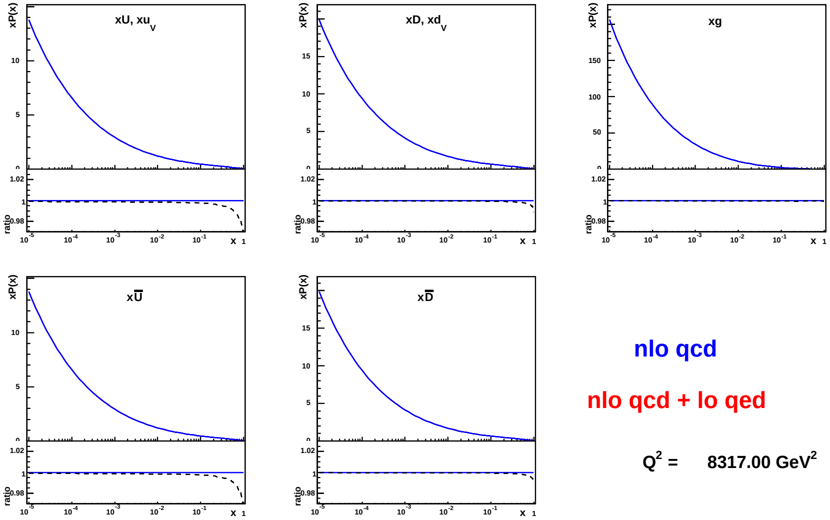
$$\begin{aligned}
P_{11} &= a_s P_{qq}^{(0)} + a_s^2 P_+^{(1)} + \frac{e_u^2+e_d^2}{2} a\tilde{P}_{qq}^{(0)}, \\
P_{12} &= \frac{n_u-n_d}{n_f} a_s^2 (P_{qq}^{(1)} - P_+^{(1)}) + \frac{e_u^2-e_d^2}{2} a\tilde{P}_{qq}^{(0)}, \\
P_{13} &= \frac{n_u-n_d}{n_f} (a_s P_{qg}^{(0)} + a_s^2 P_{qg}^{(1)}), \\
P_{14} &= \frac{n_u e_u^2 - n_d e_d^2}{n_f} a P_{q\gamma}^{(0)}, \\
P_{21} &= \frac{e_u^2-e_d^2}{2} a\tilde{P}_{qq}^{(0)}, \\
P_{22} &= a_s P_{qq}^{(0)} + a_s^2 P_{qq}^{(1)} + \frac{e_u^2+e_d^2}{2} a\tilde{P}_{qq}^{(0)}, \\
P_{23} &= a_s P_{qg}^{(0)} + a_s^2 P_{qg}^{(1)}, \\
P_{24} &= \frac{n_u e_u^2 + n_d e_d^2}{n_f} a P_{q\gamma}^{(0)}, \\
P_{31} &= 0, \\
P_{32} &= a_s P_{gq}^{(0)} + a_s^2 P_{gq}^{(1)},
\end{aligned}$$

$$\begin{aligned}
P_{33} &= a_s P_{gg}^{(0)} + a_s^2 P_{gg}^{(1)}, \\
P_{34} &= 0, \\
P_{41} &= \frac{e_u^2-e_d^2}{2} a P_{\gamma q}^{(0)}, \\
P_{42} &= \frac{e_u^2+e_d^2}{2} a P_{\gamma q}^{(0)}, \\
P_{43} &= 0, \\
P_{44} &= a P_{\gamma\gamma}^{(0)}, \\
P_{55} &= a_s P_{qq}^{(0)} + a_s^2 P_-^{(1)} + a e_d^2 \tilde{P}_{qq}^{(0)}, \\
P_{66} &= a_s P_{qq}^{(0)} + a_s^2 P_-^{(1)} + a e_u^2 \tilde{P}_{qq}^{(0)}, \\
P_{77} &= P_{99} = a_s P_{qq}^{(0)} + a_s^2 P_+^{(1)} + a e_d^2 \tilde{P}_{qq}^{(0)}, \\
P_{88} &= a_s P_{qq}^{(0)} + a_s^2 P_+^{(1)} + a e_u^2 \tilde{P}_{qq}^{(0)}.
\end{aligned}$$

Implementation in QCDNUM

- New beta version of QCDNUM program allows to solve an arbitrary number of coupled evolution equations in FFNS and VFNS
- NLO QCD + LO QED evolution equations has been implemented in this version of QCDNUM program
- We collaborate with HERAFtitter team (DESY, Hamburg). For the next release of HERAFitter package (https://www.herafitter.org) we are going to include the possibility of data fits based on QED-modified DGLAP evolution
- The results in FFNS were cross-checked with partonevolution-1.1.3 program in FFNS and with MRST2004QED PDF set and APFEL program in VFNS

The effect of QED corrections to PDFs



Summary

- With abundance of LHC high precision measurements available it become important to address the impact of QED corrections to PDFs
- QED-modified DGLAP evolution equations is realized with help of new beta version of `QCDNUM` program and cross-checked in FFNS and VFNS. It is planned to be implemented into next release of `HERAFitter` package
- The paper describing this work is available at http://arxiv.org/abs/1401.1133